A review on analytical models of brushless permanent magnet machines

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ABSTRACT The electrical machines will be analyzed with the analytical methods as well as numerical methods. However, the numerical models (such as the finite-element method (FEM)) can analyze and model the complex geometries of electrical machines that maybe include saturation effects, but these models have high computational burdens and too time-consuming which is so important in the design stage and optimization issues with too many iterations in electrical machine companies or their related R&D department. Also, the FEM models are not flexible in terms of changing the machine geometries or changing in the input values causing reconstruct the machines for new simulations. Furthermore, the users have no sense of the machine behaviors by applying the numerical models. The abovementioned challenge of the numerical methods can be overcome by analytical models. In principle, analytical methods are introduced based on the magnetic equivalent circuit (MEC) or solving Maxwell's equation. Generally, MEC models are realized as zero-dimensional (0-D) analytical models which are generic and applicable for analyzing various types of electrical machine's topologies that maybe include the saturation effects. Although the MEC models are faster than the numerical model, they are not as accurate as the numerical models for various structures of electrical machines including a great magnetic airgap. Also, the analytical models based on the Maxwell equations are faster than the numerical ones and they have the potential to obtain acceptable accuracy similar to the numerical models in electrical machines. Among all type of electrical machines, the increasing interest of the Brushless permanent-magnet (PM) machines in all low and medium power applications has led to the development of related analytical models. Therefore, this paper interprets the review of the analytical models in these PM machines to explain their recent developments in terms of the machines' quantities such as magnetic flux density components, induced voltage, inductances, electromagnetic force/torque, efficiency or unbalanced magnetic force (UMF). Also, this literature review helps the researchers to save time for determining appropriate references regarding the analytical models of the brushless PM machines. This paper mainly gives the pros and cons of the different analytical models for various PM machines, where the (0-D), (1-D), (2-D) and (3-D) analytical methods has discussed. The Maxwell and basic mathematical analysis for different PM machines has been discussed.

Index Terms *Analytical models, magnetization patterns, Maxwell equations, Numerical models.*

1- INTRODUCTION

The PM machines have been increasingly used in many industrial applications, such as railway traction, servo systems and electric vehicles. Compared to conventional electric machines, PMs provide various advantages like excellent performance, high speed, ultra-lightweight, high efficiency/ reliability, and lower manufacturing cost [1]-[8].

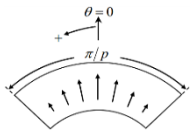
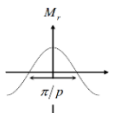
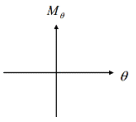
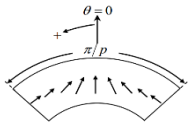
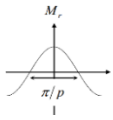
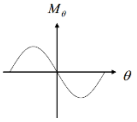
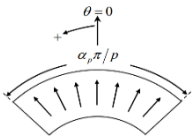
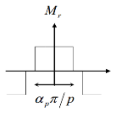
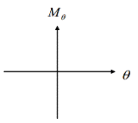
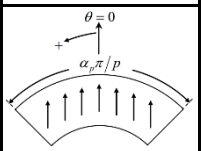
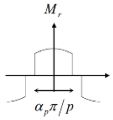
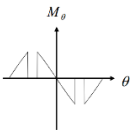
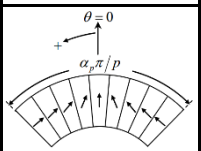
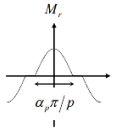
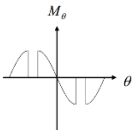
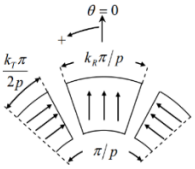
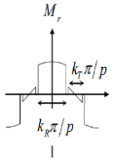
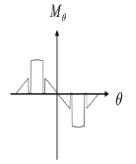
In the designing and optimization process of the PM machines with different geometrical as well as performance characteristic considerations is a challenging task. To reduce time consuming for machine design, research has been carried out to find quick and accurate modeling approach to compute and realize the behavior and performance of PM machines. Analytical models (AM) and Numerical models (NM) are used for analyzing and predicting the machine quantities. The analytical method is essential to minimize the time required for calculation, particularly during the design optimization process. NM become very powerful tools in calculating and predicting machine quantity. However, NMs such as finite element analysis (FEA) are still slow and time consuming for analyzing the characteristics of the machines [9].

Among various computational techniques to design machines, AM, if possible, plays a significant role since diverse crucial characteristics of motors can be obtained accurately and quickly based on these analytical representations. AM provide electric machine developers with a powerful tool for analysis and investigation of the machine behavior under different operational conditions [10], [11]. AM can be used in electromagnetic torque-calculation, back-EMF waveform prediction, cogging torque calculation and stator iron loss estimation.

AM has many advantages over NM, it significantly requires less computational time which is essential in the optimization goals with many iteration numbers. Also, the user can realize the machine behavior according to the related analytical expression [12]-[17] therefore, AM if possible preferred rather than NM.

Mainly, there are four AM techniques used in machine design: (1) relative permeance model [18], [19]; (2) complex permeance model [20]-[22]; (3) Schwarz–Christoffel mapping [47], [58]. Finally, (4) subdomain (SD) technique [23]-[29]. Zhu *et al.* [30] briefly explained the disadvantages and advantages of all these techniques.

TABLE I
BRUSHLESS PM MACHINES CLASSIFICATION IN TERMS OF MAGNETIZATION PATTERNS FOR SM OR SI

Magnetization Patterns	Illustrative Representation	Radial Component waveform	Tangential Component Waveform
Radial sinusoidal amplitude magnetization			
Ideal Halbach or sinusoidal angle magnetization			
Radial magnetization			
Parallel magnetization			
Bar magnets in shifting directions or multi-segment Halbach			
Two-segment Halbach			

p is number of pole-pairs
$\boldsymbol{\alpha_p}$ is magnet-arc per pole-pitch ratio
$\boldsymbol{k_R}$ and $\boldsymbol{k_T}$ are contribution factor of radial nd tangential component respectively

SD technique showed an interesting and high accuracy compared to all other techniques, where the air gap is divided into different subdomains. In this technique, the magnetic flux density in each subdomain is calculated by solving Poisson's equation and applying boundary conditions. However, the calculation of magnetic flux in the stator slot subdomain is redundant and increases the computational volume, a comparison and comprehensive analysis of these models using finite element model as a reference for comparison presented in [31].The analytical model of brushless permanent magnetic has been reported with different magnetization patterns, such as: radial magnetization; parallel magnetization; sinusoidal amplitude magnetization; ideal Halbach; and multi segment Halbach. Halbach array is a kind of magnetization pattern that magnetic field in one part is stronger than others, also magnetic field in the other point is zero. Information of the six types of magnetization patterns with the radial and tangential components have been represented in Table I, the table represent the radial and tangential components of all different types in surface magnet or inset magnet topologies.

In this paper, the review of the analytical models of the brushless PM machines provides a suitable reference for developing the related future studies that lead to saving time for researching about these models in electrical machines. The machines' quantities such as magnetic flux density components, induced voltage, inductances, electromagnetic force/torque, efficiency or unbalanced magnetic force (UMF) have been considered. Moreover, this paper provides appropriate references regarding the analytical models of the brushless PM machines to help the researchers to obtain more accurate and proper results for their future studies.

2- ANALYTICAL MODELS:

In general, if possible, AM can be divided into four analytical models that is, (0-D), (1-D), (2-D), and (3-D) analytical models.

A. (0-D) Analytical Model:

The (0-D) or Magnetic equivalent circuits (MEC) are used to find the maximum or average value of the magnetic flux, this model can be developed when the rotor has no displacement and the linear part of the magnetization curve is considered to predict the magnetic flux density, it is a suitable candidate when the saturation effects are appeared in some parts of electrical machines [32]. MEC is used to develop the (0-D) analytical model for PM machine. Fig. 1 illustrate the (0-D) formed MEC for slotless linear PMSM.

Where R_m ,R_{PM}, R_a, R_s, φ_r are respectively the reluctances of the mover, PM, airgap, stator, and the remanence magnetic flux for the PMs. The maximum magnetic flux density due to PM in the airgap can be computed as,

$$B_g = \frac{\varphi_g}{A_g} \quad \text{(1)}$$

The induced voltage can be calculated using MEC as,

$$e_a = -pN_t\omega\frac{2N_t\varphi_g}{\tau_p}y \quad \text{(2)}$$

Also, electromagnetic force component can be predicted,

$$F_y = \frac{B_g^2}{4\mu_o}A_g \quad \text{(3)}$$

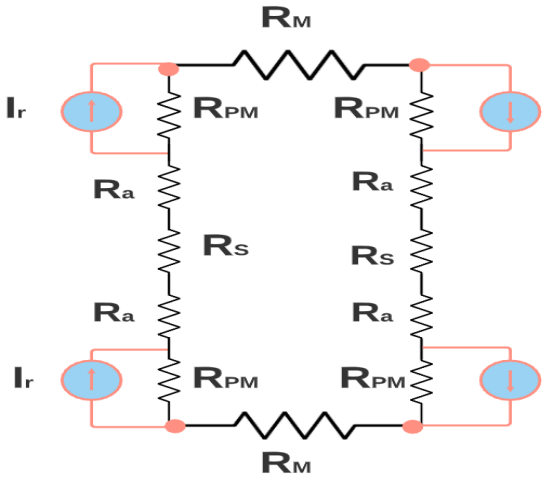


Fig. 1. MEC of slotless linear PM machine

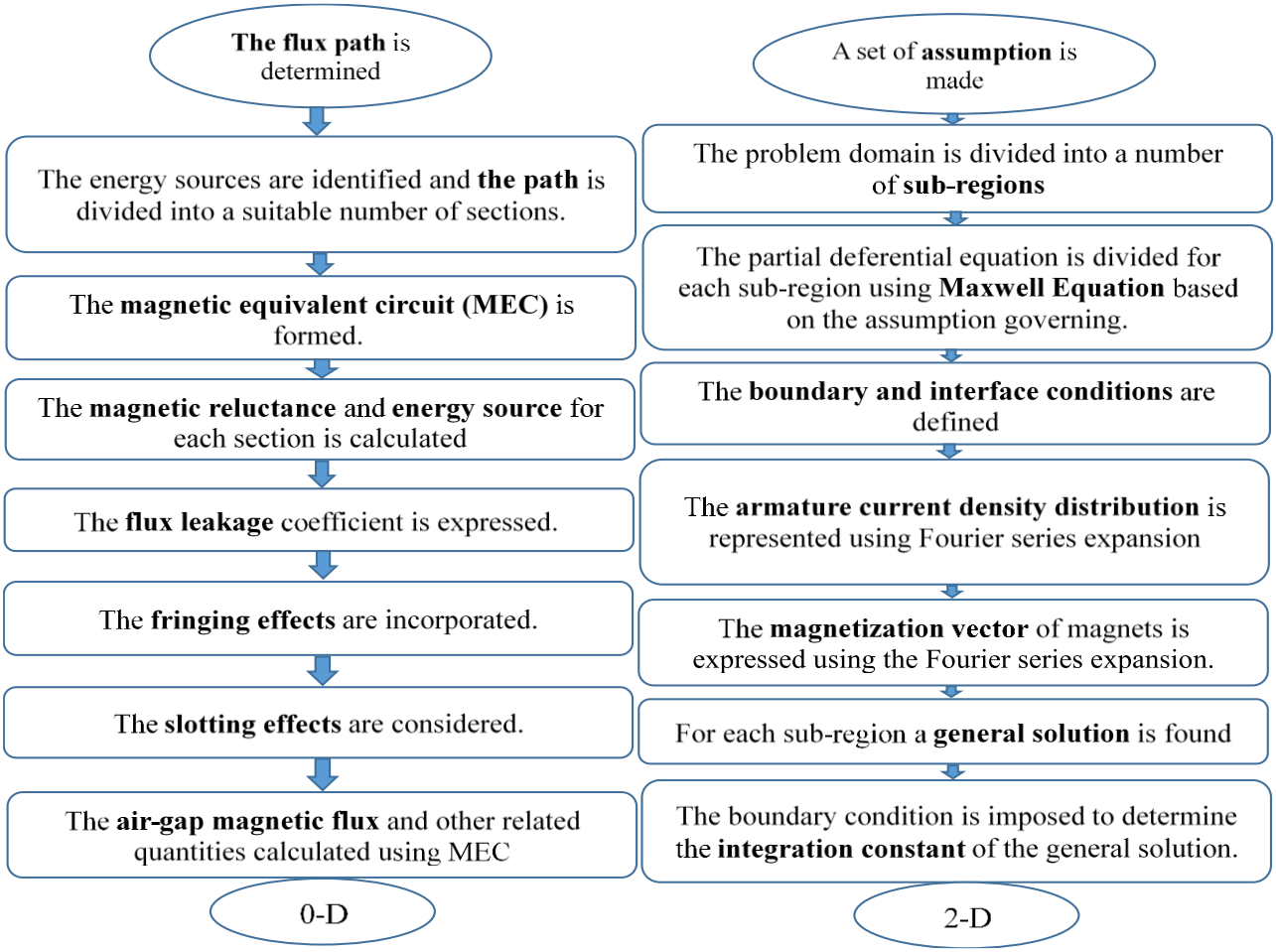


Fig. 2. The procedure of the analytical models.

