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Identification of ship's hull mathematical model with numerical methods

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Abstract The modern maritime industry is moving toward the development of technology that will allow for full or partial autonomy of ship operation. This innovation places high demands on ship performance prediction techniques at the design stage. The research work presented in the article is related to the design stage of the ship and concerns methods for prognosis and evaluation of the specific operational condition of the ship, namely the dynamic positioning (DP). The paper is an introduction to a study that seeks to assess the impact of using advanced simulation models on the accuracy of DP capability prediction. To this end, the Potential Theory and methods of Computational Fluid Dynamics (CFD) are applied to determine the mathematical model of the ship. The parameters obtained in the course of simulation studies have been compared to those obtained experimentally. The study showed that the proposed method is sufficiently accurate for the purposes of determining the added mass and damping coefficients of the ship. Consequently, it is considered that design offices could improve the accuracy of the DP prediction by using mathematical modeling and numerical methods to estimate selected ship parameters.

Key words: Dynamic Positioning, Identification, Mathematical model, Damping, Computational Fluid Dynamics

1 Introduction

Current efforts are focused on increasing ship automation. The aim is to achieve fully autonomous or remotely controlled unmanned ships. A key role in this is played by the Maritime Autonomous Steering System (MASS) along with the Dynamic Positioning (DP) system. The former constitutes a global control system. The second enables the ship to maneuver at low speeds. Combined, they provide the structural and algorithmic basis for autonomous operations.

Already at the stage of ship design a reliable prognosis of the ship performance in DP can assist designers decisions relevant for the future ship operational window

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and fuel efficiency allowing for the ship to realise its target purpose [10]. Amongst many elements that comprise a DP control system a feasible identification of the mathematical model of the ship's hull is essential for the accurate prognosis and it comes down to determination of added mass and viscous damping coefficients.

Estimation of a mathematical model with a high degree of accuracy can be done through model tests. However, they tend to be expensive and time consuming. An alternative approach is to use methods involving Computation Fluid Dynamics (CFD). In particular, the use of CFD to determine the hull damping model of a DP vessel has the potential to improve prediction accuracy compared to empirical methods. Hence, it has important implications for the prediction of DP performance in offshore operations and for supporting the decision-making process in the ship design phase.

A procedure for DP ship mathematical model identification is given in [13, 12] and it concerns a ship model in scale 1:70. The added mass was determined with analytical formulas and sway and surge damping was conducted by towing the model with constants velocities. In case of yaw damping an *adaptive estimation* was applied to a free running model with the propulsion system. The identification can also be made based on a full scale trials [9] maneuvers recordings and employment of intelligence optimization algorithms. Another branch of methods of mathematical model determination are *estimation methods* such as Kalman Filter (KF) and its extension and improvements [16], however this requires a real object where both inputs and outputs can be measured. In case of luck of the model tests data or full scale trials one has to refer to simplified analytical methods, numerical methods or use a similar case.

In [11] authors refer to analytical approach to calculate the added mass based on equivalent ellipsoid method. The authors also provide a comparison with strip theory (with Lewis transformation mapping) and experiment. The discrepancies compared to experiment are -10% and -5% for ellipsoid and strip theory methods respectively in case of sway motion and -30% and -20% in yaw motion. In surge, only ellipsoid method was compared with experiment with 3% discrepancy. In [8, 7] authors introduce added mass estimation with CFD viscous flow tool *Ansys Fluent* and provide the comparison with ellipsoid method. The discrepancies between the ellipsoid method and CFD for added mass in surge are 7%. In [2] authors also use CFD software (*STAR CCM+* and *OpenFOAM*) for added mass determination and compare it to analytical approach showing around 1% differences in surge, sway and yaw.

In [4] author proposes using current coefficients as damping coefficients in the DP ship model. Those are a result of the model tests in wind tunnel, however in the literature an extensive database of those coefficients is also available for different ship types. In [4, 13, 6] authors suggest that a linear damping model for DP simulation purposes is sufficient. However, for the purpose of controller design a nonlinear model is more adequate based on [14]. In [8, 7] authors presents results and methodology of viscus damping estimation with CFD tool *Ansys Fluent* of a simple-shaped barge. However, they propose time efficient 2D analysis and extrapolation to 3D.

A certain gap in the literature is the lack of description of a comprehensive approach to CFD analysis related to the estimation of the ship's damping model. In addition, comparisons with experimentally obtained data are rarely presented.

