Results in Physics 43 (2022) 106048

Contents lists available at ScienceDirect

Results in Physics

journal homepage: www.elsevier.com/locate/rinp

# Some new soliton solutions to the higher dimensional Burger–Huxley and Shallow water waves equation with couple of integration architectonic

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ARTICLE INFO

*Keywords:*
Integrability
$F$-expansion scheme
Jacobi elliptic function

ABSTRACT

In this paper, we retrieve some traveling wave, periodic solutions, bell shaped, rational, kink and anti-kink type and Jacobi elliptic functions of Burger's equation and Shallow water wave equation with the aid of various integration schemes like improved $F$-expansion scheme and Jacobi elliptic function method respectively. We also present our solutions graphically in various dimensions.

## Introduction

Nonlinear evolution equations (NLEEs) arising in nonlinear sciences play an important role in understanding nonlinear phenomenon. Solitons consist of huge applications in physics, communication systems, optical science, applied mathematics and engineering problems. The most arising and fast growing area of research is the optical solitons. A soliton or solitary wave is a self reinforcing wave packet that maintains its shape while it propagates at constant velocity. Solitons are caused by a cancellation of nonlinear and dispersive effects in the medium. Solitons are the solutions of a well-known class of weakly nonlinear dispersive partial differential equations describing physical systems. Soliton solutions have very interesting properties of some nonlinear partial differential equations. A soliton is a localized traveling wave solution of a nonlinear PDE that is remarkably stable [1–8]. In optical fibers, the solitons are the attractive field of research in applied mathematics, telecommunication systems and in distinctive branches of physics. A lot of research has been done on solutions of the solitons in nonlinear optics [9–14]. Many high-dimensional nonlinear evolution equations also contain some nonlinear structures such as the breath wave and lump solutions. In low-dimensional models, nonlinear waves can be classified from different aspects [15–17]. It is well known that the nonlinear Schrödinger (NLS) equation is a fundamental model in nonlinear physical systems, which plays a prominent role in a wide range of physical subjects such as plasma physics. This work not only reveals the characteristics of single bright–bright soliton by theoretical analysis, but also describes a wealth of new phenomena of two, three, and four bright–bright solitons, including elastic collision, soliton reflection, parallel propagation, time-periodic propagation, and (space, time)-periodic propagation [18–20].

The objective of this article is to execute the Improved F-expansion method in accomplishing the traveling wave arrangements to NLEEs within the mathematical physics through the DSW condition and the Burgers condition in terms of functions that fulfill the Riccati condition $F(\xi) = k + F^2(\xi)$.

The Burgers condition is the least order estimation for the one-dimensional proliferation of weak waves in a fluid. It is additionally utilized in vehicle frequency in high way activity. It is one of the fundamental PDEs in fluid mechanics. Burgers condition is totally fundamental. The wave arrangements of Burgers equation are single and multiple-front arrangements (Wazwaz, 2009). The DSW condition is an imperative wave demonstrate in material science (Inc, 2006).

Burgers condition was to begin with presented at Bateman within 1915 and afterward analyzed at Burgers within 1948. The condition is utilized as a demonstrate in numerous areas such as, ceaseless stochastic forms [21] dissipate water [22] stun waves, warm [23]. Burgers condition can onto considered to a rearranged frame of the N-S condition onto frame item or the event consistency term [24]. Unused correct arrangements of the common Burgers condition are

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https://doi.org/10.1016/j.rinp.2022.106048
Received 16 August 2022; Received in revised form 7 October 2022; Accepted 11 October 2022
Available online 21 October 2022
2211-3797/

independently determined at a coordinate strategy or only condition to the Bernoulli condition come the only condition [25] individually. The couple correct arrangements are appeared will be equal [26]. In expansion, the recently determined arrangement can be effectively decreased to straight [27] whereas the nonlinear term coefficient equal to zero [28]. At last, inferred arrangement is compared with the irritation arrangement and some existing correct arrangements [29]. A few numerical comes about are displayed and outlined [30,31].

The generalized Burger–Huxley (gBH) equation is a first-order nonlinear partial differential equation (NPDE) in time $t$ that allows for the propagation of a single-wave (single-mode) solution. The generalized Burger–Huxley condition may be a non-linear fractional differential condition of first arrange in time $t$ and peruses [32]:

$$U_t - U_{xx} - \mu U U_x - \nu U(1-U)(U-\gamma) = 0 \tag{1}$$

By wave transformation

$$U = U(\xi), \quad \xi = x + ct \tag{2}$$

where $\phi = \phi(x,t)$ and $\mu$, $\nu$, $\gamma$ are common genuine scalars.

For $\mu = 0$ and $\gamma = 1$, gBH is reduced to the Huxley model, representing nerve pulse propagation in nerve fibers and liquid crystal wall motion (Wang et al. 1990). Eq. (1) is the Burgers' equation, which describes the field of wave propagation in nonlinear dissipative models for $\nu = 0$ and $\gamma = 1$. Burger–Huxley Eq. (1) describes how convection terms interact with diffusion transmission (Wang et al. 1990).

We will look at a partial differential equation with non-linearity, a weakly nonlinear shallow water wave equation used to study wave propagation in dispersive and weakly nonlinear media.

The model under consideration is:

$$U_{tt} - \lambda\sigma U_{xx} + \frac{1}{2\sigma}(U_t^2)_x - \frac{\sigma^2}{3}U_{xxtt} = 0, \tag{3}$$

By wave transformation

$$U = U(\xi), \quad \xi = x + ct \tag{4}$$

where $U = U(x,t)$ represents the unification of the displacement and velocity of the water particles, the gravitational force is represented by $\lambda$. The wave height is represented by $\sigma$. The Euler equation of motion, the equation of dynamics and mass conservation, and kinematics constraints on the boundary of the domain consisting of the free surface have essentially illustrated the water problem for gravity waves.

We applied two integration schemes, namely IFE scheme [33] and JEF method [34] in this paper. IEF method is applicable to get some solitary wave, periodic wave and rational function solutions and JEF method provide Jacobi elliptic function solutions.

The paper is written in the following sequence: in Section "Contents of Integration Schemes", we summarize IFE and JEF scheme. In Section "Mathematical analysis", we use IFE and JEF schemes to obtain some solitary, periodic wave, bell shaped, kink and anti kink type solutions, rational solutions and Jacobi elliptic solutions in terms of hyperbolic and trigonometric functions with their graphical representations. in Section "Results and discussion" we discuss our results and lastly in Section "Conclusion", we conclude our results.

## Contents of integration schemes

### *Analysis of IFE method*

Consider NLEE in the form [33].

$$G(U, U_x, U_y, U_t, U_{xx}, U_{xy}, U_{xt}, \dots) = 0,$$

where

$$U(x,y,t) = U(\xi), \quad \xi = \mu x + \nu y + \varepsilon t.$$

Replace in above equation to get ODE.

$$H(U, U', U'', U''', \dots) = 0, \quad U = U(\xi).$$

Now we use following transformation,

$$U(\xi) = \sum_{i=0}^{m} a_i F^i + \sum_{i=1}^{n} b_i F^{-i},$$

where $m$ can be find through homogeneous balance and $a_i$' and $b_i$'s are to be determined. We know the Riccati equation $F'(\xi) = k + F^2(\xi)$. Substitute these transformations into first equation to get periodic wave and solitary wave solutions. Now we represent the general solution of Riccati equation [35].

When $k < 0$,

$$F_1 = -\sqrt{-k}\tanh(-\sqrt{-k}\xi),$$

$$F_2 = -\sqrt{-k}\coth(-\sqrt{-k}\xi).$$

When $k > 0$,

$$F_3 = \sqrt{k}\tan(\sqrt{k}\xi),$$

$$F_4 = \sqrt{k}\cot(\sqrt{k}\xi).$$

For $k = 0$,

$$F_5 = \frac{-1}{\xi}.$$

### *Analysis of JEF method*

We have nonlinear evolution equation [14].

$$G(U, U_t, U_x, U_{xt}, \dots) = 0.$$

We use following transformations.

$$U(x,y,t) = U(\xi), \quad \xi = \alpha x + \beta y + \varepsilon t,$$

this can be transform into nonlinear ODE.

$$G_0(U, U_\xi, U_{\xi\xi}, \dots) = 0.$$

Now use these transformations.

$$U(\xi) = \sum_{i=0}^{m} d_i F^i,$$

in which $d_i$'s are to be determined where $i = 0, 1, 2, \dots, m$. By homogeneous balance we find "$m$" and $F(\xi)$ express in following substitution. $F' = \sqrt{r + dF^2 + \frac{eF^4}{2} + \frac{cF^6}{3}}$ where $r$, $c$, $d$, and $e$ are real parameters.

We insert $F'$ in previous transformations to obtain system of equations then obtain the values of $d_i$'s.

If we assume $F(\xi) = \mathrm{sn}\xi, \mathrm{cn}\xi, \mathrm{cs}\xi$, its called JEF method.

## Mathematical analysis

Putting Eq. (2) along its derivatives in Eq. (1) we get,

$$CU' - U'' - \mu UU' - \nu u^2 + \nu U\gamma + \nu U^3 - \nu U^2\gamma = 0, \tag{5}$$

By comparing highest nonlinear term with highest derivative, we get $m = 1$ through homogeneous balance.

