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A. Perelomova

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A. Perelomova[a)]

AFFILIATIONS

Gdansk University of Technology, Faculty of Applied Physics and Mathematics, 11/12 Gabriela Narutowicza Street, 80-233 Gdansk, Poland

[a)]Author to whom correspondence should be addressed: anna.perelomova@pg.edu.pl

ABSTRACT

Two-dimensional magnetosonic beams directed along a line forming a constant angle θ with the equilibrium straight magnetic field are considered. Perturbations in a plasma are described by the system of ideal magnetohydrodynamic equations. The dynamics of perturbations in a beam are different in the cases of fast and slow modes, and it is determined by θ and equilibrium parameters of a plasma. In particular, a beam divergence may be unusual in the case of parallel propagation ($\theta = 0$). Diffraction is more pronounced in the case of parallel propagation as compared to a flow without magnetic field, and less manifested in the case of perpendicular propagation. The beams propagating oblique to the magnetic field do not reveal diffraction. The dynamics of perturbations in a beam are analytically described in the cases of weak and strong nonlinearity compared to diffraction. Small magnitude perturbations at the axis of a beam in unusual cases propagate slower than that in the plane wave. Involving of thermal conduction leads to the coupling equations describing thermal self-action of a beam, which behaves differently in the ordinary and unusual cases. Self-focusing may occur in the presence of a magnetic field instead of conventional defocusing in gases.

I. INTRODUCTION

The variety of MHD (magnetohydrodynamic) modes are the main factor, which complicate analytical description of a plasma flow. There are wave modes, that is, Alfvén, fast, and slow magnetoacoustic modes, and non-wave modes including the entropy mode. The key issue is the correct definition of all modes and derivation of dynamic equations for any mode in a linear flow, that is, in the case of small magnitudes of perturbations. This is the starting point in studies of the nonlinear flow as well. Modes are determined by dispersion relations. They may be defined equivalently by the corresponding relations of perturbations between thermodynamic variables, which, in some cases, are more preferable and informative.[1] Degenerate cases yield some special kinds of links. The number of modes increases in the multi-dimensional flow. A plane wave is always an approximation to real conditions of a flow. As usual, we deal with the bounded wave beams. The behavior of real beams differs from the rays behavior and cannot be considered in the frames of geometrical optics. The reason for this difference is diffraction. Diffraction specifies wave propagation in the two- and three-dimensional flows. It is a key factor in dynamics of wave perturbations, which is responsible for the distribution of the wave energy in a plane transversal to the direction of propagation. In contrast to dissipation processes, diffraction taken alone cannot compete with the nonlinear distortion of the wave profile. Diffraction reduces magnitude of perturbations away from a beam's axis and, thus, has impact on their nonlinear distortion and on nonlinear phenomena in the field of intense beams. The variety of wave modes and dependence of wave perturbations on θ and equilibrium parameters of a plasma impose variety in the diffraction behavior of beams. We focus on slightly diverging beams propagating along z axis, which is aligned according to the constant straight magnetic field ($\theta = 0$), and these ones directed along x axis perpendicular to the magnetic field ($\theta = \pi/2$). Diffraction is insignificant in the case of oblique beam propagation. Two particular cases of parallel propagation reveal unusual diffraction of a beam. Equations that describe the evolution of quasi-plane perturbations in linear and weakly nonlinear flows are derived in Secs. III and IV, respectively. Section V is devoted to analytical description of perturbations in a beam in cases of comparatively weak and strong diffraction in relation to nonlinearity. Thermal self-action of a beam is considered in Sec. VI.

II. THE EQUATIONS OF MHD FLOW

The initial point is the system of ideal MHD equations, which make use of the single-fluid model, macroscopic equilibrium quantities of a plasma, and equation of state for an ideal gas. An ideal gas that internal energy depends exclusively on temperature consists of molecules of negligible size. MHD equations impose that the temporal and spatial scales of perturbations must be much larger than gyrokinetic scales and ignore relativity quantum mechanical effects and displacement current in Ampère's law.

A full set of ideal MHD equations for perfectly conducting fluid consists of the continuity equation, momentum equation, energy balance equation, and electrodynamic equations (e.g., Refs. 2–4):

$$\begin{gathered}\frac{\partial \rho}{\partial t}+\nabla\cdot(\rho \boldsymbol{v})=0,\\ \rho\left(\frac{\partial \boldsymbol{v}}{\partial t}+(\boldsymbol{v}\cdot\nabla)\boldsymbol{v}\right)=-\nabla p+\frac{1}{\mu_0}(\nabla\times\mathbf{B})\times\mathbf{B},\\ \frac{\partial p}{\partial t}+(\boldsymbol{v}\cdot\nabla)p+\gamma p(\nabla\cdot\boldsymbol{v})=0,\quad \frac{\partial \boldsymbol{B}}{\partial t}=\nabla\times(\boldsymbol{v}\times\boldsymbol{B}),\\ \nabla\cdot\boldsymbol{B}=0,\end{gathered} \tag{1}$$

where p, ρ, $\boldsymbol{v}$, and $\boldsymbol{B}$ are hydrostatic pressure and density of a plasma, its velocity, and the magnetic field, respectively, μ_0 is the permeability of free space, and γ denotes the ratio of specific heats under constant pressure and constant density $\gamma, \gamma = C_P/C_V$.

