

Article citation info:

Pyrchla K, Łapiński K, Szulwic J, Jurczak W, Przyborski M, Pyrchla J, Accuracy of marine gravimetric measurements in terms of geodetic coordinates of land reference benchmark, *Eksploracja i Niezawodność – Maintenance and Reliability* 2024; 26(3) <http://doi.org/10.17531/ein/188592>

Accuracy of marine gravimetric measurements in terms of geodetic coordinates of land reference benchmark

Indexed by:



Krzysztof Pyrchla^a, Kamil Łapiński^b, Jakub Szulwic^b, Wojciech Jurczak^c, Marek Przyborski^d, Jerzy Pyrchla^{b,*}

^a Faculty of Electronics, Telecommunications and Informatics, Gdansk University of Technology, Poland

^b Faculty of Civil and Environmental Engineering, Gdansk University of Technology, Poland

^c Faculty of Mechanical and Electrical Engineering, Polish Naval Academy, Poland

^d Faculty of Navigation and Naval Weapons, Polish Naval Academy, Poland

Highlights

- Reliable pier binding is critical to the precision of marine gravity campaigns.
- GNSS static measurements are suitable for monitoring absolute gravity network points.
- Regional data about disturbances of gravity is useful for plication in ultra-precise INS.

Abstract

The article presents how the values of (3D) coordinates of land reference points affect the results of gravimetric measurements made from the ship in sea areas. These measurements are the basis for 3D maritime inertial navigation, improving ships' operational safety. The campaign verifying the network absolute point coordinates used as a reference point for relative marine gravity measurements was described. The obtained values were compared with catalogue values. In verification of network points 3D position the satellite data Global Satellite Navigation System (GNSS) and ground supporting systems (GBAS) was used. In this example, the height difference of the land reference point was 0.32 m. As a consequence, the offset budget of the marine campaign was affected in the range of up to 0.35 mGal. The influence on gravity free-air anomaly was not constant over the entire area covered by the campaign.

Keywords

inertial navigation, GNSS measurement, gravity measurement, shipborne gravimetry

This is an open access article under the CC BY license (<https://creativecommons.org/licenses/by/4.0/>)

1. Introduction

In recent years, efforts to create high-resolution and precision maps of gravity field functionals have been made in various research facilities around the world. The results of these efforts are dedicated to application in ultra-precise inertial navigation (INS) [1–3]. The precision and resolution requirements for gravity measurements are largely dependent on the purpose of these measurements: geological survey or inertial navigation. In this article, we focus on gravity data collection, which is utilised in inertial navigation systems. The compensation of gravity disturbances plays an important role in the improvement of

positioning accuracy in algorithms of inertial navigation. The requirements for gravity data quality are highest in that case since the few mGal's of error in gravity value can be attributed to a horizontal position error of a few meters after the time span of minutes. It should be noted that all errors in INS have a tendency to accumulate (growth is logarithmic). Because of that, the mentioned few meter inaccuracies will quickly rise to the level potentially dangerous for marine navigation safety. In case satellite navigation in the Baltic Sea area is successively disrupted, the reliability of alternative navigation systems will

(*) Corresponding author.

E-mail addresses:

K. Pyrchla (ORCID: 0000-0001-6027-7341) krzpyrch@pg.edu.pl, K. Łapiński (ORCID: 0000-0003-1927-4047) kamlapin@pg.edu.pl, J. Szulwic (ORCID: 0000-0003-2573-4492) szulwic@pg.edu.pl, W. Jurczak (ORCID: 0000-0002-1608-7249) w.jurczak@amw.gdynia.pl, M. Przyborski (ORCID: 0000-0001-5354-1407) m.przyborski@amw.gdynia.pl, J. Pyrchla (ORCID: 0000-0002-4829-3784) jerpyrch@pg.edu.pl

play a more important role in providing safety on the sea.

The presented manuscript describes the influence of precision and reliability of land reference points on three-dimensional marine gravity data. Considering the current theoretical accuracy of ship-borne gravity campaigns (the uncertainty 1 mGal or lower), the consistency and reliability of pier binding are crucial.

The issue of linking measurements at the point of gravimeter placement on the ship to the absolute value of gravity in the port pier is presented in the literature [4,5]. However, the description of the verification of the coordinate values of the gravimetric matrix points from which the gravity acceleration values are transferred to points on the harbour pier is omitted in the literature. From this perspective, the presented analysis results are an important contribution to obtaining high-quality gravimetric measurements [6]. Gravimetric measurements in sea areas are most often performed with the use of relative gravimeters, whose measurements need to be linked/connected to points located onshore [7]. The gravity force values of these points are transferred from the points of the national gravimetric control network and constitute the basis for the analysis and calculations [8].

While conducting gravimetric measurements in marine areas, the team encountered a case of an incorrectly stated gravity reference station catalogue value. The study of this case allows us to present a real case in which the error of the land reference point's ellipsoidal height can influence the transferred gravity value. Despite the fact that for relative measurements, the pair of relative CG5 (Fig. 1A) gravimeters was used, it has no influence on the reduction of systematic error of gravity network reference point. The only yet important effect of such a procedure was lowering the uncertainty of gravity difference estimation between the point of gravity absolute network and reference point on the pier, in close proximity to the moored measurement vessel. By relating the pier reference point with a few gravity absolute network control points the final gravity value for the reference point was established. The spirit levelling between the reference pier point and the marine gravimetric sensor on the vessel board was performed to ensure the control of ellipsoidal heights estimated for the marine gravimeter by the vessel's GNSS positioning system. The gravity disturbance along the measurement vessel trajectory has

been calculated based exclusively on the value of the Gdynia pier reference point. The marine gravimeter utilised for the measurements at sea was MGS-6 Micro-g LaCoste & Romberg.

In this communication, it is presented how, in the marine area covered by gravimetric measurements, the erroneous parameter of the reference point propagates to the results of the recorded signals. The influence of the bias in the reference point value has a complex effect on the values on the measurement lines because it interacts with other campaigns (which are vastly bonded to independent piers values) during the data processing and line leveling.

Experience in performing marine gravity campaigns shows that the error resulting from incorrect referencing parameters during offshore measurements is not detectable. It is because it does not create a uniform offset in the campaign data, as one may expect, but it spreads nonuniformly over the campaign area. That situation is particularly difficult when data is planned to be used in the products dedicated to usage in ultra-precise inertial navigation which is sensitive to regional field disturbances [9]. To the best of our knowledge, the methodological validation of the reference point of the marine gravity campaign, as performed by our team, has not yet been described in the literature. We show that it is worth checking the other values assigned to the gravity absolute reference points to eliminate errors resulting from this. Such a procedure should be a rule, especially when data collected during marine campaigns is dedicated to being used in ultra-precise inertial navigation systems.

