# The Way One Defines Specification Matters: On the Performance Criteria for Efficient Antenna Optimization in Aggregated Bi-Objective Setups

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***Abstract*—Design of antenna structures for real-world applications is a challenging task that often involves addressing multiple design requirements at a time. Popular solution approaches to this class of problems include utilization of composite objectives. Although configuration of such functions has a significant effect on the cost and performance of the optimization, their specific structure is normally determined based on engineering experience and does not involve auxiliary investigations oriented towards adjustment of process efficiency. In this work, the effects of used functions and their composition on performance and cost of the bi-objective optimization process are investigated. The balance between the requirements is tailored to the problem at hand based on visual inspection of functional landscapes. The analyses are performed on a case study basis using a planar, multi-parameter antenna optimized for minimization of footprint and reflection within the 3.1 GHz to 10.6 GHz range. The numerical results show significant differences between the performance of the obtained solutions, as well as the computational cost of the optimization. The best geometry found using one of the considered objective functions is characterized by an in-band reflection of –9.6 dB and the footprint of only 171 mm$^2$.**

***Index Terms*—bi-objective design, composite objective functions, gradient optimization, objective function adjustment, performance analysis.**

## I. Introduction

The development of contemporary communication systems is subject to multiple requirements concerning, among others, access to wireless medium, electromagnetic compatibility, but also geometrical dimensions of transceiver components [1]-[3]. The design of antennas is oriented towards addressing many of the mentioned criteria at a time, which renders conventional design approaches—based on manual or semi-manual adjustment of radiator topologies followed by visual inspection of its responses—unsuitable for generation of high-performance radiators [4]. Consequently, numerical optimization techniques are indispensable for adjustment of antenna geometries in multi-objective setups [5]-[7].

Dealing with multi-objective problems often involves balancing the quality of solutions and numerical cost of the design process. The latter might be bounded from above by the computational budget (i.e., the maximum number of expensive full-wave electromagnetic (EM) simulations allowed for the algorithm that seeks for an acceptable solution) [7], [8]. The quality of attainable designs highly depends on the selected objective function and the utilized optimization strategy [5], [6]. Another problem is that, for multi-criteria tasks, the number of optimal solutions is infinite. They represent a compromise between the selected performance requirements [5]. From this perspective, appropriate composition of objective functions is crucial for obtaining solutions that comply with the imposed specifications. Having in mind complex and multi-modal mappings between the search and feature spaces, the decision making oriented towards selection of suitable objectives and their tuning is a challenging task [6], [7].

Approaches for dealing with multi-objective problems fall into two main categories that involve either genuine optimization, or the use of goal aggregation mechanisms [4]-[8]. The first class of methods is useful when priorities of the designer are not clearly defined, or thorough validation of the antenna capabilities for a range of specifications is considered [4], [7]. Genuine multi-objective optimization is often handled using population-based metaheuristic algorithms that require tremendous number of EM simulations to find the set of compromise solutions [8], [9]. Consequently, they might be infeasible from the standpoint of the computational budget [7]. The second group of techniques is preferred when the goal is to find a single solution that fulfills a set of defined specifications. In such a case, the optimization can be undertaken using a standard (e.g., gradient-based [10], or derivative-free [11]) method and an objective function that represents suitable combination of the requirements. Popular goal-construction methods include aggregation of individual objectives using appropriate scaling coefficients [5], or the use of composite functions tailored to adjust the primary objective while controlling the auxiliary figures using penalty components [7]. The latter might be implemented in a multi-stage framework, where a sequence of optimizations is performed to fulfill requirements w.r.t. each objective while maintaining the remaining ones at the acceptable levels [4], [7], [12].