Following Botha *et al.*,[5] we reduce the analysis to two dimensions, considering these equations in Cartesian coordinates and assuming dependence of all perturbations on x and z. All equilibrium quantities are subscripted by 0, and perturbations are superscripted by apostrophe. The bulk flow is absent ($\boldsymbol{v}_0 = \mathbf{0}$, apostrophes by components of velocity are dropped). The equilibrium magnetic field is aligned along z axis, $\boldsymbol{B}_0 = (0, 0, B_0)$, and equilibrium pressure and density are constant. The MHD equations may be written in the following form:[5,6]

$$\begin{gathered}\frac{\partial \rho'}{\partial t}+\rho_0\left(\frac{\partial v_x}{\partial x}+\frac{\partial v_z}{\partial z}\right)=N_1,\\ \frac{\partial v_x}{\partial t}+\frac{1}{\rho_0}\frac{\partial p'}{\partial x}-\frac{B_0}{\rho_0\mu_0}\left(\frac{\partial B'_x}{\partial z}-\frac{\partial B'_z}{\partial x}\right)=N_2,\\ \frac{\partial v_y}{\partial t}-\frac{B_0}{\rho_0\mu_0}\frac{\partial B'_y}{\partial z}=N_3,\\ \frac{\partial v_z}{\partial t}+\frac{1}{\rho_0}\frac{\partial p'}{\partial z}=N_4,\\ \frac{\partial p'}{\partial t}+\gamma p_0\left(\frac{\partial v_x}{\partial x}+\frac{\partial v_z}{\partial z}\right)=N_5,\\ \frac{\partial B'_x}{\partial t}-B_0\frac{\partial v_x}{\partial z}=N_6,\\ \frac{\partial B'_y}{\partial t}-B_0\frac{\partial v_y}{\partial z}=N_7,\\ \frac{\partial B'_z}{\partial t}+B_0\frac{\partial v_x}{\partial x}=N_8,\end{gathered} \tag{2}$$

where $\tilde{N} = (N_1 \ldots N_8)^T$ is a vector that consists of quadratically nonlinear terms.

A set of equations describe perturbations correct to second order in the magnetoacoustic Mach number M, which equals the ratio of amplitude of velocity to the speed of magnetosonic perturbations. The terms on the left of Eq. (2) are of the order of smallness M, while those on the right are of smallness M^2. Equation (2) determines the dispersion relations in a linear flow, if one assumes all perturbations proportional to $\exp(i\omega t - ik_x x - ik_z z)$ [$\vec{k} = (k_x, 0, k_z)$ designates the wave vector]. The dispersion equation for the magnetosonic modes takes the following well-known form:[7]

$$c_0^2(k_x^2+k_z^2)(C_A^2k_z^2-\omega^2)+\omega^2(\omega^2-C_A^2(k_x^2+k_z^2))=0, \tag{3}$$

where

$$c_0=\sqrt{\frac{\gamma p_0}{\rho_0}},\quad C_A=\frac{B_0}{\sqrt{\mu_0\rho_0}}$$

denote the acoustic speed in unmagnetized gas in equilibrium and the Alfvén speed, respectively. Any non-zero small MHD perturbation $\varphi(x, z, t)$ satisfies the equation that follows from the dispersion relation (3),

$$\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2\varphi}{\partial t^2}-C_A^2\Delta\varphi\right)-c_0^2\Delta\left(\frac{\partial^2\varphi}{\partial t^2}-C_A^2\frac{\partial^2\varphi}{\partial z^2}\right)=0, \tag{4}$$

where $\Delta = \left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)$.

III. QUASI-PLANE LINEAR DYNAMICS

The speed of sound in the plane one-dimensional wave with the wave vector forming angle θ with the z axis satisfies the equation:[8,9]

$$C^4+c_0^2C_A^2\cos^2(\theta)-C^2(c_0^2+C_A^2)=0. \tag{5}$$

We consider beams reasonably directed along axis z (or along axis x).

A. Nearly parallel wave vector and the equilibrium magnetic field

This is the case of $\theta = 0$. To be specific, a beam propagating in the positive direction of axis z is considered. There are two positive roots of Eq. (5) in this case: $C = c_0$ and $C = C_A$. If $C_A = c_0$, the roots of (3) are degenerate. This case will be considered individually in Sec. III A 3.

1. The case $c_0 \neq C_A$ and $C = c_0$

Substituting all perturbations in the form of the plane waves proportional to $\exp(i\omega t - k_x x - k_z z)$ and treating k_x/k_z as a small parameter, we arrive at the leading-order dispersion relation valid to second order of this parameter,

$$\omega=c_0k_z\left(1+\frac{c_0^2k_x^2}{2(c_0^2-C_A^2)k_z^2}\right). \tag{6}$$

The links of specific perturbations take the following form:

$$\begin{gathered}v_x=\frac{c_0^3}{(c_0^2-C_A^2)\rho_0}\int dz\frac{\partial\rho'}{\partial x},\quad v_y=0,\\ v_z=\left(\frac{c_0\rho'}{\rho_0}-\frac{c_0^3}{2(c_0^2-C_A^2)\rho_0}\int dz\int dz\frac{\partial^2\rho'}{\partial x^2}\right),\\ p'=c_0^2\rho',\quad B'_x=-\frac{c_0^2B_0}{(c_0^2-C_A^2)\rho_0}\int dz\frac{\partial\rho'}{\partial x},\\ B'_x=-\frac{c_0^2B_0}{(c_0^2-C_A^2)\rho_0}\int dz\frac{\partial\rho'}{\partial x},\quad B'_y=0,\\ B'_z=\frac{c_0^2B_0}{(c_0^2-C_A^2)\rho_0}\int dz\int dz\frac{\partial^2\rho'}{\partial x^2}.\end{gathered} \tag{7}$$

Integrals of this type $\int_z^\infty dz'\psi(x, z', t)$ are abbreviated as $\int dz'\psi$ in Eq. (7) and subsequent formulas. The links are valid at any time. Equation

(6) resembles the dispersion relation inherent to a quasi-plane beam in a unmagnetized gas,

$$\omega = c_0 k_z\left(1+\frac{k_x^2}{2k_z^2}\right), \tag{8}$$

which represents a leading-order series expansion of $\omega = c_0\sqrt{k_z^2+k_x^2}$. This suggests to make use of the known methods in the wave theory and to seek any non-zero perturbation φ inherent to this mode as a function of the retarded time $\tau = t - z/c_0$, "slow" coordinates mz, $\sqrt{m}x$ (m is a small parameter responsible for the divergence), and to substitute it in Eq. (4). This choice of slow scale suggests that the spatial variations occur more slowly along the axis z of a beam than across the beam to an observer, which moves at speed c_0 along the axis of a beam.[1,10,11] This approach assumes quasi-plane geometry of a flow. Discarding terms of the high order in smallness [that is, terms $O(m^2)$], collecting terms of order m and transforming equation from the slow scale back to x and z yield an equation, which is valid for any non-zero wave perturbation (may be substituted by ρ', v_x, v_z, p', B_x', and B_z'). It takes the following form:

