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ORIGINAL ARTICLE

Numerical simulation of temperature distribution of heat flow on reservoir tanks connected in a series

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Initial temperature distribution;
2D graphs

Abstract The flow of temperature distribution through a medium in thermodynamic studies plays an important role in understanding physical phenomena in chemical science and petroleum engineering, while temperature distribution indicates the degree of reaction that must be undergone to obtain the final product. Therefore, this paper aims to present and apply the exponential matrix algorithm (EMA), differential transformation algorithm (DTA), and Runge-Kutta (RK5) to simulate the temperature distribution in five heating tanks in series. successive preheating of multicomponent oil solutions. A mathematical model of the energy balance equations of the reservoir is considered. Two computer experiments were performed to test and investigate the relationship between two constant parameters appearing in the model. Numerical simulation of saturated steam T_{steam} temperature of 500 °C and 1000 °C used to heat the tanks and initial temperature $T_0 35^{\circ}\mathrm{C}$ and $100^{\circ}\mathrm{C}$ of the first tank feed oil are considered. The fluids in the reservoirs were considered homogeneous throughout the experiment and changes in the cell configuration at two constant parameters were presented in the 2D plot control with the use of the MAPLE 18 software package. The study revealed the nature of the temperature distribution that the higher temperature distribution is obtained when heat is transferred from the first tank to the fifth tank and the reverse reaction

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Nomenclature

T_0	The initial temperature $[(35^\circ C, 100^\circ C)]$	Cp	The specific heat capacity of the oil 2 kJ/kg°C
T_{steam}	The temperature of the saturated steam $(500^\circ C, 1000^\circ C)$	G	The overall heat transfer coefficient 2
V	Mass flow rate 160 kg/min	A	The heat transfer area in each vessel 5
M	Mass of the fluid in the tank 2000 kg		

occurs in all five reservoirs when ψ = 0.0025 and ω = 0.0025 respectively. Numerical results obtained are prototypes of oil temperature distribution performed under laboratory conditions in a thermodynamic experiment.

1. Introduction

The science of thermodynamics deals with energy and its transformations. It tells us about the direction in which heat changes take place in nature. It also defines the conditions under which a proposed change reaches equilibrium with no further change in the given conditions. The thermodynamic analysis is today applied to a wide variety of problems including the physical and biological sciences. The thermodynamics has many applications in chemical, mechanical, and petrochemical engineering [1–5]. In recent years, analytical and numerical methods have aroused the interest of researchers to find approximate solutions to ordinary differential equations, systems of differential equations, and direct equations. eigenfunctions appear in many fields of applied sciences and engineering [6]. The dynamic system has generated a great deal of interest in many areas of applied mathematics in both industry and science, leading to the study and understanding of many physical phenomena such as [7] dynamic response of a rod due to a thermal motion source under hyperbola. thermal conduction model, [8] presented simultaneous solutions for the frst and second order slips on micropolar fuid fow across a convective surface in the presence of Lorentz force and variable heat source/sink, [9] studied the influences of the viscous dissipasion on MHD flow in micropolar fluid flow a slandering stretching surface with modified heat flux model, [10] studied the impact of frictional heating on MHD radiative ferrofuid past a convective shrinking surface, [11] presented the hyperbolic heat conduction equation in anisotropic materials, [12] isotope perturbation method was used to solve the system of nonlinear linkage equations, [13] Maple 18 coded variational iteration method was formulated to solve predator–prey mode, [14] obtained the solutions of the LotkaVolterra random equation through the activity matrix, [15] the homotopy analysis method was used to obtain a precise flow of tertiary fluids through a porous plate, [16] presented a decomposition method to solve gas dynamics equations, [17] discussed the kinetic equations of gases, [18] used finite difference method to solve a system of gas dynamics equations in a kind of continuous function and [19] obtained numerical solution of chemically reactive non-Newtonian fluid flow dual to stratification.

2. Formulation of the model

The computational framework to obtain the dynamic response of five well-mixed and heated serial tanks is considered to preheat the multicomponent oil solution before it is introduced into the distillation column for separation as shown in Fig. 1. Initially, each tank is full. with 2000 kg of oil at 35 °C and saturated steam at 500 °C condensing into coils submerged in each barrel. Oil is fed into the first storage tank at a rate of 160 kg/min and overflows into the second, third, fourth, and fifth tanks at a corresponding rate. The temperature of the oil supplied to the first tank is 35 °C. The tanks are mixed so that the temperature inside the tanks is uniform. Considering the outlet stream temperature as the tank interior temperature and heat capacity, the oil Cp is 2.0 KJ/kg. The heat transfer rate for oil from the steam coil and energy balance system of differential equations for five tanks connected in series is given as follows [20].

$$\begin{cases} \frac{dT_1}{dt} = \psi(T_0 - T_1) + \omega(T_{steam} - T_1) \\ \frac{dT_2}{dt} = \psi(T_1 - T_2) + \omega(T_{steam} - T_2) \\ \frac{dT_3}{dt} = \psi(T_2 - T_3) + \omega(T_{steam} - T_3) \\ \frac{dT_4}{dt} = \psi(T_3 - T_4) + \omega(T_{steam} - T_4) \\ \frac{dT_5}{dt} = \psi(T_4 - T_5) + \omega(T_{steam} - T_5) \end{cases} \quad (1)$$

where parameters $\psi = \frac{V}{M}$ and.$\omega = \frac{GA}{MCp}$
with initial conditions:

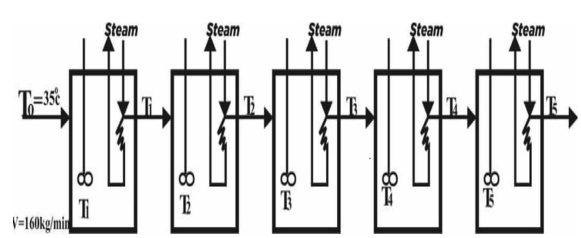

Fig. 1 Experimental setup of five tanks for oil heating connected in series.

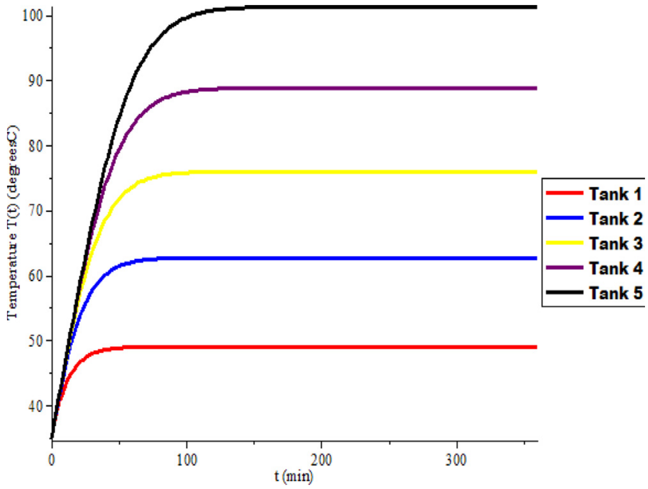

Fig. 2 Depict the simulated temperature distributions for the tanks T_1, T_2, T_3, T_4, and T_5 tanks in °C obtained when the uniform initial temperature is 35 °C and T_{steam} temperature of 500 °C saturated steam for five connected oil heating tanks in series.

$$T_1(0) = T_2(0) = T_3(0) = T_4(0) = T_5(0) = \begin{cases} 35^{\circ}C \\ 100^{\circ}C \end{cases} \tag{2}$$

Where T_1, T_2, T_3, T_4, and T_5 are the temperatures in °C in tanks 1, 2, 3, 4, and 5, respectively, and T_{steam} is the temperature of the saturated steam (500 °C, 1000°C) used to make hot tank and $T_0(35^{\circ}C, 100^{\circ}C)$ is the temperature of the oil that is put into the first tank; V is mass flow rate; M is the mass of the liquid in the container; Cp is the specific heat capacity of the oil; G is the overall heat transfer coefficient and A is the heat transfer area in each vessel (see Fig. 2).

