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Holistic collision avoidance decision support system for watchkeeping deck officers

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ABSTRACT

The paper presents a 3-stage synthesis-based Decision Support System for watchkeeping deck officers. Its functional scope covers conflict detection, maneuver selection, and maneuver execution, all phases supplemented by collision alerts. First, a customized elliptic ship domain is used for checking if both OS and TS will have enough free space. A survey-based navigators' declarative OS arena is then used to determine the time at which OOW would like to take evasive action. Next, a safety level is assigned to the current situation based on the predicted violations of the ship domain and the declarative arena. The safety levels are also attributed to potential evasive maneuvers (single actions combining course alteration and rudder deflection). For a selected maneuver, Collision Avoidance Dynamic Critical Area (CADCA) is displayed, which informs OOW about the time window when the maneuver remains feasible. All of the above contribute to a holistic system of multi-level safety assessment utilizing: empirical ship domain, survey-based declarative arena, and ship dynamics-based CADCA. These, in turn, take into account navigators' knowledge and preferences, ship maneuverability, and the impact of environmental conditions. The system is presented in three real-life scenarios located in the southern part of the Baltic Sea around the Danish straits.

1. Introduction

The research on ship collision avoidance is ongoing and a few trends can be distinguished here, mostly focused on methods used in various stages of an encounter situation. Among other lines of division, a ship encounter can be broken into conflict detection and conflict resolution phases. It is worth emphasizing here that a conflict is not synonymous with a collision. A conflict occurs if two or more vessels are on collision courses. A collision should then be avoided by means of taking an appropriate action. In the conflict detection phase, the Collision Risk Index (CRI) or other related measure is used to assess whether a target poses a threat and to quantify its level. CRI may be additionally incorporated in a Collision Alert System (CAS), which in turn informs the Officer of the Watch (OOW) about targets that either require an evasive action, or – at the very least – should be paid attention to. A CAS can also utilize the concept of a critical area around the ship, whose approaching should trigger an action to avoid an incident. As for conflict resolution, collision avoidance algorithms may differ in their functional scopes. A basic solution would only propose a certain maneuver: course alteration, speed reduction, or a combination of both. A more advanced approach would be to find a collision-free path, that would additionally include getting back to the initial course or pre-planned trajectory, once the

Abbreviations: AIS, Automatic Identification System; ARPA, Automatic Radar Plotting Aid; CADCA, Collision Avoidance Dynamic Critical Area; CAS, Collision Alert System; CD&R, Conflict Detection & Resolution; COLREG, International Regulations for Preventing Collisions at Sea; CRI, Collision Risk Index; DCPA, Distance at Closest Point of Approach; DDV, Degree of Domain Violation; DOF, Degrees of Freedom; DSS, Decision Support System; GUI, Graphical User Interface; GVO, Generalized Velocity Obstacle; MDTC, Minimum Distance to Collision; OOW, Officer Of the Watch; OS, Own Ship; PoC, Proof of Concept; TCPA, Time to Closest Point of Approach; TCR, Time-varying Collision Risk; TDE, Time to Domain Exit; TDV, Time to Domain Violation; TS, Target Ship.

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target has been safely passed. It is worth noting that there is also a concept that can be applied to both conflict detection and its resolution, namely – a ship domain. Such a domain, by its classic definition, is an area around the ship, that the OOW prefers to keep free of other ships. It must be emphasized here, that this is not synonymous with a critical area, whose entering or approaching should result in immediate action. In order to keep the ship domain unviolated a much earlier action is needed. Thus, a space-defined ship domain cannot be directly used as a criterion for determining the time when a maneuver must be performed [1]. Instead, when applied correctly, it can tell OOW if a maneuver is needed at all and whether a particular action will be safe, in terms of keeping the domain unviolated throughout the encounter situation.

While the literature is abundant in works documenting research on collision avoidance-related systems, rarely covers simultaneously conflict detection, risk assessment, and conflict resolution. The majority of available CASs are primarily focused only on conflict detection and risk assessment [2–5]. They can provide indirect conflict resolution, usually forcing an operator to observe fluctuations of the risk metrics to empirically find a safe maneuver, which is impractical. There is also another class of collision avoidance tools aiming at online assessing direct risk metrics for various possible maneuvering [6–8], thus helping the operator to choose a safe maneuver as a conflict resolution. However, these proposals do not work proactively as they do not detect conflicts and thereby cannot be considered full-range collision avoidance solutions. Lastly, there are state-of-the-art methods covering all three phases, namely conflict detection, risk assessment, and conflict resolution [9,10] but they, in turn, are not able to present an overview of possible evasive actions. A detailed description of the state-of-the-art collision avoidance solutions is provided later in Section 2 (Related works).

The limited functional scope of the previously presented solutions has been the motivation for the research documented here. Namely, the goal was to design a DSS (Decision Support System) that would cover all three above-mentioned phases of an encounter situation and would offer quantitative, deterministic, and time-efficient methods for handling each of those phases. Additionally, it should enable the operator to choose a feasible evasive maneuver presented in a meaningful way, whenever such a possibility does exist. As evidenced by Lan et al. [11] “failure to determine the risk” and “failure to take effective collision avoidance action early” are among the most frequent risk factors associated with severe ship collisions. The solution proposed here would help mitigate these factors by improving awareness of the risk associated with a particular maneuver as well as facilitating the selection of the safest and most effective one.

The rest of the paper is organized as follows. Related works are discussed in Section 2, which is followed by an extensive description of the methodology in Section 3. The results of the conducted simulation study are provided in Section 4. The system’s limitations and possible future developments are outlined in Section 5. Finally, conclusions are given in Section 6.