### *IFE method*

We use following transformation [33],

$$U(\xi) = \Sigma_{i=0}^{m} a_i F^i + \Sigma_{i=1}^{n} b_i F^{-i},$$

$$U(\xi) = a_0 + a_1 F + b_1 F^{-1}. \tag{6}$$

Putting Eq. (5) along its derivatives in Eq. (4), we get

$$C(a_1K + a_1F^2 - \frac{b_1K}{F^2} - b_1) - \mu(a_1K + a_1F^2 - \frac{b_1K}{F^2} - b_1)$$
$$.(a_0 + a_1F + b_1F^{-1}) - (2a_1FK - 2a_1F^3 - \frac{2b_1K^2}{F^3} - \frac{2b_1K}{F})$$
$$- \nu(a_0 + a_1F + b_1F^{-1})^2 + \nu\gamma(a_0 + a_1F + b_1F^{-1}) + \nu(a_0 + a_1F + b_1F^{-1})^3$$
$$- \nu\gamma(a_0 + a_1F^1 + b_1F^{-1})^2 = 0 \quad (7)$$

where $F' = k + F^2$ (see Figs. 1–20). Setting the coefficients of each power of $F(\xi)$ to zero as follows.

$$-2a_1 - \mu a_1^2 + \nu a_1^3, \quad (8)$$
$$ca_1 + 3\nu a_0 a_1^2 - \mu a_1 a_0 - \nu a_1^2 - \nu\gamma a_1^2, \quad (9)$$
$$\nu\gamma a_1 - 2a_1K - \mu(a_1K - b_1)a_1 - \mu a_1 b_1$$
$$+\nu(b_1a_1^2 + 2a_0^2a_1 + a_1(2a_1b_1 + a_0^2)) - 2\nu\gamma a_0 a_1 - 2\nu a_0 a_1, \quad (10)$$
$$\nu\gamma a_0 + c(a_1K - b_1) - \mu(a_1K - b_1)a_0 - \nu\gamma(2a_1b_1 + a_0^2)$$
$$+\nu(4a_0a_1b_1 + a_0(2a_1b_1 + a_0^2)) - \nu(2a_1b_1 + a_0^2), \quad (11)$$
$$-2\nu b_1 a_0 + \nu(b_1(2a_1b_1 + a_0^2) + 2a_0^2 b_1 + a_1 b_1^2) + \mu b_1 K a_1$$
$$-\mu(a_1K - b_1)b_1 + \nu\gamma b_1 - 2b_1K - 2\nu\gamma b_1 a_0, \quad (12)$$
$$3\nu b_1^2 a_0 - cb_1K - \beta b_1^2 + \mu b_1 K a_0 - \nu\gamma b_1^2, \quad (13)$$
$$-2b_1K^2 + \nu b_1^3 + \mu b_1^2 K. \quad (14)$$

We get,

**Set 1:**

$$K = \frac{-1}{16a_1^2}, \quad c = \frac{-1 + \nu\gamma a_1^2}{a_1}, \quad \mu = \frac{-2 + \beta a_1^2}{a_1} \quad a_1 = a_1, \quad a_0 = \frac{1}{2},$$
$$b_1 = \frac{1}{16a_1}.$$

we get

$$U_1(\xi) = \frac{1}{2} + a_1F + \frac{1}{16a_1}F^{-1}.$$

**When** $k < 0$,

$$U_1 = \frac{1}{2} - a_1\sqrt{-k}tanh(\sqrt{-k}\xi) - \frac{1}{16a_1(\sqrt{-k}tanh(\sqrt{-k}\xi))}, \quad (15)$$

and

$$U_1 = \frac{1}{2} - a_1\sqrt{-k}coth(\sqrt{-k}\xi) - \frac{1}{16a_1(\sqrt{-k}coth(\sqrt{-k}\xi))}. \quad (16)$$

**When** $k > 0$,

$$U_1 = \frac{1}{2} + a_1(\sqrt{k}tan\sqrt{k}\xi) + \frac{1}{16a_1(\sqrt{k}tan(\sqrt{k}\xi))}, \quad (17)$$

and

$$U_1 = \frac{1}{2} - a_1(\sqrt{k}cot\sqrt{k}\xi) - \frac{1}{16a_1(\sqrt{k}cot(\sqrt{k}\xi))}. \quad (18)$$

**When** $k = 0$,

$$U_1 = \frac{1}{2} - a_1(\frac{1}{\xi}) + \frac{1}{16a_1}(-\frac{1}{\xi})^{-1}. \quad (19)$$

**Set 2:**

$$k = k, \quad a_0 = (\mu^2 - \nu^2)k, \quad a_1 = 0, \quad a_2 = 0, \quad b_1 = 0, \quad b_2 = (\mu^2 - \nu^2)k^2.$$

we get

$$U_2(\xi) = (\mu^2 - \nu^2)k + (\mu^2 - \nu^2)k^2F^{-2}. \quad (20)$$

**When** $k < 0$,

$$U_2 = (\mu^2 - \nu^2)k - (\mu^2 - \nu^2)k^3(\coth(-\sqrt{-k}\xi))^2, \quad (21)$$

and

$$U_2 = (\mu^2 - \nu^2)k - (\mu^2 - \nu^2)k^3(\tanh(-\sqrt{-k}\xi))^2. \quad (22)$$

**When** $k > 0$,

$$U_2 = (\mu^2 - \nu^2)k + (\mu^2 - \nu^2)k^3(\cot(\sqrt{k}\xi))^2, \quad (23)$$

and

$$U_2 = (\mu^2 - \nu^2)k + (\mu^2 - \nu^2)k^3(\tan(\sqrt{k}\xi))^2. \quad (24)$$

**When** $k = 0$,

$$U_2 = (\mu^2 - \nu^2)k + (\mu^2 - \nu^2)k^2\frac{1}{\xi^2}. \quad (25)$$

**Set 3:**

$$k = k, \quad a_0 = 2(\mu^2 - nu^2)k, \quad a_1 = 0, \quad a_2 = \mu^2 - \nu^2, \quad b_1 = 0, \quad b_2 = (\mu^2 - \nu^2)k^2.$$

we get

$$U_3(\xi) = 2(\mu^2 - \nu^2)k + (\alpha^2 - \beta^2)F^2 + (\mu^2 - \nu^2)k^2F^{-2}. \quad (26)$$

**When** $k < 0$,

$$U_3 = 2(\mu^2-\nu^2)k - (\mu^2-\nu^2)k(\tanh(-\sqrt{-k}\xi))^2 - (\mu^2-\nu^2)k^3(\coth(-\sqrt{-k}\xi))^2, \quad (27)$$

and

$$U_3 = 2(\mu^2-\nu^2)k - (\mu^2-\nu^2)k(\coth(-\sqrt{-k}\xi))^2 - (\mu^2-\nu^2)k^3(\tanh(-\sqrt{-k}\xi))^2. \quad (28)$$

**When** $k > 0$,

$$U_3 = 2(\mu^2 - \nu^2)k + (\mu^2 - \nu^2)k(\tan(\sqrt{k}\xi))^2 - (\mu^2 - \nu^2)k^3(\cot(\sqrt{k}\xi))^2, \quad (29)$$

and

$$U_3 = 2(\mu^2 - \nu^2)k + (\mu^2 - \nu^2)k(\cot(\sqrt{k}\xi))^2 - (\mu^2 - \nu^2)k^3(\tan(\sqrt{k}\xi))^2. \quad (30)$$

**When** $k = 0$,

$$U_3 = 2(\mu^2 - \nu^2)k + (\mu^2 - \nu^2)\frac{1}{\xi^2} + (\mu^2 - \nu^2)k^2\xi^2. \quad (31)$$

**Set 4:**

$$k = k, \quad a_0 = \frac{-2}{3}(\mu^2-\nu^2)k, \quad a_1 = 0, \quad a_2 = (\mu^2-\nu^2) \quad b_1 = 0, \quad b_2 = (\mu^2-\nu^2)k^2.$$

we get

$$U_4(\xi) = \frac{-2}{3}(\mu^2 - \nu^2)k + (\mu^2 - \nu^2)F^2 + (\mu^2 - \nu^2)k^2F^{-2}. \quad (32)$$

**When** $k < 0$,

$$U_4 = \frac{-2}{3}(\mu^2-\nu^2)k - (\mu^2-\nu^2)k(\tanh(-\sqrt{-k}\xi))^2 - (\mu^2-\nu^2)k^3(\coth(-\sqrt{-k}\xi))^2, \quad (33)$$

and

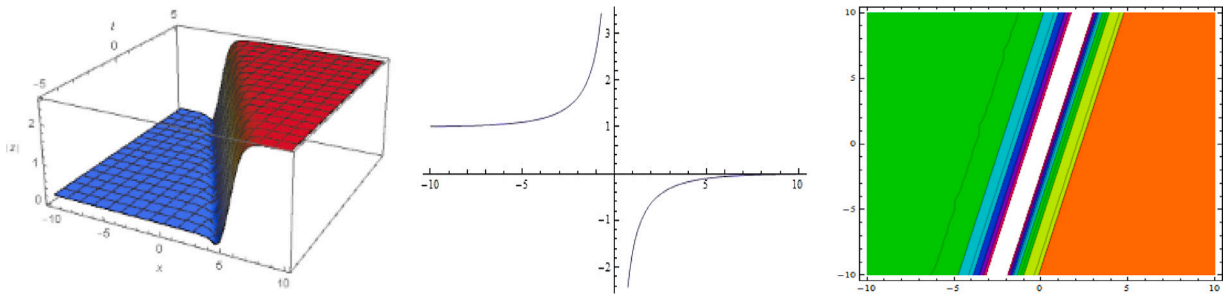
$$U_4 = \frac{-2}{3}(\mu^2-\nu^2)k - (\mu^2-\nu^2)k(\coth(-\sqrt{-k}\xi))^2 - (\mu^2-\nu^2)k^3(\tanh(-\sqrt{-k}\xi))^2. \quad (34)$$

**When** $k > 0$,

$$U_4 = \frac{-2}{3}(\mu^2 - \nu^2)k - (\mu^2 - \nu^2)k(\tan(\sqrt{k}\xi))^2 - (\mu^2 - \nu^2)k^3(\cot(\sqrt{k}\xi))^2, \quad (35)$$