The determination of 3D coordinates values of land reference network points and especially the precise determination of the ellipsoidal height offset was possible due to the application of long static GNSS measurements on the absolute gravity network point (Fig 1B) and on three other geodetic class reference points simultaneously. The technique of data acquisition during measurements and its postprocessing analysis was presented. Geodetic measurement instrument improvements ensure higher accuracy when verifying horizontal and vertical reference systems [10].

The quality of the results obtained for the three-dimensional position for benchmark 5403 (POLREF-GORA DONAS) has been achieved due to the utilisation of data collated by nearby ground-based augmentation system (GBAS) stations [11,12].



Fig. 1. Gravimeters and GNSS receivers at points of the measurement span: A – POLREF-GORA DONAS, B – Gdynia mareograph.

Acquisition of gravimetric data for the development of high-precision and high-resolution maps of gravity acceleration field functions required precise positioning of the vessel during the campaign. Positioning was obtained by recording raw GNSS data from receivers installed on the vessel and data from reference stations along the Polish coast and along the southern coast of Sweden. It must be noted the application of the Global Navigation Satellite System technique has revolutionised maritime gravimetric measurements [13,14]. This technique has not only increased the accuracy of positioning measurements but, on top of that, made it possible to increase the frequency of measurements. In addition, the GNSS technique makes it possible to obtain the data necessary to eliminate interference resulting from the Eötvös effect, Harrison effect and cross-coupling from recorded signals [15]. It should be noted that in the 1970s, measurements made with an accuracy of 2 mGal ($1 \text{ mGal} = 10^{-5} \text{ m/s}^2$) were considered truly accurate. Nowadays, gravimetric measurements at sea are feasible with an error of less than 1 mGal [16]. Achieving such accuracy requires a control check of all measurement stages, including a catalogue value of the reference station. It is extremely difficult to determine the offset error of gravimetric registration signals with such accuracy during a sea campaign.

This article is organised as follows: section 2 describes the input data, details of the field measurement campaign conducted to determine the values of the parameters of the national control

point network and the data processing methods used. Section 3 presents the results of the accuracy calculations: a comparison of the results between two independent systems, ASG-EUPOS and HxGN SmartNet, and a description of how control point values affect the distribution of gravimetric values recorded in the study area. Section 4 presents a summary and conclusions.

2. Materials and methods

The marine gravimetric measurements team of the Gdańsk University of Technology performed during the preparation of the measurement campaign, and a two-day relative gravity survey was carried. The measurements were intended to determine the force of gravity at a point located on the quay in the port of Gdynia, which was a reference point in the campaign. Gravimetric measurements carried out in the eastern part of the Southern Baltic Sea area are linked to a point in the port of Gdynia. The gravity value was transferred from the closest absolute base points of the higher-order gravimetric control network, point 5403 (POLREF-GORA DONAS) and point 363 (GDANSK ABS) (ID 5418234342.000). The benchmarks included in the measurements are concrete poles sunk 1 m into the ground. On the surface of the poles, there are concrete slabs $0.6 \times 0.6 \times 0.12 \text{ m}$ in which metal pins are mounted. The value at the control network points was obtained from the National Register of Basic Geodetic, Gravimetric and Magnetic Networks, managed by Poland's Head Office of Geodesy and

Cartography. Two Scintrex CG5 relative gravimeters were used to transfer the gravity (Fig. 1). A two-day relative gravity survey was carried out with two CG5s, starting from the benchmark 5403 (POLREF-GORA DONAS) to the point located on the quay in the port in Gdynia, where ORP "HEWELIUSZ" was moored, back and forth along the same route. As a mid-station, the tide-meter point of Gdynia port was used. Thanks to this action, we tied our relative measurement twice and obtained four measurement ranges. The same action was adopted when transferring values from the 363 (GDANSK ABS) (ID 5418234342.000) point. In this case, two intermediate points were established due to the distance. The values listed in the catalogues are assumed to be actual values, determined with a supposed accuracy, and their validity is rarely checked [17]. The difference between the values transferred from these points at the Gdynia shore point was 0.3 mGal. It was decided to carry out a land campaign to check the validity of the 5403 POLREF-GORA DONAS point, which was used as the starting point. During the measurement campaign, it was decided to compare the points 0301 EUREF = EUVN-PL04 ROZEWIE, 5403 POLREF-GORA DONAS, and mareograph points in the ports of Gdynia and Gdańsk. An ellipsoidal height was adopted for comparisons at individual points because this height is used in the post-processing of gravimetric data recorded during sea campaigns.

Reference values need to be validated whether they are still up to date. One of the control measurement techniques is the GNSS [18] measurement technique, which involves the use of the American Global Positioning System (GPS), the Russian Global'naya Navigatsionnaya Sputnikovaya Sistema (GLONASS), the European Galileo system, the Chinese BeiDou system, or the Japanese Quasi Zenit Satellite System (QZSS). These systems permit the determination of the position of a measured element in three-dimensional space by assigning coordinates in a global geodetic reference frame or after performing a special transformation into a local reference frame [9,20]. Taking into account the characteristics of the reference point, the techniques and technologies for satellite measurements were selected to achieve the highest accuracy. Due to the design of satellite navigation systems, time measurement stability has a crucial impact on accuracy [21]; thus, a measurement strategy was designed to address this issue.

GPS and GLONASS were selected from among the currently operating GNSS systems. Ground-based systems were also used in the measurements, specifically, HxGN SmartNet and ASG-EUPOS, included in the Ground-Based Augmentation System (GBAS). ASG-EUPOS was launched in 2008 and is run by the Head Office of Geodesy and Cartography. Thus, it provides the official implementation of the European Terrestrial Reference System 1989 (ETRS89) in Poland and provides observations of the four satellite systems GPS, GLONASS, Galileo and BeiDou [22]. The ASG-EUPOS system includes 107 Polish stations and 22 stations located in the territories of neighbouring countries to ensure full coverage of services in border areas. The equipment used to build the network comes from various manufacturers; the most common are Leica and Trimble receivers. Through ASG-EUPOS, it is possible to access the following services: NAWGEO, KODGIS/NAWGIS, and POZGEO/POZGEO D [23,24].