2. Related works

Various CASs and DSSs focused on maritime collision avoidance and assurance of ship safety have been introduced so far on different levels of maturity [12,13]. Concerning on-board ship CAS, the proposed concepts of the systems differ in utilized approach, their features, complexity, or application. For instance, Du et al. [2] proposed CAS where conflict severity classes based on ship intentions, conflict evolution, and COLREG (International Regulations for Preventing Collisions at Sea) rules are linked to typical encounter stages. This approach is, however, limited to the stand-on ship perspective. A more universal approach was proposed by Goerlandt et al. [3], establishing strong grounds for maritime risk-based CAS by proposing a theoretical framework. Based on the fuzzy expert system, the authors used many proximity indicators as well as the COLREG status of the vessel to validate the application of the framework proposed in four different types of encounter scenarios.

Also, many propositions working on the basis of the radar data or their visualization may be met in the literature. Exemplary solutions utilizing the radar display were proposed in [6] and further developed in [7]. In both papers, the authors proposed a complex approach to the visualization of the safe parameters of the vessel. The proposed approaches use the Collision Threat Parameters Area (CTPA) technique [14] to determine an area that is unsafe for own ship and thus should be avoided. The CAS proposed by the authors takes into account many features from analyzing safe combinations of the own course and speed, through supporting ship domain and restricted water navigation, to consideration of ship stability-related issues. Ożoga and Montewka [8] proposed the development of one of the most popular CAS used commonly at sea, namely ARPA (Automatic Radar Plotting Aid), by introducing the concept of Multi-ARPA. This aid allows visualization in a user-friendly way of direct and indirect hazards on the radar screen, raising an alarm at an early stage, and finally, ensuring maintaining safe passing distance during maneuver execution.

A quite popular approach utilized in the maritime CAS binds triggering specific action (such as raising a warning or deciding on an evasive maneuver) with crossing subsequent zones or violating specific limits by an obstacle. Such a multi-stage approach focused on violating given zones by a target is usually rooted in the CD&R (Conflict Detection and Resolution) concept which was originally used in aviation [15,16], especially in airborne CAS [17]. As often happens, certain solutions have been transferred between the modes of transportation and then adapted to their specific (user) needs. In this case the specificity of maritime transportation and vessel operation in the marine environment [9,18,19]. One of the most advanced proposals based on this pattern is MTCAS proposed by Denker et al. [9] and developed later by Steide and Hahn [10]. MTCAS combines conflict detection and resolution phases and as a result, can propose an evasive maneuver. Concerning the CD&R foundations, Huang and van Gelder [4] proposed a concept of a maritime alert system dedicated primarily to autonomous ships. The solution uses Time-varying Collision Risk (TCR) [20] along with the GVO (Generalized Velocity Obstacle). An extended version of the algorithm utilizing real-time TCR (R-TCR) considering the motion uncertainty of target ships was proposed by Li et al. [21].

Rooted in a similar approach, Szlapczynski and Szlapczynska [5] proposed an advanced two-stage CAS based on the ship domain approach and utilizing inner (domain) and outer-rescaled (arena) boundaries surrounding a vessel to raise an appropriate alert. Consequently, the authors used domain-related indicators to detect unsafe ship behavior during an encounter, and eventually, a close-quarters or near-miss situation through the violation of the area. A two-stage approach utilized in the mentioned CAS brings a significant benefit due to the possible differentiation of critical and non-critical actions to be taken during ship encounters. However, for both boundaries, the same type of (properly scaled) ship domain was used, which neither fully reflects the seafarer’s preferences (the outer one) nor the critical actions to be taken (the inner one).

Despite a variety of solutions proposed, many of them are still based on the well-known and popular concept of the ship safety domain. The notion was used for the very first time by Fuji et al. [22]. He proposed the domain as a two-dimensional area around the ship considered an area of evasion. Dimensions of the domain’s elliptic contour were defined by the distance from the central ship at which the density of other ships reaches a local maximum. The concept of ship domain was then further developed by Goodwin [23]. Based on open-water maritime traffic surveys, she proposed the domain as an area around the ship that a navigator would like to keep free from other ships and obstacles. Next to that, Coldwell in [24] proposed the domain as the effective area around a ship that a navigator actually keeps free with respect to the traffic. Apart from those definitions, Davis et al. [25] proposed a notion of a ship arena, a much bigger area surrounding the ship domain. Entering the arena by the other ship should trigger a collision-avoidance action in order to prevent violation of the actual ship domain.

In general, the main difference between the abovementioned domain and arena concepts lies in the intention of its utilization [26]. Namely the fact, a navigator wants to keep free, has to keep free, or just keeps free from other objects [27]. That is why, in the following years, a few scholars proposed differentiation between the domains depending on their usage. The purpose of their utilization may be maintaining some (safe) passing distance during routine passages or a kind of critical distance in a close-quarters situation [1]. To distinguish both purposes and the navigator's intentions, the researchers sometimes consider separately declarative and effective ship domains [28], or in other words, subjective and objective ones [29], which are sometimes also called desired and forbidden boundaries [30].

