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ON BIDIRECTIONAL PREESTIMATES AND THEIR APPLICATION TO IDENTIFICATION OF FAST TIME-VARYING SYSTEMS

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ABSTRACT

When applied to identification of time-varying systems, such as rapidly fading telecommunication channels, adaptive estimation algorithms built on the local basis function (LBF) principle yield excellent tracking performance but are computationally demanding. The subsequently proposed fast LBF (fLBF) algorithms, based on the preestimation principle, allow a substantial reduction in the complexity without significant performance losses. We propose a novel preestimator, called bidirectional, which further improves performance of the fLBF scheme.

***Index Terms*—** Identification of time-varying systems, local basis function approach, rapidly fading telecommunication channels

1. INTRODUCTION

We will consider the problem of identification (tracking) of a time-varying FIR (finite impulse response) system governed by

$$\begin{aligned} y(t) &= \sum_{i=1}^{n} \theta_i^*(t) u(t-i+1) + e(t) \\ &= \boldsymbol{\theta}^{\mathrm{H}}(t)\boldsymbol{\varphi}(t) + e(t) \end{aligned} \tag{1}$$

where $t = \ldots, -1, 0, 1, \ldots$ denotes discrete (normalized) time, $y(t)$ denotes the complex-valued output signal, $\boldsymbol{\varphi}(t) = [u(t), \ldots, u(t-n+1)]^{\mathrm{T}}$ denotes regression vector made up of past samples of the complex-valued input signal $u(t)$, $\boldsymbol{\theta}(t) = [\theta_1(t), \ldots, \theta_n(t)]^{\mathrm{T}}$ is the vector of time-varying system coefficients, and $\{e(t)\}$ denotes measurement noise. The symbol $*$ stands for complex conjugate and H – complex conjugate transpose (Hermitian transpose). Many nonstationary communication channels (terrestrial, underwater) can be well approximated by a FIR model of this form [1], [2].

We will focus on noncausal estimation, i.e., we will assume that in addition to past measurements $\{y(s), \boldsymbol{\varphi}(s), s \leq t\}$ a certain number of "future" ones (with respect to the "current" time instant t) $\{y(s), \boldsymbol{\varphi}(s), s > t\}$ are available and can be used for estimation purposes. Such is the situation, for example, in the case of identification of channels operating in the full-duplex (FD) mode [3], [4]. In this case $u(t)$ is a known, near-end signal, emitted by the transmit antenna (placed near the receive antenna) and $e(t)$ is a mixture of a far-end signal and channel noise, which should be extracted from $y(t)$ by removing from it the self-interference component (the first term on the right hand side of (1)). Note that FD systems, which – to increase channel throughput – simultaneously transmit and receive information in the same frequency bandwidth, allow one to work with a decision delay, which means that channel identification can be carried out using noncausal estimation algorithms with improved tracking capabilities, such as the ones described in this paper.

Additionally, we will assume that: (A1) $\{u(t)\}$ is a zero-mean circular white noise with variance σ_u^2, (A2) $\{e(t)\}$, independent of $\{u(t)\}$, is a zero-mean circular white noise with variance σ_e^2, and (A3) $\{\boldsymbol{\theta}(t)\}$ is a sequence independent of $\{u(t)\}$ and $\{e(t)\}$. Note that assumptions (A1)-(A3) are typical of wireless communication systems.

2. FAST LBF ESTIMATORS

The LBF identification technique is based on the assumption that in the local analysis interval $T(t) = [t-k, t+k]$ of length $K = 2k+1$, centered at t, system parameters can be expressed as linear combinations of a certain number of linearly independent complex-valued functions of time $f_1(j), \ldots, f_m(j), j \in I_k = [-k, k]$, further referred to as basis functions. The point-estimation LBF approach (generalized Savitzky-Golay filtering [5]) is a time-localized version of the interval-estimation basis function (BF) technique explored by many authors [6] - [15].