HxGN SmartNet is a commercial network of reference stations and its role is similar to that of the ASG-EUPOS system (Fig. 2). The main difference is the station density and single use of in-house hardware only. The network includes 172 reference stations in Poland and 21 stations in neighbouring countries. Using HxGN SmartNet, we have access to RTK, RTK-RTN, VRS, and post-processing services [25,26].

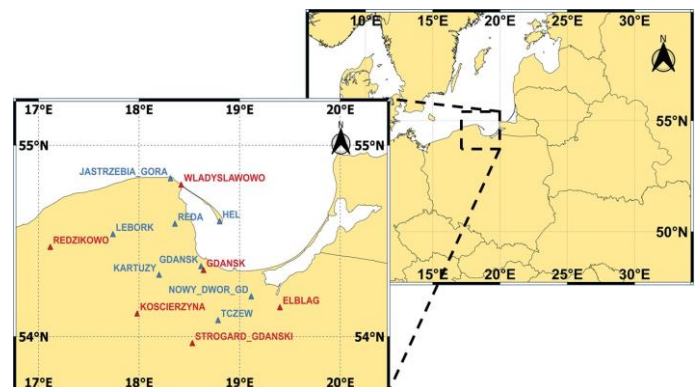


Fig. 2. The area of the Southern Baltic Sea is covered by gravimetric measurements established in the port of Gdynia. Distribution of ASG-EUPOS reference stations (red) and HxGN SmartNet (blue).

After analysing GNSS measurement methods such as real-time measurement [27–29], DGNSS measurements [30,31], and Precise Point Positioning [32–34], the static method [35], most often used to establish geodetic networks and geodynamic measurements, was selected.

Satellite observations were made with receivers placed over the points. The measurement sessions lasted several hours. This method allowed for high accuracy of a long baseline (which has a coordinate difference vector). The locations of points in global geodetic coordinates were obtained in post-processing using Leica Geo Office v8.4 and proprietary scripts. The calculation

process also used data obtained from observations made at points of known locations (GBAS) [36].

As part of the measurement campaign, GNSS signals were recorded at four points, located in Rozewie, on Góra Donas, and in Gdańsk and Gdynia, as presented in Figure 3.

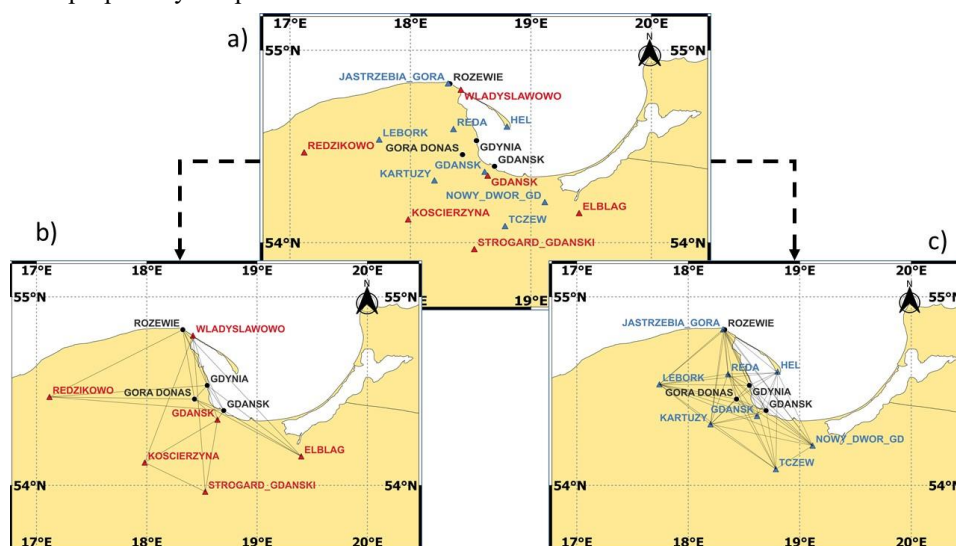


Fig. 3. Distribution of the reference stations of two service providers in relation to the measurement points (a) and the baselines obtained in the calculation process based on the ASG-EUPOS (b) and HxGN SmartNet (c) systems.

The point located in Rozewie is the EUREF-POL network point, the point on Góra Donas is the POLREF point, and the mareograph points are in the ports of Gdynia and Gdańsk. The measurements started on 27 June 2019 at 17:30 and continued until 00:00. At approximately 9 pm, the measurement was stopped and restarted, resulting in two measurement sessions, lasting approximately 3h each.

The locations of the control points were determined by calculating the coordinates in the global geodetic reference frame and then comparing them with catalogue values. To check the current catalogue data, post-processing was conducted using two independent reference station systems, ASG-EUPOS and HxGN SmartNet.

Firstly, post-processing was performed using ASG-EUPOS reference stations (Gdańsk, Elbląg, Kościerzyna, Redzikowo, Władysławowo, Starogard Gdański). Observation data (in the universal Rinex data format) for reference stations were obtained using the POZGEO D service designed for this purpose. Data for reference station antennas were also collected to check and possibly correct the Rinex files. Errors in the type and parameters of the reference station antennas of Gdańsk, Elbląg, Kościerzyna, and Redzikowo were found. Corrections

were made by supplementing the correct antenna models and introducing appropriate offsets to the phase centres of the antennas in accordance with the data contained in the files of the reference stations' antenna characteristics. We related our observations to the highly accurate state control network using the data and computational process mentioned. It is worth mentioning that the accuracy of the calculations depends on the distance of the measured points from the reference stations and on the geometry of the obtained baseline. To increase the accuracy of the calculations, precise ephemerides for the GPS and GLONASS satellite systems were obtained from the International GNSS Service (IGS). The Leica Geo Office v8.4 software was used for the post-processing of the satellite observations.

A similar post-processing procedure was performed using HxGN SmartNet reference stations. The reference stations Tczew, Reda, Nowy Dwór Gdański, Lębork, Kartuzy, Jastrzębia Góra, Hel, and Gdańsk were used. Data for reference stations in the universal Rinex data format and data from the service providers of the base station antennas were collected. Precise ephemerides from the first stage were used. The post-processing was carried out using the same parameters as in the case of the

ASG EUPOS network. As a result of the post-processing, 128 baselines were counted. The control process was conducted with the "GPS Loop Misclosure" option, resulting in the detection of 104 meshes, in which 45 vector components did not meet the assumed criteria. Strict adjustment was performed using the least squares method, which resulted in the received coordinates of points measured in the global geodetic reference frame. Figure 3 shows the baselines obtained in the calculation process using the ASG EUPOS system (Figure 3b) and HxGN SmartNet (Figure 3c).