The subjective (declarative) domains are delivered using navigators' experience and are usually classified as knowledge- or expert-based. These relied mainly on answers collected from questionnaires fulfilled by the practitioners [31–33] or from the interviews carried out [34–37]. For instance, Lee et al. [37] interviewed 125 seafarers to propose the ship domain constructed using threshold values of DCPA (Distance at the Closest Point of Approach) embedded further into the theory of situational awareness. However, despite a large data sample obtained, the values were gathered for the limited number of ship encounter scenarios. Additionally, the distances given by the practitioners were provided for the confined waters only and based on various ship models (according to the seafarer's experience without unification). Pietrzykowski and Uriasz [33] were focused on the declared safe passing distances provided by the practitioners in open waters, while a year earlier Pietrzykowski [32] considered experts' knowledge in the fuzzy ship domain dedicated to restricted waters. Wielgosz [31] asked 153 participants of the nautical courses about their opinion on the distance that should be kept between the own and target ship being located in Singapore Strait at eight different relative bearings.

Nevertheless, despite extracting valuable navigators' knowledge and providing interesting insights, the vast majority of the studies on declarative ship domains consider the ship passing distance while omitting the distance of an evasive maneuver execution. Therefore, these works do not link directly the collision avoidance process with the ship's maneuverability, and thereby the time required to perform preplanned course alteration, as well as the distance traveled toward the target, before achieving the final passing distance and desired ship course.

On the other hand, the objective (effective) domains are mainly determined through statistical analysis of maritime traffic or analytically using mathematical models. The first approach utilizes observations gathered from RADAR [22,23] or messages obtained from AIS (Automatic Identification System) [38–40]. The second group of methods usually uses computer simulations with some mathematical formulations implemented [41–45]. The variety of effective ship domains or areas bearing different names but, in fact, fulfilling a similar role is much larger than in the case of the declarative ones. This concerns not only the shape and sizes of their envelopes but also the parameters taken into account and methods of their determination. For instance, some of the proposed concepts utilize fuzzy logic [46,47], while others deal with risk-related indicators [30,48,49].

A special kind of effective ship domain is the concept called critical area [50–52]. Its limits are directly linked to the distance of an effective evasive maneuver execution and through this, the ship's maneuverability is also taken into account. In its foundations, the critical area is somehow similar to the arena concept introduced by Davis et al. [25]. However, the critical area considers the last-chance maneuver of the vessel for given operational conditions, thus the violation of the envelope leads unavoidably to the collision. In contrast, Davis's arena also informs about the distance at which the maneuver should be executed, not to indicate the last opportunity to avoid a collision but to prevent violation of the inner ship domain.

It is of note, that the abovementioned distinction between the effective and declarative domains collated with a way used for their determination is not always so obvious. This may be blurry and ambiguous depending on the ship domain definition assumed, as well as the interpretation of the method utilized for a domain determination. For instance, in the literature, some examples of studies can be noted, where direct seafarers' engagement was used to determine an effective not a declarative domain, like during collision avoidance exercises conducted on a navigational simulator [28]. That is why, Wielgosz and Pietrzykowski [53] additionally argued that an effective domain is a final result of an application by a navigator of the additional factors affecting ship maneuvers, which, in fact, differs from the initially declared values. Similarly, some of the authors raised that the traffic analysis method may not be fully compliant with an understanding of the effective or objective kind of ship domains, as the subjectivity of navigators' behavior results in the traffic flow or ship maneuvers analyzed [27,29].

To sum up, there are several available conflict detection and resolution solutions designed for maritime transportation. Table 1 presents state-of-the-art ship CD&R solutions and their features. As it may be seen, hardly any solution presented there supports all three: conflict detection, risk assessment, and direct conflict resolution. The sole exception ([9,10]) does offer direct conflict resolution, but proposes only a single maneuver without informing OOW about any alternative evasive actions. This can be identified as a gap, that the current paper aims to bridge. Moreover, different types of ship domains or arenas should be applied, depending on a particular purpose. In the case of a system, whose functions include conflict detection as well as its resolution (CD&R), a separate area is needed at each stage to properly address various phases of the maritime collision avoidance process. Until now, there has been no such system, which would utilize a dedicated area for each of the above features. While some of them have been implemented in various DSS, they have never been applied in one coherent system. Thus, the original contribution of the paper, when compared with past works, is the combination of the following features in one, 3-stage DSS.

2.1. Contribution on the functional level

The system addresses separately the issues of conflict detection, collision alerts, maneuver selection, and maneuver execution. Quantitative assessment of the conflict-related risk is visualized by means of a color-denoted safety level, which is assigned to the current encounter situation. The potential single-action solutions to the conflict (combinations of rudder deflection and course alteration) are also visualized as color-marked safety levels. The user is then able to select a particular maneuver and see when this maneuver becomes a necessity. If the selected maneuver is no longer able to be effectively executed, a special alert is displayed so that the user can choose a different solution from the proposed set. All of the above contributes to a holistic system of unique functional scope and multi-level safety assessment, whose results are provided graphically in a synthesized and intuitive way.

2.2. Contribution on the computational level

Customized elliptic ship domain of configurable dimensions is applied through the authors-designed Degree of Domain Violation (DDV) measure, which is used for quantitative assessment of both the conflict-associated risk and the maneuver safety levels. This is accompanied by a navigators' survey-based declarative arena around OS, which is used to compute the time when OOW, according to his/her preferences, would like to take evasive action or expects TS to do so in a stand-on situation. Furthermore, the authors-invented Collision Avoidance Dynamic Critical Area (CADCA) is applied to determine the time window when a particular pre-selected evasive maneuver remains feasible. Summarizing, the contribution here is a combined use of three kinds of indicators (all of them introduced by the authors of the current

Table 1
Comparison of available CD&R solutions for ships.