Regardless of differences between the available goal-aggregation strategies (and their effects on the optimization performance), cost functions are normally selected without auxiliary analyses oriented towards justification of their usefulness for the problem at hand [4], [8], [9]. Instead, the minimizer is determined based on engineering experience. Tuning of its control parameters (if considered) is realized through rudimentary estimation of function behavior for the expected close-to-optimal solutions [13]. Although such an approach might be understandable having in mind high cost of EM-driven optimization, dynamic changes of antenna behavior along the search space (especially, in terms of balancing the objectives) affects performance of the process. Consequently, the use of inappropriate functions may result in generation of sub- or even far-from-optimal results [4]. Availability of comparative studies concerning the impact of various objective functions on optimization performance might be invaluable from the standpoint of determining an appropriate minimizer for the problem at hand.

In this work, the effects of using objective functions on the performance of bi-objective optimization are investigated. The analysis involves visualization and discussion of functional landscapes generated by the three selected minimizers, as well as the impact of control coefficients values on their shape. The numerical tests are performed using a fairly complex 38-parameter antenna. The design goals include minimization of size and reflection of the structure within the frequency band from 3.1 GHz to 10.6 GHz. For the considered test cases, the number of EM simulations required for convergence varied from 277 to 712 (over 250%). The performance changes of the final designs w.r.t. size and reflection ranged from 152 mm$^2$ to 248 mm$^2$ (over 38%) and –11.3 dB to –7.7 dB (over 3.5 dB), respectively. The results indicate that the selected structure of the cost function and its setup have substantial effects on both the optimization cost and performance of the resulting designs.

## II. Design Example

Our design example is a compact planar antenna shown in Fig. 1 [13]. The structure is implemented on a Rogers RO4003 substrate ($\varepsilon_r = 3.55$, $h = 0.813$ mm, $\tan\delta = 0.0027$). The antenna comprises a driven element in the form of a spline-shaped radiator fed through a microstrip line. In order to maintain small dimensions while ensuring acceptable performance, the ground plane edge of the structure is parameterized using splines and further enhanced using an L-shaped stub. The latter promotes increasing electrical size of the antenna and improving its impedance matching over a wide frequency range [14]. The structure is represented using a 38-variable vector $\boldsymbol{x} = [\boldsymbol{x}_a\ Y{\cdot}\boldsymbol{x}_g\ S{\cdot}\boldsymbol{x}_r]^T$, where $\boldsymbol{x}_a = [X\ l_f\ l_1\ l_{2r}\ w_1\ o_r]^T$. The parameter sets $\boldsymbol{x}_g$ and $\boldsymbol{x}_r$ (each of which contains 16 elements), represent the normalized coordinates that define shapes of the driven element and the ground plane edge, respectively. The variables $Y = l_1 + w_1$ and $S = \min(X - o_r,\ Y - l_f)/2$ denote scaling coefficients for $\boldsymbol{x}_g$ and $\boldsymbol{x}_r$ vectors. Other relative parameters are $l_2 = (X - w_1)l_{2r}$ and $o = X/2 + o_r$, whereas $w_f = 1.8$ remains fixed to maintain 50 Ohm input impedance. The unit for all parameters—except the dimensionless components of vectors $\boldsymbol{x}_g$ and $\boldsymbol{x}_r$, as well as parameters with $r$ in subscript—are in mm. It should be noted that geometry of the monopole-like driven element is defined in a cylindrical coordinate system in order to maintain a constant distance between the control points used for its representation [14].

The antenna is optimized for minimization of reflection within the frequency range from 3.1 GHz to 10.6 GHz (objective $F_1$) and reduction of footprint (objective $F_2$). The lower/upper bounds $\boldsymbol{l}/\boldsymbol{u}$ for the design optimization are: $\boldsymbol{l} = [6\ 4\ 10\ 0.05\ 0.5\ {-1}\ \boldsymbol{lx}_g\ \boldsymbol{lx}_r]^T$ and $\boldsymbol{u} = [30\ 15\ 30\ 1\ 2.5\ 1\ \boldsymbol{ux}_g\ \boldsymbol{ux}_r]^T$, where $\boldsymbol{lx}_g = [0.2\ \ldots\ 0.2]^T$, $\boldsymbol{lx}_r = [0.1\ \ldots\ 0.1]^T$, $\boldsymbol{ux}_g = [0.8\ \ldots\ 0.8]^T$, and $\boldsymbol{ux}_r = [1\ \ldots\ 1]^T$, respectively. Note that the considered antenna is represented using a large number of mutually-dependent variables which makes the design problem complex and multi-modal. For more detailed discussion on the spline-parameterized structures see [8], [13].