$$\frac{\partial^2\varphi}{\partial\tau\partial z} = \begin{cases} \dfrac{D_0^2 c_0}{2}\dfrac{\partial^2\varphi}{\partial x^2}, & c_0 > C_A, \\ -\dfrac{D_0^2 c_0}{2}\dfrac{\partial^2\varphi}{\partial x^2}, & c_0 < C_A, \end{cases} \tag{9}$$

where

$$D_0^2 = \frac{c_0^2}{|c_0^2 - C_A^2|},$$

and resembles the famous equation for the perturbation of pressure in slightly diverging beams in a gas in the absence of magnetic field:[1,11]

$$\frac{\partial^2 p'}{\partial\tau\partial z} = \frac{c_0}{2}\frac{\partial^2 p'}{\partial x^2}. \tag{10}$$

The term on the right of Eq. (9) is responsible for diffraction.

Seeking a solution to Eq. (9) in the following form:

$$\phi = A(x,z)\exp(i\omega\tau), \tag{11}$$

we arrive at the parabolic equation for the complex amplitude $A(x, z)$:[1]

$$\frac{\partial A}{\partial z} = \mp i D_0^2\frac{c_0}{2\omega}\frac{\partial^2 A}{\partial x^2}. \tag{12}$$

The upper sign minus corresponds to the case $c_0 > C_A$, and the lower sign corresponds to $c_0 < C_A$. The exact solution to Eq. (12) responses to the Gaussian at a transducer ($z = 0$) beam

$$A(x,z) = \frac{A_0}{\sqrt{1\mp iz/z_{d,0}}}\exp\left(-\frac{x^2}{a^2(1\mp iz/z_{d,0})}\right), \tag{13}$$

where a is the characteristic beam's width at $z = 0$,

$$z_{d,0} = \omega a^2/(2c_0 D_0^2) \tag{14}$$

is the diffraction length, and A_0 is the amplitude of perturbation at $z = 0$ and at the axis of a beam, $x = 0$. The module of A

$$|A(x,z)| = A_0\frac{\exp\left(-\dfrac{x^2}{a^2((z/z_{d,0})^2+1)}\right)}{\sqrt{(z/z_{d,0})^2+14}}, \tag{15}$$

gets smaller away from the axis of a beam in both cases. Since $D_0^2 > 1$, the divergence is more manifested in the presence of a magnetic field. This corresponds to the shorter diffraction length $z_{d,0}$ compared to that in the unmagnetized gas,

$$z_d = \omega a^2/(2c_0). \tag{16}$$

2. The case $c_0 \neq C_A$ and $C = C_A$

Similar manipulations yield the leading-order dispersion relation corresponding to a beam propagating in the positive direction of axis z,

$$\omega = C_A k_z\left(1+\frac{C_A^2 k_x^2}{2(C_A^2-c_0^2)k_z^2}\right), \tag{17}$$

and the links of specific perturbations

$$\begin{gathered} \rho' = \frac{C_A\rho_0}{C_A^2-c_0^2}\int dz\frac{\partial v_x}{\partial x}, \quad v_y = 0, \\ v_z = \frac{c_0^2}{C_A^2-c_0^2}\int dz\frac{\partial v_x}{\partial x}, \quad p' = \frac{c_0^2 C_A\rho_0}{C_A^2-c_0^2}\int dz\frac{\partial v_x}{\partial x}, \\ B_x' = \left(-\frac{B_0}{C_A}v_x + \frac{C_A B_0}{2(C_A^2-c_0^2)}\int dz\int dz\frac{\partial^2 v_x}{\partial x^2}\right), \\ B_y' = 0, \quad B_z' = \frac{B_0}{C_A}\int dz\frac{\partial v_x}{\partial x}. \end{gathered} \tag{18}$$

An equation that governs perturbations in a beam may be extracted from Eq. (4) by assuming perturbations as $\varphi(t - z/C_A, mz, \sqrt{m}x)$. We arrive at

$$\frac{\partial^2\varphi}{\partial\tau\partial z} = \begin{cases} \dfrac{D_A^2 C_A}{2}\dfrac{\partial^2\varphi}{\partial x^2}, & C_A > c_0, \\ -\dfrac{D_A^2 C_A}{2}\dfrac{\partial^2\varphi}{\partial x^2}, & C_A < c_0, \end{cases} \tag{19}$$

where

$$D_A^2 = \frac{C_A^2}{|C_A^2 - c_0^2|},$$

and φ may take value of any non-zero wave perturbation, which specifies the wave modes (ρ', v_x, v_z, p', B_x', and B_z'). The solution to (19) for the initially Gaussian beam is Eq. (11) with A in the following form:

$$A(x,z) = \frac{A_0}{\sqrt{1\mp iz/z_{d,A}}}\exp\left(-\frac{x^2}{a^2(1\mp iz/z_{d,A})}\right), \tag{20}$$

where

$$z_{d,A} = \omega a^2/(2C_A D_A^2). \tag{21}$$

The upper sign minus in the denominator corresponds to the case $C_A > c_0$, and the plus sign corresponds to the case $C_A < c_0$. The mode (18) resembles the Alfvén mode, which propagates in the positive direction of axis z with the speed C_A and is determined by the links,

$$\rho' = 0, \quad v_x = 0, \quad v_z = 0, \quad p' = 0, \quad B'_x = 0, \\ B'_y = -\frac{B_0}{C_A} v_y, \quad B'_z = 0, \tag{22}$$

but it is an important difference between these two modes. Namely, the mode with links (22) propagates as a plane wave without diffraction due to exact dispersion relation $\omega = C_A k_z$. Links (22) are also exact.

