

# Prediction of the fatigue lifetime of PUR structural elements using a combined experimental-numerical approach

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## ABSTRACT.

This paper presents a method for estimating the fatigue life of polyurethane elastomeric components. A rubber replacement - polyurethane of hardness 80ShA commonly used in vibration damping systems, for example, in motor vehicle suspensions, was used for the study. A metal-rubber bushing component was selected for analysis, and numerical analysis was carried out along with a fatigue model proposal based on a modification of the Wang-Brown model. The results obtained indicate that the description of the durability process using the proposed relationship is also possible. A constitutive model based on Ogden's hyperelastic model was identified and verified. The proposed methodology can be used in any part analysis based on the numerical model and fatigue data. The paper also evaluates the effectiveness of other models against the proposed relationship.

**Keywords:** Fatigue behavior, rigid PUR, FEA, Fatigue lifetime prediction.

## Nomenclature

$\alpha$	- Experimentally determined constants
$\alpha_i$	- Temperature-dependent material parameters
$\Delta\gamma$	- Shear strain range
$\Delta\varepsilon_n$	- Normal strain range
$\bar{\lambda}_i$	- Deviatoric principal stretches
$\sigma_n$	- Normal stress acting in the critical plane,
$\sigma_c$	- Critical rupture stress
$\theta$	- Critical plane position angle of the plane
$\mu_i$	- Shear modulus
BM	- Brown-Miller criterion
$D_i$	- Incompressible parameter
$K_0$	- Bulk modulus
JWB	- Junik-Wang-Brown model
PUR	- Polyurethane
RP	- Reference point
S	- Experimentally determined constants
ShA	- Shore A Hardness Scale
N	- Order of the model
WB	- Wang-Brown model

## 1. Introduction

Polyurethane (PUR) is a widely used plastic with a variety of properties and functionalities. It plays a key role in energy conservation in construction, transport, and food preservation. In modern homes and offices, they are widely used in the form of flexible foam. The furniture made of it is soft and, at the same time, sufficiently durable, keeping its shape. Polyurethane foams fill seats and mattresses, and the possibility of using different material densities ensures high application potential. The flexible foams follow the shape of the body and support it, maintaining the correct posture when sitting and lying down. Polyurethane is perfectly suited for use as a component of modern furniture coatings, automotive and rail vehicles, power cables, floors, walls, roads, and bridges. Due to its properties, it protects exposed surfaces against the negative influence of pollution and environmental factors. Durability, corrosion resistance, and weather conditions make polyurethane suitable for covering various types of surfaces.

Polyurethanes are also interesting as a composite material for various applications [1], [2] including mining machinery [3] and the automotive industry [4] as replacement for rubber materials. Typically, PUR with the 80 in Shore scale A is a material used as a conventional rubber-like hardness in a suspension system. Elastomers represent a highly non-linear elastic material with large deformations. Their behavior is complex and dependent on many factors, such as the crosslinking state, loading conditions,



32 the stress softening effect (also called Mullins effect), and time-dependent effects [5]. PUR materials  
33 have excellent energy absorption and dynamic damping capabilities [6].

34 The aforementioned complexity of the behavior forces engineers and scientists to make an effort to  
35 reflect the mechanical performance of the material properly. There are already some modelling ap-  
36 proaches that estimate static mechanical behavior, such as the Ogden, Mooney-Rivlin, Yeoh, or Kilian  
37 model [7]. The pioneering work in the mathematical description of rubber materials (and thus elasto-  
38 mers) is the work of Mooney [8] and Rivlin [9]. They demonstrated that the commonly used linear  
39 theory and Hooke's law is an inadequate tool to describe hyperelastic materials. It was not until the  
40 development of the idea of elasticity in the range of large deformations that contributed, among others,  
41 thanks to the work of [10]–[12] to a better description and understanding of the mechanics of elastomeric  
42 materials. The authors have extensively researched the applicability of fracture mechanics and static  
43 modeling of PUR material using available models in papers [13]–[15]

44 The nature of polyurethane component loading requires a comprehensive analysis of behaviour under  
45 static and cyclic loading conditions defined by the application. Since static behaviour can be reasonably  
46 described by the models above and is proved in the previous article by the authors [15], it is not the main  
47 objective of this investigation. Thus, the focus is placed on the fatigue lifetime estimation of the com-  
48 ponents made from PUR material.