## III. Objective Functions for Antenna Optimization

In this section, the methods and algorithms used to investigate the effects of the objective functions of choice on the performance and results of the bi-objective optimization are outlined. The numerical tests and discussions are provided in Section IV.

### A. Problem Formulation

Let $\boldsymbol{R}(\boldsymbol{x})$ denote the response of the antenna structure obtained for the given input parameter vector $\boldsymbol{x}$ (cf. Section II). The multi-objective design problem can be defined as [6]:

$$\boldsymbol{x}^* \in \arg\min_{x \in X} \boldsymbol{F}(\boldsymbol{x}) \tag{1}$$

where $\boldsymbol{x}^*$ denote the optimal design and $\boldsymbol{X}$ is set that defines the feasible region of the search space. The objective function vector for a $q$-objective problem is $\boldsymbol{F}(\boldsymbol{x}) = [F_1(\boldsymbol{x})\ \ldots\ F_q(\boldsymbol{x})]^T$. Here, a design task with two requirements ($q = 2$) is considered. The goals for the discussed antenna are given as:

$$F_1(\boldsymbol{x}) = \max\left(\left|\boldsymbol{S}_{11}(f)\right|\right)\Big|_{f_L \le f \le f_H} \tag{2}$$

$$F_2(\boldsymbol{x}) = X \cdot Y \tag{3}$$

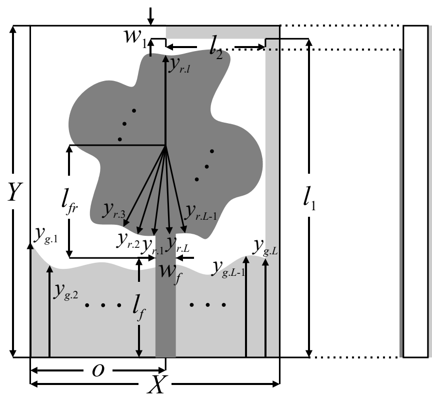


Fig. 1. Geometry of the considered spline antenna [13]. From the left: top- and cross-section view. Dark and light grey represent metallization of the radiator and the ground plane, respectively.

Here, $|\boldsymbol{S}_{11}(f)| = \boldsymbol{R}(\boldsymbol{x}, f)$ represents the antenna reflection (in dB) over the frequency sweep $f$ (the lower and upper bounds are $f_L = 3.1$ GHz and $f_H = 10.6$ GHz). The parameters $X$ and $Y$ correspond to width and length of the substrate the structure is implemented on (see Fig. 1).

Direct solving of (1) can be performed using a genuine multi-objective optimization which is numerically expensive and produces a set of trade-off solutions between the objectives [5]-[7]. Obtaining a number of optimal designs is impractical when the goal is to generate a structure that fulfills a strictly defined set of requirements. In such a case, the task (1) can be substituted by [7], [12]:

$$\boldsymbol{x}^* = \arg\min_{\boldsymbol{x}\in\boldsymbol{X}} U(\boldsymbol{x}) \tag{4}$$

where $U(\boldsymbol{x}) = U(\boldsymbol{R}(\boldsymbol{x}, f))$ is a scalar function that aggregates the objectives $F_1$ and $F_2$. Composition of the design function in (4) is important from the standpoint of optimization performance and quality of the attainable solutions.