3. The case C=c_0=C_A

This case is degenerate in the one-dimensional flow with $k_x = 0$. It is not a limiting case of the general case if C_A tends to c_0. The leading-order dispersion relations in the quasi-planar geometry sound

$$\omega = c_0 k_z \pm \frac{k_x}{2} c_0.$$

There is no solution of the type

$$\varphi(\tau, mz, \sqrt{m}x)$$

apart from physically insignificant case $\frac{\partial^4 \varphi}{\partial^2 \tau \partial^2 x} = 0$. Any non-zero perturbation

$$\varphi(\tau, mz, mx)$$

is a solution with accuracy up to $O(m^2)$ terms. The diffraction is negligible in this case.

B. Nearly perpendicular wave vector and the equilibrium magnetic field

This is the case $\theta = \pi/2$ and a beam propagating along axis x. We consider k_z/k_x as a small parameter. The leading-order dispersion relation and corresponding links in a beam directional along the positive direction of axis x are as follows ($C_\perp = \sqrt{c_0^2 + C_A^2}$):

$$\omega = C_\perp k_x \left(1 + \frac{(c_0^4 + C_A^4 + c_0^2 C_A^2) k_z^2}{2 C_\perp^4 k_x^2}\right) \tag{23}$$

and

$$v_x = \frac{C_\perp \rho'}{\rho_0} - \frac{(c_0^4 + c_0^2 C_A^2 - C_A^4)}{2 C_\perp^3 \rho_0} \int dx \int dx \frac{\partial^2 \rho'}{\partial z^2}, \\ v_y = 0, \quad v_z = \frac{c_0^2}{C_\perp \rho_0} \int dx \frac{\partial \rho'}{\partial z}, \\ p' = c_0^2 \rho', \quad B'_x = -\frac{B_0}{\rho_0} \int dx \frac{\partial \rho'}{\partial z}, \quad B'_y = 0, \\ B'_z = \frac{B_0}{\rho_0} \rho' - \frac{c_0^2 B_0}{C_\perp^2 \rho_0} \int dx \int dx \frac{\partial^2 \rho'}{\partial z^2}. \tag{24}$$

Making use of the method of multiple scales for perturbations and imposing all non-zero perturbations as functions of $\tau = t - x/C_\perp$, mx, and $\sqrt{m}z$, we arrive at the leading-order equation for the non-zero specific perturbation,

$$\frac{\partial^2 \varphi}{\partial \tau \partial x} = \frac{D_\perp^2 C_\perp}{2} \frac{\partial^2 \varphi}{\partial z^2}, \tag{25}$$

where

$$D_\perp^2 = \frac{(c_0^4 + C_A^4 + c_0^2 C_A^2)}{(c_0^2 + C_A^2)^2}$$

is positive. The solution to Eq. (25) in the form (11) is Eq. (13) with the amplitude

$$A(x, z) = \frac{A_0}{\sqrt{1 - ix/x_d}} \exp\left(-\frac{z^2}{a^2(1 - ix/x_d)}\right),$$

where

$$x_d = \omega a^2/(2 D_\perp^2 C_\perp) \tag{26}$$

is the diffraction length. $D_\perp$ is larger than $\sqrt{3/4}$ ($c_0 = C_A$) and smaller than 1 ($C_A \gg c_0$, $C_A \ll c_0$). Hence, the diffraction manifests itself weaker compared to unmagnetized gas in the case of perpendicular propagation and stronger in the case of parallel one.

IV. QUASI-PLANE NONLINEAR DYNAMICS

The nonlinear phenomena have a key impact on the wave dynamics. They lead to the second and higher harmonics excitation and self-interacting stationary formations in plasmas.[12,13] As usual, the dynamic equations correct to the second order of perturbations are considered. This may be done making use of the initial system (2) and links specifying the magnetosonic mode. As for the case $C = c_0$ ($c_0 \neq C_A$), we may rearrange the leading-order equations in terms of specific perturbation of density and note that the linear left-hand sides of equations are identical. Hence, we equate the right-hand sides of equations following from $N_1, \ldots, N_8$, which contain terms only of second order of smallness. All nonlinear terms are expressed in the terms of perturbation of density by use of links (7). This is a standard procedure in nonlinear acoustics.[1,10] The parameter of nonlinearity ε in the quasi-plane geometry coincides with that in the plane one-dimensional flow. It has been evaluated by Nakariakov *et al.* for any C except for the case $C = c_0 = C_A$,[14]

$$\varepsilon = \frac{3c_0^2 + (\gamma + 1) C_A^2 - (\gamma + 4) C^2}{2(c_0^2 - 2C^2 + C_A^2)}. \tag{27}$$

So, we simply use the result to generalize the dynamic equations to the weakly nonlinear case, considering the Mach number M and diffraction parameter m of comparative smallness. Equation (9) ($C = c_0$, $c_0 \neq C_A$) in terms of perturbation of density may be rearranged as

$$\frac{\partial}{\partial \tau}\left(\frac{\partial \rho'}{\partial z} - \frac{\gamma + 1}{2 \rho_0 c_0} \rho' \frac{\partial \rho'}{\partial \tau}\right) = \begin{cases} \dfrac{D_0^2 c_0}{2} \dfrac{\partial^2 \rho'}{\partial x^2}, & c_0 > C_A, \\ -\dfrac{D_0^2 c_0}{2} \dfrac{\partial^2 \rho'}{\partial x^2}, & c_0 < C_A, \end{cases} \tag{28}$$

Equation (19) ($C = C_A$, $c_0 \neq C_A$) in terms of v_x may be transformed into

$$\frac{\partial}{\partial \tau}\left(\frac{\partial v_x}{\partial z} - \frac{3}{2C_A^2} v_x \frac{\partial v_x}{\partial \tau}\right) = \begin{cases} \dfrac{D_A^2 C_A}{2}\dfrac{\partial^2 v_x}{\partial x^2}, & c_0 < C_A, \\ -\dfrac{D_A^2 C_A}{2}\dfrac{\partial^2 v_x}{\partial x^2}, & c_0 > C_A, \end{cases} \tag{29}$$

and Eq. (25) ($C = C_\perp$) in terms of perturbation of density is converted to the following form:

$$\frac{\partial}{\partial \tau}\left(\frac{\partial \rho'}{\partial x} - \frac{3C_A^2 + (\gamma+1)c_0^2}{2\rho_0 C_\perp^3}\rho' \frac{\partial \rho'}{\partial \tau}\right) = \frac{D_\perp^2 C_\perp}{2}\frac{\partial^2 \rho'}{\partial z^2}. \tag{30}$$