Literature reference	Conflict detection	Risk assessment	Conflict resolution	Overview of possible evasive actions	Feasibility of conflict resolution	
					Considering bathymetry data	Modeling maneuver dynamics
[2]	yes	yes	**indirect** (operator can postpone the evasive action until the risk measure indicates it is safe)	no	no	yes
[3]	yes	yes	**indirect** (operator can postpone the evasive action until the risk measure indicates it is safe)	no	no	yes
[6]	no	yes (safe motion parameters)	**indirect** (indication of safe motion parameters per each manually selected maneuver)	yes	no	no
[7]	no	yes (safe motion parameters)	**indirect** (indication of safe motion parameters per each manually selected maneuver)	yes	no	no
[8]	no	yes	**indirect** (safe headings are depicted)	yes	no	no
[19]	yes	yes	no	N/A	N/A	N/A
[9,10]	yes	yes	**direct**(evasive maneuver proposed, maneuver auto-negotiation enabled)	no	yes	yes
[4,21]	yes	yes	**indirect** (operator can postpone the evasive action until the risk measure indicates it is safe)	yes	no	yes
[5]	yes	yes	**indirect** (operator can postpone the evasive action until the risk measure indicates it is safe)	no	no	yes

paper): DDV, a survey-based declarative arena, and CADCA, which are used as complementary elements of the proposed system's computational layer.

3. Methodology

In this section, the proposed system's architecture is presented in more detail. The system's diagram and overview are provided in Section 3.1, followed by a description of the declarative arena around the ship in Section 3.2. Notions regarding a ship domain that are utilized in the presented research are recalled in Section 3.3. In Section 3.4 maneuvering meta-model is briefly outlined. Bathymetric data taken into account in the research is depicted in Section 3.5. Next to that, details on maneuver safety levels and coloring codes are provided in Section 3.6. Finally, the critical area – CADCA is described in Section 3.7.

3.1. Overview - 3-stage synthesis-oriented DSS

An overview of the proposed Decision Support System is presented in Fig. 1. The proposed DSS applies three different concepts of a safe area around the ship. The first of them is the classic ship domain [24], which is herein used in two stages:

a) for assessing whether the target ship will be passed at a safe distance (whose value depends on relative bearing on her),
b) for assigning a safety level to a certain, single evasive maneuver depending on the passing distance and relative bearing when passing.

The authors' designed Degree of Domain violation (DDV) measure is applied here at both stages. The second of the applied concepts is that of a declarative arena around the own ship, whose violating would motivate most navigators to initiate evasive action. This area is used for deciding when to display an alert and information on potential maneuvers with their safety levels. The exact shape and size of this area are a result of a poll carried out among active navigators. Finally, the third and last of the applied concepts is the critical area around the ship – CADCA [54]. If two ships are on collision courses, a maneuver should be initiated before this area is entered, otherwise, it may not be possible to avoid a collision.

The system's Graphical User Interface (GUI), implemented in Python ver.3.9 and the standard tkinter package, is shown in Fig. 2. As can be seen, the GUI is divided into two window panels – the Maneuver Selection Display and the CADCA display.

The **Maneuver Selection Display** works as follows. The system monitors both the own ship's and the other vessels' positions, courses, and speeds. If another vessel is on collision course with own ship (OS) then, depending on the current distance, navigators-determined declarative arena, and predicted Degree of Domain Violation (DDV) [44], the safety level assigned to the current OS course is determined and updated. It is denoted by the color of the indicator in the middle of the coordinate system in the left panel. This is the first stage of the proposed DSS – **conflict detection**. Similarly, all potential single-action maneuvers (combinations of course alteration and rudder deflection values) are also denoted by colorful indicators representing their associated safety levels, resulting from predicted DDV. This is the second major phase – the **maneuver selection**, which contributes to conflict resolution. As soon, as the user selects any of the indicators representing potential maneuver, a specific critical area is shown in the right display, the so-called **CADCA display** for this particular combination of course alteration and rudder deflection. An envelope of CADCA informs OOW when a selected maneuver has to be started at the latest, to avoid a collision. The moment when the target enters the CADCA is the last one when there is still enough time and space to perform successfully a particular evasive maneuver. CADCA is thus an essential feature for the second part of conflict resolution – the **maneuver execution** phase.

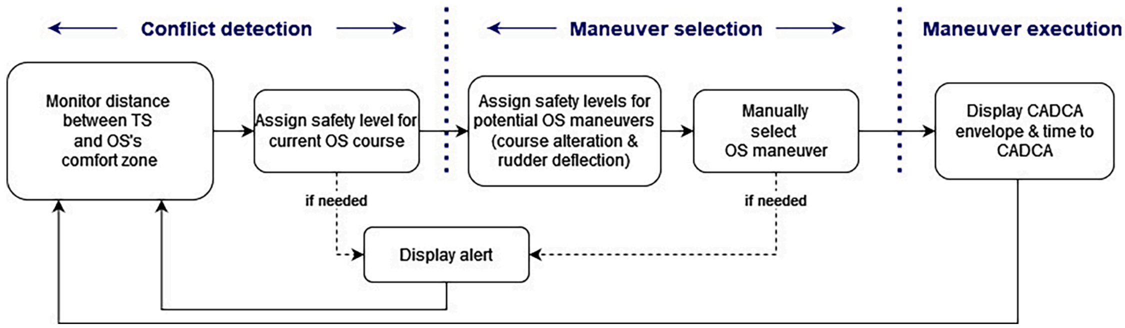

Fig. 1. Overview of the proposed Decision Support System.

3.2. Declarative arena according to navigators

In order to account for navigators' preferences concerning an expected distance to the target ship when taking evasive action, an expert elicitation was carried out. To this end, an online survey containing an interactive questionnaire has been distributed among practicing seafarers. A total of 43 participants holding at least the Officer of the Watch (OOW) diploma were invited to share their professional knowledge, including:

- Officers of the Watch (19 persons, 44.2% of respondents),
- Chief Officers (11 persons, 25.6% of respondents),
- Master Mariners (7 persons, 16.3% of respondents),
- Sea Pilots (6 persons, 13.9% of respondents).