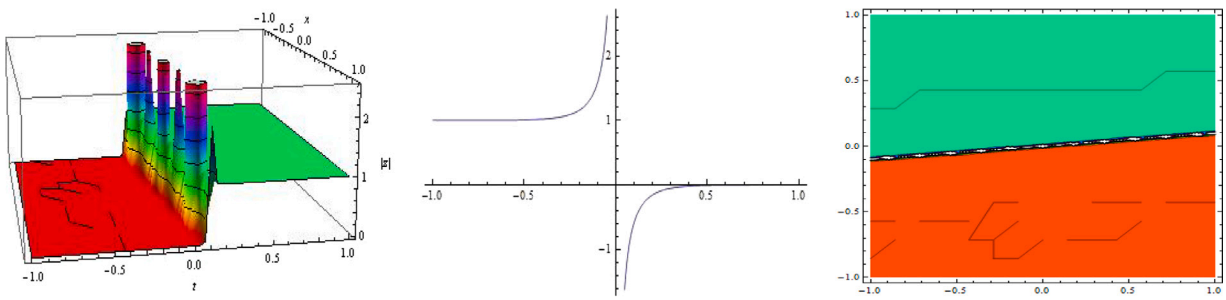
and

$$U_4 = \frac{-2}{3}(\mu^2 - \nu^2)k - (\mu^2 - \nu^2)k(\cot(\sqrt{k}\xi))^2 - (\mu^2 - \nu^2)k^3(\tan(\sqrt{k}\xi))^2. \quad (36)$$



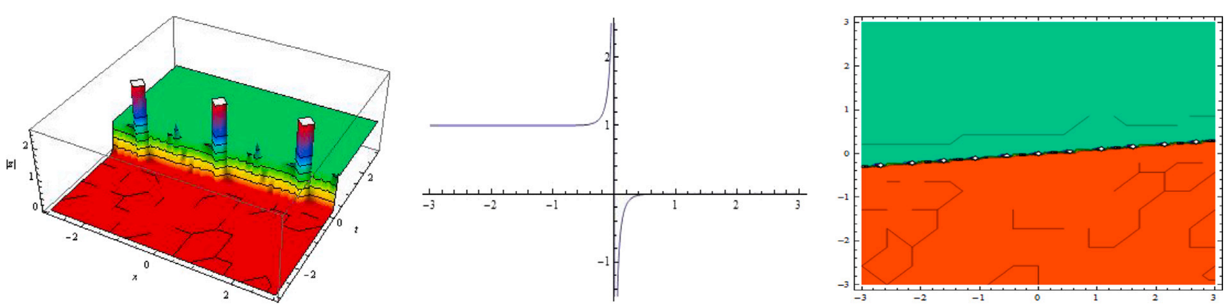
(a) (b) (c)

**Fig. 1.** The graphical presentation of $U_1(\xi)$ given by Eq. (13).



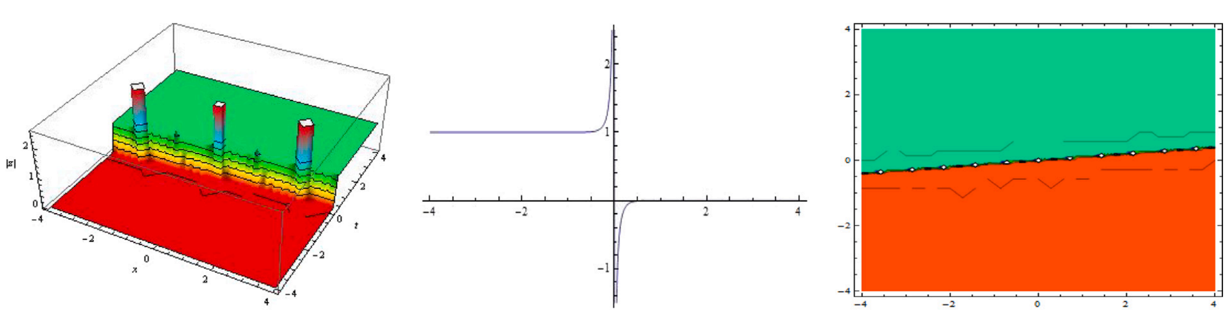
(a) (b) (c)

**Fig. 2.** The dynamical behavior of the solution $U_1(\xi)$ given by Eq. (14).



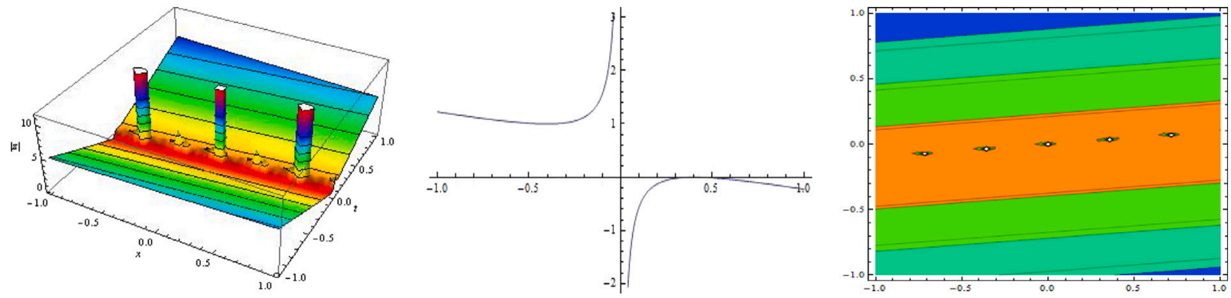
(a) (b) (c)

**Fig. 3.** The shape profile of $U_1(\xi)$ given by Eq. (15).



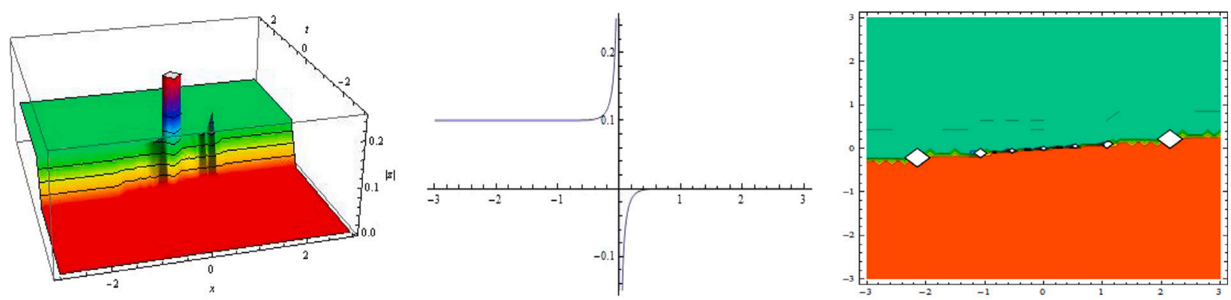
(a) (b) (c)

**Fig. 4.** The graphical presentation of $U_1(\xi)$ given by Eq. (16).



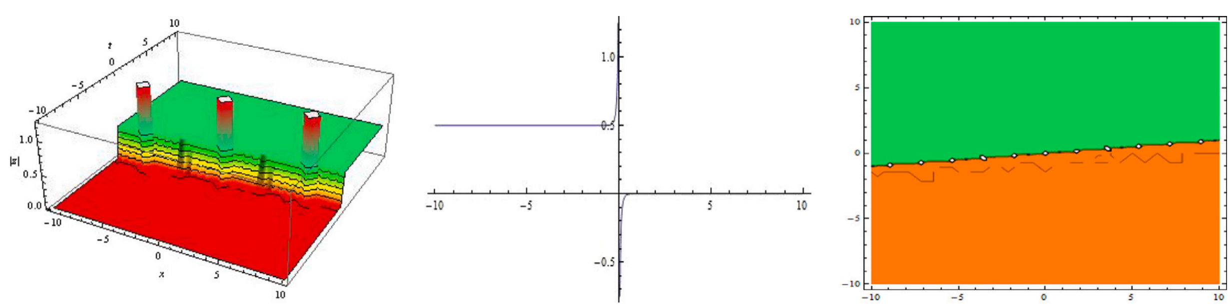
(a) (b) (c)

**Fig. 5.** The shape profile of $U_2(\xi)$ given by Eq. (17).



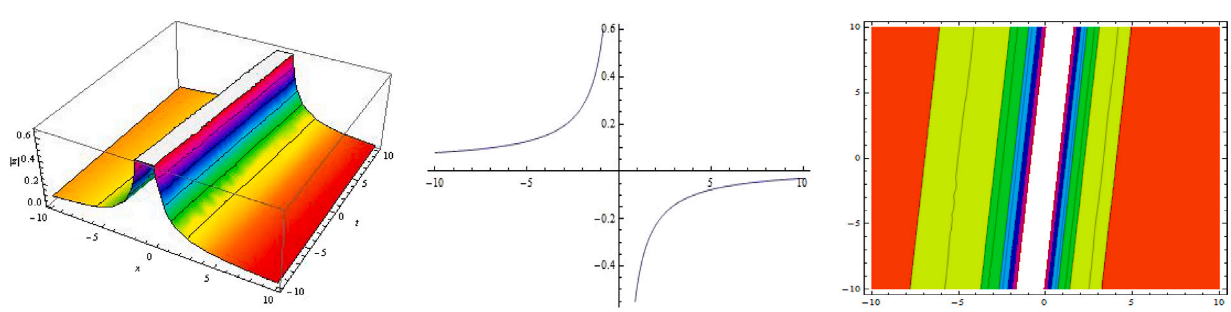
(a) (b) (c)

**Fig. 6.** The shape profile of $U_2(\xi)$ given by Eq. (19).



(a) (b) (c)

**Fig. 7.** The graphical presentation of $U_2(\xi)$ given by Eq. (20).



(a) (b) (c)