49 The state-of-the-art presented in [16]–[18] shows that several fatigue modelling approaches can be found  
50 in the scientific literature and can be grouped for crack nucleation and the crack growth approach. This  
51 division reflects the general manner of fracture of elastomers under cycling load. Such materials often  
52 fail due to crack nucleation (initiation phase) and their growth. Within the first approach group, criteria  
53 can be found based on the maximum principal strain and strain energy density. On the other hand, the  
54 energy release rate is introductory in analysing the propagation phase. Of the available approaches, only  
55 two proposed by Brown et al. [19], [20] are considered and modified to assess the useful life of the PUR  
56 component subjected to cycling load. Evaluation of fatigue life is crucial from a design perspective. A  
57 properly calibrated fatigue model can assess the durability of the designed element. Simultaneously, it  
58 increases safety and reduces the required experimental campaign.

59 This research concerns the experimental–numerical fatigue lifetime assessment of PUR components  
60 used in vehicle suspension subjected to complex loading conditions. Presented within the investigation,  
61 the modified fatigue model shows off the innovative nature of the manuscript.

## 62 **2. Materials and Methods**

63 This section includes all necessary information about the material, specimens, and conducted experi-  
64 mental tests. The research framework is shown in the flow chart form in Figure 1.



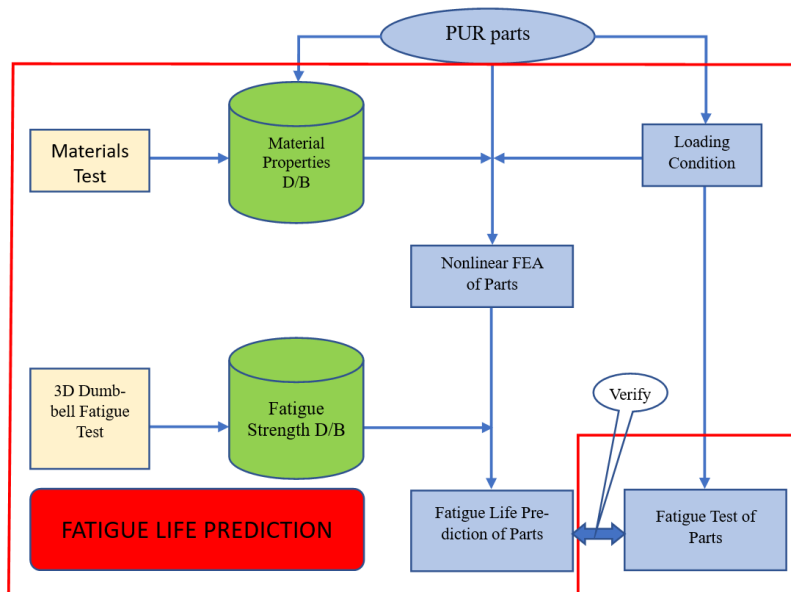


Figure 1 Flow chart of the proposed fatigue lifetime analysis of components

Analysis has been done for a polyurethane metal bushing commonly used in vehicle suspension systems. A technical drawing of the component is shown in Figure 2. As was mentioned before, commonly used for such an application is PUR material with a hardness of 80 on a Shore A scale. This material is a substitute for rubber parts. However, PUR material exhibits greater durability with respect to rubber-made parts. Due to cost, it is used mainly in sport vehicles, where performance is significantly more important than cost.

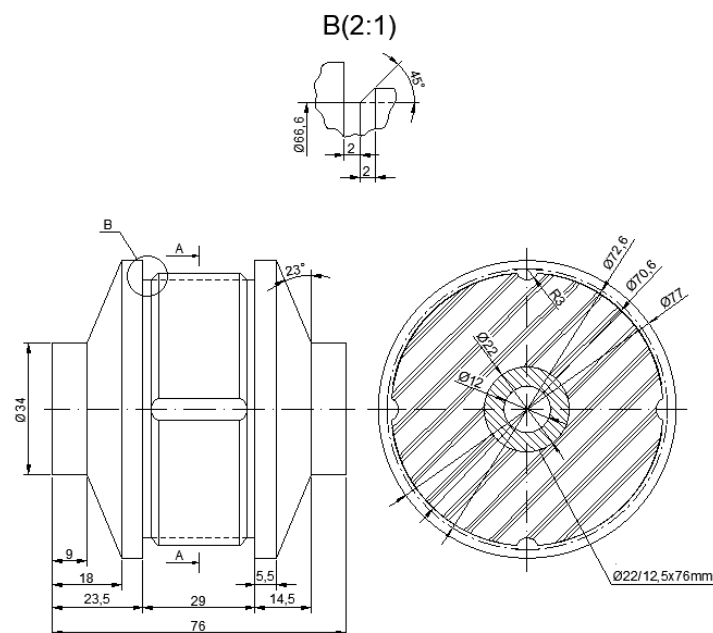


Figure 2 Scheme and dimensions of the components designed for the fatigue test (dimensions in mm).