In the paper a full hull model identification is presented, both added mass and damping, however since added mass estimation is clearly stated in the literature, attention will be paid to the damping model determination. The added mass is derived from the potential theory simulation.

The identified mathematical model of the ship will be further utilised for the validation of the DP simulator which is to be developed for the ship design purposes in Maritime Advanced Research Centre. The model will also be used for evaluation of accuracy of the static methods for DP capability assessment presented in [10] and is a continuation of the cited work.

The paper is organized as follows. The hull mathematical model is described in Sec. 2, the methodology of launching simulations for added mass and damping coefficients calculations are given in Sec. 3 and finally results are presented in Sec. 4 together with the discussion, conclusion and suggestions for future research in Sec. 5.

2 Problem formulation

Based on [4, 17, 1], the following mathematical model of the ship in DP operation is adopted with the assumptions presented below.

Assumption 1. *Considering ship motion in 3DOF: surge, sway and yaw is sufficient for DP capability assessment.*

Assumption 2. *A vessel operating in DP mode is maneuvering at relatively low speeds (around 2 m/s) [4].*

Assumption 3. *The position of the vessel operating in the presence of waves can be calculated by composition of high-frequency and low-frequency wave induced motion [1].*

Assumption 4. *Following assumption (3) only low frequency ship mathematical model is adopted.*

Based on Assumption (1) – (4) the mathematical model presented in Eq. (1) is to be used for simulation of the ship's position in time due to low-frequency wave excitation forces in presence of current, waves and wind. To account for the motion due to first-order wave forces one has to determine motion based RAO's (Response Amplitude Operators) experimentally or using potential theory respective software, transform to the time-domain and apply superposition [4, 3].

$$\begin{gathered}\dot{\eta} = R(\psi)\nu,\\ M_{RB}\dot{\nu} + M_A\dot{\nu}_r + C_{RB}(\nu)\nu + C_A(\nu_r)\nu_r + B\nu_r + D(\nu_r)\nu_r + K\eta = \tau_{thr} + \tau_{env},\end{gathered} \tag{1}$$

where $\eta = [x, y, \psi]^T \in R^3$ is position vector in the NED (North East Down) fixed reference frame; $\nu = [u, v, r]^T \in R^3$ is velocity vector in BODY fixed reference frame; $\nu_r = \nu - \nu_c$, where ν_c is current velocity; $R(\psi)$ is rotation matrix from BODY to NED; M_{RB} and C_{RB} are inertial and Coriolis - centripetal matrices of a rigid body; M_A and C_A are frequency dependant added mass inertia and Coriolis - centripetal matrices; $B(\omega) = B(0) = 0$ is frequency dependant potential damping; $K = 0$ is stiffness; D is viscous damping matrix containing linear and quadratic damping; τ_{thr} is thruster forces vector; $\tau_{env} = \tau_{wind} + \tau_{waves}$ is environmental disturbances (wind and waves) vector.

For the simplicity of the calculations the center of the BODY fixed frame is positioned at the centre of gravity of the ship. For simplification of the notation no current condition is considered: $\nu_c = 0 \rightarrow \nu_r = \nu$. Matrices that appear in the Eq. (1) are given in Eq. (2) – (6).

$$M_{RB} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_z \end{bmatrix} \tag{2}$$

where m is ship's mass and I_z is the ship's moment of inertia about z axis.

$$M_A(\omega) = \begin{bmatrix} A_{11}(\omega) & 0 & 0 \\ 0 & A_{22}(\omega) & A_{26}(\omega) \\ 0 & A_{62}(\omega) & A_{66}(\omega) \end{bmatrix} \tag{3}$$

where: A_{11} [kg], A_{22} [kg], A_{66} [kg m^2] are frequency dependant added mass coefficients in surge (along x axis), sway (along y axis) and yaw (about z axis) motion; $A_{26} = A_{62}$ [kg m] [17, 15] is frequency dependant added mass coefficient in sway motion due to rotation about z axis.

Moreover, the following assumption based on assumption (4) and [1, 4, 3] is adopted.