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Without any loss of generality we will assume that the basis $\mathcal{F}_m = \{f_1(j), \ldots, f_m(j), j \in I_k\}$ is orthonormal, namely $\sum_{j=-k}^{k} \mathbf{f}(j)\mathbf{f}^{\mathrm{H}}(j) = \mathbf{I}_m$ where $\mathbf{f}(j) = [f_1(j), \ldots, f_m(j)]^{\mathrm{T}}$ and $\mathbf{I}_m$ denotes the $m \times m$ identity matrix. We will also assume that the first basis function is constant: $f_1(j) = 1/\sqrt{K} = f_0, j \in I_k$. As an example of a basis set (which has some physical justification in telecommunication applications [1]) we will consider the complex exponential basis of the form

$$\mathcal{F}_m = \left\{(1/\sqrt{K})e^{\mathrm{i}j\omega_1}, \ldots, (1/\sqrt{K})e^{\mathrm{i}j\omega_m}, j \in I_k\right\} \tag{2}$$

where $\mathrm{i} = \sqrt{-1}$, $\omega_1 = 0$, $m = 2m_0 + 1$ and $\omega_{2l} = -(2\pi l)/K, \omega_{2l+1} = (2\pi l)/K, l = 1, \ldots, m_0$.

Computational complexity of LBF estimators is $\mathcal{O}(m^3 n^3)$ complex flops (complex multiply-add operations) per time update. This cost can be significantly reduced if the iterative dichotomous coordinate descent (DCD) technique is used [16]. The computationally much cheaper fast LBF estimators, proposed in [17], take the form

$$\widehat{\boldsymbol{\theta}}(t) = \sum_{j=-k}^{k} h^{\mathrm{LBF}}(j)\widetilde{\boldsymbol{\theta}}(t+j) \tag{3}$$

where

$$h^{\mathrm{LBF}}(j) = \mathbf{f}^{\mathrm{H}}(0)\mathbf{f}(j), \quad j \in I_k \tag{4}$$

denotes the impulse response of a FIR filter associated with the LBF estimator and $\{\widetilde{\boldsymbol{\theta}}(t)\}$ is a sequence of "preestimates" – maximum bandwidth parameter estimates [18], which are approximately unbiased (no matter how the system parameters change) but might have a large variance.

Unidirectional (forward-time) preestimates, which were originally proposed in [19] and further developed in [17], [20] can be obtained by means of "inverse filtering" the estimates yielded by the short-memory exponentially weighted least squares (EWLS) algorithm

$$\begin{aligned}\widehat{\boldsymbol{\theta}}^{\mathrm{EWLS}}(t) &= \arg\min_{\boldsymbol{\theta}} \sum_{j=1}^{t} \lambda^{t-j}|y(j) - \boldsymbol{\theta}^{\mathrm{H}}\boldsymbol{\varphi}(j)|^2 \\ &= \mathbf{G}^{-1}(t)\mathbf{g}(t)\end{aligned} \tag{5}$$

where λ, $0 < \lambda < 1$, denotes the so-called forgetting constant. The $n \times n$ matrix $\mathbf{G}(t) = \sum_{j=1}^{t} \lambda^{t-j}\boldsymbol{\varphi}(j)\boldsymbol{\varphi}^{\mathrm{H}}(j)$ and the $n \times 1$ vector $\mathbf{g}(t) = \sum_{j=1}^{t} \lambda^{t-j}\boldsymbol{\varphi}(j)y^*(j)$. can be computed in a recursive way using

$$\begin{aligned}\mathbf{G}(t) &= \lambda\mathbf{G}(t-1) + \boldsymbol{\varphi}(t)\boldsymbol{\varphi}^{\mathrm{H}}(t) \\ \mathbf{g}(t) &= \lambda\mathbf{g}(t-1) + \boldsymbol{\varphi}(t)y^*(t) \\ \mathbf{G}(0) &= 0, \;\; \mathbf{g}(0) = 0.\end{aligned} \tag{6}$$