76 *Experimental procedure*

77 Fatigue tests were carried out on an MTS 810 testing machine using a specially designed clamping  
78 system to simulate the conditions in service in the suspension system. An example of a component man-  
79 ufactured for testing (80 ShA) and a designed gripping system is shown in Figure 3. The specimen was  
80 loaded with a force amplitude of 20 kN, 15 kN, and 10 kN, respectively, at a frequency of 1 Hz to avoid  
81 the temperature effect. For this purpose, temperature was monitored periodically using thermal imaging  
82 cameras – FLIR TGXXX series. During the experiment, the proper phase of fatigue testing, the temper-  
83 ature span does not exceed 25°C for a wide range of strain levels. As in the case of test interruption,  
84 more than a 50 % drop in specimen stiffness was used as the stop criterion.

85 The validation of the raw PUR material was confirmed by the tensile test performed according to ASTM  
86 D412 [21] using flat dumbbell samples of 2 mm thickness. Based on the experimental results, a numer-  
87 ical model was performed to analyze the strains and stresses that act on the components tested. It was  
88 fundamental to determine and estimate fatigue life.



Figure 3 Specimen during fatigue test (80 ShA)

89 **3. Experimental results and numerical modeling**

90 Initially, it was necessary to verify the tensile properties of the used PUR material. It is fundamental for  
91 the calibration of models. It was already done by the authors and presented in [15]. Table 1 gives the  
92 data obtained for the static tensile test.

93

Table 1 Analysis of the results of the tensile test for the 80 ShA material configuration

SPECIMEN ID	UTS - Ultimate Tensile Strength (MPa)	A – elongation at break (%)
PU80_#1	17.9	749.2
PU80_#2	19.4	646.4
PU80_#3	21.6	651.0
PU80_#4	19.0	710.4
PU80_#5	23.7	711.0

95 Subsequently, the fatigue tests were performed and plotted in terms of the S-N curve fitted using linear  
 96 regression, which is performed by means of power law. The results of the fatigue test are shown in