**Fig. 8.** The shape profile of $U_2(\xi)$ given by Eq. (21).

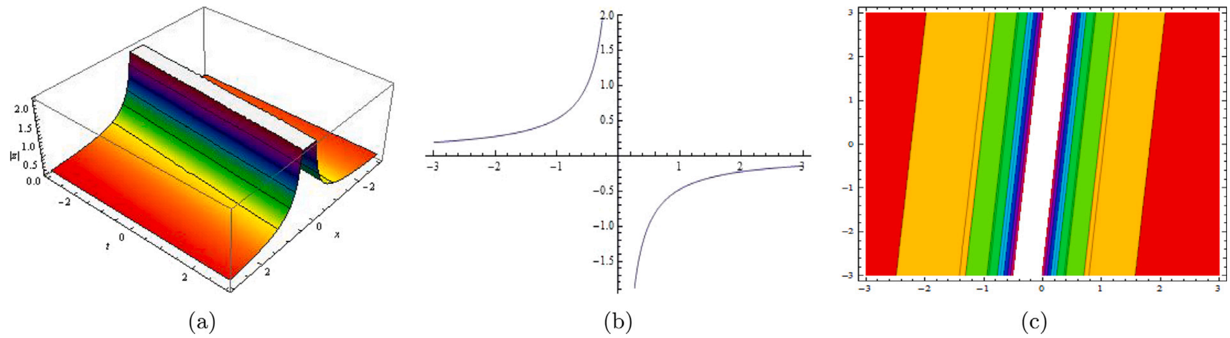



**Fig. 9.** The graphical presentation of $U_2(\xi)$ given by Eq. (22).

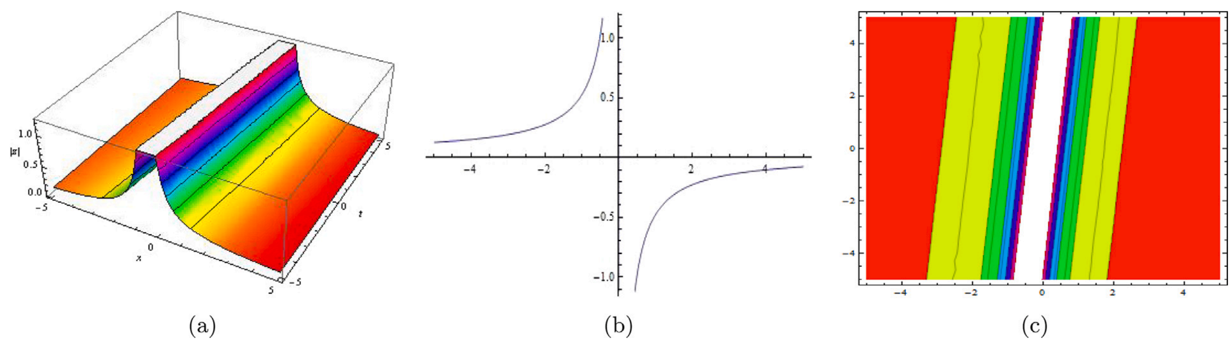



**Fig. 10.** The graphical presentation of $U_2(\xi)$ given by Eq. (23).

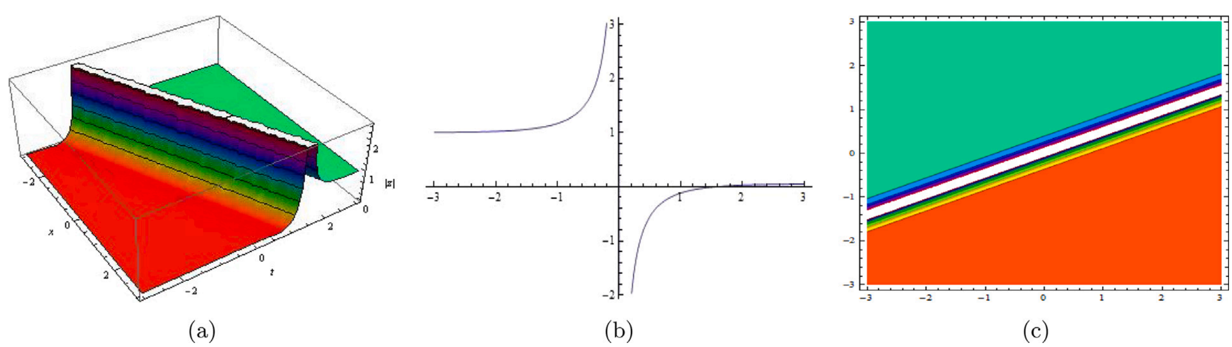



**Fig. 11.** The shape profile of $U_3(\xi)$ given by Eq. (25).

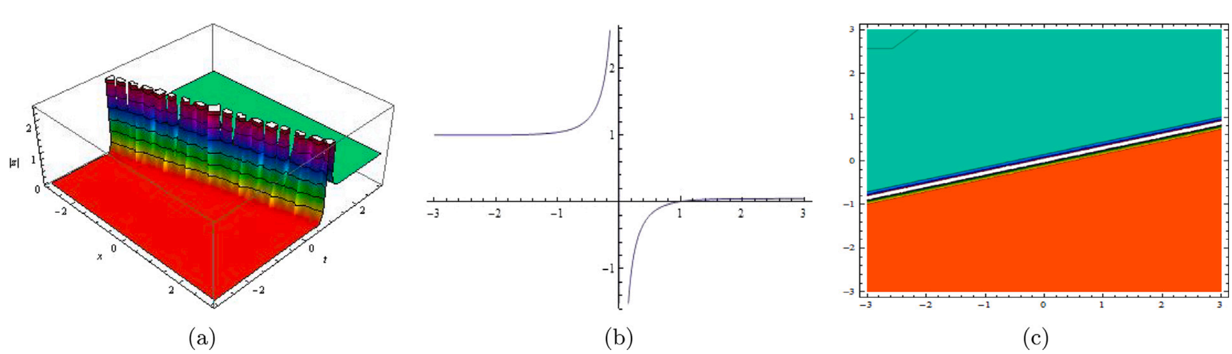
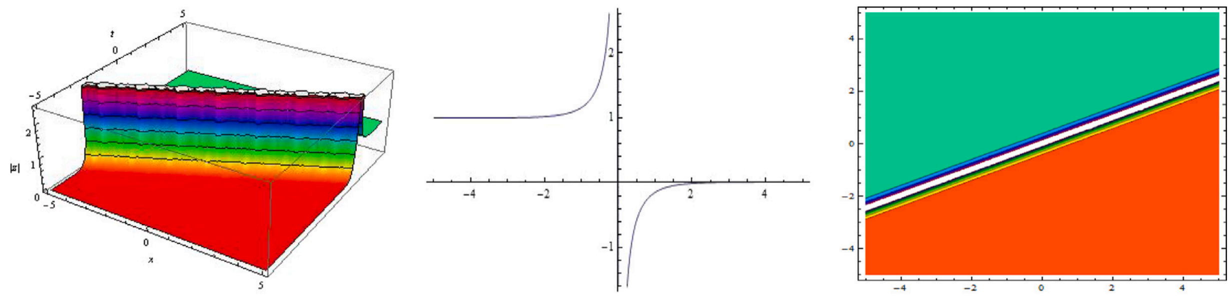
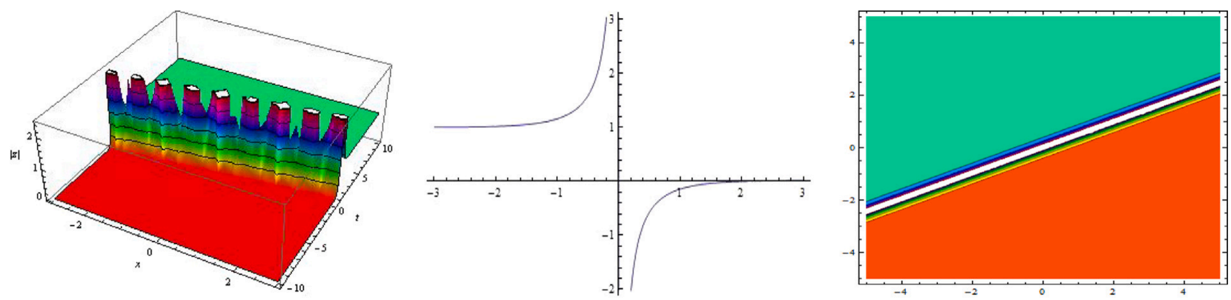



**Fig. 12.** The graphical presentation of $U_2(\xi)$ given by Eq. (26).



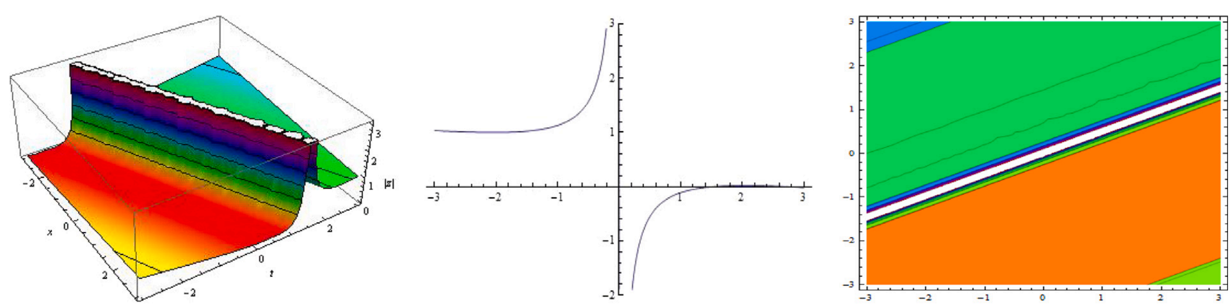
(a) (b) (c)

**Fig. 13.** The graphical presentation of $U_2(\xi)$ given by Eq. (27).



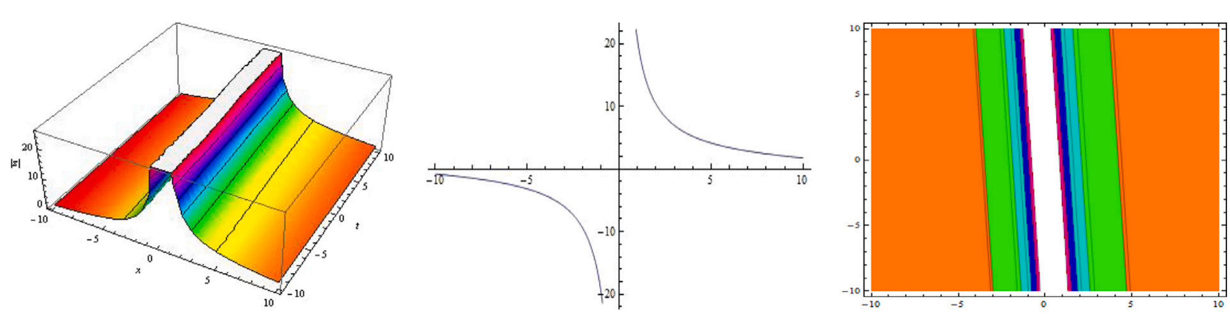
(a) (b) (c)