The inverse filtering formula derived and analyzed in [17], which can be used to obtain forward-time preestimates, has the form

$$\widetilde{\boldsymbol{\theta}}(t) = M(t)\widehat{\boldsymbol{\theta}}^{\mathrm{EWLS}}(t) - \lambda M(t-1)\widehat{\boldsymbol{\theta}}^{\mathrm{EWLS}}(t-1) \tag{7}$$

where $M(t)$ denotes the effective width of the exponential window which can be evaluated using

$$M(t) = \sum_{i=1}^{t} \lambda^{t-i} = \lambda M(t-1) + 1 \tag{8}$$

with initial condition $M(0) = 0$. The recommended choice of the forgetting factor is $\lambda = \max\{0.9, 1 - 2/n\}$ [21].

It can be shown that under (A1) - (A3) the unidirectional preestimates are approximately unbiased, i.e., $\mathrm{E}[\widetilde{\boldsymbol{\theta}}(t)] \cong \boldsymbol{\theta}(t)$ where the expectation is carried out over $\{e(t)\}$ and $\{\boldsymbol{\varphi}(t)\}$.

3. BIDIRECTIONAL PREESTIMATES

Bidirectional preestimation is a new concept. While unidirectional preestimates can be obtained by filtering EWLS estimates (causal), as a basis for evaluation of bidirectional preestimates we will use noncausal double exponentially weighted least squares (E^2WLS) estimates of the form

$$\begin{aligned}\widehat{\boldsymbol{\theta}}^{\mathrm{E^2WLS}}(t) &= \arg\min_{\boldsymbol{\theta}} \sum_{j=1}^{t+\tau} \lambda^{|t-j|}|y(j) - \boldsymbol{\theta}^{\mathrm{H}}\boldsymbol{\varphi}(j)|^2 \\ &= \mathbf{H}^{-1}(t)\mathbf{h}(t)\end{aligned} \tag{9}$$

where $\mathbf{H}(t) = \sum_{j=1}^{t+\tau} \lambda^{|t-j|}\boldsymbol{\varphi}(j)\boldsymbol{\varphi}^{\mathrm{H}}(j)$, $\mathbf{h}(t) = \sum_{j=1}^{t+\tau} \lambda^{|t-j|}$ $\boldsymbol{\varphi}(j)y^*(j)$, and τ denotes the truncation point of the double exponential window. It is recommended to set $\lambda = \max\{0.8, 1 - 4/n\}$. It can be shown that if the condition $\tau \geq 8/(1-\lambda)$ is met (which is adviced), the preestimates obtained using the truncated double exponential window are practically indistinguishable from those obtained for $\tau \longrightarrow \infty$, i.e., the truncation effects are negligible.

Note that the $n \times n$ matrix $\mathbf{H}(t)$ and the $n \times 1$ vector $\mathbf{h}(t)$ can be obtained by backward-time processing of $\mathbf{G}(\cdot)$ and $\mathbf{g}(\cdot)$

$$\begin{aligned}\mathbf{H}(j) &= \lambda\mathbf{H}(j+1) + (1-\lambda^2)\mathbf{G}(j) \\ \mathbf{h}(j) &= \lambda\mathbf{h}(j+1) + (1-\lambda^2)\mathbf{g}(j) \\ j &= t+\tau-1, \ldots, t\end{aligned} \tag{10}$$

with initial conditions $\mathbf{H}(t+\tau) = \mathbf{G}(t+\tau)$ and $\mathbf{h}(t+\tau) = \mathbf{g}(t+\tau)$.

The effective width of the double exponential window is given by

$$L(t) = \sum_{j=1}^{t+\tau} \lambda^{|t-j|} = \lambda L(t+1) + (1-\lambda^2)M(t) \tag{11}$$

with initial condition $L(t+\tau) = M(t+\tau)$.