Based on the equivalent circuit and neglecting the core reluctances the self-inductance is computed as,

$$L_{aa} = \frac{N_c N_t^2}{R_a + R_{PM}} \quad \text{..... (4)}$$

B. (1-D) Analytical Model:

The (1-D) analytical model is suitable when the magnetic flux contains only one component (i.e., radial, or tangential components), such as when the air-gap length is so small that the tangential component of the flux in the airgap can be ignored. But when the air-gap length is such that the tangential component of the flux cannot be neglected, the (1-D) analytical model is no longer appropriate, for example in case of *Ss* machines [5], [6], [8], [13], [33].

C. (2-D) Analytical Model:

(2-D) analytical model is used to estimates the waveform of the components and calculation of the magnetic flux density, electromagnetic torque, back-electromotive force (EMF) [34], [35]. The (2-D) analytical model is not only fast but also gives physical insight to the problem. Fig. 2 represent the steps for analytical computation of magnetic field for (0-D) and (2-D), the first step is to make some assumptions to relatively simplify the solution of the problem and most importantly to make the analytic solution possible. Secondly divide the machine into regions based on the Maxwell equations and by using the magnetic vector potential. Finally, after applying boundary condition all machine quantities can be computed [36], [37], and [39]-[46], [49]-[67] and [80]-[86].

Consider a 3-phase Ss brushless machine, the flux linked to the winding of phase k due to magnetic flux produced by the winding of phase j can be expressed as:

$$\emptyset_{j,k}(t) = pN_t L_s R_x \int_{\frac{\theta_c}{2p}+2\pi(k-1)/pq}^{\frac{\theta_c}{2p}+2\pi(k-1)/pq} B_{r,j}^{x}(R_x,\theta,t)d\theta \quad \text{..... (5)}$$

For k= 1, 2, 3 and j=1, 2, 3 where $B_{r,j}^{x}$ is the radial component of the flux density, θ_c is the coil pitch angle and R_x is the radius at the middle of the winding region. the magnetic flux density in the airgap, the electromagnetic torque can be computed as,

$$T(t) = \frac{L_S R_C^2}{\mu} \int_{-\pi}^{\pi} \frac{1}{\mu} \left(B_{R,PM}^{a} B_{T,PM}^{a} + B_{R,PM}^{a} B_{T,PM}^{a} + B_{R,PM}^{a} B_{T,AR}^{a} + B_{T,AR}^{a} B_{T,AR}^{a} \right) \quad \text{..... (6)}$$

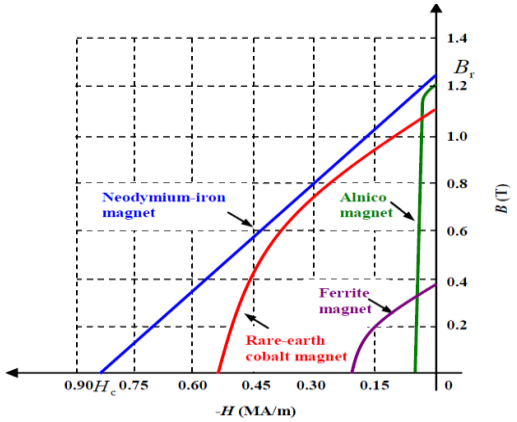
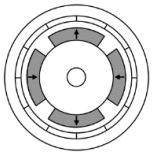
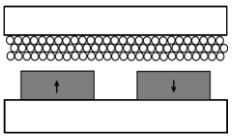
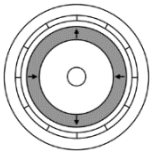
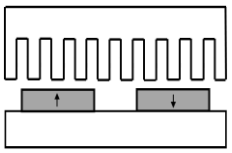
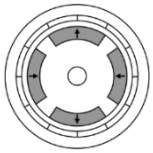
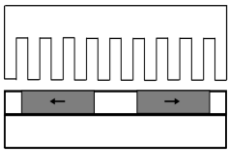
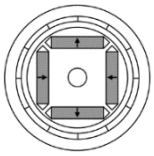
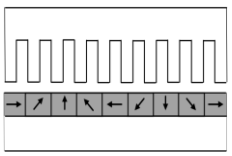
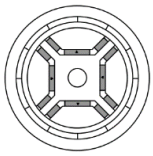
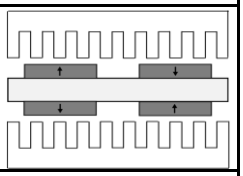
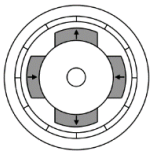
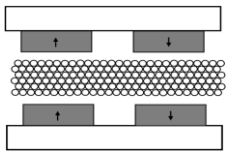
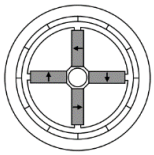
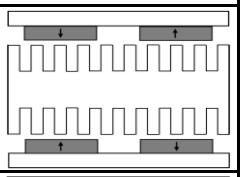
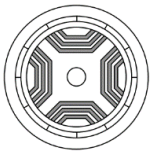
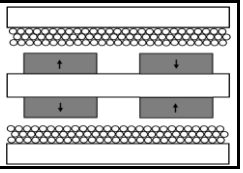


Fig. 3. Demagnetization curve of four types of permanent magnets

TABLE II: BRUSHLESS PM MACHINES CLASSIFICATION IN TERMS OF MAGNET AND MACHINE STRUCTURES

ROTARY STRUCTURES		LINEAR STRUCTURES	
Surface-mounted magnet		Slotless and surface-mounted parallel magnet	
Ring magnet		Slotted and surface-mounted parallel magnet	
Surface-inset magnet		Slotted and buried or interior magnet	
Buried or interior magnet		Slotted with Halbach array magnet	
Multi-segment interior magnet		Double sided and slotted outer armature	
Surface-mounted magnet with parallel edges		Double sided and slotless with one inner armature	
Spoke Magnet		Double sided and slotted inner armature	
Multilayer Interior Magnet		Double sided and slotless outer armature	

D. (3-D) Analytical Model:

The (3-D) analytical models are not used widely because of the difficulty and complexity of the deriving (3-D) analytical equations, a (3-D) AM is highly time consuming specially in designing structure of PM machines. Sometimes, it is necessary to apply assumptions to simplify the obtained model. It may also be necessary to combine the analytical and numerical (3-D) models [38], [47], [68], [75], [78], [92], and [100].

From the literature review (2-D) analytical model is a potential candidate to analyze and predict various machine quantities, in the terms of the accuracy of the magnetic flux density, induced voltage, self and mutual inductances as well as the tangential and normal electromagnetic forces. Researchers prefer to use this approach due to its speed and shorter computational time compared to other SD techniques.

PMs can be classified according to PM topology as; surface inset (*SI*) [39]-[43], surface mounted (*SM*) [44]-[48], spoke PM (*SPM*) and buried or interior PM (*BPM*) [49]-[51]. Fig. 3 demonstrates comparison of four types of permanent magnet. The cost of manufacturing interior magnet is very high [52], compared to surface-mounted, surface-inset provide a compromise with several advantages such as: lower PM eddy current losses, higher rotor mechanical robustness, higher quadrant-per direct-axis reactance, better filed weakening region [53], [54]. Table II above illustrate different types of magnet structure for linear and rotary machines.

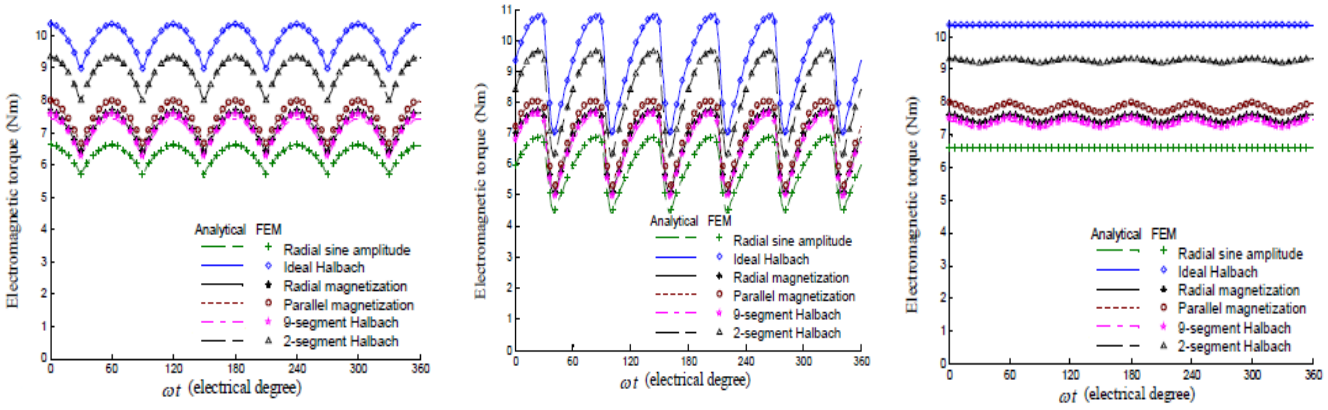
The stator structure of PM machine can be classified as *Sd* and *Ss* structures *Sd* stator structure has more air- gap flux density due to less magnetic airgap and includes better heat removal compared with the *Ss* structure. On the other hand, the *Ss* stator structure reduces the cogging torque and cost of winding. Also, the suitable space for winding exists in the *Ss* structures. Common types of The PM machines include NdFeB, Sm2Co17, SmCo5, Alnico-5 and Ferrite, in which three processes of providing these materials are sintering, injection molding, compression bonding and casting. Table III compare between *Sd* and *Ss* structure of stator.

TABLE III: A COMPARISON BETWEEN SLOTTED AND SLOTLESS STATOR STRUCTURE

	Slotted (*Sd*)	**Slotless (*Ss*)**
Cogging torque	Exist	Almost nothing
Cost of winding	Higher	Lower
Magnetic airgap	Smaller	Lower
Air-gap flux density	Higher	Lower
Inductance	Higher	Lower
Heat removal	Better	Worse
Winding space	Lower	Higher

2- PROCEEDINGS AND ANALYTICAL PROCEDURES

Electromagnetic devices can be modeled and analyzed either analytically or numerically. However generally, numerical approaches, (i.e., finite element method (FEM)), are widely employed to analyze the performance and present an accurate modeling of electrical machines, but this method has some limitations and takes a long computation time to achieve the result. Therefore, the requirement to reduce pre-design stages duration and time-consuming process by analytical solutions, if possible, are preferred frequently. Several types of analytical methods such as magnetic equivalent circuits (MEC), conformal mapping (i.e., Schwarz–Christoffel) and sub-domain method are often used so that any of them have different techniques. Based on the mentioned methods, the sub-domain [55] technique due to the high speed and precision is more famous. In this way, analytical models are divided to regions according to the shape and material characteristics, (i.e., magnet, air gap, and winding) and according to partial differential equations (PDEs) that are derived from Maxwell's equations, due to a set of assumptions, the problem is simplified and a general solution, which provides the PDEs and boundary



(a) (b) (c)

Fig. 5 Electromagnetic torque in the case of the internal rotor motor; a) With the ideal rectangular current waveform; b) With the six-step rectangular current waveform; c) With the sinusoidal current waveform.

conditions for each region. Fig. 4 illustrate the sub-domains of PM machines with Q slots and *p* pole-pairs.