**Fig. 14.** The graphical presentation of $U_2(\xi)$ given by Eq. (28).



(a) (b) (c)

**Fig. 15.** The graphical presentation of $U_2(\xi)$ given by Eq. (29).



(a) (b) (c)

**Fig. 16.** The shape profile of $U_4(\xi)$ given by Eq. (31).

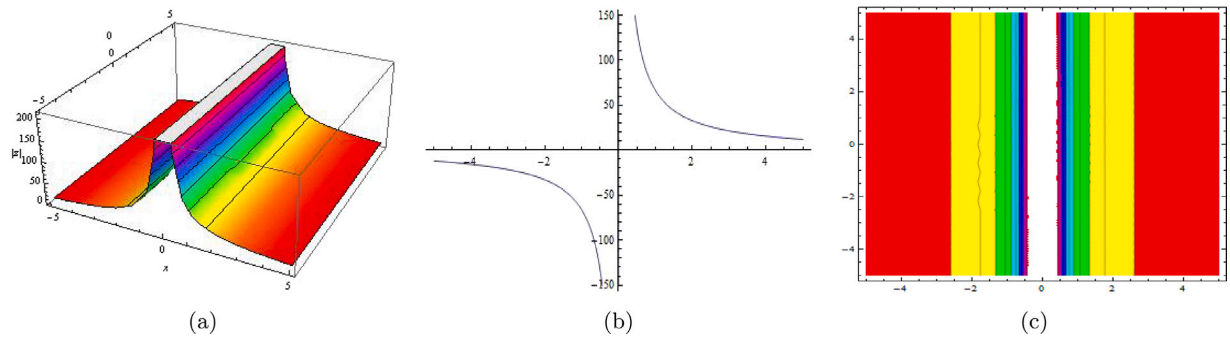



**Fig. 17.** The graphical presentation of $U_2(\xi)$ given by Eq. (32).

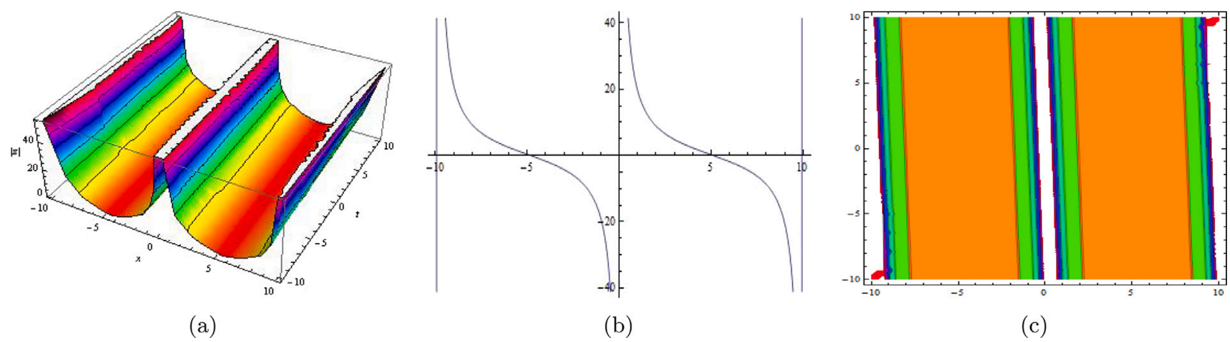



**Fig. 18.** The graphical presentation of $U_2(\xi)$ given by Eq. (33).

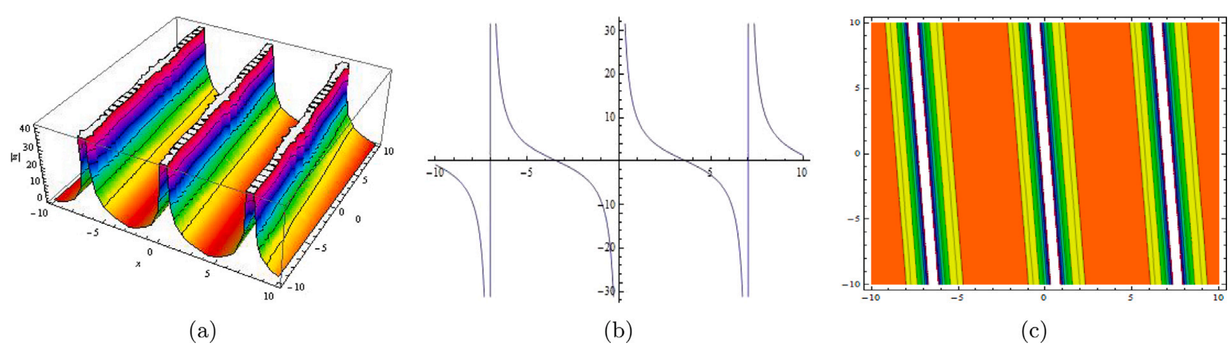



**Fig. 19.** The graphical presentation of $U_2(\xi)$ given by Eq. (34).

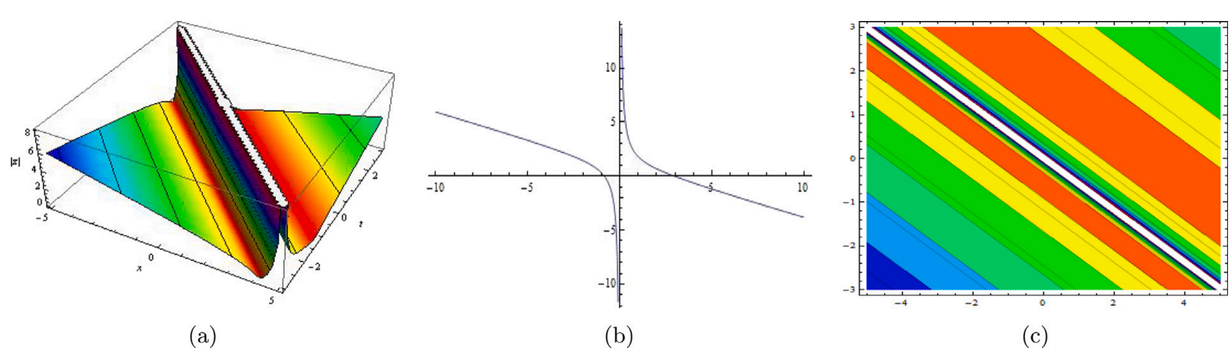
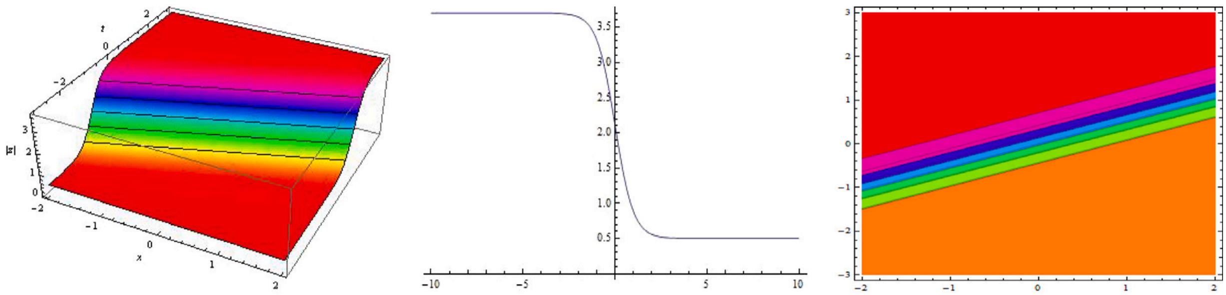



**Fig. 20.** The graphical presentation of $U_2(\xi)$ given by Eq. (35).



(a) (b) (c)

**Fig. 21.** The graphical presentation of $U(\xi)$ given by Eq. (44).

**When** $k=0$,

$$U_4=\frac{-2}{3}(\mu^2-\nu^2)k-(\mu^2-\nu^2)\frac{1}{\xi^2}+(\mu^2-\nu^2)k^2\xi^2. \tag{37}$$

*JEF method*

For JEF method, we use this transformation [36].

$$U(\xi)=\Sigma_{i=0}^{m}a_iF^i$$

$$U(\xi)=a_0+a_1F+a_2F^2 \tag{38}$$

Putting Eq. (4) along its derivatives in Eq. (3) and set $F'=\sqrt{r+aF^2+\frac{bF^4}{2}+\frac{cF^6}{3}}$ in Eq. (3), then we get,

$$a_1aF+a_1bF^3+a_1cF^5+2a_2r+4a_2aF^2+3a_2bF^4+\frac{8}{3}a_2cF^6-b_1aF^{-1}-b_1F^3c-b_1Fb$$

$$-\tau(a_0+a_1F+a_2F^2)-\xi(a_0+a_1F+a_2F^2)^2=0. \tag{39}$$

Setting the coefficients of each power of $F(\xi)$ to zero as follows.

$$3a_2b-\xi a_2^2, \tag{40}$$

$$-b_1+a_1b-2\xi a_1a_2, \tag{41}$$

$$-\xi(2a_0a_2+a_1^2)+4a_2a-\tau a_2, \tag{42}$$

$$-2\xi a_0a_1-b_1b+a_1a-\tau a_1, \tag{43}$$

$$-\xi a_0^2-\tau a_0+2a_2r. \tag{44}$$

By solving simultaneously above equations we get,

$$c=\frac{b^2}{r},\quad r=r,\quad \tau=0,\quad \xi=0,\quad a_0=a_0,\quad a_1=\frac{b_1b}{a},\quad a_2=0.$$

So,

$$U(\xi)=U=a_0+\frac{b_1b}{a}F. \tag{45}$$

Depending on $a$, $b$, $c$, $r$ in Eq. ($F'$), we obtained different type of traveling wave solutions.