3. Methods of solution

In this section, we discuss solution techniques for the simulation of the energy balance system of the differential equation (1). Three computational algorithms are formulated and applied to solve equation (1) coupled with assumptions given in the nomenclature.

3.1. Exponential matrix algorithm (EMA)

The exponential matrix method is an analytic technique to a solve system of differential equations, first, we build matrix forms of exponential functions and their derivatives, then replace the arranged points into matrix forms, and the basic matrix equations are formed. This matrix equation corresponds to a system of linear algebraic equations. By solving this system, the unknown coefficients are determined and thus approximate solutions are obtained. To simplify the procedures involved in this technique, we hereby formulate and apply-four steps algorithm for the numerical solution of the energy balance system of differential equations (1) as follows:

restart :**Step 1:**
with(*linalg*) : *with*(*plots*) : $N := 5$: *Digits* := 10 : ***fori toNdo***
Simulite[*i*] : = *diff*(((*T*[*i*]) (*t*), *t*) = *eval*(ψ ∗ (T[i − 1] (t) − *T*[*i*] (*t*)) + ω ∗ (T_{steam} − T [i](t))))**end do:**
par := (*T*[0](*t*) = [35°C, 100°C], *T*[*steam*] = [500, 1000]) : *var* : = [*seq*(*T*[*i*](*t*), *i* = 1..*N*)] : *systemequ* : = [*seq*(*rhs*(*simulate*[*i*], *i* = 1 ⋯ *N*))] :**Step 2:**
A := *genmatrix*(*systemequ*, *var*, *B*) : *evalm*(*B*) :
b := *matrix*(*N*.1) : ***forit oNdob***[*i*, 1] : = −*B*[*i*] :**end do:**
evalm(*b*) : *mat* := *exponential*(*A*, *t*) :
Y[0] := *matrix*(5, 1, [35, 35, 35, 35, 35]) : *sl* := *evalm*(*Y*[0] +*inverse*(*A*)& ∗ *b*) :**Step 3:**
sol := *evalm*(*mat*& ∗ *sl* − *inverse*(*A*) & ∗ *b*) : ***forit oNdo****T*[*i*] : = *sol*[*i*, 1] :**end do:**
pars := $\{\Psi = \frac{160}{2000}, \omega = \frac{10}{2000*2}\}$:
*sol*1 := *subs*(*pars*, *T*[1]) : *sol*2 := *subs*(*pars*, *T*[2]) :
*sol*3 := *subs*(*pars*, *T*[3]) : *sol*4 := *subs*(*pars*, *T*[4]) :
*sol*5 := *subs*(*pars*, *T*[5]) :**Step 4:**
*forifrom*0*by*5*to*80*doT*[1] : = *evalf*(*eval*(*sol*1, *t* = *i*)) :
T[2] := *evalf*(*eval*(*sol*2, *t* = *i*)) : *T*[3] := *evalf*(*eval*(*sol*3, *t* = *i*)) :
T[4] := *evalf*(*eval*(*sol*4, *t* = *i*)) : *T*[5] := *evalf*(*eval*(*sol*5, *t* = *i*)) :**end do:**
[2*Dplot*] := *plot*([*sol*1, *sol*2, *sol*3, *sol*4, *sol*5], *t* = 0 ⋯ 360, *color*[*red*, *blue*, *yellow*, *purple*, *balck*], *axes* = *boxed*, *title* = 5*X*5energy balance system of differe ntial equatio n); [2*Dplot*] : = *logplot*([*sol*1, *sol*2, *sol*3, *sol*4, *sol*5], *t* = 0 ⋯ 360, *color*[*red*, *blue*, *yellow*, *purple*, *balck*], *axes* = *boxed*, *title* = 5*X*5energy balance system of differen tial equati on);*Output* : *SeeTables*1*and*2 and Figures 2, 3, 4, … 15.

3.2. Differential transform algorithm (DTA)

The differential transformation method is an iterative procedure for solving ordinary differential equations (ODEs), a system of ordinary differential equations (SODEs), and partial differential equations (PDEs) was proposed by Zhou [21] and it has been in used to solve various problems in applied mathematics such as application of Taylor transformation to nonlinear predictive control problem was presented by [22], authors [23] applied differential transformation method to solve eigenvalue problems, [24] applied differential transformation method for a reliable treatment of the nonlinear biochemical reaction model and solving system of first-order linear and nonlinear differential equations in applied mathematics was presented [25]. To obtain approximate solutions of energy balance systems of differential equations (1), we formulate five steps differential transformation algorithm as follows:

$restart$:**Step 1:**
$with(plots) : N := 10 : Digits := 15 : \Psi := \frac{160}{2000}$:
$\omega := \frac{10}{2000*2} : T(steam) : = [500^{\circ}C, 1000^{\circ}C]\, T_1(0) :$
$= [35^{\circ}C,\ 100^{\circ}C] : T_2(0) : = [35^{\circ}C,\ 100^{\circ}C] : T_3(0) :$
$= [35^{\circ}C,\ 100^{\circ}C] : T_4(0) : = [35^{\circ}C,\ 100^{\circ}C] : T_5(0) :$
$= [35^{\circ}C,\ 100^{\circ}C] : \delta[0] := 1;\ \delta[-1] := 1;$ ***fornfrom***
1***toNdo***$\delta[n] := 0$**end do:**
Step 2:
forkfro m0***toNdo***$T_1[k+1] : = \frac{1}{(k+1)} * (\Psi * T_0[k] * \delta[k] - \Psi * T_1[k]$
$+\omega * T(steam) * \delta[k] - \omega * T_1[k]); T_2[k+1] := \frac{1}{(k+1)} * (\Psi * T_1[k] -$
$\Psi * T_2[k] + \omega * T(steam) * \delta[k] - \omega * T_2[k]) : T_3[k+1] :$
$= \frac{1}{(k+1)} * (\Psi * T_2[k] - \Psi * T_3[k] + \omega * T(steam) * \delta[k] -$
$\omega * T_3[k]) : T_4[k+1] : = \frac{1}{(k+1)} * (\Psi\Psi * T_3[k] -$
$\Psi * T_4[k] + \omega * T(steam) * \delta[k] - \omega * T_4[k]) : T_5[k+1] :=$
$\frac{1}{(k+1)} * (\Psi * T_4[k] - \Psi * T_5[k] + \omega * T(steam) * \delta[k] -$
$\omega * T_5[k])$ **:end do:**