Suppose that the sequence $\{\boldsymbol{\varphi}(t)\}$ is (locally) stationary and $\boldsymbol{\Phi}_0 = \mathrm{E}[\boldsymbol{\varphi}(t)\boldsymbol{\varphi}^{\mathrm{H}}(t)]$. Our derivation will be based on the following approximation $\mathbf{H}^{-1}(t) \cong (\mathrm{E}[\mathbf{H}(t)])^{-1} = \frac{1}{L(t)}\boldsymbol{\Phi}_0^{-1}$ which holds true for sufficiently large values of $L(t)$

(under (A1) one can easily show that the matrix $\mathbf{H}(t)/L(t)$ converges in the mean square sense to $\mathbf{\Phi}_0$ when $t \longrightarrow \infty$ and $\lambda \longrightarrow 1$).

Replacing $\mathbf{H}^{-1}(t)$ with its approximation, one obtains

$$\widehat{\boldsymbol{\theta}}^{\mathrm{E^2WLS}}(t) \cong \frac{1}{L(t)} \mathbf{\Phi}_0^{-1} \mathbf{h}(t) \tag{12}$$

and consequently, according to (10)

$$\begin{aligned} &L(t)\widehat{\boldsymbol{\theta}}^{\mathrm{E^2WLS}}(t) - \lambda L(t+1)\widehat{\boldsymbol{\theta}}^{\mathrm{E^2WLS}}(t+1) \\ &\quad \cong \mathbf{\Phi}_0^{-1}[\mathbf{h}(t) - \lambda \mathbf{h}(t+1)] = (1-\lambda^2)\mathbf{\Phi}_0^{-1}\mathbf{g}(t) \end{aligned} \tag{13}$$

which can be written down in the form

$$\mathbf{\Phi}_0^{-1}\mathbf{g}(t) \cong \frac{L(t)\widehat{\boldsymbol{\theta}}^{\mathrm{E^2WLS}}(t) - \lambda \widehat{\boldsymbol{\theta}}^{\mathrm{E^2WLS}}(t+1)}{1-\lambda^2}. \tag{14}$$

We will show that the following quantity

$$\widetilde{\boldsymbol{\theta}}_{\pm}(t) = \frac{\boldsymbol{\delta}(t)}{1-\lambda^2} \tag{15}$$

where

$$\begin{aligned} \boldsymbol{\delta}(t) &= (1+\lambda^2)L(t)\widehat{\boldsymbol{\theta}}^{\mathrm{E^2WLS}}(t) - \lambda L(t-1)\widehat{\boldsymbol{\theta}}^{\mathrm{E^2WLS}}(t-1) \\ &\quad - \lambda L(t+1)\widehat{\boldsymbol{\theta}}^{\mathrm{E^2WLS}}(t+1) \end{aligned} \tag{16}$$

can be used as a bidirectional counterpart of (7). Actually, according to (14) and (6), it holds that $\widetilde{\boldsymbol{\theta}}_{\pm}(t) \cong \mathbf{\Phi}_0^{-1}\boldsymbol{\varphi}(t)y^*(t)$. Hence, under assumptions (A1)-(A3), one obtains

$$\begin{aligned} \mathrm{E}[\widetilde{\boldsymbol{\theta}}_{\pm}(t)] &\cong \mathrm{E}[\mathbf{\Phi}_0^{-1}\boldsymbol{\varphi}(t)y^*(t)] = \mathrm{E}[\mathbf{\Phi}_0^{-1}\boldsymbol{\varphi}(t)\boldsymbol{\varphi}^{\mathrm{H}}(t)\boldsymbol{\theta}(t)] \\ &\quad + \mathrm{E}[\mathbf{\Phi}_0^{-1}\boldsymbol{\varphi}(t)e^*(t)] = \boldsymbol{\theta}(t) \end{aligned} \tag{17}$$

i.e., just like unidirectional preestimates, bidirectional preestimates are (approximately) unbiased.