**Case 1:**
$a=-(1+m^2),\quad b=2m^2,\quad r=1,\quad c=0.$
We get,

$$U=a_0-\frac{2b_1m^2}{1+m^2}sn(\zeta,m),$$

As $m\to 1$ this generate shock wave (see Fig. 21).

$$U=a_0-\frac{2b_1m^2}{1+m^2}tanh(\zeta). \tag{46}$$

**Case 2:**
$a=2m^2-1,\quad b=2,\quad r=-m(1-m^2),\quad c=0.$
We obtained,

$$U=a_0+\frac{2b_1}{2m^2-1}ds(\zeta,n)$$

As $m\to 1$ this generate (see Fig. 22).

$$U=a_0+\frac{2b_1}{2m^2-1}cosech(\zeta) \tag{47}$$

**Case 3:**
$a=2-m^2,\quad b=2,\quad r=1-m^2,\quad c=0.$
We get,

$$U=a_0+\frac{2b_1}{2-m^2}cs(\zeta,n),$$

As $m\to 1$ this generate (see Fig. 23).

$$U=a_0+\frac{2b_1}{2-m^2}csch(\zeta). \tag{48}$$

**Case 4:**
$d=2m^2-1,\quad e=-2m^2,\quad r=(1-m^2),\quad c=0.$
We obtained,

$$U(\xi)=\frac{-2(2m^2-1)\beta^3+2(2m^2-1)\alpha^2\beta-2\epsilon}{6}-\frac{-2m^2(\alpha^2-\beta^2)}{2}\mathrm{cn}^2(\xi,m).$$

As $m\to 1$ this generate (see Fig. 24).

$$U(\xi)=\frac{-2\beta^3+2\alpha^2\beta-2\epsilon}{6}+(\alpha^2-\beta^2)\,\mathrm{sech}^2(\xi). \tag{49}$$

**Case 5:**
$d=2-m^2,\quad e=-2,\quad r=(m^2-1),\quad c=0.$
We get,

$$U(\xi)=\frac{-2(2-m^2)\beta^3+2(2-m^2)\alpha^2\beta-2\epsilon}{6}-\frac{-2(\alpha^2-\beta^2)}{2}\mathrm{dn}^2(\xi,m).$$

As $m\to 1$ this generate (see Fig. 25).

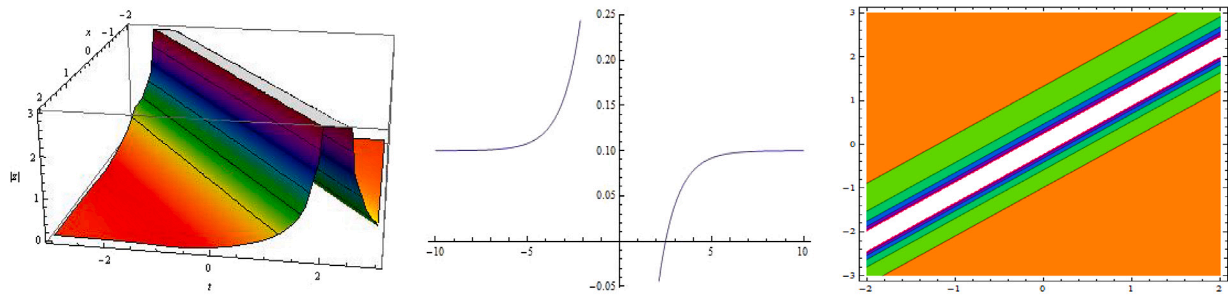
$$U(\xi)=\frac{-2\beta^3+2\alpha^2\beta-2\epsilon}{6}+(\alpha^2-\beta^2)\,\mathrm{sech}^2(\xi). \tag{50}$$

**Case 6:**
$d=\frac{m^2-2}{2},\quad e=\frac{m^2}{2},\quad r=\frac{1}{4},\quad c=0.$
We get,

$$U(\xi)=\frac{-2(\frac{m^2-2}{2})\beta^3+2(\frac{m^2-2}{2})\alpha^2\beta-2\epsilon}{6}-\frac{\frac{m^2}{2}(\alpha^2-\beta^2)}{2}\frac{\mathrm{sn}^2(\xi,m)}{(1\pm\mathrm{dn}(\xi,m))^2}.$$

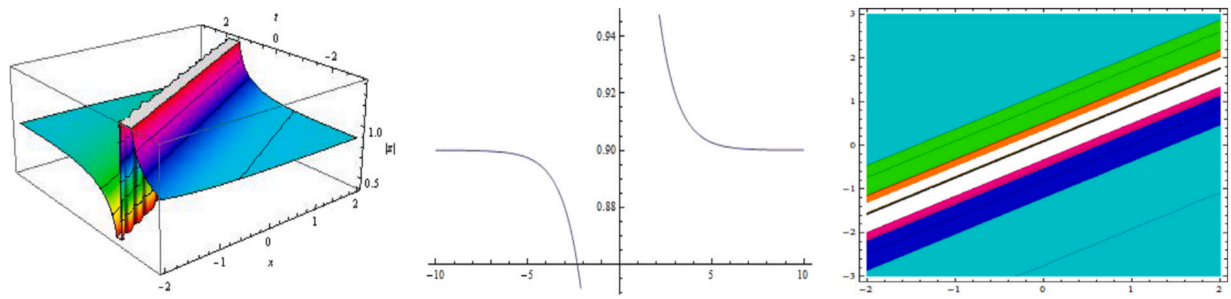
As $m\to 1$ this generate (see Figs. 26 and 27).

$$U(\xi)=\frac{\beta^3-\alpha^2\beta-2\epsilon}{6}-\frac{(\alpha^2-\beta^2)}{4}\frac{\tanh^2(\xi)}{(1\pm\mathrm{sech}(\xi))^2}. \tag{51}$$



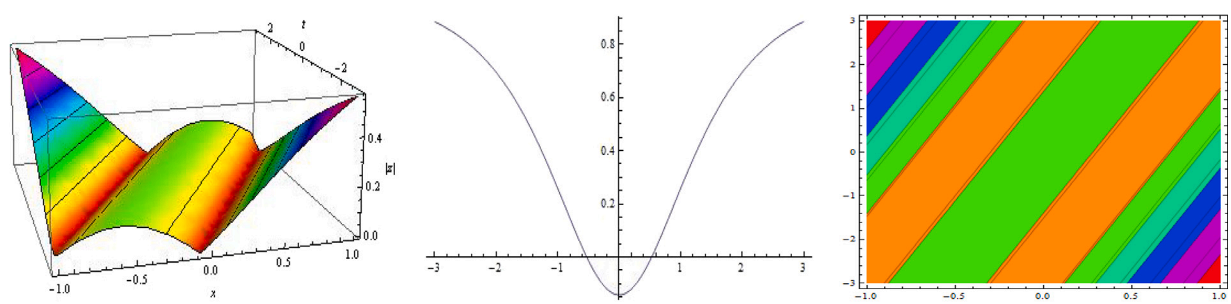
(a) (b) (c)

**Fig. 22.** Graphical presentation of $U(\xi)$ given by Eq. (45).



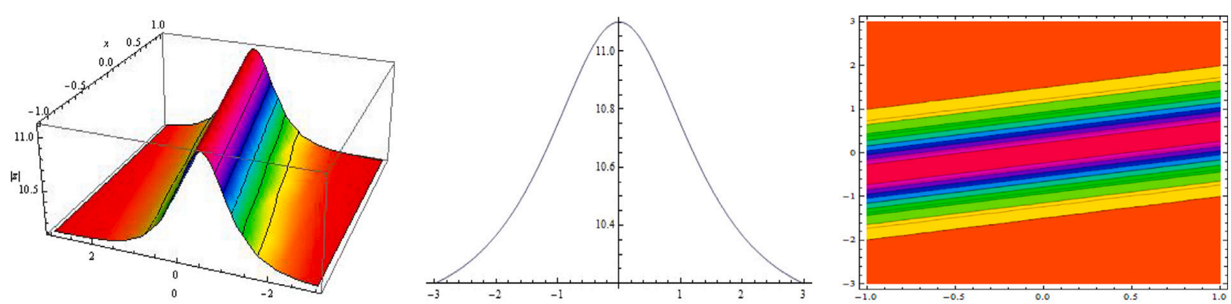
(a) (b) (c)

**Fig. 23.** The shape profile of $U(\xi)$ given by Eq. (46).



(a) (b) (c)

**Fig. 24.** The graphical presentation of $U(\xi)$ given by Eq. (47).



(a) (b) (c)

**Fig. 25.** The shock wave of $U(\xi)$ given by Eq. (48).

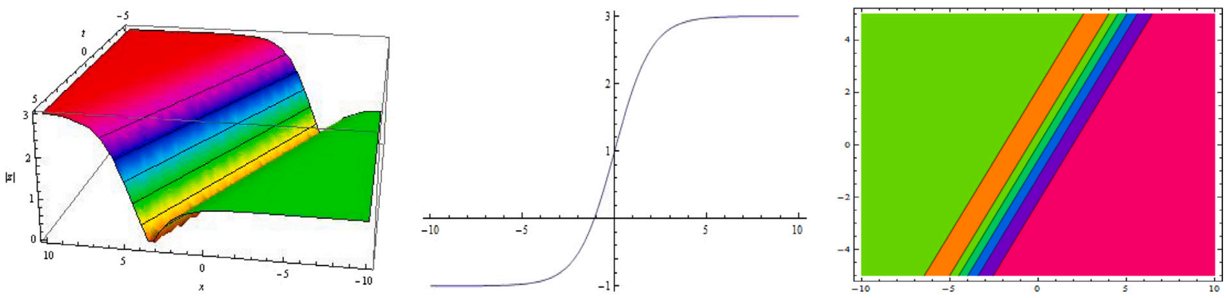
**Case 7:**
$d = \frac{m^2-2}{2}, \quad e = \frac{m^2}{2}, \quad r = \frac{m^2}{4}, \quad c = 0.$
We get,

$$U(\xi) = \frac{-2(\frac{m^2-2}{2})\beta^3 + 2(\frac{m^2-2}{2})\alpha^2\beta - 2\epsilon}{6} - \frac{\frac{m^2}{2}(\alpha^2-\beta^2)}{2}$$

$$\times \frac{\mathrm{dn}^2(\xi,m)}{(m^2+1)^2(\mathrm{sn}^2(\xi,m) \pm \mathrm{dn}(\xi,m))^2}.$$