## B. Optimization Algorithm

Direct solving of (4) is impractical due to high cost associated with a large number of EM simulations required by the optimization algorithm to converge [15]. Instead, the final design can be iteratively approximated ($i = 0, 1, 2, \ldots$) using a trust-region (TR) framework [16]:

$$\boldsymbol{x}^{(i+1)} = \arg\min_{\boldsymbol{x}:\left\|\boldsymbol{x}-\boldsymbol{x}^{(i)}\right\|<\rho^{(i)}} U\left(\boldsymbol{G}^{(i)}(\boldsymbol{x})\right) \tag{5}$$

Here, $\boldsymbol{x}^{(i)}$ represents an approximation of the design $\boldsymbol{x}^*$ obtained in $i$th iteration of (5), $\rho^{(i)}$ is the TR radius, and $\boldsymbol{G}^{(i)}$ denotes the linear model of the form [12]:

$$\boldsymbol{G}^{(i)}(\boldsymbol{x}) = \boldsymbol{R}\left(\boldsymbol{x}^{(i)}\right) + \boldsymbol{J}\left(\boldsymbol{x}^{(i)}\right)\left(\boldsymbol{x}-\boldsymbol{x}^{(i)}\right) \tag{6}$$

The Jacobian $\boldsymbol{J}$ is generated around the $\boldsymbol{x}^{(i)}$ design using a large-step finite differences [13]. The optimization engine for (5) is a gradient-based algorithm [10]. The initial TR radius is set to $\rho^{(0)} = 1$. It is controlled based on a gain ratio using a set of standard rules [16]. The algorithm is terminated when either the TR radius, or Euclidean distance between the $\boldsymbol{x}^{(i+1)}$ and $\boldsymbol{x}^{(i)}$ designs are below the specified threshold (here, $10^{-2}$). The computational cost of the algorithm (5) is $d + 1$ EM simulations for successful designs ($\rho^{(i)} > 0$; $d$ stands for the problem dimensionality). Additional evaluations are needed for unsuccessful steps ($\rho^{(i)} < 0$). For more detailed discussion on the optimization framework, see [12], [13], [16].

## C. Design Functions for Bi-Objective Optimization

The objective functions considered in this study are based on the weighted sum concept [5], [6] and the penalty function approach (composition of primary and auxiliary goals where the latter are activated upon violation of the specific criteria) [7], [17]. The functions are as follows:

$$U_1(\boldsymbol{x}) = \alpha F_1(\boldsymbol{x}) + \beta F_2(\boldsymbol{x}) \tag{7}$$

$$U_2(\boldsymbol{x}) = \left[\alpha F_1(\boldsymbol{x}) + \beta F_2(\boldsymbol{x})\right]^3 \tag{8}$$

$$U_3(\boldsymbol{x}) = F_2(\boldsymbol{x}) + \gamma \max\left(\frac{F_1(\boldsymbol{x}) - F_{\max}}{|F_{\max}|}, 0\right)^2 \tag{9}$$

The first two objectives represent variants of weighted sum method, whereas (9) implements the penalty term activated when the objective $F_1(\boldsymbol{x})$ violates the user-defined performance threshold on in-band reflection $F_{\max}$ (here, $F_{\max} = -10$ dB). Another considered example is a variant of $U_3$, where a bi-stage optimization is performed, i.e., the antenna is first optimized for minimization of (2) starting from $\boldsymbol{x}^{(0)}$ and the resulting intermediate design $\boldsymbol{x}^{\#}$ is used as a starting point for minimization of (9) [12], [13]. The rationale behind sequential optimization is that the $F_1(\boldsymbol{x}^{\#}) < F_{\max}$, and thus the penalty component is inactive at the beginning of the miniaturization-oriented design. The consequence is that the starting point for optimization is located in the more linear region of the functional landscape which should promote generation of smaller designs as compared to the ones obtained from direct application of (9) starting from $\boldsymbol{x}^{(0)}$ [4].