Step 3:
$T_1(t) := sum(T_1[j] * t^j, j = 0,$
$\cdots, N+1) : T_2(t) := sum(T_2[j] * t^j, j = 0,$
$\cdots, N+1) : T_3(t) := sum(T_3[j] * t^j, j = 0,$
$\cdots, N+1) : T_4(t) := sum(T_4[j] * t^j, j = 0,$
$\cdots, N+1) : T_5(t) := sum(T_5[j] * t^j, j = 0,$
$\cdots, N+1) : Sol1 := T_1(t) :\ Sol2 := T_2(t) :$
$Sol3 := T_3(t) : Sol4 := T_4(t) : Sol5 := T_5(t)$ **:Step 4:**
for i from 0 by 5 to 80 do$T_1(i) := evalf(eval(sol1, t = i))$:
$T_2(i) := evalf(eval(sol2, t = i)) : T_3(i) := evalf(eval(sol3, t = i)) :$
$T_4(i) := evalf(eval(sol4, t = i)) :$
$T_5(i) := evalf(eval(sol5, t = i))$ **:end do:**
Step 5:
$[2Dplot] := plot([sol1, sol2, sol3, sol4, sol5],$
$t = 0 \cdots 360, color[red, blue, yellow, purple, black],$
$axes = boxed, title = 5X5$ene rgy bal ance system of dif feren
*t*ial equati on); $[2Dplot]$:
$= logplot([sol1, sol2, sol3, sol4, sol5], t = 0$
$\cdots 360, color[red, blue, yellow, purple, black],$
$axes = boxed, title = 5X5$energy balance system of diffe
rential equa tion);$Output$: *See* Tables 1 and 2 and Figures 2, 3, 4, … 15

3.3. Runge-Kutta (RK5)

The Runge-Kutta method is step by step numerical method for obtaining the approximate solutions of ordinary differential equations and was proposed Carl Runge (1856–1927) in which the original idea was formulated by Wilhelm Kutta. It has been used extensively to obtain approximate numerical solutions of differential equations of first, second, and higher orders. It transforms second and higher orders into a system of equations of first-order. In the last one decade, several authors have applied Runge-Kutta to solve many applied problems that arise in biological and physical sciences such as [26] applied fifth-order Runge-Kutta-Nystrom methods for solving linear second-order oscillatory problems, [27] introduced the direct explicit integrators of RK type for solving special fourth-order ordinary differential equations with an application, [28] applied of the Euler and Runge-Kutta generalized methods for FDE and symbolic packages in the analysis of some fractional attractors, [29] employed Runge-Kutta scheme for the numerical results of Group theoretical analysis for MHD flow fields, [30] presented symmetry analysis on thermally magnetized fluid flow regime with heat source/sink, [31] applied Runge-Kutta (RK5) and new iterative method (NIM) for numerical comparison of or solving metastatic cancer model. In other to apply the Runge-Kutta (RK5), we present two steps algorithm as follows:

restart:
Step 1:
$Digits := 10 : h := 4.0 : t[0] := 0 : \Psi := \frac{160}{2000} : \omega := \frac{10}{2000*2}$:
$T(steam) := [500^{\circ}C, 1000^{\circ}C]$:
$T_1[0] := [35^{\circ}C,\ 100^{\circ}C] : T_2[0] := [35^{\circ}C,\ 100^{\circ}C] : T_3[0] :=$
$[35^{\circ}C,\ 100^{\circ}C] : T_4[0] := [35^{\circ}C,\ 100^{\circ}C] : T_5[0] :=$
$[35^{\circ}C,\ 100^{\circ}C] : \{\ T_1[t] :=$
$\rightarrow \Psi * (T_0 - T_1) + \omega * (T_{steam} - T_1) : T_2[t] :=$
$\rightarrow \Psi * (T_1 - T_2) + \omega * (T_{steam} - T_2) : T_3[t] :=$
$\rightarrow \Psi * (T_2 - T_3) + \omega * (T_{steam} - T_3) : T_4[t] :=$
$\rightarrow \Psi * (T_3 - T_4) + \omega * (T_{steam} - T_4) : T_5[t] :=$
$\rightarrow \Psi * (T_4 - T_5) + \omega * (T_{steam} - T_5)$: .**Step 2:**
$fornfrom1to10dot[n] := n * h : k1 := T[j](t[n-1], t[n-1]);$
$k2 := T[j](t[n-1] + \frac{h}{3}, t[n-1] + \frac{h}{3} * k1);$
$k3 := T[j](t[n-1] + \frac{2*h}{5}, t[n-1] + \frac{1}{25} * (4 * k1 + 6 * k2);$
$k4 := T[j](t[n-1] + h, t[n-1] + \frac{1}{4} * (k1 - 12 * k2 + 15 * k3));$
$k5 := T[j](t[n-1] + \frac{2*h}{3}, t[n-1]$
$+ \frac{1}{81} * (6 * k1 + 90 * k2 - 50 * k3 + 8 * k4));$
$k6 := T[j](t[n-1] + \frac{4*h}{5}, t[n-1]$
$+ \frac{1}{75} * (6 * k1 + 36 * k2 + 10 * k3 + 8 * k4)); T[j] : = T[n-1] + \frac{h}{192}$
$*(23 * k1 + 125 * k3 - 81 * k5 + 125 * k6)$;od;$jfrom1to5$
$[2Dplot] := plot([sol1, sol2, sol3, sol4, sol5],$
$t = 0 \cdots 360, color[red, blue, yellow, purple, black], axes = boxed,$
$title = 5X5$energy ba lance syst em of diffe rential equ
ation); $[2Dplot] : = logplot([sol1, sol2, sol3, sol4, sol5],$
$t = 0 \cdots 360, color[red, blue, yellow, purple, black],$
$axes = boxed, title = 5X5$energy balance sy stem of dif ferenti
al equa tion);$Output : SeeTables1and2$ and Figures 2, 3, 4, … 15

4. Computational experiments, results and plots presentation

In this section, we applied the proposed techniques to examine and investigate the relationship between ψ and ω parameters that appeared in the energy balance system of differential equations for five tanks connected in series (1). We considered a test case $\Psi = 0.0800$ greater than $\omega = 0.0025$ for the two experiments and the results obtained are presented in Table 1 and Table 2 as follows:

2DPlots representation.