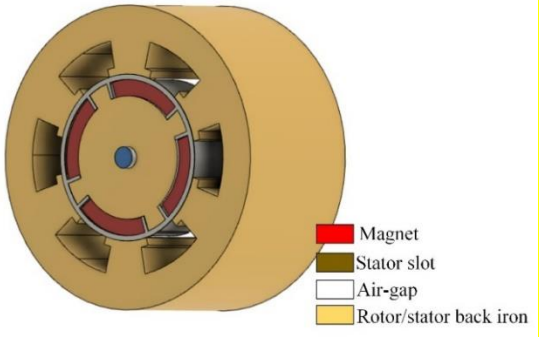


Fig. 4 Illustrative representation of PMSM sub-domains

A. List of Assumptions:

The important assumption in the subdomain technique is related to iron parts, the following assumptions are made to either enable or simplify the analytical model:

- End effects are ignored, that is, the motor is assumed to have infinite axial length.
- The magnetic flux density vector has only radial and tangential components
- The media have finite permeability and linear magnetization characteristic. The saturation effects of the media are neglected.
- The airspace between the magnets has the same permeability as the magnets.
- Eddy current reaction field is neglected.

It is noted that importance of accurate determination of the magnetic field distribution (as for PM and armature reaction) in the air gap of permanent magnet machines is identified to evaluate the machine performance meanwhile, the air gap magnetic field computation can be assessed by numerical (i.e., FEM) or analytical methods. In the most analytical papers, machine quantities such as magnetic field distributions, a back electromotive force (Back-EMF) and electromagnetic torque (cogging torque and load torque) are computed with the proposed analytical method and verified by finite element analysis. Fig. 5 (a-c) illustrate the ideal rectangular, six-step rectangular and sinusoidal current waveforms, for electromagnetic torque [56].

B. Governing PDEs:

Based on Maxwell's equations, partial differential equations (PDEs) can be represented, and the armature current density distribution can also be expressed using its Fourier series expansion. A set of assumptions for the designed machine must be made to simplify the calculations.

A methodology for study the problem resolution for magnetism is presented by Ampere's law $\nabla . \boldsymbol{B} = 0$ and Gauss's law $\nabla \times \boldsymbol{H} = \boldsymbol{J}$, wherein $\boldsymbol{H}$ is the magnetic field intensity vector and $\boldsymbol{J}$ is the electric current density vector. The corresponding relationship between magnetic field density vector and magnetic field intensity vector is expressed as follow:

$$\boldsymbol{B} = \mu_0 \mu_r \boldsymbol{H} + \mu_0 \boldsymbol{M} \quad \text{.....} \quad (7)$$

Where μ_0 is the free space permeability, μ_r is the relative permeability and $\boldsymbol{M}$ is the magnetization vector in A/m. with substituting equation (1) in Ampere's circuital law, yields:

$$\nabla \times \boldsymbol{B} = \mu_0 \mu_r \nabla \times \boldsymbol{H} + \mu_0 \nabla \times \boldsymbol{M} \quad \text{.....} \quad (8)$$

According to gauss's law, magnetic flux is determined as follow:

$$\boldsymbol{B} = \nabla \times \boldsymbol{A} \quad \text{.....} \quad (9)$$

With substituting (3) into (2) and considering $\nabla . \boldsymbol{A} = 0$, governing equation is achieved as follow:

$$\nabla^2 \boldsymbol{A} = -\mu_0 \mu_r \boldsymbol{J} - \mu_0 \nabla \times \boldsymbol{M} \quad \text{.....} \quad (10)$$

By the use of separated variables technique, the general solution of the Laplace and Poisson equation for each region can be obtained.

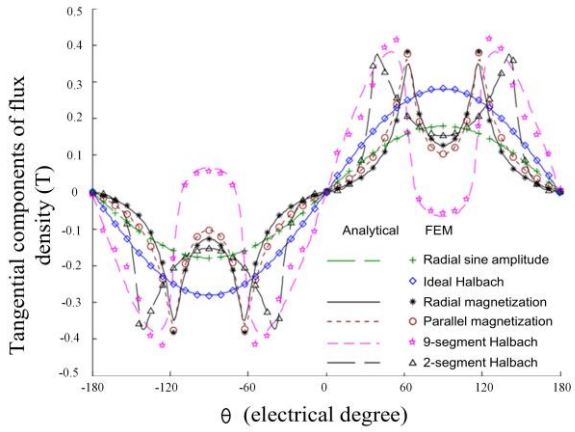
$$\nabla^2 \boldsymbol{A}^w = -\mu_0 \boldsymbol{J} \quad \text{.....} \quad (11)$$

$$\nabla^2 \boldsymbol{A}^m = -\mu_0 \nabla \times \boldsymbol{M} \quad \text{.....} \quad (12)$$

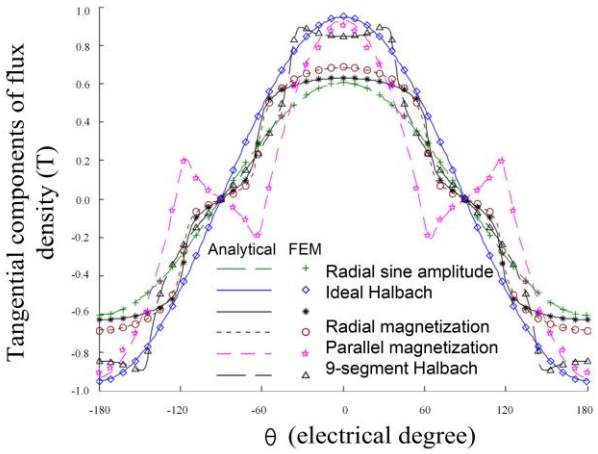
$$\nabla^2 \boldsymbol{A}^i = 0 \quad \text{.....} \quad (13)$$

Where superscripts {*w*, *m*} designate the winding and magnet, respectively and {*i*}is representative of other regions such as exterior, stator, airgap, retaining sleeve, rotor and shaft regions that are indicated by {*a*, *sl*, and *so*} respectively. Fig. 6 [57] demonstrate the value of radial

and tangential open circuit flux density for internal rotor in the winding region and Fig. 7 [53] show flux density due to armature field in the middle of air gap for different electrical degrees.



(a)



(b)

Fig. 6 (a) Flux density of the internal rotor in tangential component, (b) Flux density of the internal rotor in radial component

C. Boundary Conditions:

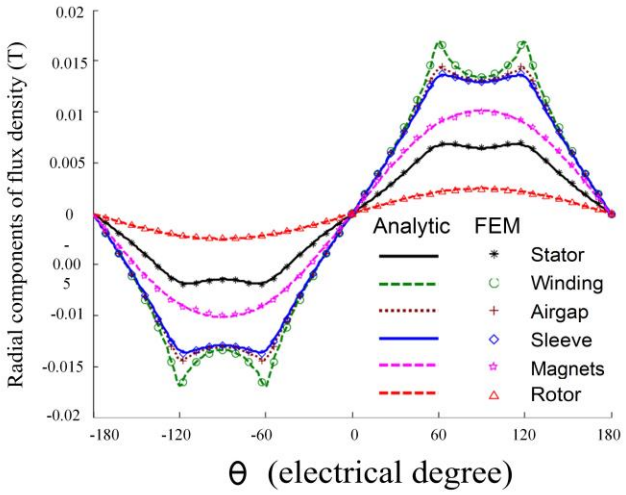
According to the extracted PDEs, the boundary conditions play substantial roles in solving these equations and obtaining the value of each variable in the magnetic flux density components. The boundary conditions can be categorized into two main groups as follows [58]-[60]:

1. the normal component of the magnetic flux density vector is continuous at the interfaces between two adjacent media ($\boldsymbol{B}_{\perp}^{\boldsymbol{i}} = \boldsymbol{B}_{\perp}^{\boldsymbol{i+}}$) where *i* and *i+* are two adjacent sub-region).
2. In the case of the source-free interface, the tangential component of the magnetic field intensity vector is continuous at that interface ($\boldsymbol{H}_{\parallel}^{\boldsymbol{i}} = \boldsymbol{H}_{\parallel}^{\boldsymbol{i+}}$).

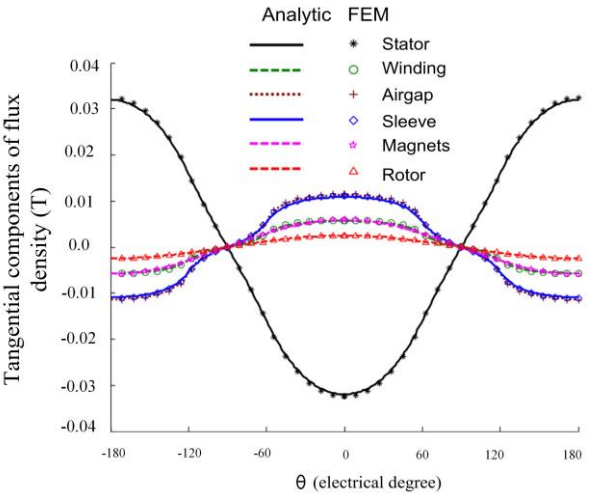
After imposing the boundary interface conditions given in Table IV of Appendix A (internal/external rotor machine), a set of simultaneous algebraic equations with the number of defined unknown variables can be formed to solve PDEs.

E. Extracting the Magnetic Model:

The machine divided into several sub-regions, and partial differential equation extracted for each part based on Maxwell's equations. In this section mathematical equations for slotted/slotless or inner/outer rotor machine represented, these equations can be used to design any PMSM machine of any number of phases, slots, and poles. Also, the equations mentioned in this paper is suitable for internal or external rotor machines.



(a)



(b)

Fig. 7 (a) Flux density in the middle of air gap due to AR field in radial component, (b) Flux density in the middle of air gap due to the AR in tangential component.