The main aim of the survey was to determine the preferred distance, at which a navigator would like to execute an evasive maneuver against a single and clearly dangerous echo under favorable conditions concerning ship operation, weather, and maritime traffic. In the cases where according to COLREG, the own ship is a stand-on vessel, the participant assessed the distance at which (s)he would expect the target ship to execute the maneuver. The distances were specified using interactive sliders while a decision was made based on the ARPA display. The radar screen depicted the encounter situation along with relative vectors and typical proximity indicators, such as DCPA and TCPA.

Prior to the survey, each participant was familiarized with the objective of the study, research procedure, and maneuvering characteristics of the model vessel selected as a case study (i.e. pilot card and wheelhouse poster of the Ro-Pax ship). According to the research assumptions, the conditions in the navigational area surrounding own ship, both weather and traffic, were favorable. There was always only one threatening target, while the encounter situation allowed for its resolution according to the navigator's preference instead of making sudden or critical decisions. Each of the respondents was asked to assess the distances of maneuver execution for 12 different encounter situations, whereby the target was located at different bearings (every 30°). Additionally, the entire study was conducted twice for different types of navigational areas, namely for open sea and restricted waters.

Ultimately, the gathering and processing of experts' responses allowed for designing a ship arena reflecting navigators' preferences in

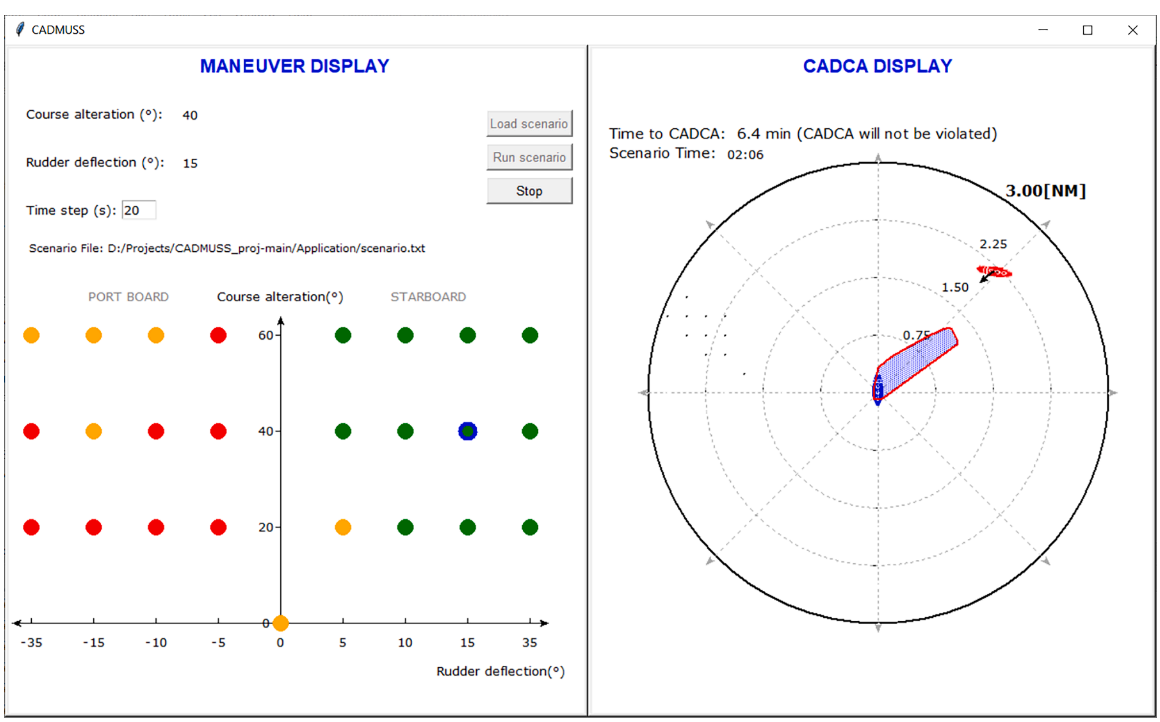

Fig. 2. GUI (Python 3.9, tkinter) of the proposed Decision Support System: The left panel (Maneuver Display) depicts color-coded safety levels of current OS situation (origin of the coordinate system) and possible maneuvers (selected maneuver depicted by a blue circle); right panel (CADCA Display) presents OS (blue icon) and TS (red icon) positions, areas with insufficient depth or land (grey dotted area) together with CADCA envelope (blue region with red borderline) for the currently selected maneuver.

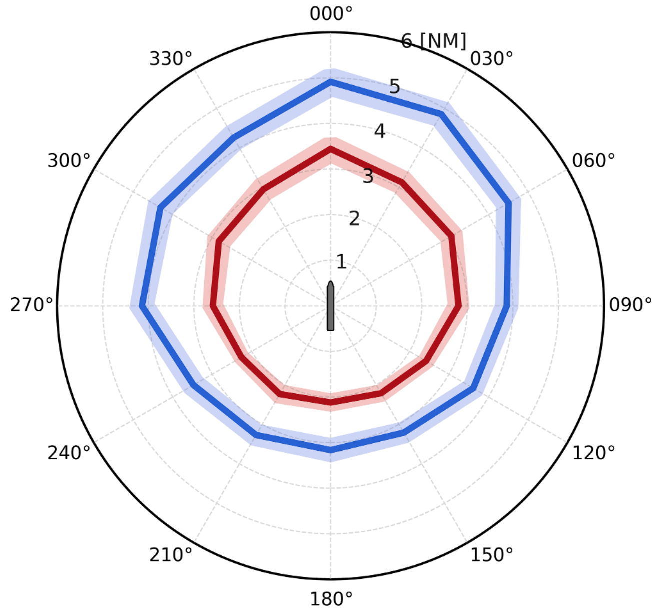

Fig. 3. The declarative arenas reflecting navigators' preferences obtained from expert elicitation.