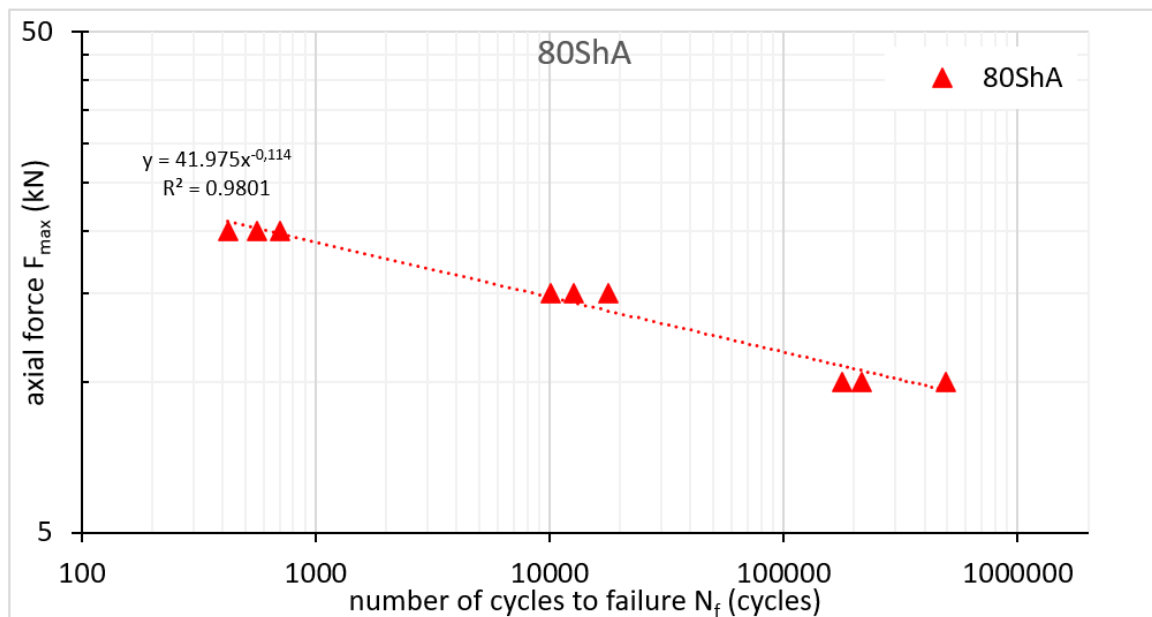


Figure 4 Fatigue curve obtained for 80 ShA polyurethane bushes (based on [22])

97 The samples after fatigue tests are shown in Figure 5 – 6. Figure 5 presents a damaged component after  
 98  $N_f = \text{approximately } 170 \text{ thousand cycles}$  [22]. A large major longitudinal crack is evident. Another dam-  
 99 age mechanism (80 ShA) is shown in Figure 6 and is dominated by ductile deformation, corresponding  
 100 to  $F_{\max} = 20 \text{ kN}$  [22]. An important conclusion from the observations was that the location of the damage  
 101 plane was identified. This plane included the location of the main crack that led to fatigue failure.



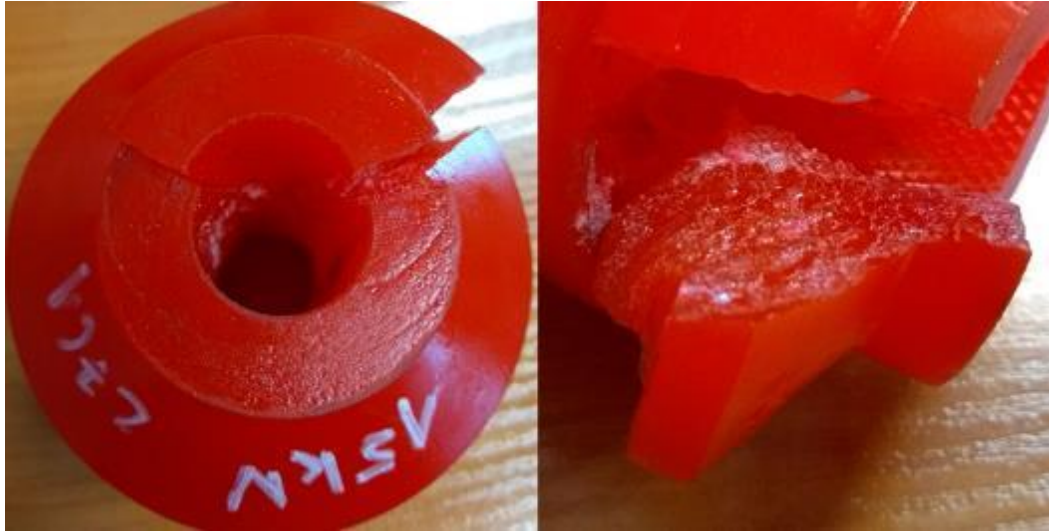


Figure 5 Damaged 80 ShA polyurethane sample (load level 15kN)



Figure 6 Damaged 80 ShA polyurethane sample (load level 20kN)

The fracture nature observed under service conditions is used to assess the results obtained by numerical analysis, performed simultaneously.

#### *Numerical analysis*

Commercially available finite element software offers a tool for model calibration. Several models are available in the software. Based on the results and detailed analysis conducted by the authors in [15]. The Ogden material model was chosen and calibrated because it reflects the mechanical behavior with the highest precision under tension load. The Ogden [23] model is a very general hyperelasticity model

110 with a Helmholtz free energy per reference volume that is expressed in terms of the principal stretches  
 111 applied and presented in Eq. (1).

$$W = \sum_{i=1}^N \frac{2\mu_i}{\alpha_i^2} (\bar{\lambda}_1^{\alpha_i} + \bar{\lambda}_2^{\alpha_i} + \bar{\lambda}_3^{\alpha_i} - 3) \quad (1)$$

Where  $\bar{\lambda}_i$  denote the deviatoric principal stretches.  $N$  – order of the model.  $\mu_i, \alpha_i$  temperature-dependent material parameters. The initial shear modulus and the bulk modulus are given by:

$$\mu_0 = \sum_{i=1}^N \mu_i, K_0 = \frac{2}{D_i} \quad (2)$$

112 The calibration procedure was described in the previous article of the authors [24], for 80 ShA material,  
 113 the Ogden model with 3 model parameters fits the appropriate parameters presented in Table 2.