1. Extracting FS of AR Currents:

The current of each phase of PMSM can be represented as

$$i_j(t) = \sum_u I_m \sin\left[u(pwt - \gamma_j) + \theta_m\right] \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

The current density distribution of a 3-phase winding

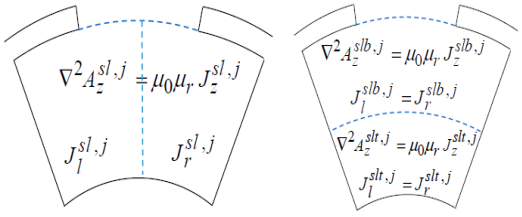


Fig. 8 Winding configuration

depends on winding configuration, there are two common types of configurations, overlapping and nonoverlapping winding, the distribution of current density represented in Fig. 8. Using equation (11), current density is determined by its Fourier series as follows:

$$J(\theta, t) = j_0^j(t) + \sum_{v=1}^{V} j_v^j(t) \cos\left(\frac{\pi v}{\delta}(\theta - \theta_j + \delta/2)\right) \ldots\ldots (14)$$

$\theta_j = 2\pi(j-0.5)/Q$ The angle of the center of slot j w.r.t. x-axis.

where $j_0^j(t)$ and $j_0^j(t)$ are as follows:

$$j_0^j(t) = \frac{j_\ell^j(t)+j_r^j(t)}{2} \quad \& \quad j_v^j(t) = \frac{j_\ell^j(t)-j_r^j(t)}{\pi v/2}\sin(\pi v/2)$$

2. Extracting FS of PM Magnetization Pattern:

In (2-D) AM considering polar coordinates coordinate, the magnetization vector can be expressed as,

$$M = M_R \mathrm{r} + M_T\theta$$

The radial and tangential components for radius independent magnetization are expressed as,

$$M_R^k(\theta) = \sum_{x=1,3,5\ldots\ldots}^{X} M_{Rx}^k \sin\left(\frac{xp}{\alpha_r}\left(\theta-\alpha-\frac{2k\pi}{p}+\frac{\alpha_r\pi}{2p}\right)\right)\ldots.(15)$$

$$M_T^k(\theta) = \sum_{x=1,3,5\ldots\ldots}^{X} M_{Tx}^k \cos\left(\frac{xp}{\alpha_r}\left(\theta-\alpha-\frac{2k\pi}{p}+\frac{\alpha_r\pi}{2p}\right)\right) \ldots (16)$$

Depending on different magnetisation patterns illustrative in Table I, M_{Rx}^k & M_{Tx}^k can be defined as follows:

In the case of *radial* magnetization:

$$\left.\begin{aligned} M_{Rx}^k &= \frac{4\,B_{rem}}{\mu_o x\pi}\sin\left(\frac{x\pi\alpha_p}{2\alpha_r}\right) \\ M_{Tx}^k &= 0 \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (17)$$

In the case of *Parallel* magnetization:

$$\left.\begin{aligned} M_{Rx}^k &= \frac{4\,B_{rem}\alpha_p}{\mu_o\,\alpha_r}\left[A_{1x}(\alpha_p,\alpha_r) + A_{2x}(\alpha_p,\alpha_r)\right] \\ M_{Tx}^k &= \frac{4\,B_{rem}\alpha_p}{\mu_o\,\alpha_r}\left[A_{1x}(\alpha_p,\alpha_r) - A_{2x}(\alpha_p,\alpha_r)\right] \end{aligned}\right\}\ldots\ldots\ldots.. (18)$$

Where

$$A_{1x}(\alpha_p,\alpha_r) = \frac{\sin\left((xp+\alpha_p)\dfrac{\pi\alpha_p}{2p\alpha_r}\right)}{(xp+\alpha_p)\dfrac{\pi\alpha_p}{2p\alpha_r}}$$

$$A_{2x}(\alpha_p,\alpha_r) = \begin{cases} \dfrac{\sin\left((xp+\alpha_p)\dfrac{\pi\alpha_p}{2p\alpha_r}\right)}{(xp+\alpha_p)\dfrac{\pi\alpha_p}{2p\alpha_r}}, & xp \neq \alpha_r \\ 1, & xp = \alpha_r \end{cases}$$

In the case of *Halbach* magnetization:

$$M_{Rx}^k = -\frac{4\,B_{rem}\alpha_p}{\mu_o\,\alpha_r}\frac{\cos\left(\frac{\pi\alpha_p}{2\alpha_r}\right)}{\left(\frac{x\alpha_p}{\alpha_r}\right)^2-1}, \quad x\alpha_p \neq \alpha_r \ldots\ldots\ldots\ldots\ldots (19)$$

$$M_{Tx}^k = \frac{B_{rem}\alpha_p}{x\,\mu_o}, \qquad x\alpha_p = \alpha_r \ldots\ldots\ldots\ldots\ldots (20)$$

The expected waveform of radial and tangential components of all different types of magnetization pattern presented in Table I.

3. Extracting equations to calculate flux density:

Based on infinite permeability assumption of the machine stator and rotor back iron, four different regions are defined to extract flux density due to both AR and PM: 1) magnet; 2) airgap; 3) slot 4) slot-opening.

The flux density calculation for all sub-regions due to AR and PM for internal/external rotor machine can be represented in Table V of Appendix A

By applying the interfacing conditions (section C) and using the correlation technique [107], [157], the integral constants are obtained for both internal/ external rotor machine in Appendix B. These equations can be used to design slotted/ slotless machine with any combination of pole-pairs.

3-MACHINE QUANTITIES:

Accurate magnetic field calculations of brushless PM machines are essential to compute other electromagnetic quantities. Mainly there are two sources of the magnetic field distribution: the permanent magnets (PMs) and the armature winding current. Based on the superposition principle, the solution of the electromagnetic problem is equal with a linear combination of the open-circuit magnetic field problem ($\boldsymbol{J} = \mathbf{0}$) [62], and the armature reaction magnetic field problem ($\boldsymbol{M} = \mathbf{0}$) [67].

Based on the open circuit flux density, flux linked with each coil, induced back-emf in the armature winding, stator eddy current losses, winding eddy current losses, local traction and cogging torque can be obtained [101], [107].

A. Flux Linkage and Induced Voltage:

The first phase winding flux linkages are estimated from the calculated flux density of the subregions in the same winding and the presence of due to the PMs is given below,

$$\lambda_a = N_t N_c \int B^w . \mathrm{dS} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

Where λ_a is the phase-*a* flux linkages and B^w is the flux density component because of PM in the middle of winding linked with the phase "*a*". With the help of Faraday's law, the induced voltage of phase "*a*" can be deduced using the following equation,

$$E_a = \frac{d\lambda_a}{dt} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (22)$$

B. Inductance:

The inductances of the analytical machine model are not depending on the armature current where the saturation effects are neglected. The self and mutual inductance are calculated as the following expression:

$$L_{aa} = \frac{\lambda_{aa}}{i_a} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

$$L_{ab} = \frac{\lambda_{ab}}{i_a} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

Here L_{aa} is phase-*a* self-inductance, L_{ab} is the mutual inductance of phase-a and phase-b, λ_{ab} flux linkages of phase-b due to the phase-a and i_a phase-*a* current.

C. Torque:

By employing both flux densities (i.e., produced by armature reaction and PMs), the electromagnetic torque and unbalanced magnetic forces are computed. The instantaneous developed torque consists of cogging, electromagnetic and reluctance components.

$$T(t) = T_{cog}(t) + T_{em}(t) + T_{rel}(t) \ldots\ldots\ldots\ldots (25)$$

$$T(t) = L_s \int_{-\pi}^{\pi}\frac{1}{\mu_o}(B_{R,PM}^a + B_{R,AR}^a)(B_{T,PM}^a + B_{T,AR}^a)\Big|_{r=R_c} R_c^2 d\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots... (26)$$

Where $R_c = \frac{R_a+R_{sl}}{2}$

D. Unbalance Magnetic Force (UMF):

Another important quantity in PM machines is unbalance magnetic force, based on Maxwell stress tensor the radial and tangential components of the magnetic local traction acting on each rotor surface can be obtained as

$$F_x(t) = L_s \int_{-\pi}^{\pi} (f_r cos\theta - f_\theta sin\theta)\, rd\theta \quad \text{..... (27)}$$

$$F_y(t) = L_s \int_{-\pi}^{\pi} (f_r sin\theta + f_\theta cos\theta)\, rd\theta \quad \text{..... (28)}$$

Where

$$F_r = \frac{1}{2\mu_o}(B_n^2 - B_t^2) \quad \& \quad F_\theta = \frac{1}{\mu_o}(B_r B_\theta)$$

The magnitude of the unbalanced force can be obtained as

$$|F| = \sqrt{\left(F_x^2(t) + F_y^2(t)\right)} \quad \text{..... (29)}$$

4. Review of Analytical Modeling for Various Permanent Magnet Machines:

Table VI in Appendix C represent summary of the references published on the analytical modeling of brushless permanent magnet synchronous machines (PMSM).

Over the past 30 years, a large contribution has been made on AM to solve and design PMSM. A summarized review of AM techniques used to solve the magnetic field on PMSM presented in Table VI. In this table the calculation of flux density and all other machine quantities for *Sd*/*Ss*, internal/external rotor machine using different magnetization pattern has been addressed [1-250].

The third and fourth column of Table VI, the direction of motion and flux for all publication is mentioned, the most common configuration used to design brushless PMSM are Axial and Radial configurations, in terms of power density, stator manufacturing cost and number of poles, axial flux configuration is a potential candidate that produces higher flux, but this configuration failed to be used in case of high power PMSMs.

In fifth column, the structure of PMSM stator has been classified, there are mainly two types of stators, *Sd* or *Ss* stator. Slotted machines preferred due to its higher airgap flux density and provide better heat removal, the comparison between *Ss* and *Sd* given in Table III. The calculation of slotting effects has been considered in some papers using either conformal transformation techniques [1], [4], [6], [13], [19]- [22], [66] or subdomain technique (SD) [7], [23]-[30], [39- [42], [52]-[56], [96], [128], [137], [145], [156], [165], [176], [189], [200], [205], [209], [213], [226], [228], [229]. The advantage and disadvantages of conformal transformation technique has been explained briefly by *Zhu et al.* [30].

In brushless PMSM there are two sources to generate the magnetic field, the first one is armature reaction (AR), by predicting armature field distribution PMSM, it's easy to calculate the self and mutual inductances and computing the eddy current loss in all different part of the machine. The second one is PM which can be used to calculate the magnetic flux linked with each coil, the electromotive force (EMF) induced in each phase, cogging torque and unbalance magnetic forces (UMFs). There are some quantities of PMSM require both AR and PM, like electromagnetic torque.

In sixth column, the location and topology of PM has been presented for PMSM machines as illustrated in Table II (in introduction section), the most used are topologies are *SM*, *SI* and *SPM* and *BPM*. In terms of cost *SM* and *SI* show low cost but for *BPM* the cost is very high but in case of magnetic eddy current losses, *SM* showing high cost compared to other topologies.

There are different types of magnetization pattern considered when designing PMSM, such as: radial; ideal Halbach; parallel; sinusoidal amplitude; and segmented-Halbach, as shown in the seventh column of the table.

Another criterion of the classification is the number of dimensions, the formulation dimensions carried out in polar or rectangular (Cartesian) coordinates, as indicated for each reference in the eighth column of Table VI. Last column of Table V represents the various consideration and calculations of each reference [1] – [75].

5. CONCLUSION

This literature review has investigated the analytical models of the brushless PM machines to provide a suitable reference for developing the related future studies that lead to saving time for researching these models in electrical machines. For the aim, such as flexible in terms of changing the machine geometries or changing the input values, low computational burdens and low time-consuming, the analytical models are recommended to overcome the mentioned challenges.

The review of the analytical models in these PM machines explains their recent developments in terms of the machines' quantities such as magnetic B components, induced voltage, Inductances, electromagnetic F/T, η and UMF. Also, one of the applicable methods for studying different types of the brushless PM machines classification in terms of magnet and machine structures are considered in this review. Comparison of characteristics such as flux direction, stator structure, permanent magnet configuration, magnetization pattern, number of dimensions, coordinate, magnetic potential, PM, and AR reaction effects are performed. By comparing the used methods and obtained results in the articles, the accuracy and speed of the analytical expressions and the efficacy of the analytical approach have been confirmed. This paper summarizes [250] publications to help the researchers to save time for determining appropriate references regarding the analytical models of the brushless PM machines.