collision avoidance rather than minimum safety/operational requirements. The data provided by each participant was aggregated using arithmetic mean and then analyzed concerning the type of navigational area as presented in Fig. 3. To account for potential uncertainties arising from the statistical inference using a relatively small sample of the respondents, the standard error was additionally calculated and considered within arenas' envelopes. For the navigational DSS proposed herein, the preference-based declarative arena enlarged by the band of standard error determined for open waters has been implemented and utilized as a part of the Conflict Detection (CD) module of the system.

Table 2
Summary of the model vessel and scenario parameters for delivered ship trajectories.

Own ship parameters	
Ship type	Ro-Pax
Length overall (LOA)	221.5 m
Beam (B)	32.0 m
Draft (T)	7.2 m
Scenario parameters (operational)	
Initial ship speed	$V_s \in \{16\ kts,\ 20\ kts\}$,
Rudder settings	$\delta \in \{\pm 5^{\circ}, \pm 10^{\circ}, \pm 15^{\circ}, \pm 35^{\circ}\}$
Course alterations	$\Delta\psi \in \{\pm 20^{\circ}, \pm 40^{\circ}, \pm 60^{\circ}\}$
Scenario parameters (environmental)	
Significant wave height	$H_S \in \{0.9\ m,\ 1.8\ m,\ 4.3\ m\}$
Wave peak period	$T_p \in \{4.5\ s,\ 5.5\ s,\ 6.5\ s\}$
Initial wave angle	$\mu \in \{0^{\circ},\ \pm 45^{\circ}, \pm 90^{\circ},\ \pm 135^{\circ},\ 180^{\circ}\}$

3.3. Ship domain and degree of domain violation

In [44] an approach factor f_{min} is defined for an encounter of two ships. It is a factor, by which one ship's domain has to be multiplied for the other ship to pass on the boundary of the f_{min}-scaled ship's domain (constant courses and speeds of both ships are assumed here).

$f_{min} \geq 1$ means safe passages and $f_{min} < 1$ represents domain violations. Additionally, the degree of domain violation (DDV) has been defined as:

$$DDV = \max(1 - f_{min}, 0) \tag{1}$$

An example illustrating f_{min} and DDV for an elliptic ship domain is given in Fig. 4. As one can see there, a target is approaching a central ship's domain and will violate it resulting in DDV equal to 0.26 and *fmin* – 0.74. Detailed formulas for determining f_{min} for such a domain have been provided in [44] while application of this factor to CAS in [5].

3.4. Trajectory-based meta-model of course alterations

In order to efficiently model own ship maneuvers in real-time, a meta-model for her trajectory prediction was required. To this end, the advanced LaiDyn ship motion model [55] was used as an exemplary

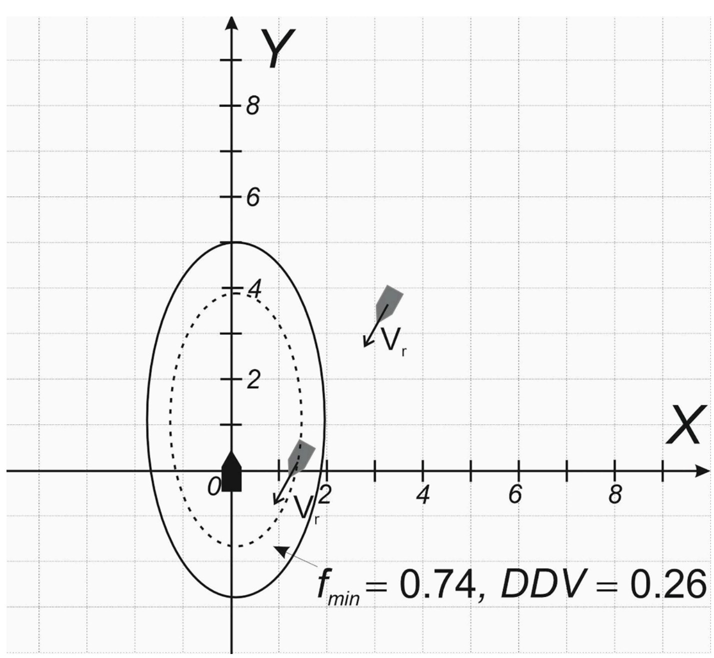

Fig. 4. A violation of a ship's domain presented in a relative coordinate system (Vr – relative speed).

source of ship motion data, in order to generate an extensive set of input OS trajectories delivered for a variety of operational and environmental conditions. To these belong: the magnitude of rudder angle (δ), ship speed (V_s), course alteration ($\Delta\psi$), significant wave height (H_s), wave peak period (T_p), and initial wave direction (μ). In this study, Ro-Pax ship type was used as a case study vessel in the OS role. The ranges of the considered parameters' values along with the dimensions of the model ship are summarized in Table 2.

The LaiDyn motion model utilized herein as the case study source of ship motion data allows for simulating the ship's response in irregular, long-crested waves with respect to 6DOF (degrees of freedom). Therefore, the assumed wave conditions played an important role in both the shape of OS trajectories and the overall contribution to the concept of holistic DSS, by considering ship maneuverability and the impact of environmental disruptions.