V. ANALYTICAL SOLUTIONS

We will consider the result of Sec. III A 1, Eq. (28), and associate φ with a perturbation of density. It has a lot in common with the results of Sec. III A 2 and Sec. III B [Eqs. (29) and (30)], and the analytical solutions can be easily generalized to these cases. Equations (28) and (29) include cases of unusual diffraction. Equation (28) describes a magnetosonic beam, an only mode that may excite heating (considered in Sec. VI B). The module of the complex amplitude (13) for both signs equals

$$|A(x,z)| = A_0 \frac{\exp\left(-\dfrac{X^2}{1+Z^2}\right)}{\sqrt[4]{1+Z^2}}, \tag{31}$$

where $Z = z/z_{d,0}$ and $X = x/a$. The magnitude of the wave perturbation at the axis of a beam $|A(0,z)|$ is proportional to $1/\sqrt[4]{1+Z^2}$, and the width of a beam increases as $\sqrt{1+Z^2}$. The real function Φ that satisfies Eq. (4) takes the following form:

$$\Phi = A_0 \frac{\exp\left(-\dfrac{X^2}{1+Z^2}\right)\sin\left(\omega\tau \mp \dfrac{X^2 Z}{1+Z^2} \pm \dfrac{1}{2}\arctan(Z)\right)}{\sqrt[4]{1+Z^2}}. \tag{32}$$

The upper signs correspond to the case $c_0 > C_A$, and the lower signs correspond to $c_0 < C_A$. At the axis of a beam ($X = 0$), the phase of perturbations equals $\omega\tau \pm 0.5\arctan(Z)$. An excess speed $\Delta c/c_0$ equals approximately $\pm c_0/(2z_{d,0}\omega)$ for $Z \ll 1$. Usually, the perturbations at the axis of a beam propagate faster than that in the plane wave (sign plus, $c_0 > C_A$), but they propagate slower in the unusual case $c_0 < C_A$.

An extreme case of very strong nonlinearity compared to diffraction is described by Eq. (28) with zero right-hand side. It may be of importance in some cases of a magnetohydrodynamic flow.[15,16] This is the case of the small ratio of diffraction to nonlinear effects, that is, the high-frequency case, $z_{nl} \ll z_{d,0}$, where

$$z_{nl} = \frac{2\rho_0 c_0}{(\gamma+1)\omega A_0} \tag{33}$$

is the discontinuity formation distance in the case of a plane wave. The dynamics are the same as in a plane wave with nonlinear distortions in the course of propagation. In particular, for the initially Gaussian harmonic perturbation at $z = 0$, the specific perturbation of density equals[1,10]

$$\begin{aligned} \frac{\rho'}{A_0} &= \exp(-X^2)\sin\left(\omega\tau + \frac{\rho'}{A_0}\frac{z_{d,0}}{z_{nl}}Z\right) \\ &= \sum_{n=1}^{\infty} \frac{J_n(n\exp(-X^2)(z_{d,0}/z_{nl})Z)}{n(z_{d,0}/z_{nl})Z}\sin(n\omega\tau), \end{aligned} \tag{34}$$

where J_n is the Bessel function of the nth order. The details of nonlinear dynamics are considered by Rudenko and Soluyan.[1] The width of a beam of nth harmonics equals $a_n = a/\sqrt{n}$ in the vicinity of $z = 0$ and $z \ll z_{nl}$, and the width of a beam of average energy per unit volume

$$E = \rho_0\langle v_z^2\rangle = \rho_0 c_0^2 \left\langle \left(\frac{\rho'}{\rho_0}\right)^2 \right\rangle = \frac{A_0^2 c_0^2}{2\rho_0}\exp(-2X^2), \tag{35}$$

remains constant until the formation of the discontinuity (the angle brackets denote averaging over the period of perturbations). The discontinuity formation distance in a beam depends on the distance from a beam's axis, $z_{nl,beam} = z_{nl}\exp(X^2)$. Since the shock wave is formed earlier near the axis ($z_{nl} = 1$), its intense damping starts in the paraxial domain and leads to a decrease in the average energy. The profile of axial distribution of E becomes distorted.

The analytical description of Eq. (28) in regard to the unmagnetized gas ($C_A = 0$, $D_0 = 1$) is still an unresolved problem. A special analytical method that combines the parabolic approximation and nonlinear geometrical acoustics has been developed to model nonlinear and diffraction effects in the paraxial zone of a finite amplitude axial symmetric sound beam in Ref. 17. The numerical results for the initially Gaussian sinusoidal signal reveal that the positive half-period is shorter in duration than the negative one. The peak value of the disturbance in the compressional phase is larger than that in the rarefaction phase.[1,10] Analysis of Eqs. (29) and (30) is very similar. It makes no sense to bring it; the last equation combines nonlinearity and usual somewhat scaled diffraction.

VI. MODEL EQUATIONS AND POSSIBLE APPLICATIONS

In this section, we derive the model equations and mention the issues where the unusual divergence may be of importance.

A. Dynamics of a finite-magnitude beam in a viscous medium

The dynamic equations can be readily specified including damping terms. The damping terms depend, in general, on θ and equilibrium parameters of a plasma and may be evaluated in the plane geometry of a flow. In particular, the dispersion relation for the plane magnetosonic modes with an account of thermal conduction (in the non-degenerate case, if $\theta = 0$, $c_0 \neq C_A$) follows from the momentum equation:

$$\frac{\partial p}{\partial t} + (\boldsymbol{v}\cdot\nabla)p + \gamma p(\nabla\cdot\boldsymbol{v}) - (\gamma-1)\nabla(\kappa\nabla T) = 0, \tag{36}$$

and the thermal equation of state for an ideal gas

$$T = \frac{\gamma p}{(\gamma-1)\rho C_P}.$$

It takes the following form:[9]

$$\omega = Ck + i\frac{C^2 - C_A^2}{2(2C^2 - c_0^2 - C_A^2)}\frac{(\gamma-1)\kappa}{C_P\rho_0}k^2. \tag{37}$$