Appendix A

TABLE IV The Boundary Conditions of PM machine

Region 1	Region 2	Boundary	Interface	Range
Magnet k	Rotor iron	$H_T^{m,k}(r,\theta)=0$	$r=R_r$	$\left\|\theta-\alpha-\frac{2k\pi}{p}\right\|\le\frac{\alpha_r\pi}{2p}$
Magnet k	Both sides iron-pole	$H_R^{m,k}(r,\theta)=0$	$\theta=\alpha-\frac{2k\pi}{p}\pm\frac{\alpha_r\pi}{2p}$	$R_r\le r\le R_m$
Airgap	Magnet k	$B_R^{a}(r,\theta)=B_R^{m,k}(r,\theta)$	$r=R_m$	$\left\|\theta-\alpha-\frac{2k\pi}{p}\right\|\le\frac{\alpha_r\pi}{2p}$
Airgap	Magnets & iron-poles	$H_T^{a}(r,\theta)=\begin{cases}\sum_{k=0}^{p-1}H_T^{m,k}(r,\theta)\\ 0\end{cases}$	$r=R_m$	$\begin{cases}\left\|\theta-\alpha-\frac{2k\pi}{p}\right\|\le\frac{\alpha_r\pi}{2p}\\ else\ where\end{cases}$
Slot-opening j	Both sides tooth-tip	$H_T^{so,j}(r,\theta)=0$	$\theta=\theta_j\pm\beta$	$R_s\le r\le R_{so}$
Airgap	Slot-opening j	$B_R^{a}(r,\theta)=B_R^{so,j}(r,\theta)$	$r=R_s$	$\begin{cases}\theta_j-\frac{\delta}{2}\le\theta<\theta_j-\frac{\beta}{2}\\ \theta_j-\frac{\delta}{2}\le\theta\le\theta_j-\frac{\beta}{2}\\ \theta_j-\frac{\delta}{2}<\theta\le\theta_j-\frac{\beta}{2}\end{cases}$
Airgap	Slot-openings &teeth	$H_T^{a}(r,\theta)=\begin{cases}\sum_{j=0}^{Q}H_T^{so,j}(r,\theta)\\ 0\end{cases}$	$r=R_s$	$\left\|\theta-\theta_j\right\|\le\frac{\beta}{2}$
Slot j	Slot-opening j	$B_R^{sl,j}(r,\theta)=B_R^{so,j}(r,\theta)$	$r=R_{so}$	$\begin{cases}\left\|\theta-\theta_j\right\|\le\frac{\beta}{2}\\ else\ where\end{cases}$
Slot j	Tooth-tip Slot-opening j Tooth-tip	$H_T^{sl,j}(r,\theta)=\begin{cases}0\\ H_T^{so,j}(r,\theta)\\ 0\end{cases}$	$r=R_{so}$	$\left\|\theta-\theta_j\right\|\le\frac{\beta}{2}$
Slot j	Both sides tooth	$H_R^{sl,j}(r,\theta)=0$	$\theta=\theta_j\pm\delta_j$	$R_s\le r\le R_{so}$
Slot j	Stator back-iron	$H_T^{sl,j}(r,\theta)=0$	$r=R_{sl}$	$\left\|\theta-\theta_j\right\|\le\frac{\delta}{2}$

TABLE V Radial and Tangential Components of flux density for internal/external rotor slotted machine

Regions	B_R & B_T Components	limits
Magnet k	$B_R^{m,k}(r,\theta)=-\sum_{t=1}^{T}\bar{t}\left\{\frac{a_t^{m,k}}{R_m}\left[\left(\frac{r}{R_m}\right)^{\bar{t}-1}+\left(\frac{R_r}{R_m}\right)^{t-1}\left(\frac{R_r}{r}\right)^{\bar{t}+1}\right]+\xi_{t1}^{k}\left(\frac{R_r}{r}\right)^{\bar{t}-1}+k_t^{k}r\right\}\sin\left(\bar{t}(\theta-\alpha-\frac{2k\pi}{p}+\frac{\alpha_r\pi}{2p})\right)$ $B_T^{m,k}(r,\theta)=-\sum_{t=1}^{T}\left\{\bar{t}\frac{a_t^{m,k}}{R_m}\left[\left(\frac{r}{R_m}\right)^{\bar{t}-1}-\left(\frac{R_r}{R_m}\right)^{\bar{t}-1}\left(\frac{R_r}{r}\right)^{\bar{t}+1}\right]+\xi_{t1}^{k}\left(\frac{R_r}{r}\right)^{\bar{t}-1}+k_t^{k}r\right\}\cos\left(\bar{t}(\theta-\alpha-\frac{2k\pi}{p}+\frac{\alpha_r\pi}{2p})\right)$	$R_r\le r\le R_m$ $\left\|\theta-\alpha-\frac{2k\pi}{p}\right\|\le\frac{\alpha_r\pi}{2p}$
Airgap	$B_R^{a}(r,\theta)=-\sum_{x=1}^{X}x\left\{\left[\frac{a_x^{a}}{R_s}\left(\frac{r}{R_m}\right)^{x-1}+\frac{b_x^{a}}{R_m}\left(\frac{R_m}{r}\right)^{x+1}\right]\sin(x\theta)-\left[\frac{c_x^{a}}{R_s}\left(\frac{r}{R_m}\right)^{x-1}+\frac{d_x^{a}}{R_m}\left(\frac{R_m}{r}\right)^{x+1}\right]\cos(x\theta)\right\}$ $B_T^{a}(r,\theta)=-\sum_{x=1}^{X}x\left\{\left[\frac{a_x^{a}}{R_s}\left(\frac{r}{R_m}\right)^{x-1}-\frac{b_x^{a}}{R_m}\left(\frac{R_m}{r}\right)^{x+1}\right]\cos(x\theta)-\left[\frac{c_x^{a}}{R_s}\left(\frac{r}{R_m}\right)^{x-1}-\frac{d_x^{a}}{R_m}\left(\frac{R_m}{r}\right)^{x+1}\right]\sin(x\theta)\right\}$	$R_m\le r\le R_s$
Slot-opening j	$B_R^{so,j}(r,\theta)=-\sum_{y=1}^{Y}\bar{y}\left\{\frac{a_y^{so,j}}{R_{so}}\left[\left(\frac{r}{R_{so}}\right)^{\bar{y}-1}+\frac{b_y^{so,j}}{R_s}\left(\frac{R_s}{r}\right)^{\bar{y}+1}\right]\right\}\sin\left(\bar{y}(\theta-\theta_j+\frac{\beta}{2})\right)$ $B_T^{so,j}(r,\theta)=-\frac{b_y^{so,j}}{r}-\sum_{y=1}^{Y}\bar{y}\left\{\frac{a_y^{so,j}}{R_{so}}\left[\left(\frac{r}{R_{so}}\right)^{\bar{y}-1}-\frac{b_y^{so,j}}{R_s}\left(\frac{R_s}{r}\right)^{\bar{y}+1}\right]\right\}\cos\left(\bar{y}(\theta-\theta_j+\frac{\beta}{2})\right)$	$R_s\le r\le R_{so}$ $\left\|\theta-\theta_j\right\|\le\frac{\beta}{2}$
Slot j	$B_R^{sl}(r,\theta)=-\sum_{z=1}^{Z}\bar{z}\left\{\frac{b_z^{sl,j}}{R_{so}}\left[\left(\frac{R_{so}}{R_{sl}}\right)^{\bar{z}+1}\left(\frac{r}{R_{sl}}\right)^{\bar{z}-1}+\left(\frac{R_{so}}{r}\right)^{\bar{z}+1}\right]+\frac{\mu_0J_v^{j}}{\bar{z}^2-4}\left[r-\frac{2R_{sl}}{\bar{z}}\left(\frac{r}{R_{sl}}\right)^{\bar{z}-1}\right]\sin\left(\bar{z}(\theta-\theta_j+\frac{\delta}{2})\right)\right\}$ $B_R^{sl}(r,\theta)=-\frac{\mu_0J_0^{j}}{2}\left[\frac{R_{sl}^2}{r}-r\right]-\sum_{z=1}^{Z}\left\{\bar{z}\frac{b_z^{sl,j}}{R_{so}}\left[\left(\frac{R_{so}}{R_{sl}}\right)^{\bar{z}+1}\left(\frac{r}{R_{sl}}\right)^{\bar{z}-1}-\left(\frac{R_{so}}{r}\right)^{\bar{z}+1}\right]+\frac{2\mu_0J_v^{j}}{\bar{z}^2-4}\left[r-R_{sl}\left(\frac{r}{R_{sl}}\right)^{\bar{z}-1}\right]\right\}\cos\left(\bar{z}(\theta-\theta_j+\frac{\delta}{2})\right)$	$R_{so}\le r\le R_{sl}$ $\left\|\theta-\theta_j\right\|\le\frac{\delta}{2}$

Appendix B

The solutions for the integrals are given below:

For $\alpha_r = \frac{xp}{t}$

$$\rho_s(t,x,k) = -\frac{1}{4t\pi}\left[\cos\left(\left(\frac{3x\pi}{2}+t\alpha+\frac{kt\pi}{p}\right)\right)-\cos\left(\left(\frac{x\pi}{2}-t\alpha-\frac{kt\pi}{p}\right)\right)\right]-\frac{\alpha_r}{2p}\sin\left(\left(\frac{x\pi}{2}-t\alpha-\frac{kt\pi}{p}\right)\right) \quad (30)$$

$$\rho_c(t,x,k) = \frac{1}{4t\pi}\left[\sin\left(\left(\frac{3x\pi}{2}+t\alpha+\frac{kt\pi}{p}\right)\right)+\sin\left(\left(\frac{x\pi}{2}-t\alpha-\frac{kt\pi}{p}\right)\right)\right]-\frac{\alpha_r}{2p}\cos\left(\left(\frac{x\pi}{2}-t\alpha-\frac{kt\pi}{p}\right)\right) \quad (31)$$

$$6_s(t,x,k) = -\frac{1}{2t\pi}\left[\sin\left(\left(\frac{3x\pi}{2}+t\alpha+\frac{kt\pi}{p}\right)\right)+\sin\left(\left(\frac{x\pi}{2}-t\alpha-\frac{kt\pi}{p}\right)\right)\right]-\cos\left(\left(\frac{x\pi}{2}-t\alpha-\frac{kt\pi}{p}\right)\right) \quad (32)$$

$$6_c(t,x,k) = -\frac{1}{2t\pi}\left[\cos\left(\left(\frac{3x\pi}{2}+t\alpha+\frac{kt\pi}{p}\right)\right)-\cos\left(\left(\frac{x\pi}{2}-t\alpha-\frac{kt\pi}{p}\right)\right)\right]+\sin\left(\left(\frac{x\pi}{2}-t\alpha-\frac{kt\pi}{p}\right)\right) \quad (33)$$

And for $\alpha_r \neq \frac{xp}{t}$ we have:

$$\rho_s(t,x,k) = \frac{\alpha_r}{2\pi}\left\{\frac{-\cos\left(\left(x\pi+\frac{t\pi\alpha_r}{2p}+t\alpha+\frac{kt\pi}{p}\right)\right)+\cos\left(\left(\frac{t\pi\alpha_r}{2p}-t\alpha-\frac{kt\pi}{p}\right)\right)}{\alpha_r t+xp}-\frac{\cos\left(\left(x\pi-\frac{t\pi\alpha_r}{2p}-t\alpha-\frac{kt\pi}{p}\right)\right)-\cos\left(\left(\frac{t\pi\alpha_r}{2p}-t\alpha-\frac{kt\pi}{p}\right)\right)}{\alpha_r t+xp}\right\} \quad (34)$$

$$\rho_s(t,x,k) = \frac{\alpha_r}{2\pi}\left\{\frac{-\cos\left(\left(x\pi+\frac{t\pi\alpha_r}{2p}+t\alpha+\frac{kt\pi}{p}\right)\right)+\cos\left(\left(\frac{t\pi\alpha_r}{2p}-t\alpha-\frac{kt\pi}{p}\right)\right)}{\alpha_r t+xp}-\frac{\cos\left(\left(x\pi-\frac{t\pi\alpha_r}{2p}-t\alpha-\frac{kt\pi}{p}\right)\right)-\cos\left(\left(\frac{t\pi\alpha_r}{2p}-t\alpha-\frac{kt\pi}{p}\right)\right)}{\alpha_r t+xp}\right\} \quad (35)$$

$$6_s(t,x,k) = \frac{p}{\pi}\left\{\frac{-\sin\left(\left(x\pi+\frac{t\pi\alpha_r}{2p}+t\alpha+\frac{kt\pi}{p}\right)\right)-\sin\left(\left(\frac{t\pi\alpha_r}{2p}-t\alpha-\frac{kt\pi}{p}\right)\right)}{\alpha_r t+xp}-\frac{\sin\left(\left(x\pi-\frac{t\pi\alpha_r}{2p}-t\alpha-\frac{kt\pi}{p}\right)\right)-\sin\left(\left(\frac{t\pi\alpha_r}{2p}-t\alpha-\frac{kt\pi}{p}\right)\right)}{\alpha_r t+xp}\right\} \quad (36)$$

$$6_c(t,x,k) = \frac{p}{\pi}\left\{\frac{-\cos\left(\left(x\pi+\frac{t\pi\alpha_r}{2p}+t\alpha+\frac{kt\pi}{p}\right)\right)+\cos\left(\left(\frac{t\pi\alpha_r}{2p}-t\alpha-\frac{kt\pi}{p}\right)\right)}{\alpha_r t+xp}+\frac{\cos\left(\left(x\pi-\frac{t\pi\alpha_r}{2p}-t\alpha-\frac{kt\pi}{p}\right)\right)-\cos\left(\left(\frac{t\pi\alpha_r}{2p}-t\alpha-\frac{kt\pi}{p}\right)\right)}{\alpha_r t+xp}\right\} \quad (37)$$