3. Results

An analysis of the results of the alignment process was carried out to examine the accuracy achieved. A fundamental aspect of this process is the degree of redundancy in the observed network as a result of the measurement. An excessive number of observations has a direct impact on the quality of the adjustment, the achievable accuracy, and, thus, the results obtained. The graphs below show the redundancies in the adjustment process using the ASG EUPOS (Fig. 4a) and HxGN SmartNet (Fig. 4b) systems.

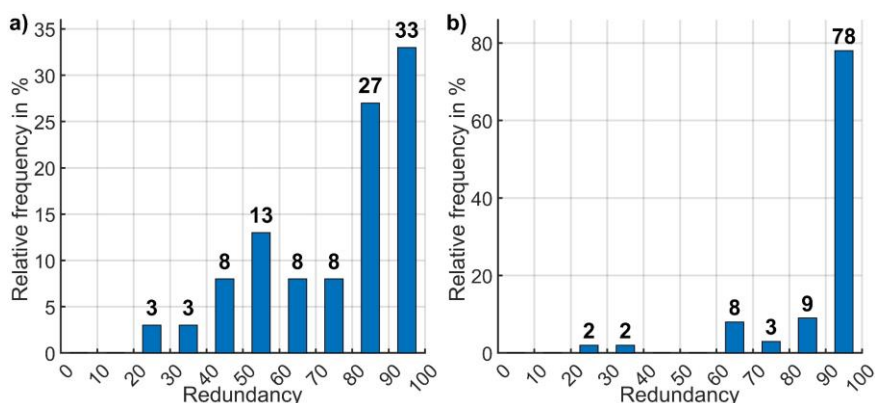
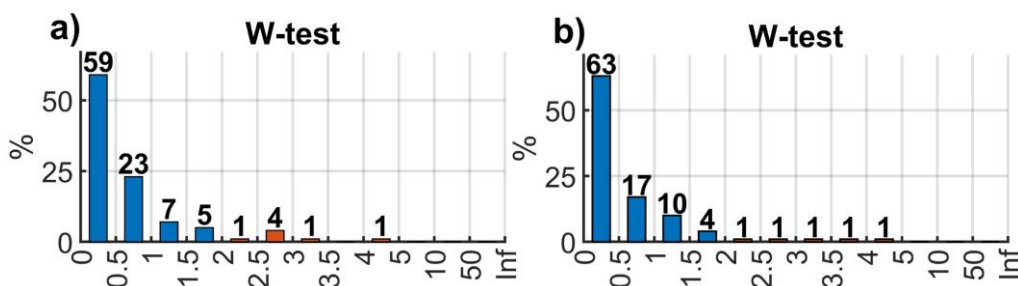


Fig. 4. Redundancy based on the following systems: (a) ASG Eupos, (b) HxGN SmartNet.

The graphs in Fig. 4 indicate that in the calculation process, a higher relative frequency of baselines with redundancy numbers of 90%-100% was observed for the HxGN SmartNet stations, compared to only 33% relative frequency for the same redundancy class in the case of the ASG EUPOS system. Many elements influence the redundancy factor. Taking into account the analysis factors, the baselines between the reference stations and the points observed, the geometry and interdependencies between these points and base stations, the number of baselines obtained, control "GPS loop misclosure" allowed some baselines eliminated because they do not meet certain accuracy criteria. The mentioned elimination of baselines leads to different redundancy of measurement points.

The Leica Geo Office program and proprietary scripts were used to determine the accuracy of the calculations. Statistical

tests such as the F-test [37], the Test-W [38,39] and the Test-T [40-43] were used. The null hypothesis of the Test-F was that there were no errors in the observations and thus the measurements. The test results indicated that the hypothesis could be rejected. In this case, it was necessary to establish the reasons for this rejection. This was done by checking the observations individually with the Test-W, using the so-called conventional alternative hypothesis, to find one-dimensional observations that were significant outliers from the others that were assumed to be correct (ΔX or ΔY or ΔH). Since GNSS observations are represented by pseudo-ranges, it was considered appropriate to perform the Test-T, which is a multidimensional Test-W (two-dimensional, 2D, and three-dimensional, 3D). The results of these tests are presented in Fig. 5.



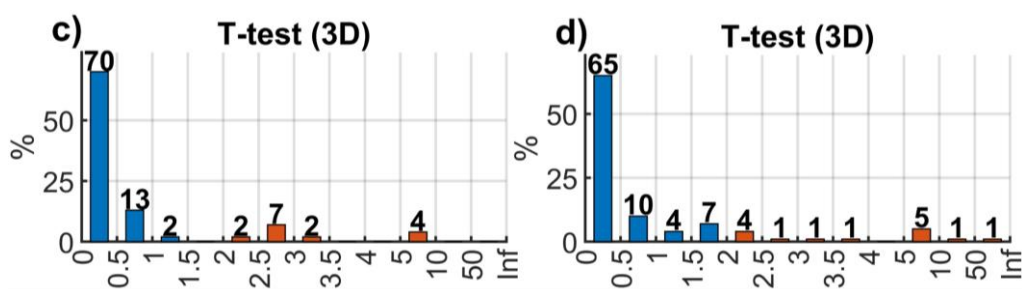


Fig. 5. Results of statistical Test-W (a) ASG EUPOS, (b) HxGN SmartNet, and three-dimensional Test-T (c) ASG EUPOS, (d)HxGN SmartNet.

The above graphs show that the test results were similar for the ASG EUPOS and HxGN SmartNet systems. The Test-W pass rate was 93.5% for ASG EUPOS and 94.5% for HxGN SmartNet. The pass rate of the multivariate Test-T was 84.8% for ASG EUPOS and 83.8% for HxGN SmartNet.

the systems used in the reference stations. The tables present the measurement results for the control points counted based on the ASG EUPOS and HxGN SmartNet systems. The accuracy of the values are presented in Tables 1. The estimated 3D coordinates of the control points are shown in Table 2.

The results obtained were analysed and compared between

Table 1. Accuracy values obtained.

Point Id	Horizontal accuracy [m]		Height accuracy [m]	
	ASG-EUPOS	HxGN SmartNet	ASG-EUPOS	HxGN SmartNet
Measurement session no. 1				
GORA DONAS 1	0.0172	0.0023	0.0272	0.0036
GDANSK 1	0.0172	0.0022	0.0272	0.0034
GDYNIA 1	0.0173	0.0025	0.0274	0.0039
ROZEWIE 1	0.0173	0.0026	0.0274	0.0041
Measurement session no. 2				
GORA DONAS 2	0.0142	0.0029	0.0220	0.0044
GDANSK 2	0.0141	0.0025	0.0219	0.0039
GDYNIA 2	0.0144	0.0032	0.0223	0.0049
ROZEWIE 2	0.0144	0.0033	0.0223	0.0050

Table 2. Geodetic coordinates.