5. Discusion and conclusion

5.1. Discussion

The temperature distribution plays an important role in the thermodynamic properties of heat transferred through the fluid flow in a given medium. Therefore, efficient and simple computational algorithms are required to simulate the temperature distribution of a given environment which this article aims to do. Simulation solutions were presented for the two

Table 1 Simulated temperature distribution at T_0 initial temperature 35°C and T_{steam} temperature of the saturated steam 500°C five tanks for oil heating connected in series.

t (mins)		$T(1)$°C	$T(2)$°C	$T(3)$°C	$T(4)$°C	$T(5)$°C
s0	Exact	35.00000000	35.00000000	35.00000000	35.00000000	35.00000000
	EMA	35.00000000	35.00000000	35.00000000	35.00000000	35.00000000
	DTA	35.00000000	35.00000000	35.00000000	35.00000000	35.00000000
	RK(5)	35.00000000	35.00000000	35.00000000	35.00000000	35.00000000
5.0	Exact	39.76282314	40.65008392	40.76421114	40.77538038	40.77626117
	EMA	39.76282314	40.65008392	40.76421113	40.77538038	40.77626120
	DTA	39.76282314	40.65008392	40.76421114	40.77538038	40.77626118
	RK(5)	39.76282380	40.65008151	40.76421420	40.77537887	40.77626124
10.0	Exact	42.91577965	45.65158364	46.32844306	46.45784722	46.47794116
	EMA	42.91577965	45.65158363	46.32844305	46.45784722	46.47794114
	DTA	42.91577964	45.65158364	46.32844307	46.45784721	46.47794117
	RK(5)	42.91578100	45.65157936	46.32844734	46.45784631	46.47794054
15.0	Exact	45.00301541	49.79743674	51.50328912	51.98013563	52.08933822
	EMA	45.00301541	49.79743672	51.50328910	51.98013563	52.08933824
	DTA	45.00301542	49.79743674	51.50328912	51.98013562	52.08933820
	RK(5)	45.00301771	49.79743039	51.50329370	51.98013638	52.08933710
20.0	Exact	46.38475129	53.09465763	56.13735149	57.24043879	57.57113770
	EMA	46.38475129	53.09465762	56.13735147	57.24043877	57.57113766
	DTA	46.38475128	53.09465763	56.13735148	57.24043877	57.57113771
	RK(5)	46.38475618	53.09464763	56.1373540	57.2404427	57.57113752
25.0	Exact	47.29945104	55.64327533	60.15134035	62.13418661	62.86264125
	EMA	47.29945104	55.64327531	60.15134032	62.13418659	62.86264126
	DTA	47.29945105	55.64327532	60.15134034	62.13418660	62.86264122
	RK(5)	47.29945526	55.64326685	60.15134119	62.13419138	62.86264187
30.0	Exact	47.90497605	57.57265291	63.53188269	66.57814004	67.89265270
	EMA	47.90497605	57.57265289	63.53188267	66.57814002	67.89265271
	DTA	47.90497601	57.57265291	63.53188269	66.57814004	67.89265269
	RK(5)	47.90498125	57.57264481	63.53187991	66.57814520	67.89265571
35.0	Exact	48.30582948	59.01022912	66.31274094	70.52161994	72.59231595
	EMA	48.30582948	59.01022910	66.31274091	70.52161993	72.59231596
	DTA	48.30582947	59.01022910	66.31274092	70.52161994	72.59231592
	RK(5)	48.30583688	59.01022165	66.31273268	70.52162341	72.59232219
40.0	Exact	48.57119173	60.06803969	68.55554542	73.94750430	76.90539068
	EMA	48.57119173	60.06803967	68.55554540	73.94750428	76.90539069
	DTA	48.57119171	60.06803960	68.55554535	73.94750434	76.90539064
	RK(5)	48.57120013	60.06803455	68.55553341	73.94750393	76.90539843
45.0	Exact	48.74685973	60.83857029	70.33442554	76.86719981	80.79421623
	EMA	48.74685973	60.83857027	70.33442551	76.86719980	80.79421623
	DTA	48.74685970	60.83857030	70.33442540	76.86719986	80.79421608
	RK(5)	48.74686824	60.83856819	70.33441193	76.86719467	80.79422322
50.0	Exact	48.86315076	61.39517271	71.72536975	79.31310832	84.24149320
	EMA	48.86315076	61.39517269	71.72536973	79.31310830	84.24149320
	DTA	48.86315060	61.39517260	71.72536960	79.31310834	84.24149334
	RK(5)	48.86315841	61.39517316	71.72535690	79.31309962	84.24149791
55.0	Exact	48.94013462	61.79443328	72.79971088	81.33089987	87.24891309
	EMA	48.94013462	61.79443326	72.79971085	81.33089986	87.24891309
	DTA	48.94013450	61.79443320	72.79971000	81.33089975	84.24149324
	RK(5)	48.94014098	61.79443499	72.79969961	81.33088945	87.24891586
60.0	Exact	48.99109742	62.07912617	73.62071754	82.97283499	89.83388047
	EMA	48.99109742	62.07912615	73.62071750	82.97283498	89.83388047
	DTA	48.99109750	62.07912640	73.62071650	82.97283489	89.83388145
	RK(5)	48.99110356	62.07912999	73.62070742	82.97282094	89.83387899
65.0	Exact	49.02483444	62.28108574	74.24230322	84.29262223	92.02541759
	EMA	49.02483444	62.28108572	74.24230319	84.29262222	92.02541759
	DTA	49.02483460	62.28108550	74.24230320	84.29262194	92.02541976
	RK(5)	49.02484132	62.28109344	74.2422954	84.29260281	92.02540726
70.0	Exact	49.04716812	62.42371507	74.70905375	85.34184050	93.86004507
	EMA	49.04716812	62.42371505	74.70905372	85.34184049	93.86004507
	DTA	49.04716790	62.42371500	74.70905200	85.34184067	93.86004628
	RK(5)	49.04717442	62.42372517	74.70905123	85.34182131	93.86002703
75.0	Exact	49.06195287	62.52404862	75.05699007	86.16772140	95.37812789

(continued on next page)

Table 1 (*continued*)

t (mins)		$T(1)^{\circ}$C	$T(2)^{\circ}$C	$T(3)^{\circ}$C	$T(4)^{\circ}$C	$T(5)^{\circ}$C
	EMA	49.06195287	62.52404860	75.05699004	86.16772139	95.37812789
	DTA	49.06195230	62.52404500	75.05698740	86.16771078	95.37813780
	RK(5)	49.06195736	62.52405703	75.05699000	86.16770657	95.37811051
80.0	Exact	49.07174027	62.59438371	75.31467258	86.81199954	96.62092484
	EMA	49.07174027	62.59438369	75.31467255	86.81199953	96.62092484
	DTA	49.07173932	62.59438800	75.31466100	86.81199959	96.62092243
	RK(5)	49.07174426	62.5943923	75.31467476	86.81198582	96.62090451

Table 2 Simulated temperature distribution at T_0 initial temperature 100°C and T_{steam} temperature of the saturated steam 1000°C five tanks for oil heating connected in series.