Table 2 Ogden model parameters for 80ShA

	$\mu_i$	$\alpha_i$
1	4.34400372	-0.380731347
2	0.210081339	3.47276528
3	4.921467147E-03	-6.93803394

116  
 117 Moreover, the stability assessment says that the model is stable for the entire range of strains in each  
 118 test (uniaxial, biaxial, planar, volumetric). Further analysis concerns the comparison of the obtained  
 119 FEM numerical results with the experimental results. The uniaxial test was conducted to provide a force  
 120 vs. displacement relationship. The test was carried out on the MTS Bionix hydraulic testing machine  
 121 system with a 50 mm/min displacement control mode. The numerical results show good agreement with  
 122 the experimental data, as presented in Figure 7.



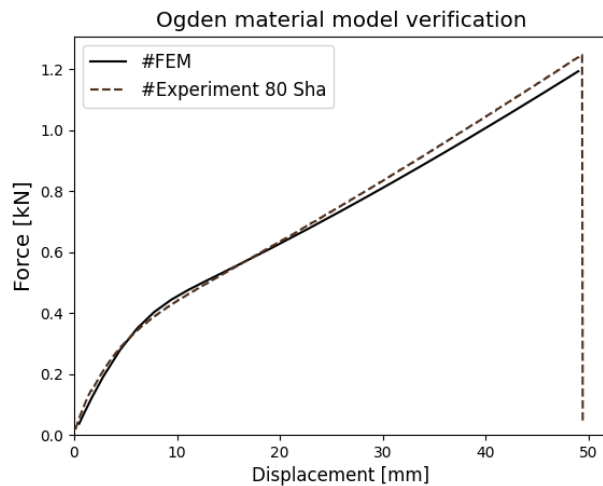
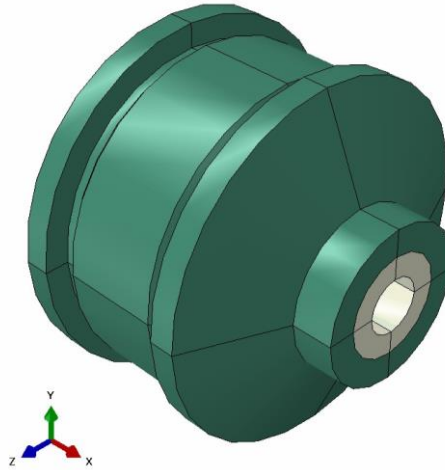


Figure 7 Comparison of the results obtained from numerical simulation and experiment

A 3D solid geometry represents a numerical model of the PUR bushing according to Figure 8. Finite element analysis was performed using Abaqus software in a static term. The model consists of two sub-parts, respectively: bushing made from PUR 80 ShA, and cylindrical insert made from steel S235 Figure 8. The previously defined PUR material model was applied, where, for steel, the following elastic properties presented in Table 3 were used.

Table 3 Typical elastic properties of S235 steel

	$E$	Poisson's ratio
Steel S235	210 GPa	0.3



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18 *Figure 8 Geometry of the analyzed assembly made of PUR material (green section) with steel insert (gray section)*  
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22 132 The purpose of this FE modeling is to obtain the distribution of the stress hot spots and to calculate the  
23 stress and strain tensor components in the critical plane position for the following loads: 10 kN, 15 kN,  
24 133 and 20 kN. The following boundary conditions shown in Figures 9 - 10 were applied to provide such  
25 loading conditions. The load transfer to the sample is carried out by coupling the reference points to the  
26 134 inner surfaces of the steel insert (Figure 9). The outer surface of the bushing was used to provide fixed  
27 support (translation and rotation in all three directions are pinned), and the load is provided by a dis-  
28 placement boundary condition applied to the reference point (RP-1) (Figure 10).  
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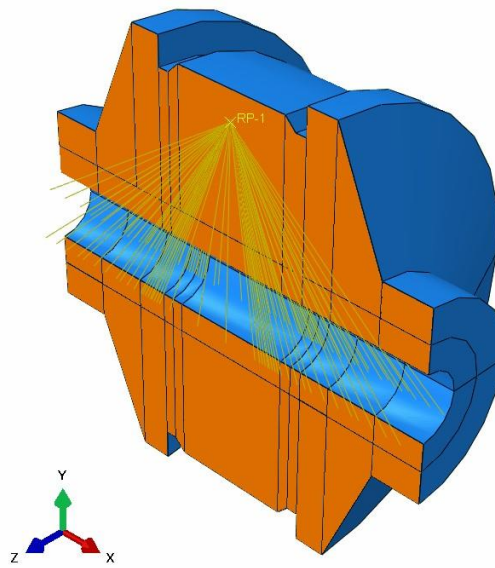