Point	ASG Eupos			HxGN SmartNet		
	Latitude	Longitude	Ellipsoidal height [m]	Latitude	Longitude	Ellipsoidal height [m]
Measurement session no. 1						
5403 – GORA DONAS	54°27'38.552017"N	18°25'55.970531" E	201.9770	54°27'38.551875"N	18°25'55.970091"E	201.9723
Mareograph GDANSK	54°23'58.826685"N	18°41'51.947558" E	30.7622	54°23'58.826356"N	18°41'51.946867"E	30.7823
Mareograph GDYNIA	54°32'0.185512" N	18°32'52.479986" E	31.5059	54°32'0.185391"N	18°32'52.479265"E	31.4961
301 EUREF ROZEWIE	54°49'39.016063"N	18°19'35.359820" E	70.8005	54°49'39.015891"N	18°19'35.359295"E	70.7940
Measurement session no. 2						
403 – GORA DONAS	54°27'38.552067"N	18°25'55.970153" E	201.9703	54°27'38.551761"N	18°25'55.970078"E	201.9759
Mareograph GDANSK	54°23'58.826843"N	18°41'51.946929" E	30.7592	54°23'58.826465"N	18°41'51.947012"E	30.7800
Mareograph GDYNIA	54°32'0.185942"N	18°32'52.479490" E	31.4837	54°32'0.185598"N	18°32'52.479446"E	31.4891
301 EUREF ROZEWIE	54°49'39.016241"N	18°19'35.359537" E	70.7532	54°49'39.015915"N	18°19'35.359558"E	70.7583

The differences between the coordinates calculated using data from the ASG EUPOS and HxGN SmartNet systems were very small differences, ranging from 0,05 to 0,21 m in ellipsoidal height, 0,03 to 0,020 m in Latitude and 0,02 to 0,013 m Longitude.

4. Discussion

After the measurement, calculation and control process, a detailed analysis of the results and a comparison of the catalogue values with the values of control measurements was carried out. At three points (EUREF Rozwie, mareograph of Gdynia, mareograph of Gdansk), slight discrepancies between the measured values (0.02 m) were noted within the limits of measurement and calculation errors. On the other hand, at point 5403 (POLREF-GORA DONAS), during the first and second measurement sessions, a significant difference was noted between the ellipsoidal heights from the catalogue and of the control measurement. The catalogue value states a height of 201.653 m, whereas the height based on the ASG EUPOS reference stations network was 201.977 m in session 1 and 201.970 m in session 2. The values based on the network of HxGN Smartnet reference stations were 201.972 m in session 1 and 201.976 m in session 2. The average difference in the ellipsoidal height from the two campaigns in relation to the data

from the ASG EUPOS reference stations network was 0.3205 m, and the average difference in relation to the data from the HxGN Smartnet reference stations network was 0.321 m. After averaging the results obtained from both systems, differences in the ellipsoidal height of 0.321 m were used. The value of the discrepancy of the ellipsoidal height is so significant that it appeared essential to check how it is transferred to gravimetric measurements recorded in the sea campaign. Gravimetric signals recorded during the measurement campaign carried out on the ORP "Heweliusz" ship from 07 to 10 June 2021 were used as a reference in the analysis. The measurement campaign was carried out in the eastern part of the Southern Baltic Sea. During the implementation of the campaign, gravimetric measurements were made on planned measurement profiles. Analyses were carried out consisting of performing distribution of free-air anomalies located in the area covered by the gravimetric measurement campaign for two values - the ellipsoidal height of the POLREF-GORA DONAS benchmark. For the first distribution, benchmark values $g = 981405.9225$ mGal and $H = 201.653$ m were assumed, and for the second distribution, $g = 981405.9225$ mGal and $H = 201.974$ m were taken. Subsequently, the differences between these values were calculated, as shown in Figure 7.

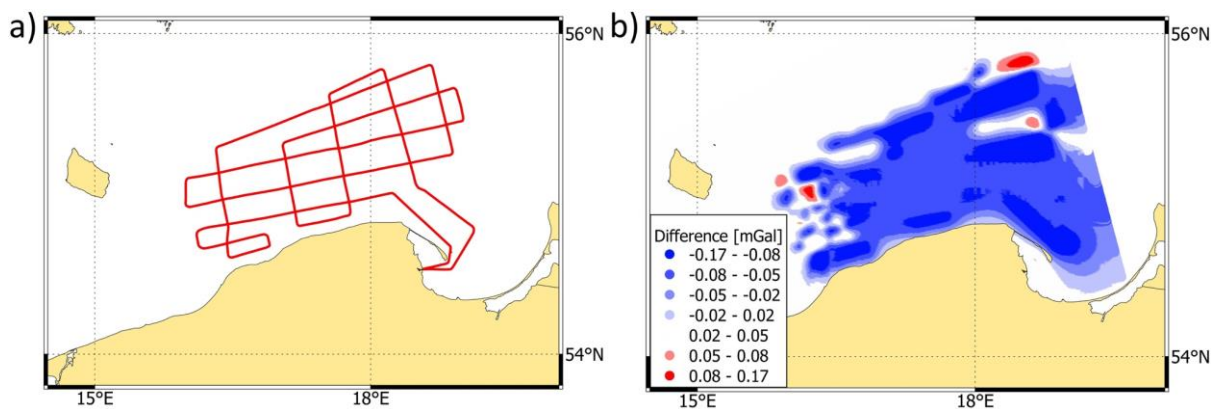


Fig. 7. ORP "Heweliusz" measurement campaign carried out from 07 to 10 June 2021: (A) measurement profiles, (B) free-air anomaly difference distribution.

Used in the analyses of the EGG2015 quasigeoid full resolution (1.0' x 1.0'). The values refer to GRS80 and the zero tide system. The entire data analysis was performed assuming the accuracy of measurements in individual phases of the campaign obtained by the Gdańsk University of Technology team. The analyses included offset measurements of the gravimeter, IMU, and GNSS antenna made on the ship using a tachymeter with

less than 0.01 m precision. The determination of the difference in the height of the ship's offset point located on the side and the reference point located on the quay was made one hour before the ship's departure and one hour upon arrival at the port. A leveller was used for measurements.