t (mins)		$T(1)^{\circ}$C	$T(2)^{\circ}$C	$T(3)^{\circ}$C	$T(4)^{\circ}$C	$T(5)^{\circ}$C
0	Exact	100.0000000	100.0000000	100.0000000	100.0000000	100.0000000
	EMA	100.0000000	100.0000000	100.0000000	100.0000000	100.0000000
	DTA	100.0000000	100.0000000	100.0000000	100.0000000	100.0000000
	RK(5)	100.0000000	100.0000000	100.0000000	100.0000000	100.0000000
5.0	Exact	109.2183674	110.9356463	111.1565377	111.1781556	111.1798603
	EMA	109.2183674	110.9356463	111.1565376	111.1781557	111.1798604
	DTA	109.2183674	110.9356463	111.1565377	111.1781555	111.1798603
	RK(5)	109.2183705	110.9356359	111.1565496	111.1781504	111.1798604
10.0	Exact	115.3208639	120.6159684	121.9260188	122.1764785	122.2153700
	EMA	115.3208639	120.6159683	121.9260188	122.1764786	122.2153700
	DTA	115.3208638	120.6159683	121.9260188	122.1764785	122.2153699
	RK(5)	115.3208697	120.6159511	121.9260341	122.1764761	122.2153678
15.0	Exact	119.3606750	128.6402002	131.9418499	132.8647786	133.0761385
	EMA	119.3606750	128.6402001	131.9418498	132.8647787	133.0761385
	DTA	119.3606750	128.6402001	131.9418499	132.8647786	133.0761385
	RK(5)	119.3606841	128.6401780	131.9418623	132.8647827	133.0761360
20.0	Exact	122.0350025	135.0219180	140.9110029	143.0460105	143.6860730
	EMA	122.0350025	135.0219179	140.9110028	143.0460107	143.6860730
	DTA	122.0350025	135.0219180	140.9110029	143.0460105	143.6860729
	RK(5)	122.0350104	135.0218989	140.9110114	143.0460176	143.6860708
25.0	Exact	123.8053891	139.9547265	148.6800136	152.5177805	153.9276927
	EMA	123.8053891	139.9547264	148.6800135	152.5177806	153.9276928
	DTA	123.8053891	139.9547264	148.6800136	152.5177805	153.9276927
	RK(5)	123.8054021	139.9547036	148.6800124	152.5177925	153.9276961
30.0	Exact	124.9773730	143.6890057	155.2229988	161.1189807	163.6631987
	EMA	124.9773730	143.6890056	155.2229987	161.1189808	163.6631988
	DTA	124.9773730	143.6890057	155.2229988	161.1189807	163.6631988
	RK(5)	124.9773928	143.6889795	155.2229846	161.1189951	163.6632106
35.0	Exact	125.7532184	146.4714112	160.6053050	168.7515224	172.7593211
	EMA	125.7532184	146.4714111	160.6053049	168.7515226	172.7593212
	DTA	125.7532184	146.4714113	160.6053051	168.7515225	172.7593212
	RK(5)	125.7532404	146.4713891	160.6052802	168.7515324	172.7593397
40.0	Exact	126.2668227	148.5187864	164.9462170	175.3822664	181.1072077
	EMA	126.2668227	148.5187864	164.9462168	175.3822665	181.1072078
	DTA	126.2668227	148.5187866	164.9462170	175.3822665	181.1072078
	RK(5)	126.2668452	148.5187718	164.9461847	175.3822665	181.1072292
45.0	Exact	126.6068253	150.0101361	168.3892107	181.0332899	188.6339668
	EMA	126.6068253	150.0101360	168.3892106	181.0332900	188.6339669
	DTA	126.6068253	150.0101361	168.3892106	181.0332901	188.6339670
	RK(5)	126.6068482	150.0101302	168.3891740	181.0332756	188.6339857
50.0	Exact	126.8319047	151.0874311	171.0813608	185.7673064	195.3061158
	EMA	126.8319047	151.0874310	171.0813607	185.7673065	195.3061158
	DTA	126.8319047	151.0874312	171.0813609	185.7673070	195.3061161
	RK(5)	126.8319258	151.0874334	171.0813264	185.7672815	195.3061266
55.0	Exact	126.9809058	151.8601935	173.1607307	189.6727094	201.1269285
	EMA	126.9809058	151.8601934	173.1607306	189.6727095	201.1269286
	DTA	126.9809057	151.8601948	173.1607304	189.6727092	201.1269295

Table 2 (*continued*)

t (mins)		$T(1)^{\circ}$C	$T(2)^{\circ}$C	$T(3)^{\circ}$C	$T(4)^{\circ}$C	$T(5)^{\circ}$C
	RK(5)	126.9809222	151.8601990	173.1607029	189.6726817	201.1269330
60.0	Exact	127.0795434	152.4112120	174.7497759	192.8506483	206.1300912
	EMA	127.0795434	152.4112119	174.7497758	192.8506484	206.1300912
	DTA	127.0795432	152.4112120	174.7497762	192.8506490	206.1300925
	RK(5)	127.0795576	152.4112204	174.7497521	192.8506162	206.1300885
65.0	Exact	127.1448409	152.8021015	175.9528449	195.4050753	210.3717760
	EMA	127.1448409	152.8021014	175.9528448	195.4050754	210.3717760
	DTA	127.1448408	152.8021022	175.9528464	195.4050719	210.3717810
	RK(5)	127.1448567	152.8021187	175.9528262	195.4050304	210.3717534
70.0	Exact	127.1880674	153.0781582	176.8562331	197.4358203	213.9226678
	EMA	127.1880674	153.0781581	176.8562330	197.4358204	213.9226678
	DTA	127.1880677	153.0781586	176.8562223	197.4358268	213.9226613
	RK(5)	127.1880821	153.0781815	176.8562268	197.4357754	213.9226263
75.0	Exact	127.2166830	153.2723522	177.5296582	199.0342995	216.8608927
	EMA	127.2166830	153.2723521	177.5296581	199.0342996	216.8608927
	DTA	127.2166845	153.2723488	177.5296621	199.0342828	216.8608988
	RK(5)	127.2166932	153.2723711	177.5296574	199.0342650	216.8608534
80.0	Exact	127.2356264	153.4084846	178.0283985	200.2812894	219.2663061
	EMA	127.2356264	153.4084845	178.0283984	200.2812895	219.2663061
	DTA	127.2356255	153.4084905	178.0283872	200.2812908	219.2662679
	RK(5)	127.2356358	153.4085053	178.0284037	200.2812570	219.2662579

experiments carried out for the heat transfer rate of the oil steam coil of the system of energy balance differential equations for five tanks connected in series, the principal outcomes are listed below:

i. That the five tanks are well mixed so that the temperature inside the buckets is evenly distributed for the two calculated experiments at the initial temperature (35 °C, 100 °C) and the T_{steam} temperature of saturated steam (500°C, 1000°C) respectively (see Figs. 3 and 5).

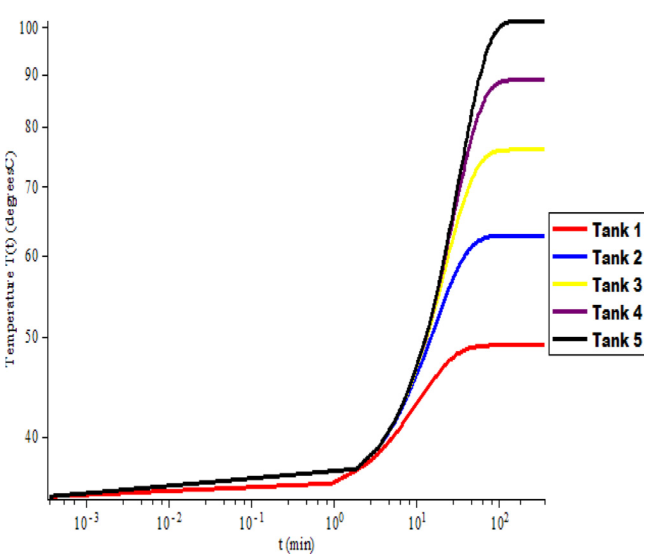

Fig. 3 Simulated logarithm functions of the temperature distribution for the tanks T_1, T_2, T_3, T_4, and T_5 in °C when the uniform initial temperature is 35 °C and temperature T_{steam} saturated steam is 500 °C five tanks for heating oil in series.