For $\pi z \neq \beta t$

$$\varepsilon_s(t,z,j) = 2\pi z\frac{(-1)^{z+1}\sin\left(t\left(\theta_j+\frac{\beta}{2}\right)\right)+\sin\left(t\left(\theta_j-\frac{\beta}{2}\right)\right)}{\pi^2z^2-\beta^2t^2} \quad (38)$$

$$\varepsilon_c(t,z,j) = 2\pi z\frac{(-1)^{z+1}\cos\left(t\left(\theta_j+\frac{\beta}{2}\right)\right)+\cos\left(t\left(\theta_j-\frac{\beta}{2}\right)\right)}{\pi^2z^2-\beta^2t^2} \quad (39)$$

$$\eta_s(t,z,j) = \frac{\beta^2t}{\pi}\frac{(-1)^z\cos\left(t\left(\theta_j+\frac{\beta}{2}\right)\right)-\cos\left(t\left(\theta_j-\frac{\beta}{2}\right)\right)}{\pi^2z^2-\beta^2t^2} \quad (40)$$

$$\eta_c(t,z,j) = \frac{\beta^2t}{\pi}\frac{(-1)^{z+1}\sin\left(t\left(\theta_j+\frac{\beta}{2}\right)\right)+\sin\left(t\left(\theta_j-\frac{\beta}{2}\right)\right)}{\pi^2z^2-\beta^2t^2} \quad (41)$$

And for $\pi z = \beta t$ we have:

$$\varepsilon_s(t,z,j) = \cos\left(t\left(\theta_j-\frac{\beta}{2}\right)\right)-\frac{\sin\left(t\left(\theta_j+\frac{\beta}{2}\right)\right)-\sin\left(t\left(\theta_j-\frac{\beta}{2}\right)\right)}{2t\beta} \quad (42)$$

$$\varepsilon_c(t,z,j) = -\sin\left(t\left(\theta_j-\frac{\beta}{2}\right)\right)-\frac{\cos\left(t\left(\theta_j+\frac{\beta}{2}\right)\right)-\cos\left(t\left(\theta_j-\frac{\beta}{2}\right)\right)}{4t\pi} \quad (43)$$

$$\eta_s(t,z,j) = \frac{-\sin\left(t\left(\theta_j-\frac{\beta}{2}\right)\right)}{\frac{2\pi}{\beta}}-\frac{\cos\left(t\left(\theta_j+\frac{\beta}{2}\right)\right)-\cos\left(t\left(\theta_j-\frac{\beta}{2}\right)\right)}{4t\pi} \quad (44)$$

$$\eta_c(t,z,j) = \frac{\cos\left(t\left(\theta_j-\frac{\beta}{2}\right)\right)}{\frac{2\pi}{\beta}}+\frac{\sin\left(t\left(\theta_j+\frac{\beta}{2}\right)\right)-\sin\left(t\left(\theta_j-\frac{\beta}{2}\right)\right)}{4t\pi} \quad (45)$$

For $\delta \neq \frac{\beta y}{z}$

$$\gamma_s(z,y) = \frac{2\delta^2z}{\pi}\frac{(-1)^{z+1}\sin\left(\frac{\pi y}{2\delta}(\delta+\beta)\right)+\sin\left(\frac{\pi y}{2\delta}(\delta-\beta)\right)}{\delta^2z^2-\beta^2y^2} \quad (46)$$

$$\gamma_c(z,y) = \frac{2\beta^2y}{\pi}\frac{(-1)^{z+1}\sin\left(\frac{\pi y}{2\delta}(\delta+\beta)\right)+\sin\left(\frac{\pi y}{2\delta}(\delta-\beta)\right)}{\delta^2z^2-\beta^2y^2} \quad (47)$$

And for $\delta = \frac{\beta y}{z}$ we have:

$$\gamma_s(z,y) = \frac{2\pi z\cos\left(\frac{\pi}{2}(z-y)\right)-\sin\left(\frac{\pi}{2}(3z+y)\right)-\sin\left(\frac{\pi}{2}(z-y)\right)}{2\pi z} \quad (48)$$

$$\gamma_c(z,y) = \frac{2\pi z\cos\left(\frac{\pi}{2}(z-y)\right)+\sin\left(\frac{\pi}{2}(3z+y)\right)+\sin\left(\frac{\pi}{2}(z-y)\right)}{2\pi z} \quad (49)$$

$$\xi_{x1}^k = \xi_{x3}^k = -\mu_0\left[\frac{M_{Rx}^k-\frac{xp}{\alpha_r}M_{Tx}^k}{\left(\frac{xp}{\alpha_r}\right)^2-1}\right] \quad (50)$$

$$\xi_{x2}^k = -\mu_0\left[\frac{\frac{xp}{\alpha_r}M_{Rx}^k-M_{Tx}^k}{\left(\frac{xp}{\alpha_r}\right)^2-1}\right] \quad (51)$$

Internal Rotor Machine:

The simultaneous equations for integral constant calculations are summarized as following matrix (for PM & AR):

$$\begin{bmatrix} A^{11} & A^{12} & A^{13} & A^{14} & A^{14} & 0 & 0 & 0 \\ A^{21} & A^{22} & A^{23} & 0 & 0 & 0 & 0 & 0 \\ 0 & A^{32} & A^{33} & 0 & 0 & A^{36} & A^{37} & 0 \\ A^{41} & 0 & 0 & A^{44} & A^{45} & 0 & 0 & 0 \\ 0 & 0 & 0 & A^{54} & A^{55} & A^{56} & A^{57} & 0 \\ 0 & A^{62} & A^{63} & 0 & A^{64} & A^{65} & A^{66} & A^{67} \\ 0 & 0 & 0 & 0 & 0 & A^{76} & A^{77} & A^{78} \\ 0 & 0 & 0 & 0 & 0 & A^{86} & A^{87} & A^{88} \end{bmatrix}\begin{bmatrix} a^m \\ a^a \\ b^a \\ c^a \\ d^a \\ a^{so} \\ b^{so} \\ b^{sl} \end{bmatrix} = \begin{bmatrix} \Gamma^{1,PM} \\ \Gamma^{1,PM} \\ \Gamma^{1,AR} \\ \Gamma^{1,PM} \\ \Gamma^{1,AR} \\ 0 \\ \Gamma^{1,AR} \\ \Gamma^{1,AR} \end{bmatrix}$$

The elements of the simultaneous equations for non-overlapping consequent-pole are as follows:

$$A_{x,x}^{11} = \bar{x}\left[1+\left(\frac{R_r}{R_m}\right)^{2\bar{x}}\right] \quad (52)$$

Where $\bar{x} = \frac{xp}{\alpha_r}$

$$A^{12}_{x+kx,t} = -t\left(\frac{R_m}{R_s}\right)^t 6_s(\mathrm{t,x,k}) \quad (53)$$

$$A^{13}_{x+kx,t} = -t\ 6_s(\mathrm{t,x,k}) \quad (54)$$

$$A^{14}_{x+kx,t} = t\left(\frac{R_m}{R_s}\right)^t 6_c(\mathrm{t,x,k}) \quad (55)$$

$$A^{15}_{x+kx,t} = t\ 6_c(\mathrm{t,x,k}) \quad (56)$$

$$\Gamma^{1,PM}_{x+kx,1} = -R_m\bar{x}\left[\xi^k_{x1}\left(\frac{R_r}{R_m}\right)^{\bar{x}+1} + \xi^k_{x1}\right] \quad (57)$$

$$A^{21}_{t,x+kx} = \frac{\bar{x}}{\mu_r}\left[\left(\frac{R_r}{R_m}\right)^{2\bar{x}} - 1\right]\rho_c\ (\mathrm{t,x,k}) \quad (58)$$

$$A^{22}_{t,t} = t\left(\frac{R_m}{R_s}\right)^t \quad (59)$$

$$A^{23}_{t,t} = -t \quad (60)$$

$$\Gamma^{2,PM}_{t,1} = \sum_{k=0}^{p-1}\sum_{x=1}^{X}\frac{\bar{x}}{\mu_r}R_m\left[-\xi^k_{x1}\left(\frac{R_r}{R_m}\right)^{\bar{x}+1} + \xi^k_{w3}\right]\rho_c\ (\mathrm{n,w,k}) \quad (61)$$

$$A^{32}_{t,t} = t \quad (62)$$

$$A^{33}_{t,t} = -t\left(\frac{R_m}{R_s}\right)^t \quad (63)$$

$$A^{36}_{t,z} = -\bar{z}\left(\frac{R_s}{R_{so}}\right)^{\bar{z}}\eta_c(\mathrm{t,z,j}) \quad (64)$$

Where $\bar{z} = \frac{zp}{\alpha_r}$

$$A^{36}_{t,z} = \bar{u}\eta_c(\mathrm{t,z,j}) \quad (65)$$

$$\Gamma^{3,AR}_{t,1} = \sum_{j=0}^{Q}\eta_c(\mathrm{t,0,j})b_0^{so,j} \quad (66)$$

$$A^{41}_{t,x+kx} = \frac{\bar{x}}{\mu_r}\left[\left(\frac{R_r}{R_m}\right)^{2\bar{x}} - 1\right]\rho_s\ (\mathrm{t,x,k}) \quad (67)$$

$$A^{44}_{t,t} = t\left(\frac{R_m}{R_s}\right)^t \quad (68)$$

$$A^{45}_{t,t} = -t \quad (69)$$

$$\Gamma^{4,PM}_{t,1} = \sum_{k=0}^{p-1}\sum_{x=1}^{X}\frac{\bar{x}}{\mu_r}R_m\left[-\xi^k_{x1}\left(\frac{R_r}{R_m}\right)^{\bar{x}+1} + \xi^k_{x3}\right]\rho_s\ (\mathrm{t,x,k}) \quad (70)$$

$$A^{54}_{t,t} = t \quad (71)$$

$$A^{55}_{t,t} = -t\left(\frac{R_m}{R_s}\right)^t \quad (72)$$

$$A^{56}_{t,z} = -\bar{z}\left(\frac{R_s}{R_{so}}\right)^{\bar{z}}\eta_s(\mathrm{t,z,j}) \quad (73)$$

$$A^{57}_{t,z} = \bar{u}\eta_s(\mathrm{n,u,j}) \quad (74)$$

$$\Gamma^{5,AR}_{t,1} = \sum_{j=0}^{Q}\eta_s(\mathrm{t,0,j})b_0^{so,j} \quad (75)$$

$$A^{62}_{t,t} = -t\mathcal{E}_s(\mathrm{t,z,j}) \quad (76)$$

$$A^{63}_{z,t} = -t\left(\frac{R_m}{R_s}\right)^t \mathcal{E}_s(\mathrm{t,z,j}) \quad (77)$$

$$A^{64}_{t,t} = t\mathcal{E}_c(\mathrm{t,z,j}) \quad (78)$$

$$A^{65}_{z,t} = t\left(\frac{R_m}{R_s}\right)^t \mathcal{E}_c(\mathrm{t,z}) \quad (79)$$

$$A^{66}_{t,z} = \bar{z}\left(\frac{R_s}{R_{so}}\right)^{\bar{z}} \quad (80)$$

$$A^{67}_{z,z} = \bar{z} \quad (81)$$

$$A^{76}_{z,z} = \bar{z} \quad (82)$$

$$A^{77}_{z,z} = \bar{z}\left(\frac{R_s}{R_{so}}\right)^{\bar{z}} \quad (83)$$

$$A^{78}_{x,x} = -\bar{y}\left[\left(\frac{R_{so}}{R_{sl}}\right)^{2\bar{y}} + 1\right]\gamma_c(\mathrm{z,y}) \quad (84)$$