Figure 9 Representation of boundary conditions applied to the specimen - coupling connection of the reference point

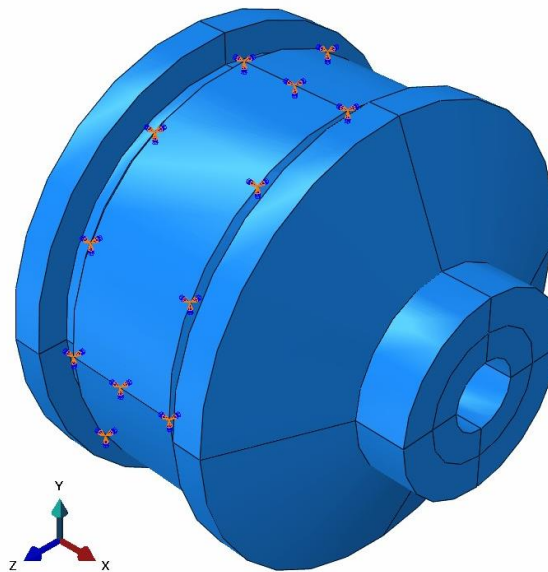


Figure 10 Representation of boundary conditions applied to the specimen, fixed support.

The presented geometry with the applied loading conditions was merged into a finite continuum object. The mesh applied to the object consists of 40 356 quadratic hexahedral elements of type C3D20H. The set of element sizes for this model is 1.5 mm; however, some regions were enriched with additional nodes. Figure 11 shows the final meshed geometry; other areas with the most nodes are noticeable. This enhanced region allowed one to obtain more adequate results.

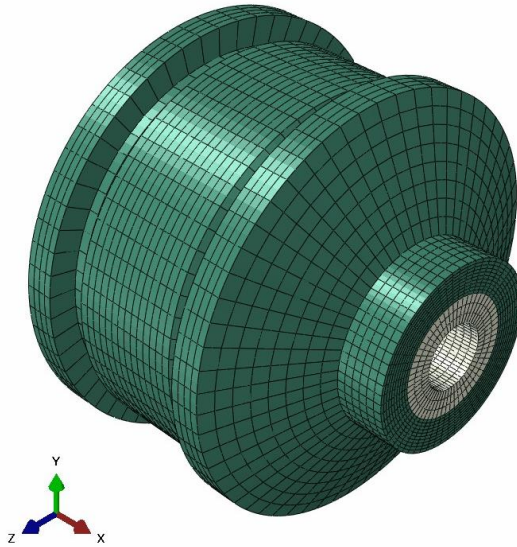


Figure 11 Discrete model of the bushing system.

144 An exemplary principal stress distribution is shown in Figure 12. As it is noticeable, the local concen-  
 145 tration appears below the outer surface. Those spots may suggest the vicinity of the crack nucleation.