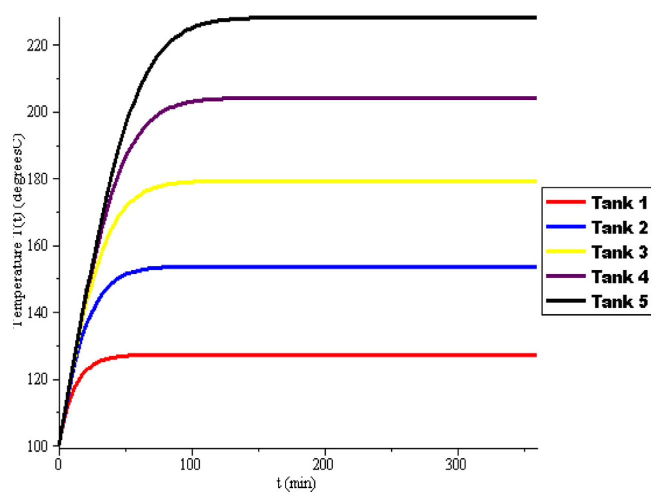

Fig. 4 Depict the simulated temperature distributions for the tanks T_1, T_2, T_3, T_4, and T_5 tanks in °C obtained when the uniform initial temperature is 100 °C and T_{steam} temperature of 1000 °C saturated steam for five connected oil heating tanks in series.

ii. The variation of the parameters are considered separately from tank 1 to tank 5 (Figs. 6–15).
iii. The higher temperature distribution is obtained when heat is transferred from the first to the fifth tank (see Figs. 2 and 4).
iv. The higher the initial temperature of 100 °C and the T_{steam} temperature of the saturated steam of 1000 °C, the higher the temperature distribution is compared to the initial temperature of 35 °C and the T_{steam} temperature of the saturated steam of 500 °C (see Table 1 and Table 2).

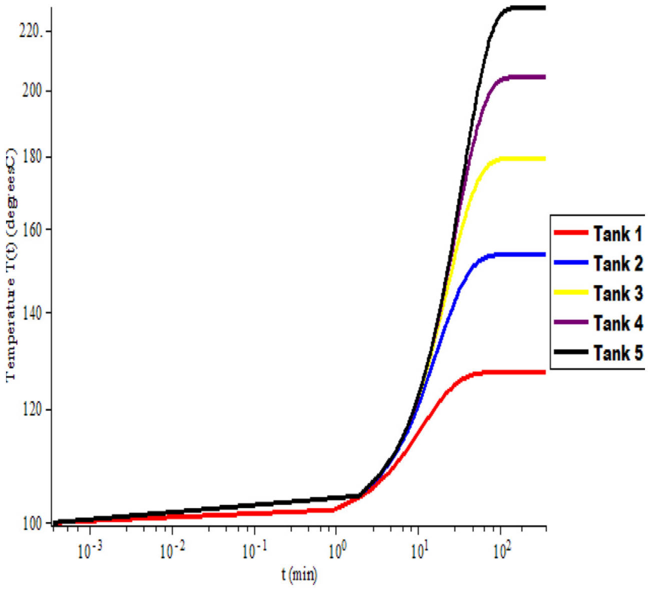

Fig. 5 Simulated logarithm functions of the temperature distribution for the tanks T_1, T_2, T_3, T_4, and T_5 in °C when the uniform initial temperature is 100 °C and temperature T_{steam} saturated steam is 1000 °C five tanks for heating oil in series.

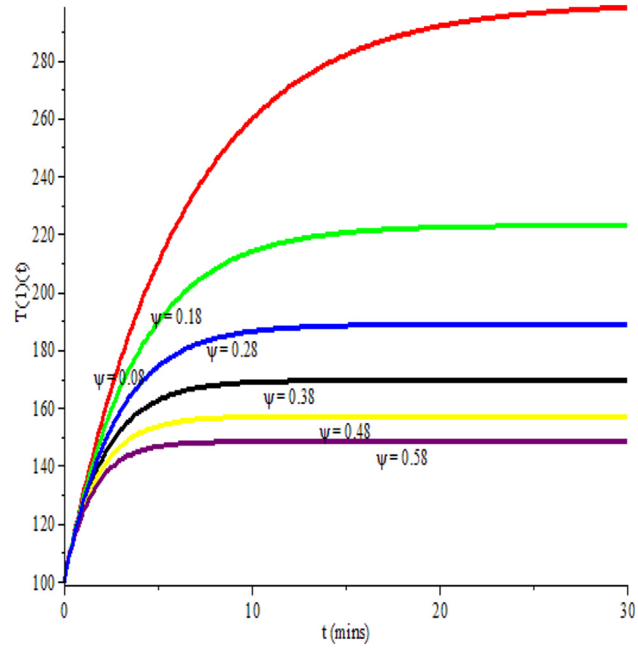

Fig. 7 Present the temperature distribution profile of Tank 1 when parameter $\omega = 0.0025$ is fixed and varies. $\Psi = [0.08, 0.18, 0.28, 0.38, 0.48, 0.58]$.

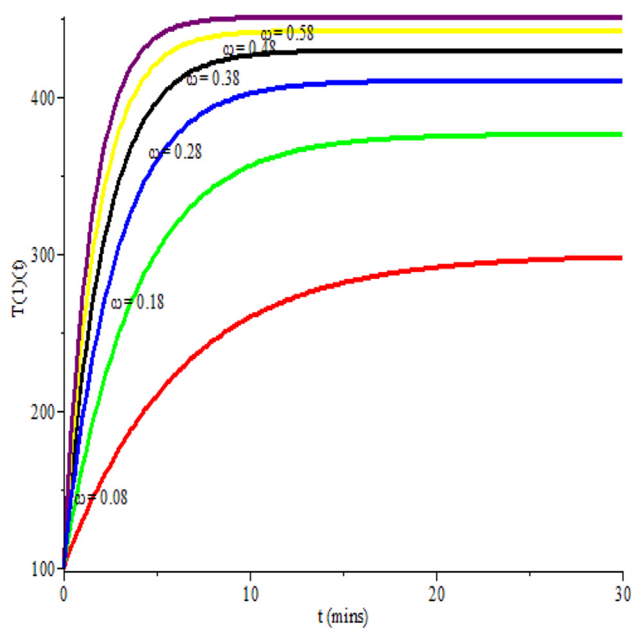

Fig. 6 Present the temperature distribution profile of Tank 1 when parameter $\Psi = 0.0025$ is fixed and varies $\omega = [0.08, 0.18, 0.28, 0.38, 0.48, 0.58]$.