Assumption 5. *At low-frequency motion the added mass coefficient for zero frequency can be assumed,* $M_A(\omega) = M_A(0)$.

$$C_{RB}(\nu) = \begin{bmatrix} 0 & 0 & -mv \\ 0 & 0 & mu \\ mv & -mu & 0 \end{bmatrix} \tag{4}$$

$$C_A(\nu) = \begin{bmatrix} 0 & 0 & A_{22}v - A_{26}r \\ 0 & 0 & A_{11}u \\ A_{22}v + A_{26}r & -A_{11}u & 0 \end{bmatrix} \tag{5}$$

$$D(\nu) = \begin{bmatrix} -X_u - X_{|u|u}|u| & 0 & 0 \\ 0 & -Y_v - Y_{|v|v}|v| - Y_{|r|v}|r| & -Y_r - Y_{|v|r}|v| - Y_{|r|r}|r| \\ 0 & -N_v - N_{|v|v}|v| - N_{|r|v}|r| & -N_r - N_{|v|r}|v| - N_{|r|r}|r| \end{bmatrix} \tag{6}$$

where: X_u [kg/s], Y_v [kg/s], N_r [kg m^2/s] are hydrodynamic linear coefficients components on x, y and ψ; Y_r [kg m/s] and N_v [kg m/s] are transverse linear coefficients of drag in sway (v) and moment in yaw (r) resulting from the movement in yaw and sway respectively; $X_{|u|u}$ [kg/m], $Y_{|v|v}$ [kg/m], $N_{|r|r}$ [kg m^2] are hydrodynamic quadratic coefficients components on x, y and ψ; $Y_{|r|r}$ [kg m], $N_{|v|v}$ [kg] are transverse quadratic coefficients of drag in sway (v) and moment in yaw (r) resulting from the movement in yaw and sway respectively. Terms $Y_{|r|v}$ [kg], $Y_{|v|r}$ [kg], $N_{|r|v}$ [kg m] and $N_{|v|r}$[kg m] are force and moment coefficients due to coupled motions at both sway velocity and yaw rate simultaneously.

In [4] author proposes application of current coefficients instead of the damping coefficients, Eq. (6). The former are derived for only sway (v) and surge (u) velocities which implies that the coupled motion of sway (v) and yaw (r) is not considered and thus neglected. Furthermore, based on damping model identification of a model Cybership II given in [13, 12] the analysis of the influence of the coupled motion damping coefficients can be made. Such analysis have been performed within this study for a reasonable range of sway and yaw velocities. Based on the analysis the coupled motions damping is minor compared to absolute values. Based on both references [4, 13] and the analysis the conclusion lead to the following assumption.

Assumption 6. *The damping model can be simplified to a form given in Eq.* (7) *with a satisfactory accuracy.*

$$D(\nu) = \begin{bmatrix} -X_u - X_{|u|u}|u| & 0 & 0 \\ 0 & -Y_v - Y_{|v|v}|v| & -Y_r - Y_{|r|r}|r| \\ 0 & -N_v - N_{|v|v}|v| & -N_r - N_{|r|r}|r| \end{bmatrix} \tag{7}$$

In the formulated matrices only m and I_z are assumed to be known parameters. The problem to be solve in this paper is to determine the remain - unknown parameters for the purpose of time-domain DP simulations with the improved accuracy, based on numerical methods.

3 Methodology

For the purpose of determination of unknown parameters of the model formulated in Eq. (1) the numerical tools are employed: *Ansys Aqwa* and *STAR CCM+*. In the following subsections the simulation setup will be given for estimating added mass Eq. (3) and damping coefficients Eq. (6).

The model parameters resulting from the application of the following methodology is later compared in Sec. 4 to the model parameters obtained through the experimental methods of the same ship model. However, the identification procedure with model tests will not be extensively described here. A reader may refer to [5] and similar procedure given in [13, 12].

3.1 Frequency dependant added mass

As stated in Sec. 2 only added mass due to frequency dependant wave excited motion is considered as unknown Eq. (3). Added mass calculations are performed in full scale with potential theory simulation software *Ansys Aqwa*. A default settings are used with 5176 elements mesh and maximum wave frequency of 0.361Hz. The results are scaled down for comparison with the model tests. The results for $M_A(\omega \approx 0)$ (Assumption 5) are adopted.

3.2 Viscous damping

With the simplified damping model given by Eq. (7) the CFD analyses reduce to only three kinds of simulations with the following assumptions.

Assumption 7. *The ship damping model can be estimated based on the forces and moments generated on the hull in a steady state. Considering the steady state being a state in which those are stable for a given condition (constant speed) and no acceleration occurs (in accordance with Newton's first law).*

Assumption 8. *The damping model given by Eq.* (7) *can be estimated based on simulation of the isolated motions of a ship - surge, sway or yaw.*

By conducting simulations for a several constant velocities in surge, sway and yaw separately, a nonlinear characteristic (damping curve) results. It can be further approximated with a quadratic polynomial by least-square method which in turn determines the coefficients given in Eq. (7).