The positioning of the ship was obtained by raw GNSS data post-processing recorded during the campaign. Data were

obtained from reference stations of the HxGN Smartnet network along the Polish coast and the SWEPOS network along the southern coast of Sweden. The calculation process included precision ephemeris of GPS and GLONASS satellites shared by IGS (International GNSS Service)[44]. Post-processing of GNSS data was carried out using original scripts. Points with height accuracies ranging from 0.01 m to 0.12 m were assumed to be adequate for the processing of gravimetric measurements.

The presented case allowed us to consider the issue of using gravimetric data to compensate for ultra-precise inertial navigation. Until the end of the 20th century, gravity data mainly used ultra-precise inertial navigation systems (INS) in defence applications. In the 21st century, due to the intensive evolution of autonomous vehicles, these systems began to be used to support this technology [45,46]. The constant improvement in inertial sensors reliability [47] significantly impacts the development of this technology. INS gravity compensation is related to the accuracy and resolution of the horizontal gravity disturbance. In this context, the requirements for the resolution and precision of gravity data in marine areas are important to consider. It should be noted that the error of such data should not exceed 1 mGal [48]. Let us consider our case in the context of meeting such a requirement: how precisely does the accuracy of the gravity at a reference point located on land determine the distribution of this value over the areas covered by the marine survey campaign? This distribution is illustrated using the example of free-air anomalies.

The relation between gravity on the geoid and on the earth's surface, according to [49], can be approximated by the Taylor expansion 1.

$$g_0 = g - \frac{\partial g}{\partial H} H + \dots \quad (1)$$

Where g_0 is the gravity on the geoid, g is the gravity on the earth's surface at the point at the height H over the geoid. The

$\frac{\partial g}{\partial H}$ is the vertical gradient of the gravity.

For small values of H , the linear term in equation 1 is sufficient, and the rest of the terms can be neglected. If we assume that there are no masses (or existing mass can be neglected) between the geoid and the point of measurement, equation 1 can be rewritten into equation 2.

$$g_0 = g + F \quad (2)$$

where F is the free-air reduction defined by the equation 3.

$$F = -\frac{\partial g}{\partial H} H \quad (3)$$

By the assumption that the $\frac{\partial g}{\partial H}$ is the normal free air gradient, the free air anomaly can be defined by equation 4.

$$\Delta g_F = g + F - \gamma \quad (4)$$

Where Δg_F is the free air anomaly, g is the measured gravity, and γ is the normal gravity.

Assuming that the difference in the ellipsoidal height is 0.321 m of the gravimetric control point used to transfer the gravity to the ship's mooring berth and assuming that the free-air gravity gradient is 0.3086 mGal/m, the spread in extreme cases in the area covered by the measurements is 0.346 mGal. Difference peak-to-peak amplitude is approximately three times higher than the estimation based on a simple multiplication of height offset times standard gradient value. This difference does not create a uniform offset over the entire area, and the distribution of the free-air anomaly values is strongly differentiated.

The explanation of this effect can be attributed to the way in which the grids of free-air gravity are made. The commonly employed procedure is to perform the interpolation of the scattered gravity anomaly data by least-squares collocation (Kriging) using a 2nd-order Markov covariance model. The basic principle of Kriging is the estimation of the unknown value \hat{Z} in the grid point s_0 basing on the N values $Z(s_i)$ of known points scattered in space with equation 5.

$$\hat{Z}(s_0) = \sum_{i=1}^N \lambda_i Z(s_i) \quad (5)$$

Where λ_i are the weights. The weights are to be estimated based on the semivariogram [49] constructed from all measurement data. Because of that, the strength of the dependence of the final grid value depends on the statistical properties of all data points in the considered area. Due to the continuity of the gravitational field, a grid of free-air anomaly is never constructed from data collected during a single campaign. The data from other campaigns is used to densify/pad the current one. In such a case, the unremoved offset between campaigns will create a non-optimal semivariogram, further increasing the error from what could be suspected based on the knowledge about the offset of the pier reference point.

5. Conclusion

For gravity data that are intended to be used in ultra-precise inertial navigation systems, the data should be as accurate as possible. When the data is recorded with a relative gravimeter, each stage of field measurements must be analysed in terms of the possibility of making a measurement error. Measurement teams are not able to check all control points used as references, so they must assume catalogue values to be reliable. In the case discussed, transferring the gravity values from two points of the geodetic control network, a divergence in the results of 0.3 mGal was observed. It was decided to check the catalogue parameters of one of the control points. Analyses of the measurement results showed differences between the catalogue and actual values. Applying this data to the gravimetric naval campaign showed how the results were affected. The following conclusions are based on the results of the analyses presented in the article.

- Measurement campaigns carried out with dynamic gravimeters at sea are costly projects. For this reason, special care should be taken when establishing the reference to the national geodetic networks for 3D positions and gravity. If possible, the values obtained for reference points determined on the port should be independently verified using references to the largest possible number of points from the absolute gravity network.

- The occurrence of a gravity value offset in one

measurement campaign may be difficult to notice when the number of intersection points with other campaigns is small. It should be noted that the case discussed in the article concerns a campaign that begins and ends in one port and is related to one point. Assuming the diligence of the team's work, the offset values introduced by errors in linking are small. However, if they are not removed, they lead to non-uniform anomaly deviations in the study area, mainly related to the impact of data from neighbouring measurement campaigns on the values obtained in the resulting grid. This may be one of the sources of errors in the distribution of functionals of the gravity field. Accurate determination of these functionals is crucial for ultra-precise inertial navigation. It was noted that, according to [2], the gravity error of magnitude 0.3 mGal after one Schuler period will generate the horizontal error in INS estimated position up to 4.5m. Therefore, the problem of the existence of small gravity offsets in the links should not be underestimated for future applications of the data in inertial navigation.

- Assuming the diligence of the team's work, the offset values introduced by errors in linking are small. However, if they are not removed, they lead to anomaly deviations in the study area, mainly related to the impact of data from neighbouring measurement campaigns on the values obtained in the resulting grid. The demonstrated effect of changing the vertical coordinate of the gravimetric reference point in relation to the distribution of the free-air correction did not show an equal distribution trend.

Acknowledgements

The measurement campaigns were co-financed under the projects European Union from the European Regional Development Fund under the 2014-2020:

Operational Programme Smart Growth, the project entitled "Development of technology for acquisition and exploration of gravimetric data of foreshore and seashore of Polish maritime areas" was implemented as part of the National Centre for Research and Development competition: 1/4.1.4/2018 "Application projects"

Operational Programme of the Pomeranian Voivodeship, the project entitled "Development of a multimodal platform based on the functioning of electro-optical sensors" was implemented as part of the Pomerania Development Agency competition: RPPM.01.01.01-2-0010/17 "Sub-activity 01.01.01 Expansion through innovation - grant support".