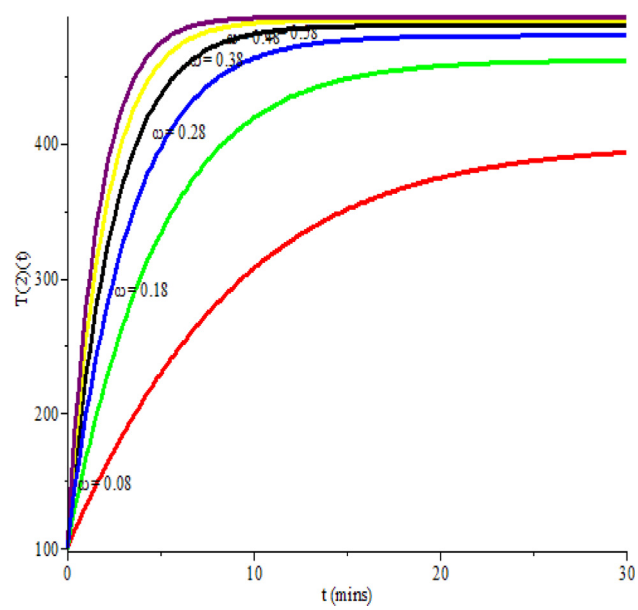

Fig. 8 Present the temperature distribution profile of Tank 2 when parameter $\Psi = 0.0025$ is fixed and varies $\omega = [0.08, 0.18, 0.28, 0.38, 0.48, 0.58]$.

v. The two constant parameters in equation (1) are fixed and tested where $\Psi = 0.0025$ gives a larger temperature distribution across tank 1 to tank 5 than $\omega = 0.0025$ (see Figs. 6–15).
vi. The reverse reaction occurs in all five reservoirs when $\psi = 0.0025$ and $\omega = 0.0025$ respectively (see Figs. 6–15 indicated in red).
vii. Steady (convrges) temperatures are recorded within the time interval 60mins to 80 mins for all five tanks in consideration (see Table 1 and Table 2).
viii. The novelty of the constructed algorithms is shown by good agreement with the exact solutions given in Table 1 and Table 2, which shows a close comparison of our proposed algorithms.

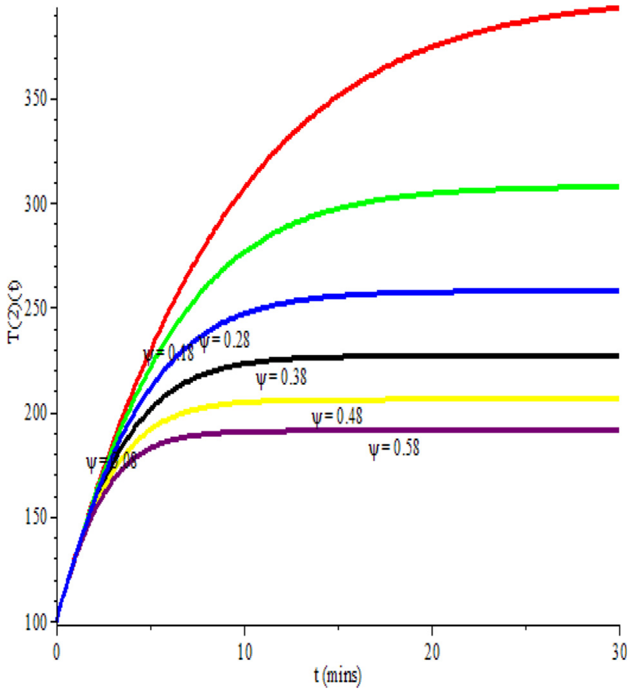

Fig. 9 Present the temperature distribution profile of Tank 2 when parameter $\omega = 0.0025$ is fixed and varies. $\psi = [0.08, 0.18, 0.28, 0.38, 0.48, 0.58]$.

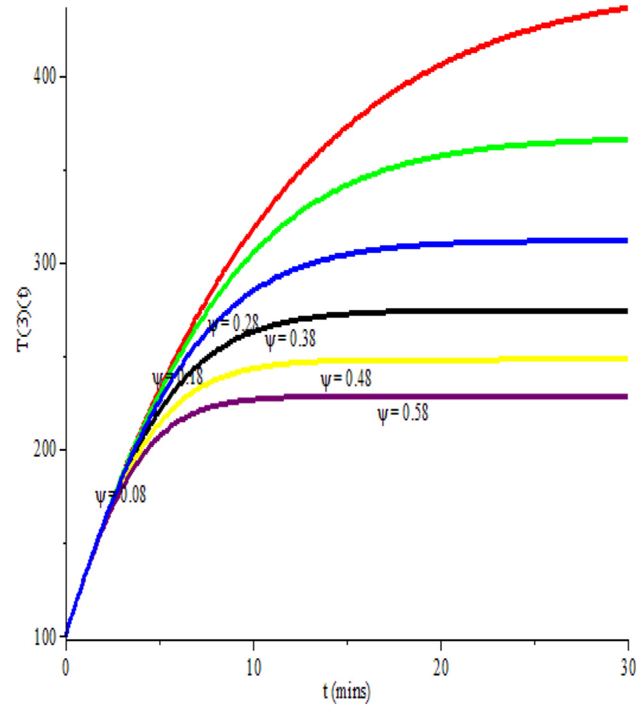

Fig. 11 Present the temperature distribution profile of Tank 3 when parameter $\omega = 0.0025$ is fixed and varies. $\Psi = [0.08, 0.18, 0.28, 0.38, 0.48, 0.58]$.

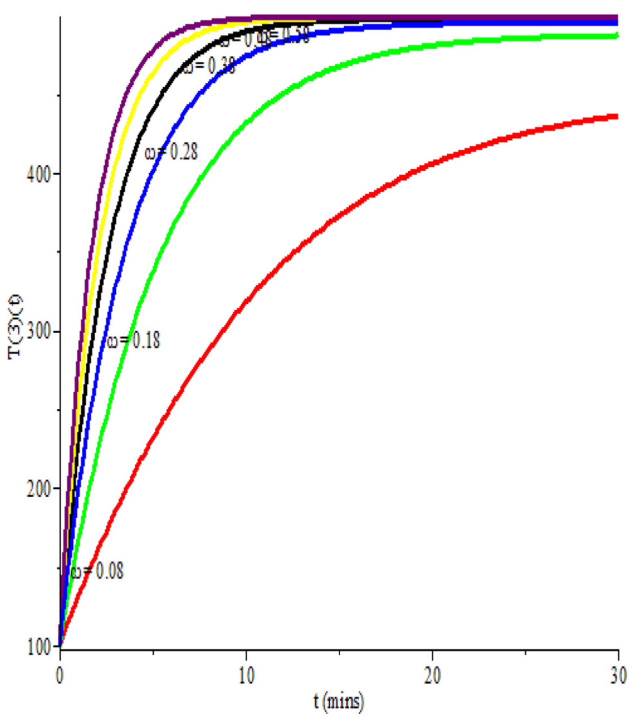

Fig. 10 Present the temperature distribution profile of Tank 3 when parameter $\Psi = 0.0025$ is fixed and varies $\omega = [0.08, 0.18, 0.28, 0.38, 0.48, 0.58]$.