Under assumptions (A1)-(A3) the unidirectional and bidirectional preestimates can be written down in the form

$$\widetilde{\boldsymbol{\theta}}(t) \cong \boldsymbol{\theta}(t) + \mathbf{z}(t), \quad \widetilde{\boldsymbol{\theta}}_{\pm}(t) \cong \boldsymbol{\theta}(t) + \mathbf{z}_{\pm}(t)$$

where $\{\mathbf{z}(t)\}$ and $\{\mathbf{z}_{\pm}(t)\}$ denote zero-mean preestimation noise sequences with large covariance matrices. Hence the fLBF estimate (4) can be regarded as a result of "denoising" preestimates using the basis function approach [17] (the same formula can be used to postfilter $\{\widetilde{\boldsymbol{\theta}}_{\pm}(t)\}$). The most important difference between the two preestimation schemes is the noise intensity: for bidirectional preestimates the preestimation noise is usually much "smaller" than for the unidirectional ones - see Fig. 1.

4. COMPUTATIONAL ASPECTS

4.1. Preestimation

The EWLS estimates can be computed using the well-known recursive algorithm [22] at the cost of $\mathcal{O}(n^2)$ flops per time update. Alternatively, to reduce the computational cost to $\mathcal{O}(n)$ flops, the EWLS estimates can be computed using the fast transversal filter algorithm [22].

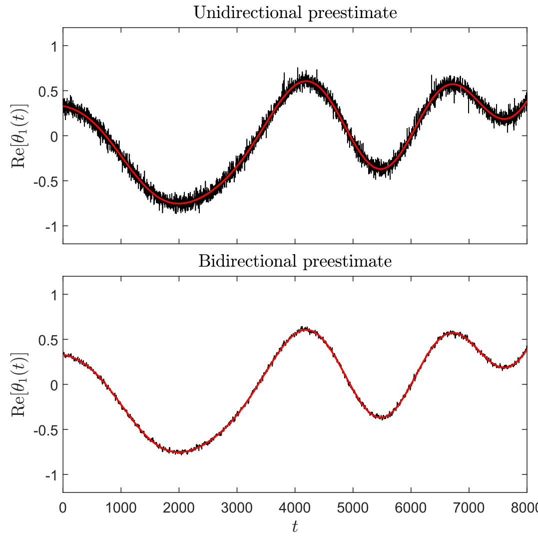

Fig. 1. Typical preestimation results obtained for a simulated channel ($n = 20$, SNR=30 dB): unidirectional (forward-time) preestimates (top figure, $\lambda = 0.9$) and bidirectional preestimates (bottom figure, $\lambda = 0.8, \tau = 41$). Forgetting constants were chosen so as to make the "information content" identical in both cases. Preestimates (black lines) are superimposed on true parameter trajectories (red lines).

Recursive computability of E^{2}WLS estimates is less obvious. To demonstrate that this is the case, note that both $\mathbf{H}(t)$ and $\mathbf{h}(t)$ can be written down as sums of two components

$$\mathbf{H}(t) = \mathbf{G}(t) + \mathbf{S}(t), \quad \mathbf{h}(t) = \mathbf{g}(t) + \mathbf{s}(t) \tag{18}$$

where $\mathbf{S}(t) = \sum_{j=t+1}^{t+\tau} \lambda^{j-t}\boldsymbol{\varphi}(j)\boldsymbol{\varphi}^{\mathrm{H}}(j)$, $\mathbf{s}(t) = \sum_{j=t+1}^{t+\tau} \lambda^{j-t} \boldsymbol{\varphi}(j)y^*(j)$. Note that

$$\begin{aligned} \mathbf{S}(t) &= \frac{\mathbf{S}(t-1)}{\lambda} - \boldsymbol{\varphi}(t)\boldsymbol{\varphi}^{\mathrm{H}}(t) + \lambda^{\tau}\boldsymbol{\varphi}(t+\tau)\boldsymbol{\varphi}^{\mathrm{H}}(t+\tau) \\ \mathbf{s}(t) &= \frac{\mathbf{s}(t-1)}{\lambda} - \boldsymbol{\varphi}(t)y^*(t) + \lambda^{\tau}\boldsymbol{\varphi}(t+\tau)y^*(t+\tau). \end{aligned}$$