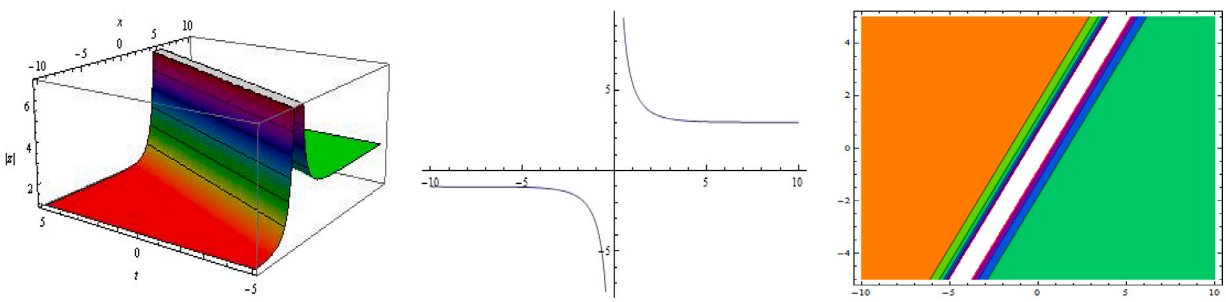
As $m \to 1$ this generate (see Figs. 28 and 29).

$$U(\xi) = \frac{\beta^3 - \alpha^2\beta - 2\epsilon}{6} - \frac{(\alpha^2-\beta^2)}{4}\frac{\mathrm{sech}^2(\xi)}{(\tanh^2(\xi) \pm \mathrm{sech}(\xi))^2}. \tag{52}$$



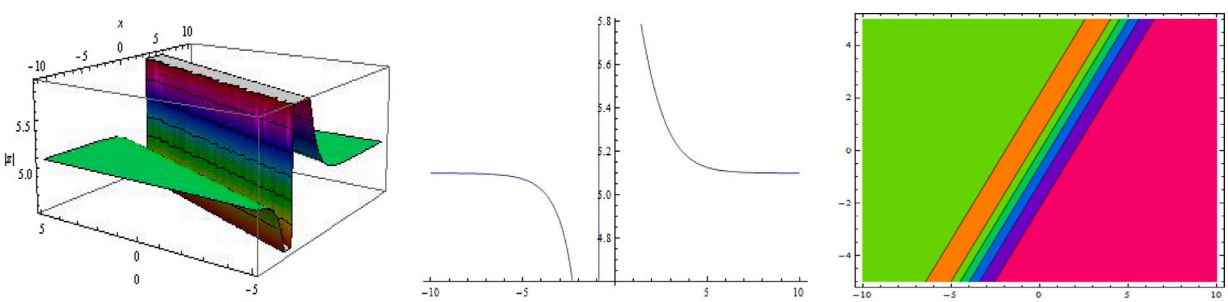
(a) (b) (c)

**Fig. 26.** The shock wave of $U(\xi)$ given by Eq. (49).



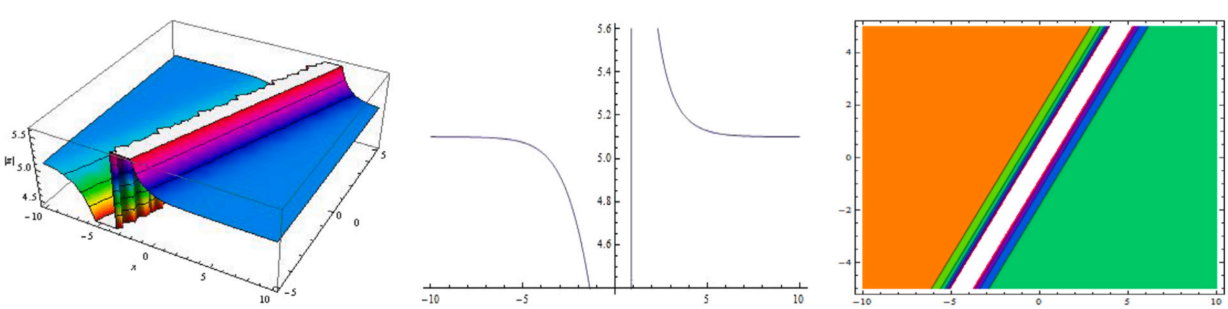
(a) (b) (c)

**Fig. 27.** The shock wave of $U(\xi)$ given by Eq. (49).



(a) (b) (c)

**Fig. 28.** The shock wave of $U(\xi)$ given by Eq. (50).



(a) (b) (c)

**Fig. 29.** The shock wave of $U(\xi)$ given by Eq. (50).

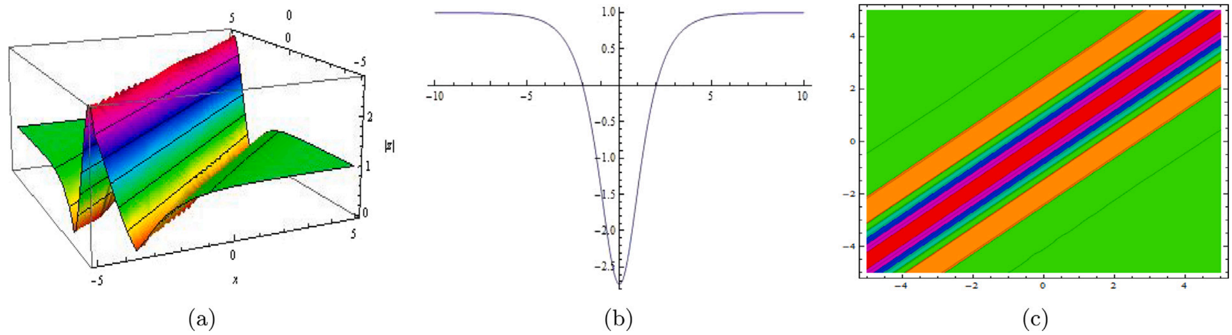


**Fig. 30.** The shock wave of $U(\xi)$ given by Eq. (51).

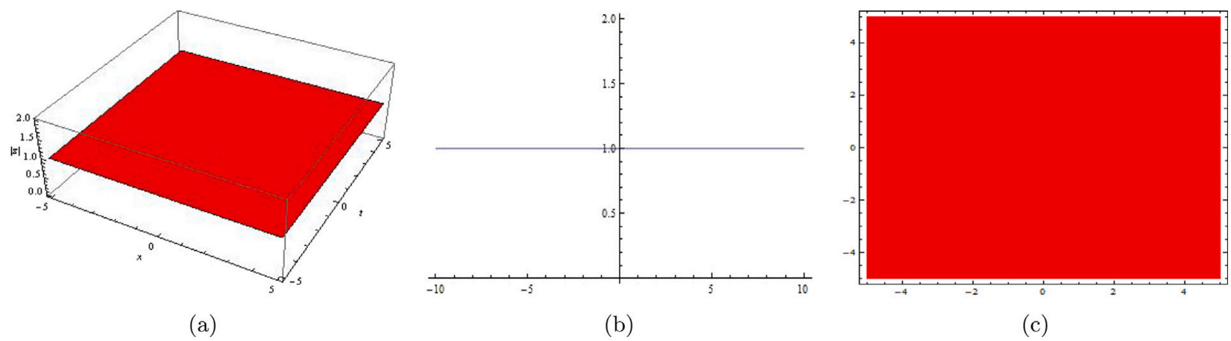


**Fig. 31.** The shock wave of $U(\xi)$ given by Eq. (51).

**Case 8:**
$d=\frac{m^2+1}{2},\quad e=\frac{-1}{2},\quad r=\frac{-(1-m^2)^2}{4},\quad c=0.$
We get,

$$U(\xi)=\frac{-2(\frac{m^2+1}{2})\beta^3+2(\frac{m^2+1}{2})\alpha^2\beta-2\epsilon}{6}-\frac{\frac{-1}{2}(\alpha^2-\beta^2)}{2}(m\mathrm{cn}(\xi,m)\pm \mathrm{dn}(\xi,m))^2.$$

As $m\to 1$ this generate (see Figs. 30 and 31).

$$U(\xi)=\frac{-2\beta^3+2\alpha^2\beta-2\epsilon}{6}+\frac{(\alpha^2-\beta^2)}{4}(\mathrm{sech}(\xi)\pm \mathrm{sech}(\xi))^2. \tag{53}$$

**Case 9:**
$d=\frac{m^2+1}{2},\quad e=\frac{m^2+1}{2},\quad r=\frac{m^2-1}{4},\quad c=0.$
We get,

$$U(\xi)=\frac{-2(\frac{m^2+1}{2})\beta^3+2(\frac{m^2+1}{2})\alpha^2\beta-2\epsilon}{6}-\frac{(\frac{m^2+1}{2})(\alpha^2-\beta^2)}{2}\frac{\mathrm{dn}^2(\xi,m)}{(1\pm m\mathrm{sn}(\xi,m))^2}.$$

As $m\to 1$ this generate (see Figs. 32 and 33).

$$U(\xi)=\frac{-2\beta^3+2\alpha^2\beta-2\epsilon}{6}-\frac{(\alpha^2-\beta^2)}{2}\frac{\mathrm{sech}^2(\xi,m)}{(1\pm\tanh(\xi))^2}. \tag{54}$$

**Case 10:**
$d=\frac{m^2+1}{2},\quad e=\frac{1-m^2}{2},\quad r=\frac{1-m^2}{4},\quad c=0.$
We obtain,

$$U(\xi)=\frac{-2(\frac{m^2+1}{2})\beta^3+2(\frac{m^2+1}{2})\alpha^2\beta-2\epsilon}{6}-\frac{(\frac{1+m^2}{2})(\alpha^2-\beta^2)}{2}\frac{\mathrm{cn}^2(\xi,m)}{(1\pm m\mathrm{sn}(\xi,m))^2}.$$

As $m\to 1$ this generate (see Figs. 34 and 35).

$$U(\xi)=\frac{-2\beta^3+2\alpha^2\beta-2\epsilon}{6}-\frac{(\alpha^2-\beta^2)}{2}\frac{\mathrm{sech}^2(\xi,m)}{(1\pm\tanh(\xi))^2}. \tag{55}$$

**Case 11:**
$d=\frac{m^2+1}{2},\quad e=\frac{(1-m^2)^2}{2},\quad r=\frac{1}{4},\quad c=0.$

We get,

$$U(\xi)=\frac{-2(\frac{m^2+1}{2})\beta^3+2(\frac{m^2+1}{2})\alpha^2\beta-2\epsilon}{6}-\frac{(\frac{(1+m^2)^2}{2})(\alpha^2-\beta^2)}{2}\times\frac{\mathrm{sn}^2(\xi,m)}{(\mathrm{dn}(\xi,m)\pm m\mathrm{sn}(\xi,m))^2}.$$

As $m\to 1$ this generate (see Fig. 36).

$$U(\xi)=\frac{-2\beta^3+2\alpha^2\beta-2\epsilon}{6}-\frac{(\alpha^2-\beta^2)}{2}\frac{\tanh^2(\xi)}{(\mathrm{sech}(\xi)\pm sech(\xi))^2}. \tag{56}$$

## Results and discussion

In this manuscript we use IFE and JEF to get solitary wave, periodic wave, bell-shaped, kink and anti-kink type, rational and Jacobi elliptic solutions in terms of hyperbolic and trigonometric functions and plot their 3D and contours graphs.