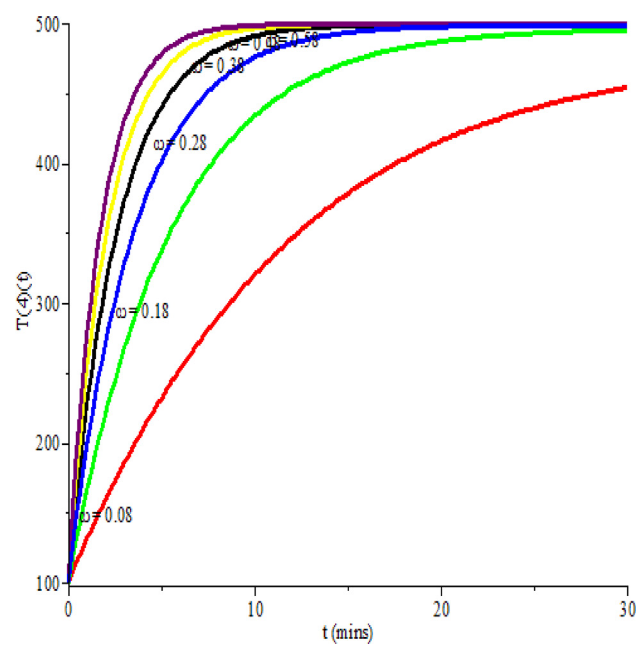

Fig. 12 Present the temperature distribution profile of Tank 4 when parameter $\Psi = 0.0025$ is fixed and varies $\omega = [0.08, 0.18, 0.28, 0.38, 0.48, 0.58]$.

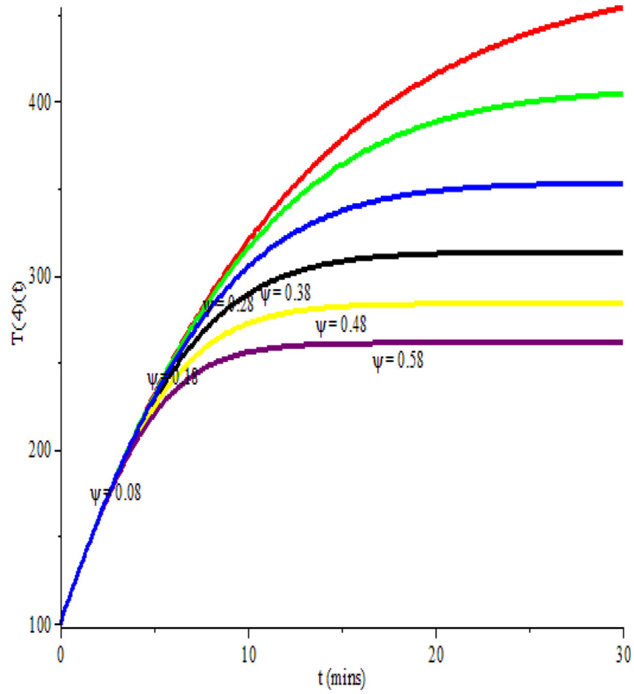

Fig. 13 Present the temperature distribution profile of Tank 4 when parameter $\omega = 0.0025$ is fixed and varies. $\Psi = [0.08, 0.18, 0.28, 0.38, 0.48, 0.58]$.

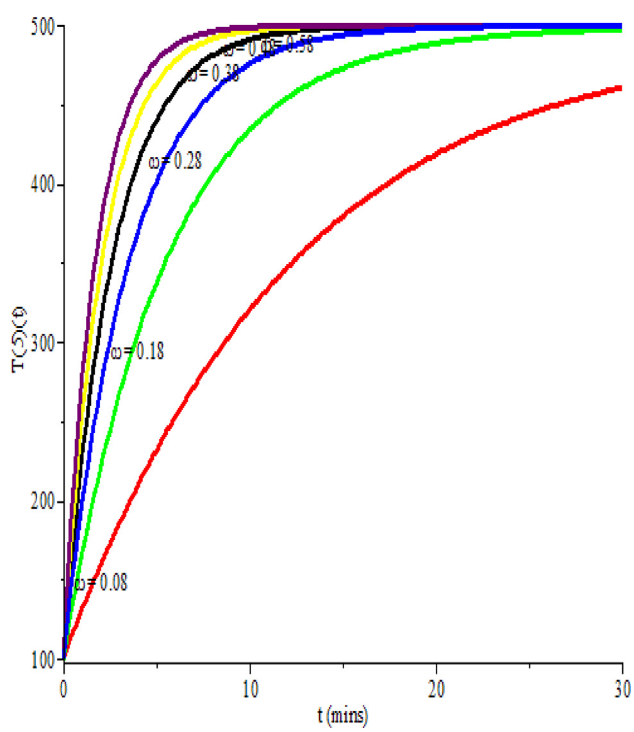

Fig. 14 Present the temperature distribution profile of Tank 5 when parameter $\Psi = 0.0025$ is fixed and varies $\omega = [0.08, 0.18, 0.28, 0.38, 0.48, 0.58]$.

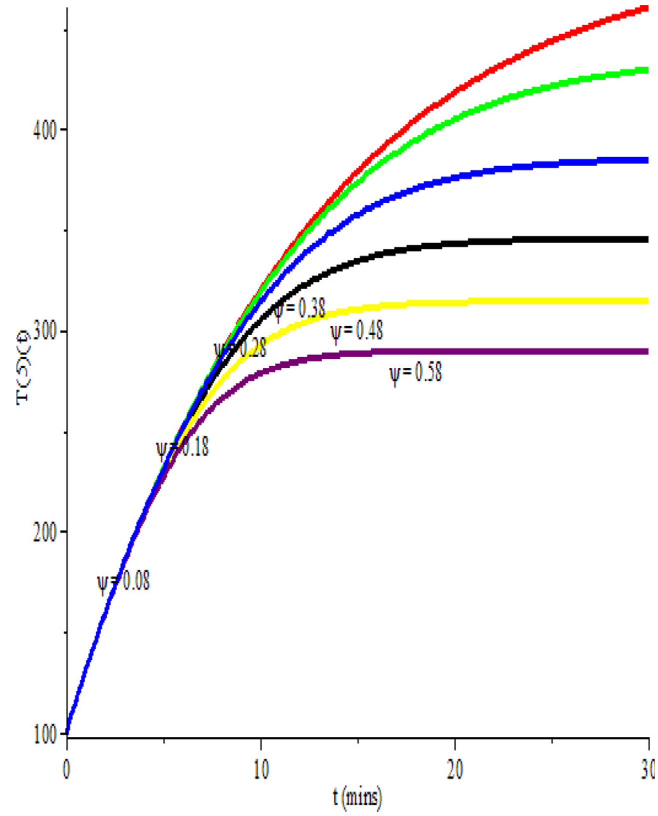

Fig. 15 Present the temperature distribution profile of Tank 5 when parameter $\omega = 0.0025$ is fixed and varies. $\Psi = [0.08, 0.18, 0.28, 0.38, 0.48, 0.58]$.

5.2. Conclusion

Our simulation results obtained from the system of energy balance differential equations for five tanks connected in series show that the built algorithms can be considered structurally simple algorithms and easy to apply to solve systems of differential equations. The two computational experiments performed have shown a significant contribution in the utilization and implementation of the MAPLE 18 software which eventually reduces the time taken to simply mathematical evaluations and simplifications involve in applying some numerical techniques in applied computational engineering sciences. A good agreement with exact solutions suggests that the techniques presented are easy, efficient, and practically feasible which can be extended to more physical and biological problems in applied mathematics. All computations and coding for the proposed algorithms are performed using the MAPLE 18 software package.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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