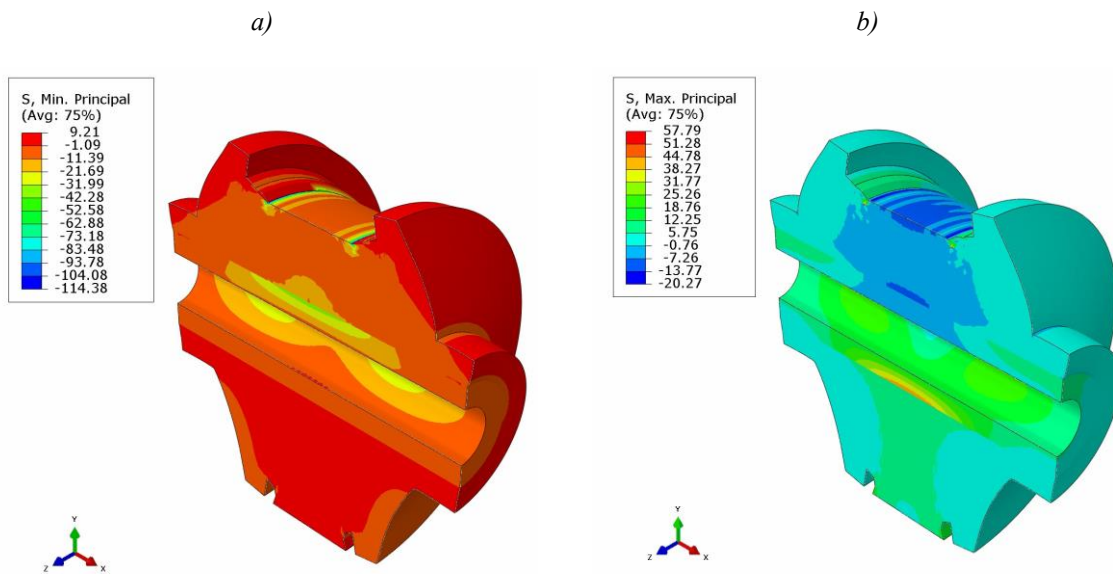


Figure 12 Minimum (a) and maximum (b) principal stress distribution for 80 ShA – load case 20kN (in MPa)

146 Having a closer look at the fracture plane defined during the fatigue experiment, the crack nucleation  
 147 given by finite element analysis seems to be of great probability. A comparison is provided and similar-  
 148 ities can be found in Figure 13.

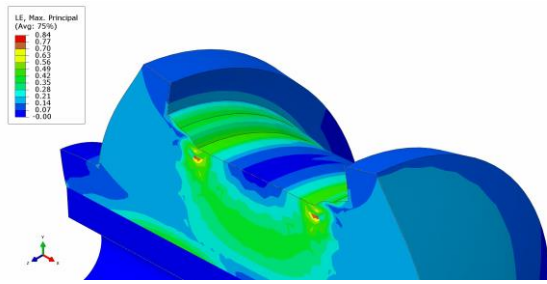


Figure 13 Comparison of results along the experiment and analysis of finite elements. Numerical results present the logarithmic strain components at the integration points (unitless).

#### 4. Fatigue lifetime prediction and discussion

The calculations were carried out in two ranges:

- First approach based on the peak strain / stress component, uniaxial loading,
- Second approach considering multiaxial loading and current stress/strain state in the damage zone.

The first approach requires only the maximum strain values as a main predictor of fatigue damage. On the other hand, to analyze the strain state, it was necessary to use commonly used criteria for multiaxial fatigue damage. The most popular Brown-Miller (BM) criterion [20]:

$$BM = (\Delta\gamma_{max}^{\alpha} + S\Delta\varepsilon_n^{\alpha})^{\frac{1}{\alpha}} \quad (2)$$

where:

$BM$  – is an equivalent strain value,

$\Delta\gamma$  – shear strain range,

$\Delta\varepsilon_n$  – is the normal strain range,

$\alpha, S$  – experimentally determined constants.

The BM criterion was based on the maximum shear stress plane. Another evolution of the BM model was the Wang-Brown (WB) modification, which allowed applications of the concept of a critical plane concept [19]

$$WB = \max_{\theta} \left( \frac{\Delta\gamma}{2} + S\Delta\varepsilon_n \right) \quad (3)$$

where:

$\theta$  - critical plane position angle of the plane.

As it seems, finding the experimental parameters - in particular the BM and WB -  $S$  parameters - has resulted in a multidimensional, nontrivial issue. In addition, there is not enough guidance on identifying this parameter in the literature. For its estimation, mathematical estimation was used by fitting the model to the experimental data. In the calculations, a constant value of  $S=0.25$  was assumed. On the other hand, observing the experimental data led to the modification and results in the JWB - Junik-Wang-Brown approach. The revision proposed by the authors is given by Eq. (4).

$$JWB = \max_{\theta} \left( \frac{\Delta\gamma}{2} + \frac{\sigma_n}{\sigma_c} \Delta\varepsilon_n \right) \quad (4)$$

where:

$\sigma_n$  – normal stress acting in the critical plane,

$\sigma_c$  – critical rupture stress.