Where $\bar{y} = \frac{yp}{\alpha_r}$

$$\Gamma^{7,AR}_{z,1} = \sum_{y=1}^{Y}\frac{\mu_0 J_v^j}{\bar{u}^2-4}\left[\bar{u}R^2_{so} - 2R^2_{sl}\left(\frac{R_{so}}{R_{sl}}\right)^{\bar{y}}\right]\gamma_s(\mathrm{z,y}) \quad (85)$$

$$A^{86}_{y,z} = -\bar{z}\ \gamma_c(\mathrm{z,y}) \quad (86)$$

$$A^{87}_{y,z} = \bar{z}\left(\frac{R_s}{R_{so}}\right)^{\bar{z}}\gamma_c(\mathrm{z,y}) \quad (87)$$

$$A^{88}_{y,y} = \bar{y}\left[\left(\frac{R_{so}}{R_{sl}}\right)^{2\bar{y}} - 1\right] \quad (88)$$

$$\Gamma^{8,AR}_{y,1} = \frac{-2\mu_0 J_v^j}{\bar{y}^2-4}\left[R^2_{so} - 2R^2_{sl}\left(\frac{R_{so}}{R_{sl}}\right)^{\bar{y}}\right] + \gamma_c(0,\mathrm{v})b_0^{so,j} \quad (89)$$

$$b_0^{so,j} = \frac{\mu_0 J_0^j}{2}[R^2_{sl} - R^2_{so}]\frac{\delta}{\beta} \quad (90)$$

External Rotor Machine:

- Due to **PM**:

$$\begin{bmatrix} C & D \\ F & F \end{bmatrix}\begin{bmatrix} a^a \\ a^m \end{bmatrix} = \begin{bmatrix} \mathrm{g} \\ \mathrm{h} \end{bmatrix}$$

Where

$$\mathrm{C}_{i,i} = yp\left[\left(\frac{R_m}{R_s}\right)^{2yp} - 1\right] \quad (91)$$

$$\mathrm{D}_{i,j} = yp\left[\left(\frac{R_r}{R_m}\right)^{\frac{2yp}{\alpha_r}} - 1\right](H^+_{yt} - H^-_{yt}) \quad (92)$$

$$\mathrm{E}_{j,i} = yp\left[\left(\frac{R_m}{R_s}\right)^{\frac{2yp}{\alpha_r}} + 1\right](H^+_{yt} + H^-_{yt}) \quad (93)$$

$$\mathrm{F}_{j,j} = -\frac{\mu_r yp}{\alpha_r}\left[\left(\frac{R_r}{R_m}\right)^{\frac{2yp}{\alpha_r}} + 1\right] \quad (94)$$

$$\mathrm{g}_i = \sum_{t=1}^{T} tp\left[\left(\frac{R_m}{R_s}\right)^{\frac{2yp}{\alpha_r}}\right](H^+_{yt} - H^-_{yt}) \quad (95)$$

$$\mathrm{h}_j = \mu_r\left[\left(\frac{R_r}{R_m}\right)^{\frac{2yp}{\alpha_r}+1} - \frac{M^k_{Rt}}{\mu_r}\right] \quad (96)$$

$H_{yt}^{+} = \frac{\sin(\alpha_r y+t)}{(\alpha_r y+t)}$, $H_{yt}^{-} = \frac{\sin(\alpha_r y-t)}{(\alpha_r y-t)}$ ……..….. (97)

- Due to **AR**:

$$\begin{bmatrix} A^{11} & 0 & A^{13} & A^{14} \\ 0 & A^{22} & A^{23} & A^{24} \\ A^{31} & A^{32} & A^{33} & 0 \\ A^{41} & A^{42} & 0 & A^{44} \end{bmatrix} \begin{bmatrix} b^{w} \\ d^{w} \\ a^{m,0} \\ a^{m,1} \end{bmatrix} = \begin{bmatrix} \Gamma^{1} \\ \Gamma^{2} \\ \Gamma^{3} \\ \Gamma^{4} \end{bmatrix}$$

$A_{t,t}^{11} = 2t\left[\left(\frac{R_m}{R_s}\right)^{2tp} - 1\right]$ ……..… (98)

$A_{t,y}^{13} = \frac{y}{\mu_r \alpha_r}\left[\left(\frac{R_r}{R_m}\right)^{\frac{2tp}{\alpha_r}} - 1\right]\varepsilon_s(t,y)$ …… (99)

$A_{t,y}^{14} = (-1)^t \frac{y}{\mu_r \alpha_r}\left[\left(\frac{R_r}{R_m}\right)^{\frac{2tp}{\alpha_r}-1}\right]\varepsilon_s(t,y)$ …….. (100)

$\Gamma_t^1 = 2t\left[\xi_{t1}^k\left(\frac{R_m}{R_s}\right)^{tp-1} - \frac{\xi_{t2}^k+\xi_{t3}^k}{2}\left(\frac{R_m}{R_a}\right)^{tp-1} + \frac{\xi_{t2}^k-\xi_{t3}^k}{2}\left(\frac{R_a}{R_m}\right)^{tp+1}\right]$ …….. (101)

$A_{t,t}^{22} = 2t\left[\left(\frac{R_m}{R_s}\right)^{2tp} - 1\right]$ …….. (102)

$A_{t,y}^{23} = \frac{y}{\mu_r \alpha_r}\left[\left(\frac{R_r}{R_m}\right)^{\frac{2tp}{\alpha_r}} - 1\right]\varepsilon_c(t,y)$ …….. (103)

$A_{t,y}^{24} = (-1)^t \frac{y}{\mu_r \alpha_r}\left[\left(\frac{R_r}{R_m}\right)^{\frac{2tp}{\alpha_r}-1} - 1\right]\varepsilon_c(t,y)$ …….. (104)

$\Gamma_t^2 = 2t\left[\xi_{t1}^k\left(\frac{R_m}{R_s}\right)^{tp-1} - \frac{\xi_{t2}^k+\xi_{t3}^k}{2}\left(\frac{R_m}{R_a}\right)^{tp-1} + \frac{\xi_{t2}^k-\xi_{t3}^k}{2}\left(\frac{R_a}{R_m}\right)^{tp+1}\right]$ …….. (105)

$A_{y,t}^{31} = t\left[\left(\frac{R_m}{R_s}\right)^{2tp} + 1\right]\gamma_c(t,y)$ …….. (106)

$A_{y,t}^{32} = -t\left[\left(\frac{R_m}{R_s}\right)^{2tp} + 1\right]\gamma_s(t,y)$ …….. (107)

$A_{y,y}^{33} = \frac{y}{\alpha_r}\left[\left(\frac{R_r}{R_m}\right)^{\frac{2tp}{\alpha_r}} - 1\right]$ …….. (108)

$\Gamma_y^3 = \sum_{t=1}^{T} t\left\{\left[\xi_{t1}^k\left(\frac{R_m}{R_s}\right)^{tp-1} - \frac{\xi_{t2}^k+\xi_{t3}^k}{2}\left(\frac{R_m}{R_a}\right)^{tp-1} + \frac{\xi_{t2}^k-\xi_{t3}^k}{2}\left(\frac{R_a}{R_m}\right)^{tp+1}\right]\gamma_c(t,y) - \left[\xi_{t1}^k\left(\frac{R_m}{R_s}\right)^{tp-1} - \frac{\xi_{t2}^k+\xi_{t3}^k}{2}\left(\frac{R_m}{R_a}\right)^{tp-1} + \frac{\xi_{t2}^k-\xi_{t3}^k}{2}\left(\frac{R_a}{R_m}\right)^{tp+1}\right]\gamma_s(t,y)\right\}$ …….. (109)

$A_{y,t}^{41} = (-1)^t t\left[\left(\frac{R_m}{R_s}\right)^{2tp} + 1\right]\gamma_c(t,y)$ …….. (110)

$A_{y,t}^{42} = (-1)^{t+1} t\left[\left(\frac{R_m}{R_s}\right)^{2tp} + 1\right]\gamma_s(t,y)$ …….. (111)

$A_{y,t}^{44} = \frac{y}{\alpha_r}\left[\left(\frac{R_r}{R_m}\right)^{\frac{2tp}{\alpha_r}} - 1\right]$ …….. (112)

$\Gamma_y^4 = \sum_{t=1}^{T}(-1)^t t\left\{\left[\xi_{t1}^k\left(\frac{R_m}{R_s}\right)^{tp-1} - \frac{\xi_{t2}^k+\xi_{t3}^k}{2}\left(\frac{R_m}{R_a}\right)^{tp-1} + \frac{\xi_{t2}^k-\xi_{t3}^k}{2}\left(\frac{R_a}{R_m}\right)^{tp+1}\right]\gamma_c(t,y) - \left[\xi_{t1}^k\left(\frac{R_m}{R_s}\right)^{tp-1} - \frac{\xi_{t2}^k+\xi_{t3}^k}{2}\left(\frac{R_m}{R_a}\right)^{tp-1} + \frac{\xi_{t2}^k-\xi_{t3}^k}{2}\left(\frac{R_a}{R_m}\right)^{tp+1}\right]\gamma_s(t,y)\right\}$ ……. (113)

Appendix C

Table VI A summary of the references published on the analytical modeling of brushless permanent magnet machines.

Reference	Year	Movement	Flux Direction	Stator Structure	Magnet Configuration SM/SI	Magnetization Pattern	Number of Dimension	Coordinate	Magnetic Potential	Field due to AR/PM	Consideration &Calculation
[1]	1984	Rotary	Radial	Sd	SM	Radial	2-D	Ca	Vr	AR, PM	B (Infinite Back-Iron Permeability)
[2]	1985	Rotary	Radial	Ss	SM	Radial, Parallel	2-D	Po	Sr	PM	B (Infinite Back-Iron Permeability)
[3]	1989	Linear	-	Sd	SM	-	2-D, 3-D	Ca	Sr	AR, PM	Magnetic field distribution, MMF (Infinite Back-Iron Permeability)
[4]	1992	Rotary	Radial	Sd	SM	Radial	2-D	Cylindrical	Sr	PM	CT (Infinite Back-Iron Permeability)
[5]-[6]	1993	Rotary	Radial	Sd, Ss	SM	Radial	1-D, 2-D	Po	Sr	AR, PM	B (Infinite Back-Iron Permeability)
[7]	2010	Rotary	Radial	Sd	SM	Radial	2-D	Po	Vr	AR, PM	P_e (Infinite Back-Iron Permeability)
[8]	1993	Rotary	Radial	Sd, Ss	SM	Radial	1-D, 2-D	Po	Sr	AR, PM	B (Infinite Back-Iron Permeability)
[9]	2000	Rotary	Radial	Ss	SM	Sinusoidal amplitude magnetization	2-D	Po	Vr, Sr	PM	B (Infinite Back-Iron Permeability)
[10]	2000	Rotary	Radial	Sd	SM	Parallel, 6-segmented magnet in shifting direction	2-D	Ca	Vr	AR, PM	B, Eddy current losses
[11]	2001	Linear	-	Ss	SM	Parallel	2-D	Ca	Vr	AR, PM	e (Infinite Back-Iron Permeability)
[12]	2002	Rotary	Radial	Sd, Ss	SM	Radial, Parallel	2-D	Po	Sr	AR, PM	B, (Infinite Core Permeability)
[13]	2003	Rotary	Radial	Sd	SM	Radial	1-D, 2-D	Po	Vr, Sr	AR, PM	B (Infinite Back-Iron Permeability)