## Conclusion

We obtained various types of solutions i.e solitary wave, periodic wave, bell shaped, kink and anti-kink type, rational solutions and Jacobi elliptic functions of Burger's and weakly nonlinear Shallow water wave differential equation. In this paper, we used IFE and JEF architectonic. First method gives solitary wave solutions, bell shaped, kink and anti-kink type solutions and second one gives Jacobi elliptic functions in terms of hyperbolic and trigonometric functions and rational solutions with their graphical representations like 3-D and contours graphs. Furthermore, our constructed solutions illustrate how simple, reliable, and consistent this method's solution process is existing solitary wave solutions, single soliton solutions, periodic solutions, and kink solutions are obtained if the parameters adopt particular values. The finding appear that the improved F-expansion approach could be a promising device since it can offer a number of solutions with different one of a kind physical contour. To the best of our knowledge, these results were obtained first time for these models.

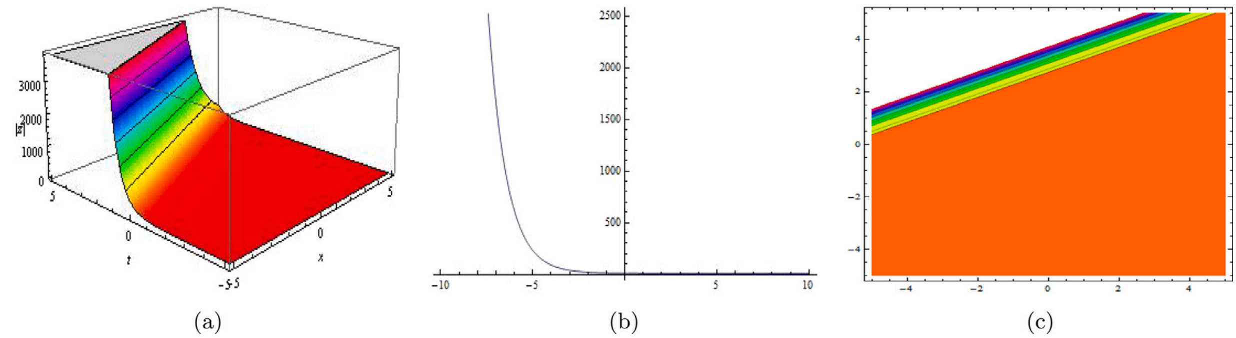


**Fig. 32.** The graphical presentation of $U(\xi)$ given by Eq. (52).

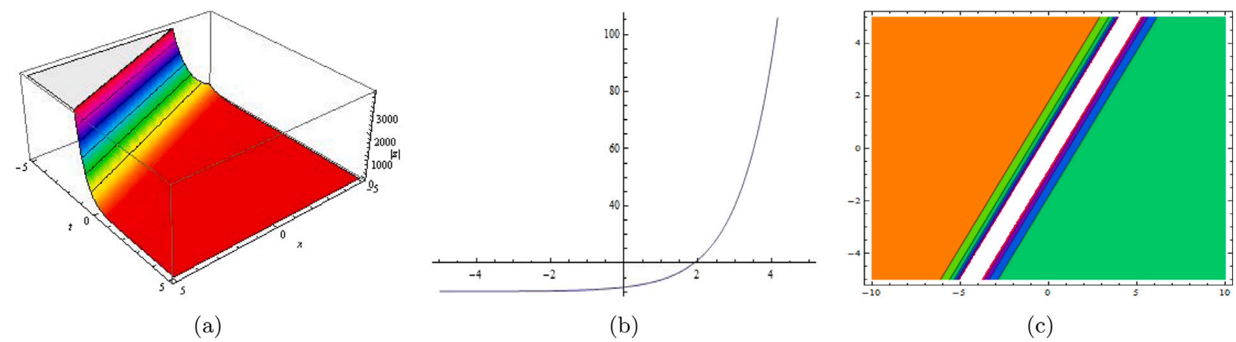


**Fig. 33.** The graphical presentation of $U(\xi)$ given by Eq. (52).

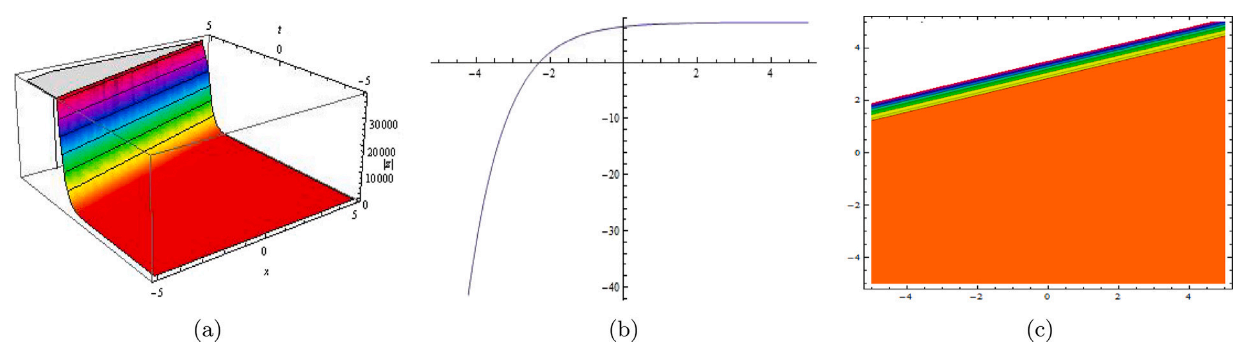


**Fig. 34.** The graphical presentation of $U(\xi)$ given by Eq. (53).

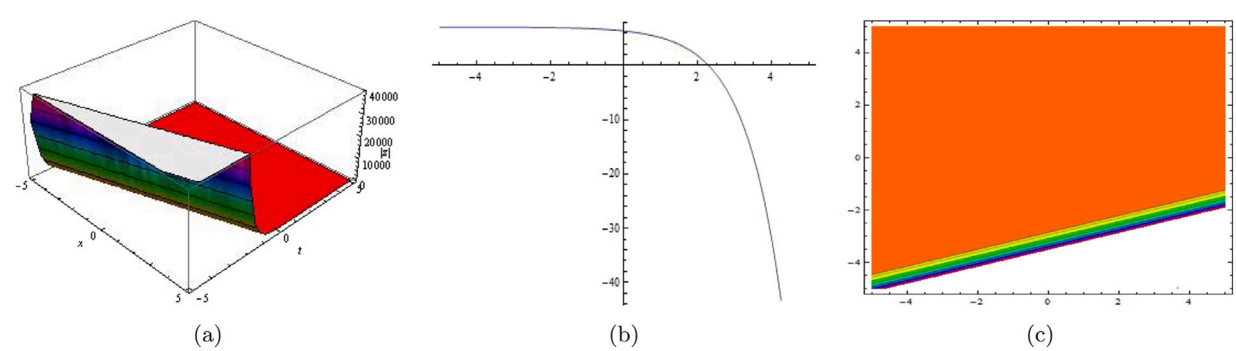


**Fig. 35.** The graphical presentation of $U(\xi)$ given by Eq. (53).

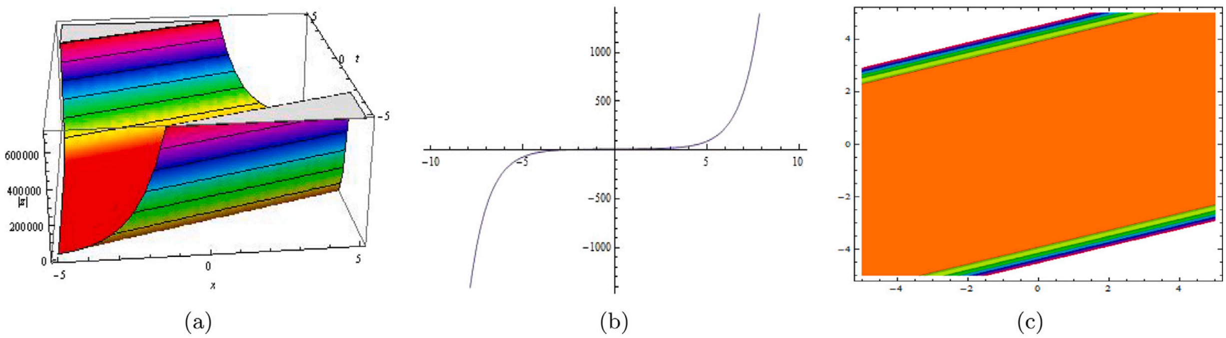


Fig. 36. The graphical presentation of $U(\xi)$ given by Eq. (54).

## CRediT authorship contribution statement

**Farrah Ashraf:** Conceptualization. **Tehsina Javeed:** Supervision. **Romana Ashraf:** Validation. **Amina Rana:** Writing first draft. **Ali Akgül:** Investigation.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Data availability

No data was used for the research described in the article.

## Acknowledgment

All authors read and approved the final manuscript.

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