The effectiveness of the strain approach (based on the peak strain concept), WB and JWB models for polyurethane materials, is shown in Figure 14, where the predictions are presented. Analysis of both graphs shows that forecasts based only on the uniaxial strain criterion are entirely incorrect. Moreover, the results obtained are underestimated and therefore not conservative, which can be particularly dangerous in the context of operation. The WB criterion correctly predicts fatigue life, resulting in five points for 80 ShA that fall outside the lifetime factor  $\times 2$  and all matters within the lifetime factor  $\times 3$ . In contrast, the JWB criterion proposed in this dissertation predicts fatigue life much better, resulting in 80 ShA having only one point outside the lifetime factor  $\times 2$  and all points within the lifetime factor  $\times 3$ . The fundamental advantage of the JWB model is that, for the materials tested, each time it only required knowledge of the static strength parameters without having to mathematically 'estimate' the value of the parameter  $S$ .

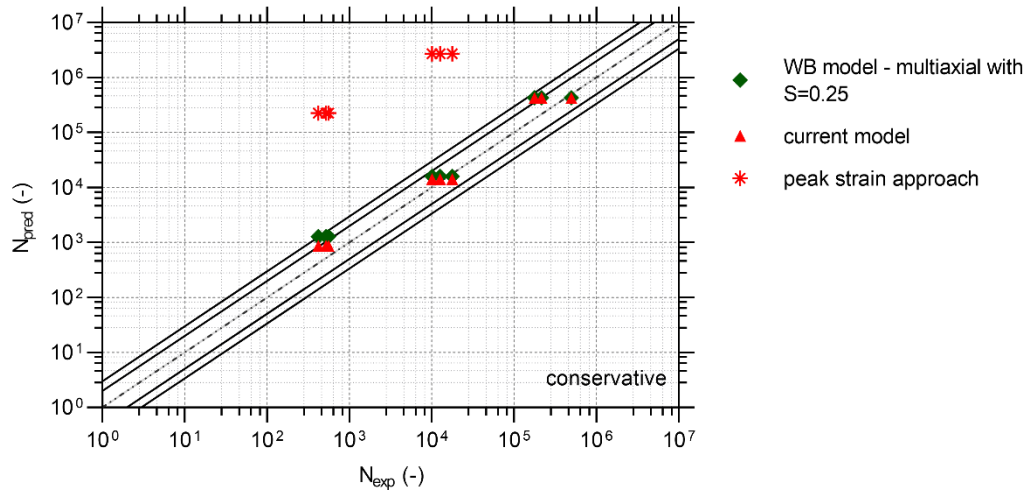


Figure 14 Predictions results for the 80 ShA component

## 5. Conclusions

The fatigue life assessment presented in the following research was based on experimental (static and fatigue tests) and numerical analysis. The calibration procedure for the material's models of suspension bushing made from PUR 80 ShA was performed. At the same time, finite element analysis was performed to obtain strains during fatigue loading. Based on that, it allows draw the following conclusions.

- The behaviour of the two polyurethane elastomeric materials with different harnesses can be successfully described by Ogden's constitutive model ( $N=3, 4$ ).
- For bushing system peak strain approach in fatigue lifetime analysis is insufficient,
- The proposed modification of the Wang-Brown model allows for more accurate fatigue life prediction (considering the multiaxial fatigue condition) of PUR components,
- On the contrary to the Wang-Brown model, better explanation of physical meaning constants  $S$  is dependent on the critical stress level and tensile properties.

### Credit authorship contribution statement

**Grzegorz Lesiuk:** execution of the analytical study, data analysis, writing, supervision.

**Krzysztof Junik:** execution of the analytical study, data analysis, writing. **Szymon Duda:** execution of the analytical study, data analysis, writing. **Tomasz Socha:** data analysis, writing, validation. **Krzysztof Kula:** data analysis, validation, writing - review. **Arkadiusz Denisiewicz:** data analysis, validation, writing - review. **Daniel Medyński:** data analysis, validation, writing - review. **Wojciech Macek:** data analysis, validation, writing - review. **José Correia:** data analysis, validation, writing - review.



## 200 Declaration of competing interest

201 The authors declare that they have no known competing financial interests or personal relation-  
 202 ships that could have appeared to influence the work reported in this paper.

203

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