Since the recursions presented above are not exponentially stable, to avoid unbounded accumulation of numerical (round-off) errors, the quantities $\mathbf{S}(t)$ and $\mathbf{s}(t)$ should be from time to time (eg. every τ sampling intervals) calculated directly instead of using the recursive formulas. Importantly from the computational viewpoint, the additional computational cost of resetting can be evenly distributed over time, which is particularly relevant when estimation is carried out in the almost-real-time mode, i.e, with a constant processing delay equal to $\Delta t = \max\{k, \tau\}$. Actually, suppose that one intends to reset $\mathbf{S}(\cdot)$ at the instant t. Since $\mathbf{S}(t) = \lambda\boldsymbol{\varphi}(t+1)\boldsymbol{\varphi}^{\mathrm{H}}(t+1) + \lambda^2\boldsymbol{\varphi}(t+2)\boldsymbol{\varphi}^{\mathrm{H}}(t+2) + \ldots + \lambda^{\tau}\boldsymbol{\varphi}(t+\tau)\boldsymbol{\varphi}^{\mathrm{H}}(t+\tau)$, evaluation of $\mathbf{S}(t)$ can be started at the instant $t - \tau + 1$ by computing its first component. Then, at the instant $t-\tau+2$, the second component can be computed and added etc. etc. Proceeding in this way, evaluation of $\mathbf{S}(t)$ will be completed at the instant t, and the process will be repeated starting from the next time instant to compute $\mathbf{S}(t+\tau)$.

Even though the matrices/vectors $\mathbf{G}(t)$, $\mathbf{g}(t)$, $\mathbf{S}(t)$ and $\mathbf{s}(t)$ are recursively computable, the last step – computation of the E^2WLS estimate

$$\widehat{\boldsymbol{\theta}}^{\mathrm{E}^2\mathrm{WLS}}(t) = [\mathbf{G}(t)+\mathbf{S}(t)]^{-1}[\mathbf{g}(t)+\mathbf{s}(t)] \qquad (19)$$

does not allow the use of fast computational techniques. Since computation of (19) requires inversion of the $n \times n$ matrix $\mathbf{G}(t)+\mathbf{S}(t)$ (either in a direct or indirect way), the associated computational burden is of order $\mathcal{O}(n^3)$ per time update.

4.2. Postfiltering

The adopted basis functions (2) are recursively computable: $\mathbf{f}(j) = \boldsymbol{\Lambda}\mathbf{f}(j+1)$, $\boldsymbol{\Lambda} = \mathrm{diag}\{e^{-\mathrm{i}\omega_1}, \dots, e^{-\mathrm{i}\omega_m}\}$. Exploiting this property, one arrives at the following recursive formula which can be used to update $\widehat{\boldsymbol{\alpha}}_i(t) = \sum_{j=-k}^{k} \mathbf{f}(j)\widetilde{\theta}_i(t+j)$ and $\widehat{\theta}_i(t)$

$$\begin{aligned}\widehat{\boldsymbol{\alpha}}_i(t) &= \boldsymbol{\Lambda}\left[\widehat{\boldsymbol{\alpha}}_i(t-1) - \widetilde{\theta}_i(t-k-1)\mathbf{f}(-k)\right] \\ &\quad + \widetilde{\theta}_i(t+k)\mathbf{f}(k) \\ \widehat{\theta}_i(t) &= \mathbf{f}^{\mathrm{H}}(0)\widehat{\boldsymbol{\alpha}}_i(t), \quad i=1,\dots,n. \end{aligned} \qquad (20)$$

The total computational cost of postfiltering is $\mathcal{O}(mn)$ and does not depend on the width of the analysis interval K.