Computations were carried out using the computers of Centre of Informatics Tricity Academic Supercomputer & Network (Grant number PT01010).

The final processed dataset. DOI 10.34808/rswn-ma26

References

Liu F, Li F, Jing X. INS/gravity gradient aided navigation based on gravitation field particle filter. *Open Physics* 2019;17:709–18.

<https://doi.org/10.1515/phys-2019-0073>.

2. Zhang P, Wu L, Bao L, Wang B, Liu H, Li Q, et al. Gravity disturbance compensation for dual-axis rotary modulation inertial navigation system. *Frontiers in Marine Science* 2023;10. <https://doi.org/10.3389/fmars.2023.1086225>.
3. Peshekhonov VG. High-Precision Navigation Independently of Global Navigation Satellite Systems Data. *Gyroscopy and Navigation* 2022;13:1–6. <https://doi.org/10.1134/S2075108722010059>.
4. Ince ES, Förste C, Barthelmes F, Pflug H, Li M, Kaminskis J, et al. Gravity Measurements along Commercial Ferry Lines in the Baltic Sea and Their Use for Geodetic Purposes. *Marine Geodesy* 2020;43:573–602. <https://doi.org/10.1080/01490419.2020.1771486>.
5. Förste C. Marine Gravimetry Activities on the Baltic Sea in the Framework of the EU Project FAMOS. *Zfv – Zeitschrift Für Geodäsie, Geoinformation Und Landmanagement* 2020:287–94. <https://doi.org/10.12902/zfv-0317-2020>.
6. Wu B, Zhang C, Wang K, Cheng B, Zhu D, Li R, et al. Marine Absolute Gravity Field Surveys Based on Cold Atomic Gravimeter. *IEEE Sensors Journal* 2023;23:24292–9. <https://doi.org/10.1109/JSEN.2023.3309499>.
7. TOMODA Y. Gravity at sea -A memoir of a marine geophysicist-. *Proceedings of the Japan Academy, Series B* 2010;86:769–87. <https://doi.org/10.2183/pjab.86.769>.
8. Rotich J. K., Gachari M.K., Mundia C.N. Satellite positioning based extension of geodetic reference network to support geospatial applications. *Journal of Applied Science, Engineering and Technology for Development* 2018. <https://doi.org/10.33803/JASETD.2017.3-1.2>.
9. Feng D. Review of Quantum navigation. *IOP Conference Series: Earth and Environmental Science* 2019;237:032027–032027. <https://doi.org/10.1088/1755-1315/237/3/032027>.
10. Hoang NH. Modernization of Height System in Vietnam Using GNSS and Geoid Model. *Lecture Notes in Civil Engineering*, vol. 108, Springer Science and Business Media Deutschland GmbH; 2021, p. 149–66. https://doi.org/10.1007/978-3-030-60269-7_8.
11. Hein GW. From GPS and GLONASS via EGNOS to Galileo – Positioning and Navigation in the Third Millennium. *GPS Solutions* 2000;3:39–47. <https://doi.org/10.1007/PL00012814>.
12. Dautermann T, Felux M, Grosch A. Approach service type D evaluation of the DLR GBAS testbed. *GPS Solutions* 2012;16:375–87. <https://doi.org/10.1007/s10291-011-0239-3>.
13. Guo J, Liu X, Chen Y, Wang J, Li C. Local normal height connection across sea with ship-borne gravimetry and GNSS techniques. *Marine Geophysical Research* 2014;35:141–8. <https://doi.org/10.1007/s11001-014-9216-x>.
14. Peshekhonov VG, Sokolov AV, Zheleznyak LK, Bereza AD, Krasnov AA. Role of Navigation Technologies in Mobile Gravimeters Development. *Gyroscopy and Navigation* 2020;11:2–12. <https://doi.org/10.1134/S2075108720010101>.
15. Peshekhonov VG, Stepanov OA, editors. *Methods and Technologies for Measuring the Earth’s Gravity Field Parameters*. vol. 5. Cham: Springer International Publishing; 2022. <https://doi.org/10.1007/978-3-031-11158-7>.
16. Pyrchla K, Pajak M, Pyrchla J, Idczak J. Analysis of Free-Air Anomalies on the Seaway of the Gulf of Gdańsk: A Case Study. *Earth and Space Science* 2020;7:e2019EA000983.
17. Bielecka E, Pokonieczny K, Borkowska S. GIScience Theory Based Assessment of Spatial Disparity of Geodetic Control Points Location. *ISPRS International Journal of Geo-Information* 2020;9:148–148. <https://doi.org/10.3390/ijgi9030148>.
18. Vivek CG, Shringeshwara TS, Jade S. GNSS and its Impact on Position Estimates. *Current Science* 2020;119:1503–1503. <https://doi.org/10.18520/cs/v119/i9/1503-1509>.
19. Luo X, Schaufler S, Branzanti M, Chen J. Assessing the benefits of Galileo to high-precision GNSS positioning – RTK, PPP and post-processing. *Advances in Space Research* 2021;68:4916–31. <https://doi.org/10.1016/j.asr.2020.08.022>.
20. Olivart I Llop JM, Moreno-Salinas D, Sánchez J. Full Real-Time Positioning and Attitude System Based on GNSS-RTK Technology. *Sustainability* 2020, Vol 12, Page 9796 2020;12:9796–9796. <https://doi.org/10.3390/SU12239796>.
1. Maciuk K. Aging of ground Global Navigation Satellite System oscillators. *Eksploatacja i Niezawodność – Maintenance and Reliability* 2022;24:371–6. <https://doi.org/10.17531/EIN.2022.2.18>.
2. Calka B, Bielecka E, Figurski M. Spatial pattern of ASG-EUPOS sites. *Open Geosciences* 2017;9. <https://doi.org/10.1515/geo-2017-0046>.
3. Krzyżek R, Skorupa B. Analysis of accuracy of determination of eccentric point coordinates of the KRAW permanent geodetic station in RTK GPS measuring mode with the application of the NAWGEO service of the ASG-EUPOS system. *Geomatics and Environmental Engineering* 2012;6:35–35. <https://doi.org/10.7494/geom.2012.6.4.35>.