One of the challenges related to the use of considered objectives is determination of the suitable $\alpha$, $\beta$, $\gamma$ coefficients. In $U_{1\text{-}2}$, the role of $\alpha$ and $\beta$ (here, $\beta = 1 - \alpha$) is to balance contribution of (2) and (3) to the functions [5], [18], [19]. On the other hand, in $U_3$, $\gamma$ is to be tailored to smooth the transition between the feasible and infeasible search space regions to prevent premature convergence of the optimization [12]. In practical design cases, the specific values of the discussed factors are determined based on rudimentary analysis of function behavior for the presumed close-to-optimal designs [13]. Alternatively, they can be selected through visual inspection of the functional landscapes. The effects of coefficients values on $U_{1\text{-}3}$ responses are shown in Fig. 2. The obtained plots indicate that adjustment of control factors is crucial to ensure balanced contribution of individual objectives to the composite function. For instance, incorrect values of $\alpha$ and $\gamma$ in Figs. 2(a) and (e) diminish the effects of in-band reflection on values of the functions. From this perspective, visualizations might be useful not only for selecting appropriate design objective but also for tailoring its coefficients so as to balance the goals w.r.t. the requirements.

# IV. Numerical Results and Discussion

The antenna of Section II was optimized using objectives $U_{1\text{-}3}$—denoted as setups (i)-(iii)—and a in multi-stage configuration (iv) that involves sequential minimization of (2) and (9), respectively. The starting point for all numerical experiments is $\boldsymbol{x}^{(0)} = [10\ 6\ 16\ 0.8\ 1\ 0.0001\ Y{\cdot}\boldsymbol{x}_g^{(0)}\ S{\cdot}\boldsymbol{x}_r^{(0)}]^T$, where $\boldsymbol{x}_g^{(0)} = [0.35\ \ldots\ 0.35]^T$ and $\boldsymbol{x}_r^{(0)} = [0.6\ \ldots\ 0.6]^T$. The design was found through manual tuning of the antenna.

The weighting coefficient for setup (i) was set to $\alpha = 0.98$ (cf. Fig. 2). The optimal design was found after 277 EM simulations that correspond to 11 iterations of (5). The final design was characterized by an in-band reflection of –11 dB and a footprint of 248 mm$^2$. The parameter $\alpha$ for (ii) was set to 0.98. The algorithm converged after 21 iterations (363 EM model evaluations). The resulting design features the in-band reflection of –9.5 dB and the size of 242 mm$^2$. To demonstrate the effect of scaling factors on the design process, the optimization (ii) was also performed with $\alpha$ = 0.9 (cf. Fig. 2(c)). The area and reflection of the final design are 152 mm$^2$ and –7.7 dB, respectively. The algorithm converged after 12 iterations (316 EM simulations). In (iii), the antenna with the footprint of 171 mm$^2$ and in-band reflection of –9.6 dB was found after 15 iterations ($\gamma$ = 1000). Finally, in (iv), the final design was found after a total of 712 EM evaluations. The first and second step completed after 20 and 8 iterations, respectively. The intermediate design—optimized w.r.t. (2)—features the in-band reflection of –12.4 dB and the footprint of 305 mm$^2$. After the second stage, the area and performance of the final design were 233 mm$^2$ and –9.3 dB, respectively. Figure 3 shows a visual comparison of the antenna characteristics obtained for the considered test cases.

The detailed breakdown of the design cost and performance of the optimized designs is given in Table I. The results indicate that the selected objective functions (as well as their settings) significantly affect the results of the optimization process. Setup (iii) yields the best balance between the size and performance of the antenna. It provides relatively smooth functional landscape with significant penalty on the solutions that violate reflection-related specification. An important—and initially counterintuitive result—is that the performance of the design obtained using (iii) is substantially better compared to the one yielded by (iv). The reason is that the intermediate design produced after the first stage of (iv) is much larger than $\boldsymbol{x}^{(0)}$ (304 mm$^2$ vs. 170 mm$^2$). Consequently, due to relatively modest slope of the landscape for the region where $F_1 < F_{\max}$—and a multi-modal character of the design problem—the algorithm converged to a local optima.