Reference	Year	Movement	Flux Direction	Stator Structure	Magnet Configuration SM/SI	Magnetization Pattern	Number of Dimension	Coordinate	Magnetic Potential	Field due to AR/PM	Consideration &Calculation
[14]	2003	Rotary	Radial	Sd	SM	Radial	2-D	Po	Sr	AR, PM	B, e (Infinite Core Permeability)
[15]	2004	Rotary	Radial	Sd	SM	-	-	Ca	-	PM	B, CT
[16]	2004	Rotary	Radial, Axial	Ss	SM	-	2-D	Po	Vr	AR	B, Eddy current (Infinite Back-Iron Permeability)
[17]	2004	Rotary	Radial	Ss	SM	Ideal Halbach	2-D	Po	Sr	PM	B (Infinite Back-Iron Permeability)
[18]	2000	Rotary	Radial	Sd	SM	Radial	2-D	Po	Vr	AR, PM	Eddy current losses, MMF (Infinite Back-Iron Permeability)
[19]	2008	Rotary	Radial	Sd, Ss	SM	Radial, Parallel, Sinusoidal amplitude magnetization, Ideal Halbach	2-D	Po	Vr	PM	B, CT, e (Infinite Back-Iron Permeability)
[20]	2006	Rotary	Radial	Sd	SM	Radial, Parallel	2-D	Po	Sr	PM	B, CT, e (Infinite Back-Iron Permeability)
[21]	2008	Rotary	Radial	Sd	SM	Radial	2-D	Po	Vr	PM	B, CT (Infinite Back-Iron Permeability)
[22]	2009	Rotary	Radial	Sd	SM	Nine-segment Halbach	2-D	Po	Vr	PM	B distribution, C, e, Electromagnetic T (Infinite Core Permeability)
[23]	2007	Linear	-	Sd	SM	Parallel	2-D	Ca	Sr	PM	B (Infinite Back-Iron Permeability)
[24]	2008	Rotary	Radial	Sd	SM	Radial	2-D	Po, Ca	Sr	PM	CT, UMF (Infinite Core Permeability)
[25]	2009	Rotary	Radial	Sd	SM	-	2-D	Po	Vr	AR	AR magnetic field (Infinite Back-Iron Permeability)
[26]	2009	Rotary	Radial	Sd	SM	Radial, Parallel	2-D	Po	Vr	PM	No-load magnetic field distribution (Infinite Back-Iron Permeability)

Reference	Year	Movement	Flux Direction	Stator Structure	Magnet Configuration SM/SI	Magnetization Pattern	Number of Dimension	Coordinate	Magnetic Potential	Field due to AR/PM	Consideration &Calculation
[27]	2011	Rotary	Radial	Sd	SM	Radial	2-D	Po	Vr	AR, PM	Magnetic field distribution (Infinite Back-Iron Permeability)
[28]	2011	Rotary	Radial	Sd	SM	-	2-D	Po	Vr	AR	AR field (Infinite Back-Iron Permeability)
[29]	2011	Rotary	Radial	Sd	SM	Radial, Parallel	2-D	Po	Vr	PM	Open-circuit magnetic field distribution (Infinite Core Permeability)
[30]	2010	Rotary	Radial	Sd	SM	Radial, Parallel	2-D	Po	Sr	PM	Magnetic field distribution, e, Electromagnetic T, CT (Infinite Core Permeability)
[31]	2012	Rotary	Radial	Sd	SM	Radial, Parallel	2-D	Po	Sr	AR, PM	B, CT, e, Electromagnetic torque, Saturation effect (Infinite Core Permeability)
[32]	2020	Linear	-	Ss	SM	Parallel	0-D,2-D	Po	Vr	PM	B, L (Infinite Iron Core Permeability)
[33]	1993	Rotary	Radial	Sd, Ss	SM	Radial	1-D, 2-D	Po	Sr	AR, PM	B (Infinite Back-Iron Permeability)
[34]	2016	Rotary	Radial	Ss	-	-	3-D	Cylindrical	Vr	AR	Magnetic field distribution, P_e (Infinite Back-Iron Permeability)
[35]	2017	Linear	-	Sd	SM	-	2-D, 3-D	Ca	Vr	AR	Ar field distribution, L (Infinite Back-Iron Permeability)
[36]	2017	Linear	-	Sd, Ss	SM	parallel	2-D	Ca	Sr	AR, PM	e (Infinite core Permeability)
[37]	2019	Linear	-	Ss	SM	Radial, Parallel	3-D	Cylindrical	Sr	PM	T, Field Distribution (Infinite core Permeability)

Reference	Year	Movement	Flux Direction	Stator Structure	Magnet Configuration SM/SI	Magnetization Pattern	Number of Dimension	Coordinate	Magnetic Potential	Field due to AR/PM	Consideration &Calculation
[38]	2020	Rotary	Axial	Sd	-	Radial	3-D	Po	Sr	PM	B, e, CT (Static Radial Deviation & Angular Eccentricity)
[39]	2009	Rotary	Radial	Sd	SI	Radial	2-D	Po	Sr	PM	Magnetic field distribution (Infinite Back-Iron Permeability)
[40]	2013	Rotary	Radial	Ss	SI	Radial, Parallel, Halbach	2-D	Po	Vr	AR, PM	Electromagnetic T, Reluctance T, L, e (Infinite Core Permeability)
[41]	2012	Rotary	Radial	Sd	SI	Radial	2-D	Po	Vr	AR, PM	Magnetic field distribution (Infinite Back-Iron Permeability)
[42]	2013	Rotary	Radial	Ss	SI	Radial, Parallel, Sinusoidal amplitude magnetization, Ideal Halbach, Halbach	2-D	Po	Vr	AR, PM	Open-circuit magnetic field distribution (Finite Core Permeability)
[43]	2013	Rotary	Radial, Axial	Sd	SI	Radial, Parallel	2-D	Ca	Vr	AR, PM	Magnetic flux distribution, Instantaneous T (Infinite Back-Iron Permeability)
[44]	2004	Rotary	Axial	Sd	SM	-	2-D, 3-D	Po, Ca	Vr	AR, PM	B (Infinite Back-Iron Permeability)
[45]	2004	Rotary	Radial	Sd	SM	Radial	2-D	Po	Vr	AR, PM	T, Magnetic Force (Infinite Core Permeability)
[46]	2005	Rotary	Radial	Sd	SM	-	2-D	Po	Vr	AR	Magnetic field distribution, Eddy-current loss, MMF (Infinite Core Permeability)
[47]	2005	Rotary	Axial	Sd	SM	Parallel	3-D	Po	Vr	PM	CT (Infinite Back-Iron Permeability)
[48]	2005	Rotary	Radial	Sd, Ss	SM	Parallel	2-D	Po	Vr	AR, PM	B, CT, P_e (Infinite Core Permeability)

Reference	Year	Movement	Flux Direction	Stator Structure	Magnet Configuration SM/SI	Magnetization Pattern	Number of Dimension	Coordinate	Magnetic Potential	Field due to AR/PM	Consideration &Calculation
[49]	2018	Linear Tubular	Radial	Sd	BPM	-	2-D	Ca	Sr	AR, PM	B, CT, Cogging force
[50]	2018	Rotary-Linear	Radial	Sd	BPM	-	2-D	Ca, Po	Sr	AR, PM	B, λ
[51]	2018	Linear	-	Ss	SM	Two-segment Halbach, Parallel	2-D	Ca	Vr	AR, PM	B distribution, λ, e (Finite core Permeability)
[52]	2011	Rotary	Radial	Ss	SI	Radial, Parallel, Halbach	2-D	Po	Sr	PM	Open-circuit magnetic field distribution (Infinite Back-Iron Permeability)
[53]	2012	Rotary	Radial	Ss	SM, SI	-	2-D	Po	Vr	AR	Ar field distribution (Infinite Back-Iron Permeability)
[54]	2012	Rotary	Radial	Sd	SI	Radial, Parallel, Halbach	2-D	Po	Vr	AR, PM	Magnetic field distribution (Infinite Core Permeability)
[55]	2015	Rotary	Radial	Sd	SM	Radial	2-D	Ca	Sr	AR, PM	Magnetic field distribution
[56]	2012	Rotary	Radial	Ss	SM	Radial, Parallel, Sinusoidal amplitude magnetization, Ideal Halbach, Halbach	2-D	Po	Vr	PM	Open-circuit magnetic field distribution, T (Infinite Back-Iron Permeability)
[57]	2010	Rotary, Linear, Tubular	Radial, Axial	Sd, Ss	SM	Radial, Parallel, Halbach	2-D	Po, Ca, Cylindrical	Vr	PM	Magnetic field distribution (Infinite Back-Iron Permeability)
[58]	1994	Rotary	Radial	Ss	SM, SI	Radial	2-D	Po	Sr	PM	B (Infinite Back-Iron Permeability)
[59]	1994	Rotary	Radial	Sd	SM	Radial	2-D	Po	Vr	PM	B (Infinite Back-Iron Permeability)
[60]	1995	Rotary	-	Sd	SM	-	2-D	Ca	Vr	AR	B, L, M (Infinite Back-Iron Permeability)

Reference	Year	Movement	Flux Direction	Stator Structure	Magnet Configuration SM/SI	Magnetization Pattern	Number of Dimension	Coordinate	Magnetic Potential	Field due to AR/PM	Consideration &Calculation
[61]	1995	Rotary	Radial	Ss	SM	Radial	-	Po	Vr	PM	i_e and thermal problems (Finite Back-Iron Permeability)
[62]	1996	Rotary	Radial	Sd	SM	Radial, Parallel	2-D	Ca	Vr	PM	Time-varying field distribution (Infinite Core Permeability)
[63]	1997	Rotary	Radial	Ss	SM	-	2-D	Po	Vr, Sr	AR	Commutation losses (Finite Back-Iron Permeability)
[64]	1998	Rotary	Radial	Sd	SM	-	2-D	Po	Vr, Sr	AR	P_e (Finite Core Permeability)
[65]	1998	Rotary	Radial	Ss	SM	Radial	2-D	Po	Vr	PM	Instantaneous magnetic field distribution (Infinite Core Permeability)
[66]	1998	Rotary	Radial	Sd	SM	Radial	2-D	Po	Sr	PM	Magnetic field distribution (Infinite Back-Iron Permeability)
[67]	1998	Rotary	Radial	Ss	SM	-	2-D	Po	Vr	AR	AR field and winding inductance (Infinite Back-Iron Permeability)
[68]	1998	Rotary	Radial	Sd	SM	Radial	3-D	-	Vr	AR, PM	Magnetic field distribution (Infinite Back-Iron Permeability)
[69]	1999	Rotary	Radial	Ss	SM	Radial	2-D	Po	Vr	PM	Magnetic field distribution (Infinite Back-Iron Permeability)
[70]	1999	Rotary	Radial	Ss	SM	Radial, Parallel	2-D	Po	Vr, Sr	AR, PM	e, Field distribution, winding inductance, iron loss (Finite Back-Iron Permeability)
[71]	1999	Tubular, Linear	Radial, Axial	Sd, Ss	SM	Radial, Parallel, Halbach	2-D	Cylindrical	Vr	AR, PM	B (Infinite Back-Iron Permeability)

[72]	2006	Linear	-	Ss	SM	-	2-D	Ca	Vr	PM	Magnetic field distribution, Optimization (Infinite Core Permeability)
[73]	2009	Rotary, Linear	Axial	Sd	SM	-	-	Ca	Vr	PM	B, CT, Optimization (Infinite Core Permeability)
[74]	2010	Linear	-	Ss	SM	Parallel	2-D	Ca	Vr	PM	B distribution, e**,** Optimization (Infinite Back-Iron Permeability)
[75]	2010	Linear	-	Sd	SM	Parallel	3-D	Po, Ca	Vr	PM	B (Infinite Back-Iron Permeability)

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