CFD analyses is conducted for the Multipurpose Support Vessel (MSV) ship model in scale 1:36 to make a fair comparison with the results from model tests of the same model. Simulations are lunched using software *STAR CCM+* at around 5 mln elements mesh (Fig. 1) and of turbulence model: realizable *K-epsilon* with boundary layer model: *Two-Layer All y+ Wall Treatment*. Three kinds of analyses are performed according to Table 1.

Table 1: CFD analysis set-up

Motion	Speed	Range	DOF	Measured forces and moments
Surge	u	-1 - 1 m/s	pitch, heave	F_u [N]
Sway	v	0.1 - 0.42 m/s	roll, heave	F_v [N], M_v [Nm]
Yaw	r	9 - 18 deg/s	roll, heave	F_r [N], M_r [Nm]

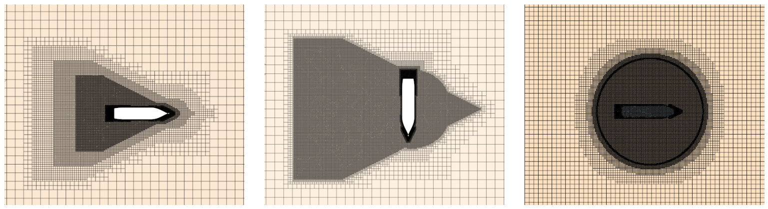

Fig. 1: Computational mesh on the free surface in surge, sway and yaw subsequently from the left

4 Results

In this section results from the numerical simulations will be presented for a Multi-purpose Support Vessel (MSV) in model scale and compared to the model parameters resulting from experimental identification of the same model in the towing tank.

The MSV is selected as a case study. The main particulars of the vessel are listed in the Table 2. Prior the numerical identification the ship model in scale 1:36 was tested at the towing tank. The model was equipped with two azimuth thrusters at the stern and two tunnel thrusters at the bow. The identification by experiment (model tests) was performed by recording the input signals (thrusters propellers revolutions and thrusters orientation) and ships position in time. Subsequently in an iterative process the mathematical model was fitted using dedicated simulation, developed for this particular purpose. Ship mass in model scale is m = 239 kg and inertia about z is I_z = 132 kg m^2.

Table 2: MSV main particulars * - full scale

Quantity	Unit	Value	Quantity	Unit	Value
L_{PP}	m	98.70	C_B	-	0.744
B	m	23.40	V	m^3	11169
T	m	6.50	L_{CG}	m	51.39

* L_{PP} is length between perpendiculars, B is beam, T is draught, C_B is block coefficient, V is volume and L_{CG} is longitudinal center of gravity from aft perpendicular.

Added mass resulting from simulation with *Ansys Aqwa* software is presented in Fig. 2 together with the values obtained from the experimental identification. The quantities are given in the Table. 3. Accounting the total mass (rigid body and added mass) the discrepancies are about 1.0% and 1.7% for surge and sway direction respectively, however for yaw direction the difference is about 30.6%. In [11] the highest discrepancy also resulted for the yaw motion, however in the opposite direction. This may be caused by inaccurate assumptions during dynamic balancing of the model in the towing tank which may resulted in slightly different moment of inertia I_z. Since the dynamic balancing of the model in yaw was not possible a value resulting from the pitch dynamic balancing was adopted. Although the longitudinal mass distribution are dominant on inertia for both pitch and yaw, this approach may not be valid even for a slender ship due to contribution of the vertical and transverse mass distribution. Moreover the parameters obtained in experiment may be influenced by the fact that the center of rotation of the model is different than the center of gravity which ultimately results in a different inertia relative to the gravity point.