5. SIMULATION RESULTS

All methods described in this paper were tested on a signal generated by an FIR system (1) with 20 time-varying parameters, simulating a FD underwater acoustic (UWA) communication system. The variance of consecutive parameters was decaying exponentially $\mathrm{var}[\theta_i(t)] = 0.69^{i-1}$, $i = 1, \dots, 20$, to reflect power delay profile due to scattering and absorption [2]. Parameter trajectories were generated as low-pass filtered complex Gaussian random sequences with a cut-off frequency equal to 1 Hz under 1000 Hz sampling (such rates of parameter changes are typical of UWA applications). The input signal was a circular complex random sequence of the form $u(t) = \pm 1 \pm \mathrm{i}$. Noise was a circular complex random Gaussian process with variance equal to $\sigma_e^2 = 3.2 \cdot 10^{-3}$ (SNR=30 dB) or $\sigma_e^2 = 3.2 \cdot 10^{-5}$ (SNR=50 dB). Note that in the self-interference cancellation case, very high values of the signal-to-noise ratio (SNR) are typical due to a small distance between the transmit and receive antennas. To avoid boundary problems, data generation was started 1000 time instants before $t = 1$ and lasted 1000 time instants after the identification was stopped.

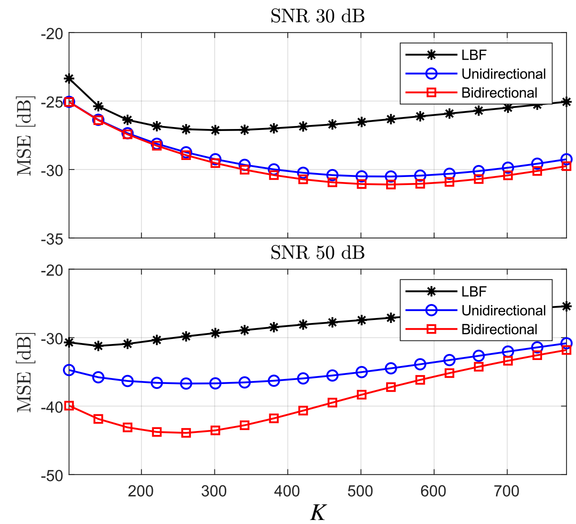

Fig. 2. The self-interference cancellation performance of the compared algorithms in the simulated FD UWA environment.

Unidirectional preestimates were computed for $\lambda = 0.9$. For bidirectional preestimates the settings were equal to $\lambda = 0.8$ and $\tau = 41$. Both fLBF and LBF schemes used $m = 5$ complex exponential basis functions (two methods of adaptive selection of k and m were proposed in [25]).

Results of our simulation experiments are summarized in Fig. 2. MSE was averaged over one long realization of data (10^5 samples). Additionally, the proposed new version of the fLBF algorithm was compared with three other algorithms: EWLS, E^2WLS and TU-RLS (time updated recursive least squares, considered the state-of-the art in UWA communication [23], [24]) and . For SNR equal to 30 dB the best results achieved using EWLS, E^2WLS and TU-RLS algorithms were -23.04 dB (for $\lambda = 0.93$), -30.01 dB (for $\lambda = 0.97$), and -24.34 dB (for $\lambda = 0.95$, $\mu = 0.003$) and -17.24 dB, respectively. For SNR equal to 50 dB the best achievable results were -29.07 dB (for $\lambda = 0.81$), -36.98 dB (for $\lambda = 0.89$), and -32.07 dB (for $\lambda = 0.83$, $\mu = 0.01$), respectively. Note that the proposed fLBF algorithm based on bidirectional preestimates outperforms all identification algorithms mentioned above.

6. CONCLUSION AND RELATION TO PRIOR WORK

The proposed method is an extension of the fast local basis function approach described in [17], recently successfully applied to identification of fast-varying underwater acoustic channels [26]. It has been shown that new, bidirectional preestimates, used in the first phase of identification, allow achieving estimation accuracy that exceeds the accuracy of all currently available tracking algorithms, including the state-of-the-art time updated recursive least squares algorithms.

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