It should be noted that majority of the obtained designs violate the requirement concerning –10 dB reflection (cf. Section III.*C*). In the case of setups (iii) and (iv), the effect could be counteracted by reducing the $F_{\max}$ to around –10.5 dB. However, for $U_1$ and $U_2$ the final reflection level is substantially affected by the specific ratio between $\alpha$ and $\beta$ (see Fig. 2). As indicated in Table I, the size and performance of the antenna design found through minimization of $U_2$ with $\alpha$ = 0.98 and $\alpha$ = 0.9 vary by 1.85 dB and 90 mm$^2$ (over 37%), respectively.

TABLE I. COST/PERFORMANCE BREAKDOWN VS. SELECTED DESIGN SETUPS

| Setup | Design step | Scale factor | $F_1(\boldsymbol{x})$ [dB] | $F_2(\boldsymbol{x})$ [mm$^2$] | Cost [$\boldsymbol{R}$] |
|---|---|---|---|---|---|
| (i) | – | $\alpha$ = 0.98 | –11.3 | 248.1 | 277 |
| (ii) | – | $\alpha$ = 0.98 | –9.52 | 242.1 | 363 |
| | – | $\alpha$ = 0.90 | –7.67 | 152.0 | 316 |
| (iii) | – | $\gamma$ = 1000 | –9.60 | 170.7 | 509 |
| (iv) | I | – | –12.4 | 304.5 | 552 |
| | II | $\gamma$ = 1000 | –9.27 | 232.7 | 160 |

Responses of the solution generated using (iii) are shown in Fig. 4. The reflection of the optimized design is improved by over 6 dB w.r.t. $\boldsymbol{x}^{(0)}$, which was achieved at a cost of only slight deterioration of $F_2$ (from 170 mm$^2$ to ~171 mm$^2$). The antenna is characterized by a high average total efficiency of around 91% [20]. Note the correspondence between increase of efficiency above 3 GHz with steep reflection improvement.