The dispersion relation that specifies the entropy mode is

$$\omega = i\frac{\kappa}{C_P\rho_0}k^2. \tag{38}$$

The dynamic equation for the magnetosonic beam propagating with the speed c_0 along the equilibrium magnetic field, which incorporates nonlinearity, diffraction, and thermal conduction, is

$$\frac{\partial}{\partial\tau}\left(\frac{\partial\rho'}{\partial z} - \frac{\gamma+1}{2\rho_0 c_0}\rho'\frac{\partial\rho'}{\partial\tau} - \frac{(\gamma-1)\kappa}{2c_0^3 C_P\rho_0}\frac{\partial^2\rho'}{\partial\tau^2}\right) = \begin{cases} \dfrac{D_0^2 c_0}{2}\dfrac{\partial^2\rho'}{\partial x^2}, & c_0 > C_A, \\ -\dfrac{D_0^2 c_0}{2}\dfrac{\partial^2\rho'}{\partial x^2}, & c_0 < C_A. \end{cases} \tag{39}$$

It resembles the famous Khokhlov–Zabolotskaya–Kuznetsov (KZK) equation for a Newtonian beam [the case $c_0 > C_A$, the KZK is, in fact, (39) with $C_A = 0$ and $D_0 = 1$]. The derivation of the KZK equation [and Eq. (39)] exploits smallness of a parameter associated with heat conduction, $\lambda = \kappa\omega/(\rho_0 C_P c_0^2)$, which is of comparative order of smallness with M and m. A generic small parameter Λ may be introduced that characterizes the smallness of both M and m, and λ. Equation (39) and the KZK equation are valid at order Λ^2. Setting $C_A = 1$ ($B_0 = 1$) and zero perturbations of the magnetic field eliminates two last equations in Eq. (1) and leads to the initial conservation equations for a thermoconducting Newtonian fluid with an account for thermal conduction, Eq. (36). The methods of derivation, including the multiscale variables in the retarded time frame and small generic parameter responsible for diffraction, attenuation, and nonlinearity, are the same in the presence or absence of a magnetic field. The derivation of the KZK equation is described in detail.[1,10,11] Equation (39) may be readily supplied by the terms responsible for mechanical damping and heating/cooling of a plasma.[18]

B. Magnetoacoustic heating and thermal self-action

The nonlinear phenomena are determined not only by nonlinearity itself but also by diffraction. Slow variations in the temperature of a medium occur due to irreversible nonlinear transfer of magnetosonic energy into the entropy mode due to some mechanism of nonadiabaticity. This phenomenon is called acoustic heating. In the context of magnetohydrodynamics, the heating is of close interest since it may indicate wave phenomena and equilibrium parameters of a plasma during remote observations.[19–21] The heating leads to the formation of thermal lenses due to heterogeneous heating in the plane perpendicular to a beam's axis. This, in turn, results to thermal self-action of a beam (due to the dependence of sound speed on temperature) and, hence, to the focusing or defocusing of a beam. The first theoretical and experimental results concerning thermal self-action of acoustic waves were reported in Refs. 22 and 23. Thermal self-action of quasi-harmonic sound waves (this is the case of weak nonlinearity) has evident counterparts in thermal self-action of optic waves. The theoretical studies[24,25] had considerable impact on the nonlinear acoustics of beams, which propagate in dispersive media. Acoustic nonlinearity in a nondispersive or weakly dispersive medium leads to spreading of the wave spectrum due to excitation of higher harmonics, and the wave cannot be considered as quasi-harmonic. In particular, the coupling system that describes the magnetoacoustic heating following a beam propagation along axis z with the speed c_0 is

$$\frac{\partial}{\partial\tau}\left(\frac{\partial\rho'}{\partial z} - \frac{\delta T'}{c_0}\frac{\partial\rho'}{\partial\tau} - \frac{\gamma+1}{2\rho_0 c_0}\rho'\frac{\partial p'}{\partial\tau} - \frac{(\gamma-1)\chi}{2c_0^3 C_P\rho_0}\frac{\partial^2\rho'}{\partial\tau^2}\right) = \pm\frac{D_0^2 c_0}{2}\frac{\partial^2\rho'}{\partial x^2}, \tag{40}$$

$$\frac{\partial T'}{\partial t} - \frac{\chi}{\rho_0 C_P}\frac{\partial^2 T'}{\partial x^2} = \frac{c_0}{C_p}F, \tag{41}$$

where $\delta = (\partial c_0/\partial T)_p/c_0$ [for an ideal gas case, $\delta = 1/2T_0 = (\gamma-1)C_P/2c_0^2$], and

$$F = \frac{(\gamma-1)\chi}{C_P c_0\rho_0^3}\left\langle\left(\frac{\partial\rho'}{\partial\tau}\right)^2\right\rangle. \tag{42}$$

where F designates the magnetosonic force of heating. The upper sign on the right-hand side corresponds to the usual case, and the lower sign, to the unusual case in Eq. (40) and subsequent equations. Equation (42) is valid for periodic or nearly periodic magnetosonic perturbations. The acoustic force is non-zero in a nonlinear flow with weakly disturbed adiabaticity, that is, in the presence of some mechanism of dissipation (mechanical viscosity and electrical resistivity) and the heating-cooling function. These factors may operate individually or together. Equations (40) and (42) can also be supplemented by these factors.[18] The instantaneous acoustic force including the impact of some heating/cooling function, which does not refer to periodicity of a source and averaging over a period, may be found in Ref. 26. If nonlinearity is not important, the "fast" time may be eliminated from Eq. (40) by substitution $\rho' = A(x,z)\exp(i\omega\tau)$,[27] which leads to a parabolic equation for the complex amplitude,

$$\frac{\partial A}{\partial Z} - i\frac{\omega T' C_P(\gamma-1)z_{d,0}}{2c_0^3}A + \frac{z_{d,0}}{z_T}A = \mp\frac{i}{4}\frac{\partial^2 A}{\partial X^2}, \tag{43}$$

where

$$z_T = \frac{2c_0^3 C_P\rho_0}{(\gamma-1)\chi\omega^2} \tag{44}$$

designates the characteristic scale of thermal diffusion. Equation (43) is related to Eq. (41) by means of a force F (42). The stationary heating imposes $\partial T'/\partial t = 0$. For the preliminary evaluations, we calculate T' assuming A as a solution to

$$\frac{\partial A}{\partial Z} + \frac{z_{d,0}}{z_T}A = \mp\frac{i}{4}\frac{\partial^2 A}{\partial X^2}. \tag{45}$$