24. Dawidowicz K, Krzan G, Świątek K. Urban area GPS positioning accuracy using ASG-EUPOS POZGEO service as a function of session duration. *Artificial Satellites* 2014;49:33–42. <https://doi.org/10.2478/arsa-2014-0003>.
25. Pyrchla K, Pyrchla J. The Use of Gravimetric Measurements to Determine the Orthometric Height of the Benchmark in the Port of Gdynia. *Proceedings - 2018 Baltic Geodetic Congress, BGC-Geomatics 2018* 2018;349–52. <https://doi.org/10.1109/BGC-Geomatics.2018.00072>.
26. Koivula H, Kuokkanen J, Marila S, Lahtinen S, Mattila T. Assessment of sparse GNSS network for network RTK. *Journal of Geodetic Science* 2018;8:136–44. <https://doi.org/10.1515/jogs-2018-0014>.
27. Mora OE, Langford M, Mislant R, Josenhans R, Chen J. Precision performance evaluation of RTK and RTN solutions: a case study. *Journal of Spatial Science* 2022;67:473–86. <https://doi.org/10.1080/14498596.2020.1837686>.
28. Riley S, Talbot N, Kirk G. A new system for RTK performance evaluation, *IEEE*; 2000, p. 231–6. <https://doi.org/10.1109/PLANS.2000.838307>.
29. El-Mowafy A. Performance Analysis of the RTK Technique in an Urban Environment. *Australian Surveyor* 2000;45:47–54. <https://doi.org/10.1080/00050353.2000.10558803>.
30. Sohn DH, Park KD. A Study on Pseudo-Range Correction Modeling in order to Improve DGNSS Accuracy. *Journal of Korean Society for Geospatial Information System* 2015;23:43–8. <https://doi.org/10.7319/kogsis.2015.23.4.043>.
31. Weng D, Ji S, Lu Y, Chen W, Li Z. Improving DGNSS Performance through the Use of Network RTK Corrections. *Remote Sensing* 2021;13:1621–1621. <https://doi.org/10.3390/rs13091621>.
32. Cai C, Gao Y, Pan L, Zhu J. Precise point positioning with quad-constellations: GPS, BeiDou, GLONASS and Galileo. *Advances in Space Research* 2015;56:133–43. <https://doi.org/10.1016/j.asr.2015.04.001>.
33. Choy S. High accuracy precise point positioning using a single frequency GPS receiver. *Journal of Applied Geodesy* 2011;5. <https://doi.org/10.1515/jag.2011.008>.
34. Tomasz H. GNSS-Warp Software for Real-Time Precise Point Positioning. *Artificial Satellites* 2015;50:59–76. <https://doi.org/10.1515/arsa-2015-0005>.
35. Paziewski J, Stepniak K. *New On-line System for Automatic Postprocessing of Fast-static and Kinematic GNSS Data*, Vilnius, Lithuania: Vilnius Gediminas Technical University Press “Technika” 2014; 2014. <https://doi.org/10.3846/enviro.2014.235>.
36. Julianto EN, Safrel I, Taveriyanto A. High Accuracy Geodetic Control Point Measurement Using GPS Geodetic With Static Methods. *Jurnal Teknik Sipil Dan Perencanaan* 2018;20:81–9. <https://doi.org/10.15294/jtsp.v20i2.16300>.
37. Taufik M, Yuwono, Cahyadi MN, Putra JR. Analysis level of accuracy GNSS observation processing using u-blox as low-cost GPS and geodetic GPS (case study: M8T). *IOP Conference Series: Earth and Environmental Science* 2019;389:012041–012041. <https://doi.org/10.1088/1755-1315/389/1/012041>.
38. Sun R, Qiu M, Liu F, Wang Z, Ochieng WY. A Dual w-Test Based Quality Control Algorithm for Integrated IMU/GNSS Navigation in Urban Areas. *Remote Sensing* 2022;14:2132–2132. <https://doi.org/10.3390/rs14092132>.
39. Rofatto VF, Matsuoka MT, Klein I. An Attempt to Analyse Baarda’s Iterative Data Snooping Procedure based on Monte Carlo Simulation. *South African Journal of Geomatics* 2017;6:416–416. <https://doi.org/10.4314/sajg.v6i3.11>.
40. Ge Z, Zhang Y, Wang F, Luo X, Yang Y. Virtual–real fusion maintainability verification based on adaptive weighting and truncated spot method. *Eksploracja i Niezawodność* 2022;Vol. 24:738–46. <https://doi.org/10.17531/EIN.2022.4.14>.
41. Zhang Q, Zhao L, Zhao L, Zhou J. An Improved Robust Adaptive Kalman Filter for GNSS Precise Point Positioning. *IEEE Sensors Journal* 2018;18:4176–86. <https://doi.org/10.1109/JSEN.2018.2820097>.
42. Kim TK. T test as a parametric statistic. *Korean Journal of Anesthesiology* 2015;68:540–540. <https://doi.org/10.4097/kjae.2015.68.6.540>.
43. Kallio U, Koivula H, Lahtinen S, Nikkonen V, Poutanen M. Validating and comparing GNSS antenna calibrations. *Journal of Geodesy* 2019;93:1–18. <https://doi.org/10.1007/s00190-018-1134-2>.
4. Tran DT, Nguyen DH, Luong ND, Dao DT. Impact of the precise ephemeris on accuracy of GNSS baseline in relative positioning technique. *VIETNAM JOURNAL OF EARTH SCIENCES* 2020;43:96–110. <https://doi.org/10.15625/0866-7187/15745>.
5. Campbell S, Naem W, Irwin GW. A review on improving the autonomy of unmanned surface vehicles through intelligent collision avoidance manoeuvres. *Annual Reviews in Control* 2012;36:267–83. <https://doi.org/10.1016/j.arcontrol.2012.09.008>.
6. Kozłowski E, Borucka A, Oleszczuk P, Jałowicz T. Evaluation of the maintenance system readiness using the semi-Markov model taking into account hidden factors. *Eksploracja i Niezawodność – Maintenance and Reliability* 2023;25. <https://doi.org/10.17531/ein/172857>.

47. Chi B, Wang Y, Hu J, Zhang S, Chen X. Reliability assessment for micro inertial measurement unit based on accelerated degradation data and copula theory. *Eksploatacja i Niezawodność – Maintenance and Reliability* 2022;24:554–63. <https://doi.org/10.17531/EIN.2022.3.16>.
48. Kwon JH, Jekeli C. Gravity Requirements for Compensation of Ultra-Precise Inertial Navigation. *Journal of Navigation* 2005;58:479–92. <https://doi.org/10.1017/S0373463305003395>.
49. Hofmann-Wellenhof B, Moritz H. *Physical geodesy*. Springer Science & Business Media; 2006.