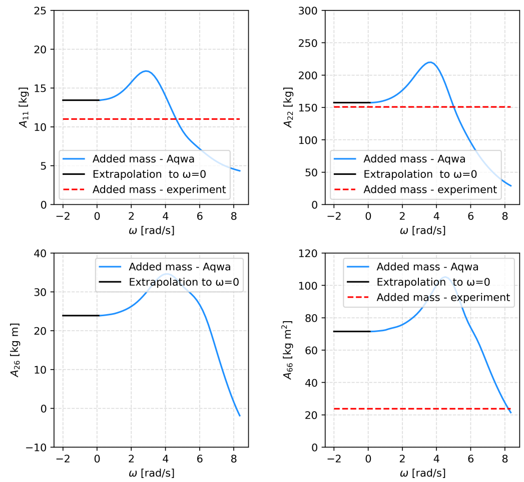

Fig. 2: Added mass in a function of angular frequency resulting from the Potential Theory simulation compared to the values resulting from the experiment

Table 3: Added mass and inertia of MSV in model scale

Coefficient	Potential theory	Model tests	Coefficient	Potential theory	Model tests
A_{11}	13	11	A_{26}	24	-
A_{22}	157	151	A_{66}	72	24

Viscous damping resulting from simulation with *STAR CCM+* software is shown in Fig. 3 together with the fitted model and model obtained from the experimental identification. The parameters quantities are given in the Table. 4. The CFD results for low speeds and force $F_r(r)$ and moment $N_v(v)$ gives a model curve which double-cross zero at each side. For better estimation of the model behaviour the analyses within a denser region around zero may improve the results. The CFD results are of higher range than the experimental identification for the purpose of the simulator architecture based on [1]. However, the typical DP ship will manoeuvre at maximum speeds around u=v=2 m/s [4], r=3 deg/s corresponding to 0.33 m/s and 18 deg/s in model scale (1:36) respectively. In that range both parameters describe a relatively close models. However, the model obtained through CFD analyses can only be fairly evaluated based on a DP simulation using both models alternatives and output comparison.

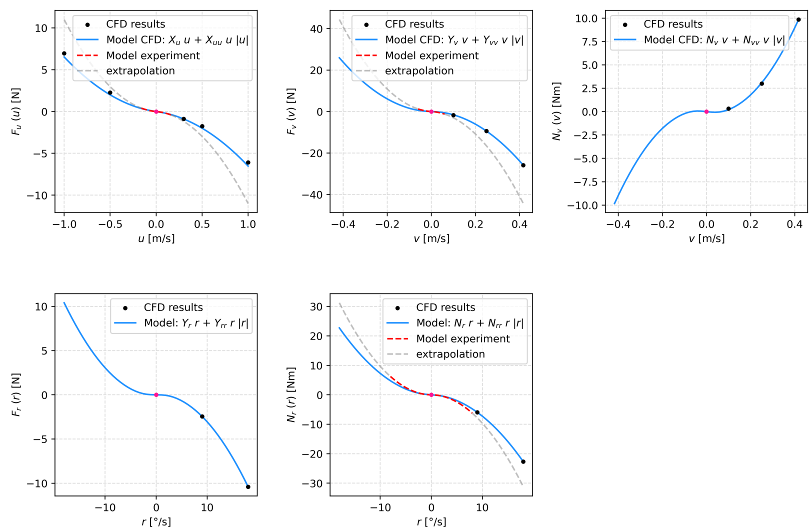

Fig. 3: Damping force and moment resulting from the CFD simulation with fitted model in comparison to the model of parameters determined by experiment

Table 4: Damping coefficients resulting from the polynomial curve fitting into the CFD results for MSV in model scale

Coefficient	CFD	Experiment	Coefficient	CFD	Experiment
X_u	-1.457	-0.9	$Y_{\|r\|r}$	-0.034	-
$X_{\|u\|u}$	-5.067	-10.1	N_v	-4.502	-
Y_v	-1.333	-6.2	$N_{\|v\|v}$	67.308	-
$Y_{\|v\|v}$	-145.613	-240.1	N_r	-0.061	-0.012
Y_r	0.033	-	$N_{\|r\|r}$	-0.067	-0.096

5 Conclusions

The paper presented an approach to identify the mathematical model of the ship in DP operation. CFD numerical methods and tools (Ansys Aqwa, ASTAR CCM+) were used to determine the added mass and damping coefficients. It has been shown, by comparison with experimental data, that the proposed solution is effective and sufficiently accurate to achieve the described objectives. Their application in a typical design office is relatively fast and allows a more accurate assessment of DP capabilities at the stage of ship design. This competes significantly with the analytical solutions in use. The added discrepancies in moment of inertia may be due to inconsistent assumptions during dynamic model balancing and testing. The damping model discrepancies are small in the speed range of DP operations.

The subsequent studies will concern testing of both models in the DP simulator, which is to be developed for further research and continuation of [10].

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