The LaiDyn model utilizes the JONSWAP wave spectrum whose input parameters are aforementioned H_S and T_p with the peakedness parameter set to $\gamma = 3.3$. Additionally, several angles of wave attack on the ship's hull (μ) at the beginning of turning were considered. This was done as different initial wave directions caused slightly different resulting ship trajectories of the ship turning. To cover main directions, the 45°-increments of the μ angle were considered in this study. As per the notion used in the LaiDyn model for the description of wave direction, 0° denotes the following while 180° head seas. Consequently, 90° stands for beam seas where "+" indicates the starboard- and "−" portside, i.e. +90° should be understood as a wave approaching from the starboard-side beam.

Utilization of the advanced motion model simulating ship response in irregular waves, allowed also for consideration of the OS instantaneous local draft when in turn. This takes into account not only the ship's static draft resulting from her loading condition but also the hull response and its momentary positioning in waves.

Nonetheless, because utilization of the 6DOF motion model is time- and resource-consuming, the meta-model employing polynomial approximation of the input trajectories was prepared to ensure efficient prediction of ship trajectory when executing evasive maneuvers. Therefore, dynamic data describing both OS coordinates and oscillatory motions imported from LaiDyn trajectories for $\Delta t = 0.2$ s timestep were determined.

The original ship trajectory delivered from the LaiDyn model contains a fixed time run-up time of straight sailing (100 s), due to a need to stabilize differential equations of ship motions and smoothly increase the wave height to a size predefined in each scenario. During this initial phase, some minor alterations of the OS course may occur due to growing waves but these were assessed negligible as usually did not exceed $\Delta\psi = 2°$ before the rudder order. After the predefined run-up time (which is not subject to further consideration and is simply cut off), the turning commenced due to rudder deflection, which realistically increased till reached one of the predefined δ angles (see Table 2). Once fully deflected, the rudder stays fixed in this position till the end of the simulation, thus single evasive action is executed in the proposed system. Several rudder angles were considered, as these caused a different ship's rate of turn, thus faster or slower reaching assumed course alteration depending on the simulation scenario.

Since in further application of the trajectory-based meta-model within DSS, the maximum considered course alteration was assumed to reach not more than $\Delta\psi = \pm 60°$ (see Table 2), each OS trajectory during its conversion into A_i array has been shrunk, taking some additional margin into account. This allowed for a significant reduction in row numbers within the ship motion array A_i which resulted in a much simpler, thus efficient approximation as well as greater accuracy.

The overall diagram of the prepared meta-model is given in Fig. 5. As can be seen, the output data being of interest due to their future utilization were OS coordinates and course: X_g, Y_g, ψ. A polynomial function is used to approximate them. The algorithm performing this polynomial approximation for all n trajectories of OS, based on the input data from ship motion array A_i (where i is the trajectory number) is the same for the, Y_g and ψ coordinates. It is shown below for X_g coordinate.

The polynomial $X_i(t)$, which approximates the data from the $A_i[X_g]$ array's column (for the i th trajectory) can be described by the formula:

$$X_i(t) = \sum_{k=0}^{m} a_k t^{m-k}, \tag{2}$$

where a_k are polynomial coefficients, m is the degree of the polynomial, k is an index ranging from 0 to m and t is the time. The polynomial coefficients a_k and the degree of the polynomial m are determined by Algorithm 1 (Fig. 6). The algorithm utilizes two Python functions:

- `numpy.polyfit(x,y,deg)`[1] – that performs the least squares polynomial fit of points `(x,y)` by a polynomial `p(x)` of degree `deg`, it returns a vector of coefficients `p` that minimizes the squared error in the order `deg`, `deg-1`, … 0;
- `sklearn.metrics.r2_score(y_true, y_pred)`[2] – that calculates the R^2 regression score indexes between the ground truth values (`y_true`) and predicted values (`y_pred`), the indexes reach 1.0 for perfect predictions and smaller values in proportion to the imperfections between the ground truth and predictions.

The algorithm starts operating with the smallest possible degree of the polynomial $m = 1$ (the m value is initialized to 0, but increased to 1 in the first loop cycle, to comply with *while* loop condition checks). The desired assumed fit tolerance $\sigma = 0.99$. First, the least squares polynomial fit of X_i for a given degree m is performed (`polyfit` function). Then, the resulting predicted values of $X_i(t)$ are compared with the trajectory data $A_i[X_g]$ by the R^2 regression score (`r2_score` function), to check if the prediction is satisfactory (fit tolerance $\sigma \geq 0.99$). If true, coefficients $a_{k,\ k\ =\ 0..m}$ and the degree of polynomial m are returned. Otherwise, the degree of the polynomial m is incremented and the *while* loop is continued. The loop is stopped once the degree of polynomial m reaches the minimal possible value for which the assumed fit tolerance ($\sigma \geq 0.99$) is achieved.

The resulting polynomial coefficients a_k are very numerous as there is an independent set of coefficients per different scenario, as presented in Table 2. They have their own degree of polynomial m assigned and are calculated separately for OS coordinates and course (Xg, Yg, ψ). Thus, for the readers interested in detailed results, there is a CSV file (polyDataFrame.txt) attached in the supplementary materials including all the coefficients and their corresponding m values.