We will consider sound beams with an initially plane wave front and a Gaussian transverse distribution of amplitude. The solution to Eq. (45) with the boundary condition

$$\frac{A(X,0)}{A_0} = \exp(-X^2) \tag{46}$$

is

$$\frac{A(X,Z)}{A_0} = \frac{\exp\left(-\frac{X^2}{1\mp iZ} - \frac{z_{d,0}}{z_T}Z\right)}{\sqrt{1\mp iZ}}. \tag{47}$$

The corresponding magnetosonic force takes the following form:

$$F = A_0^2 \frac{(\gamma-1)\chi\omega^2}{2C_P c_0 \rho_0^3} \frac{\exp\left(-\frac{2X^2}{1+Z^2} - 2\frac{z_{d,0}}{z_T}Z\right)}{\sqrt{1+Z^2}}. \tag{48}$$

Making use of (48) and (41), we rearrange (43) as

$$\frac{\partial A}{\partial Z} + i\eta \exp\left(-2\frac{z_{d,0}}{z_T}Z\right)\left(\exp\left(-\frac{2X^2}{1+Z^2}\right)\sqrt{1+Z^2} + \sqrt{2\pi}X\mathrm{erf}\left(\frac{\sqrt{2}X}{\sqrt{1+Z^2}}\right)\right)A + \frac{z_{d,0}}{z_T}A = \mp\frac{i}{4}\frac{\partial^2 A}{\partial X^2}, \tag{49}$$

where

$$\eta = \frac{(\gamma-1)^2 D_0^2}{2(\gamma+1)^2}\frac{z_{d,0}^2}{z_{nl}^2} \tag{50}$$

is a dimensionless parameter that reflects, in fact, a squared ratio of nonlinear to diffraction effects. Equation (49) is solved numerically in *Mathematica*. Zero boundary conditions are set at $X = \pm 100$, and Z varies from 0 till 100. The dimensionless magnitude at the beam's axis $|A|/A_0$ for some couples of η and $z_{d,0}/z_T$ in the unusual and ordinary cases is shown in Fig. 1. Broken curves represent $|A|/A_0$ at the axis of a beam in accordance with (47), that is, they do not consider nonlinear self-action but only linear impact of diffraction and thermal conduction. The module of magnitude is the same for both signs on the right of Eq. (47). Positive δ is responsible for the usual thermal self-defocusing of a beam in a gaseous medium. The unusual case results in larger magnitudes than that in the linear dynamics. This resembles thermal self-focusing when $\delta < 0$.

VII. CONCLUDING REMARKS

One of the main results of this study is dynamic equations for perturbations in the quasi-plane beams, which propagate parallel or perpendicular to the magnetic field (28)–(30). These equations take into account weak nonlinearity and divergence and may be readily generalized including damping. We started with the description of quasi-plane linear magnetosonic beam, which is ordered by the dispersion relation. The cases $\theta = 0$ (the beam's axis coincides with the vector of magnetic field) and $c_0 \neq C_A$ are especial and yield unusual sign of the diffraction term in a beam, which propagates with the speed C_A (c_0) if $c_0 > C_A$ ($c_0 < C_A$). The divergence is more pronounced for beams propagating with both speeds as compared to unmagnetized gas. Namely, the scale of transversal divergence is smaller $C_A/\sqrt{|C_A^2 - c_0^2|}$ ($c_0/\sqrt{|c_0^2 - C_A^2|}$) times. The analysis of dynamics of magnetosonic perturbation of density [Eq. (28)] in the limiting cases of strong and weak diffraction compared to nonlinear effects is considered in Sec. V. As usual, the small-magnitude perturbations propagate faster along a beam's axis than that in the plane wave.[1] Perturbations propagate slower in unusual cases. Section VI considers model equations and possible applications. Equation (39) incorporates nonlinear, diffraction, and damping effects due to heat conduction on a beam's dynamics and resembles the KZK equation. The analytical solution to the KZK equation is still unavailable. The exception is an analytical