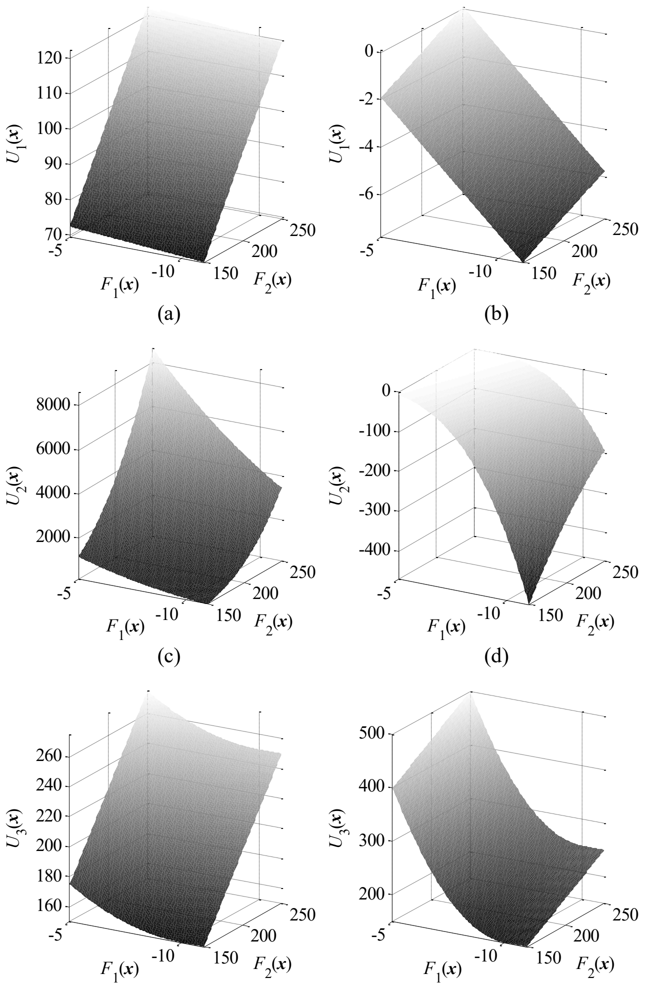

Fig. 2. Functional landscapes for the objective function $U_1$: (a) $\alpha$ = 0.5, (b) $\alpha$ = 0.98; $U_2$: (c) $\alpha$ = 0.9, (d) $\alpha$ = 0.98; and $U_3$: (e) γ = 100, (f) γ = 1000.

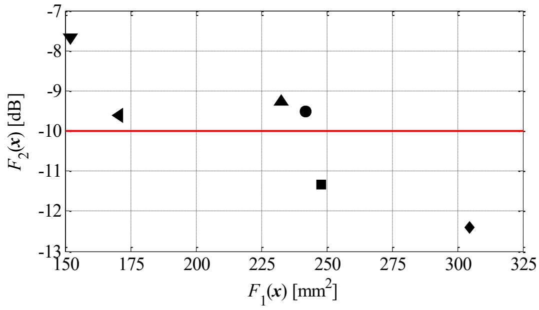

Fig. 3. Performance of the designs optimized using setup: (i) – (■), (ii) with $\alpha$ = 0.9 (▼) and with $\alpha$ = 0.98 (●), (iii) – (◀), as well as (iv) where (♦) and (▲) denote results yield after the first and second design stage. The red line denotes the desired in-band reflection (cf. Section III.*C*).

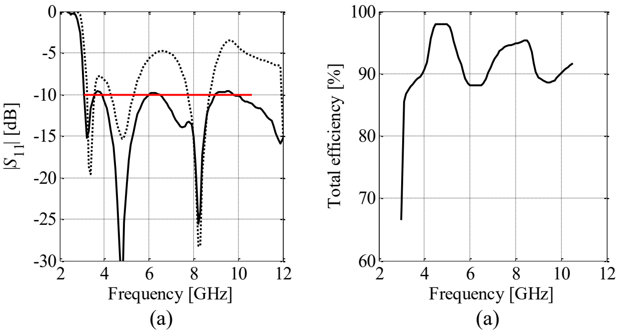


Fig. 4. Performance characteristics of the antenna optimized using setup (iii): (a) reflection responses at the initial (dotted) and final (solid) designs and (b) total efficiency at the final design.

## V. CONCLUSION

In this work, the effects of cost function on the performance and outcome of the antenna optimization in bi-objective setup have been investigated on a case study basis. The design example of choice has been a compact planar monopole represented using 38 independent parameters, optimized in a total of five test scenarios concerning the three different objective functions, a two-stage design approach oriented towards minimization of reflection followed by size reduction, as well as the design oriented towards verification of the utilized control coefficients on the optimization outcome. The visualizations and results indicate the tremendous effects of the cost function structure (along with its control parameters) on performance of the final designs (in terms of balance between the requirements) and the optimization cost. For the considered test cases, the design algorithm required from 277 to 712 EM simulations (over 2-fold difference) to converge. The performance of the obtained final solutions w.r.t. reflection (3.1 GHz to 10.6 GHz) ranged from –11.3 dB to –7.7 (a total of 3.6 dB) and from 152 mm$^2$ to 248 mm$^2$ (around 38 percent) w.r.t. size. The design that represents the best balance between the requirements features the reflection of –9.6 dB and size of 171 mm$^2$. Furthermore, its average in-band efficiency is over 91%, confirming that the structure is in fact a radiator.

Future work will focus on conducting numerical tests for a broader range of objective functions and antenna structures, as well as engineering the goals towards mitigating the risk of getting stuck in poor local optima. Determination of the guidelines for adjusting the functions control parameters to the specific design cases will also be considered.

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