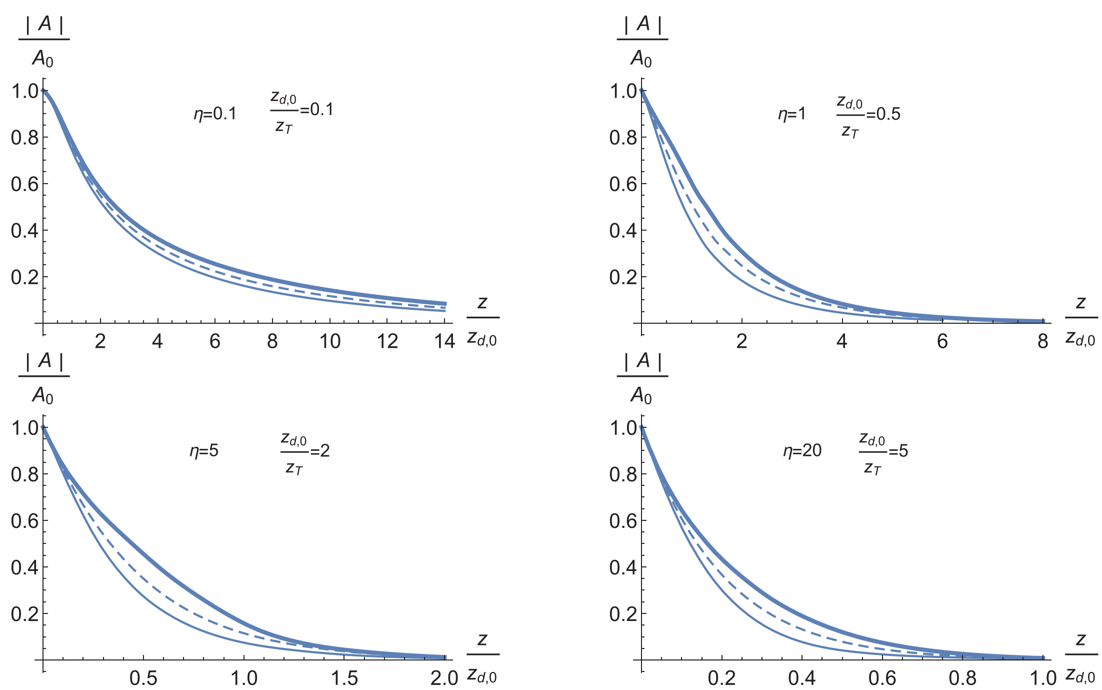

FIG. 1. The dimensionless amplitude at the axis of a beam $|A|/A_0$ for various η, $z_{d,0}/z_T$. Bold lines correspond to the unusual case [the plus sign on the right of (49)], and thin lines correspond to the ordinary case (sign minus). The broken lines represent a linear case without thermal self-action, that is, $|A|/A_0$ in accordance with (47).

method for the description of a limiting case of the KZK equation without damping (KZ equation), considering the paraxial region of a beam radiated by an axisymmetric harmonic source.[17] It is well known that the shifts of harmonics phases due to diffraction result to smoothed and extended rarefaction phase and shortened and sharpened compression phase of perturbation within each period.[10]

Section VI B brings some interesting features of stationary thermal self-action of initially Gaussian periodic beams in the case when nonlinear distortion of the magnetosonic perturbations is weak and does not lead to the broadening of frequency spectrum. That formally corresponds to the linear limit of (40), which couples with (41) by means of nonlinear excitation of temperature perturbation. The dynamics are determined by two parameters, η [Eq. (50)] and $z_{d,0}/z_T$ [Eqs. (14) and (44)], and are modeled by Eq. (49). The unusual thermal self-action leads to increase in a beam's magnitude at the axis and resembles defocusing of a beam that takes place in a medium with negative temperature coefficient δ (majority of liquids). The results can have a practical application. The character of beams propagation may be useful in analysis of plasma parameters and processes within, which are difficult for direct measurement. There are some issues that may indicate the equilibrium parameters of a plasma, the geometry of a flow, damping, and the characteristic frequency of perturbations in remote observations:

(1) Diffraction of a magnetosonic beam that takes place only in the case of parallel or perpendicular propagation of a beam in relation to the direction of the magnetic field. The divergence manifests itself stronger compared to unmagnetized gas in the case of parallel propagation and weaker in the case of perpendicular one. The characteristic scales of diffraction $z_{d,0}$, $z_{d,A}$ (parallel) or x_d (perpendicular) depend not only on the frequency of exciter and its characteristic scale but also on the ratio of c_0 and C_A (the greater the difference between c_0 and C_A, the diffraction more pronounced).
Even more unusual dynamics are in cases $\theta = 0$ and $c_0 = C_A$. Any perturbation in the form $\varphi(t - z/c_0, z, x)$ weakly (in the same order of smallness) depending on two last variables is a leading-order solution to the linear dynamic equation. Diffraction is weak in this case as well as in the case of any oblique beam propagation.
(2) A speed of propagation of perturbations at the axis of a beam ($c_0 \neq C_A$), which is directed along magnetic field. It is larger than that in the plane wave ordinarily and smaller in unusual cases. The dimensionless module of excess speed $|\Delta c|/c_0 = c_0^2 D_0^2/(\omega^2 a^2)$ [$|\Delta c|/C_A = c_A^2 D_A^2/(\omega^2 a^2)$] depends on the ratio of c_0 and C_A by means of D_0 (D_A) and enlarges proportionally to $|1 - c_0^2/C_A^2|^{-1}$ ($|1 - C_A^2/c_0^2|^{-1}$).
(3) The links specifying perturbations that determine any mode equivalent to the dispersion relation. The Alfvén mode that propagates with the speed C_A and is defined by links (22) behaves as the plane wave, but the mode (18) behaves as a beam propagating with the main speed C_A and experiences a divergence. The links (7), (18), (22), and (24) are valid at any time. The specific links are undeservedly underestimated in hydrodynamic applications. They are of importance in the wave theory and may be referred to as "the constitutive" equations or "the polarization relations." The similar relations supplied by the nonlinear terms making sound isentropic in the leading order and terms associating with damping are called "self-consistent relations" in the nonlinear acoustics.[1] The idea was exploited by Khokhlov in regard to nonlinear electromagnetic waves.[28] The links allow to project the total perturbation field into specific modes and to derive coupling dynamic equations for the interacting wave and non-wave modes in a weakly nonlinear flow.[29]
(4) Nonlinear effects of magnetosonic beam. In the case of unusual diffraction of a beam in the course of parallel propagation, thermal self-focusing of a beam takes place. Thermal self-action is determined by the ratio of characteristic thermal diffusion distance, the diffraction distance, and the shock formation distance.

Summing up, the linear (that is, small-magnitude) dynamics of a beam reveal a wide variety of behavior including unusual divergence. Diffraction of a beam is crucial also in the nonlinear dynamics since it is responsible for the variation of magnitude of wave perturbations in a beam's cross section (the nonlinear effects are proportional to the square magnitude of wave perturbations). The variety of behavior of a magnetoacoustic beam and the nonlinear effects in its field are fairly wide and do not depend only on diffraction. The heating-cooling function balances with damping and may lead to unusual properties of a beam and relative nonlinear phenomena. The magnitude of perturbations at the axis of a beam may enlarge in the course of propagation. This happens to many acoustically active flows.[30–32] The nonlinear phenomena such as acoustic heating may occur especially. A medium may get cooler, and the streaming may be excited in the unusual direction.[33–35] This concerns an acoustically active flow of a plasma.[36] The unusual diffraction may introduce new conclusions in regard to linear and nonlinear magnetohydrodynamics of confined beams in more complex geometry of a flow.

AUTHOR DECLARATIONS

Conflict of Interest

The author has no conflicts to disclose.

DATA AVAILABILITY

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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