3.5. Bathymetric data

The source of bathymetric data used herein are cells of the S-57 electronic navigational charts covering the South Baltic area, similarly as in [56]. The chart is defined by n x m grid points with resolution k. The boundary of the map is defined by its minimum and maximum latitudes and longitudes denoted as φ_1, φ_m, λ_1, λ_n, respectively. The $M_{1,\ \ldots,m,1,\ldots,n}$ matrix represents the bathymetric grid of the points. However, the raw source data were not directly applicable to this research, as the initial area was too large while the resolution was insufficient. Therefore, as shown in Fig. 7, the M matrix has been transformed into the M' matrix. First, it has been scaled down by specifying the minimum and maximum coordinates (φ_{min}, λ_{min}, φ_{max}, λ_{max}), and then its resolution has been increased twice, so the resulting matrix M' has twice as many rows and columns.

To obtain the depth for point A defined by φ_A, λ_A geographic

[1] https://numpy.org/doc/stable/reference/generated/numpy.polyfit.html

[2] https://scikit-learn.org/stable/modules/generated/sklearn.metrics.r2_score.html

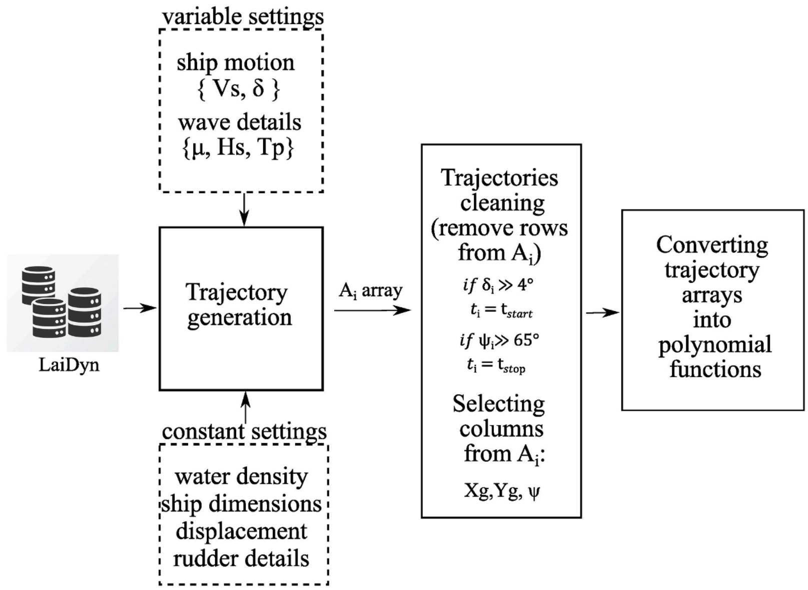

Fig. 5. The procedure for converting a trajectory into a polynomial.

Algorithm 1

Step 1: $m = 0$
Step 2: $\sigma = 0.99$
Step 3: R_squared = 0.0
Step 4: `while` (R_squared < σ)
Step 5: $m = m + 1$
Step 6: $X_i(t) =$ `numpy.polyfit`$(A_i[t], A_i[X_g], m)$
Step 7: R_squared = `sklearn.metrics.r2_score`$(A_i[X_g], X_i(t))$
Step 8: `return` $X_i(t), m$ // returns polynomial coefficients a_k and polynomial degree m

Fig. 6. Algorithm for obtaining polynomial coefficients a_k and degree of the polynomial m for OS X_g coordinate and single (i-th) trajectory.

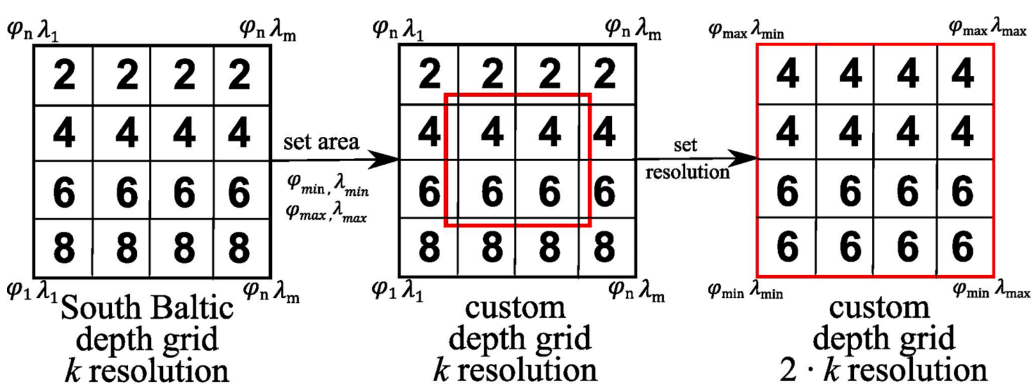

Fig. 7. Customizing bathymetric grid.

coordinates from the matrix *M'*, the following formula is utilized:

$$
\begin{aligned}
&if\ point\ A\ \in \left(\varphi_{min}, \ldots, \varphi_{max;}\lambda_{min}, \ldots, \lambda_{max}\right)\\
&y = round\left(\frac{\varphi_{max} - \varphi_A}{\varphi_{max} - \varphi_{min}}\cdot 2\cdot n\right)\\
&x = round\left(\frac{\lambda_{max} - \lambda_A}{\lambda_{max} - \lambda_{min}}\cdot 2\cdot m\right)\\
&depth_{\varphi_A\lambda_A} = M'_{x,y}
\end{aligned}
\tag{3}
$$

The visualization of the bathymetry data in matrix *M'* is presented in Fig. 8.

3.6. Maneuver safety levels

In the **Maneuver Selection Display**, one of four possible safety levels is assigned to the own ship's current course as well as to each of the potential maneuvers. Those levels are:

- *Unsafe* (a red indicator),
- *Barely Safe* (an orange indicator),

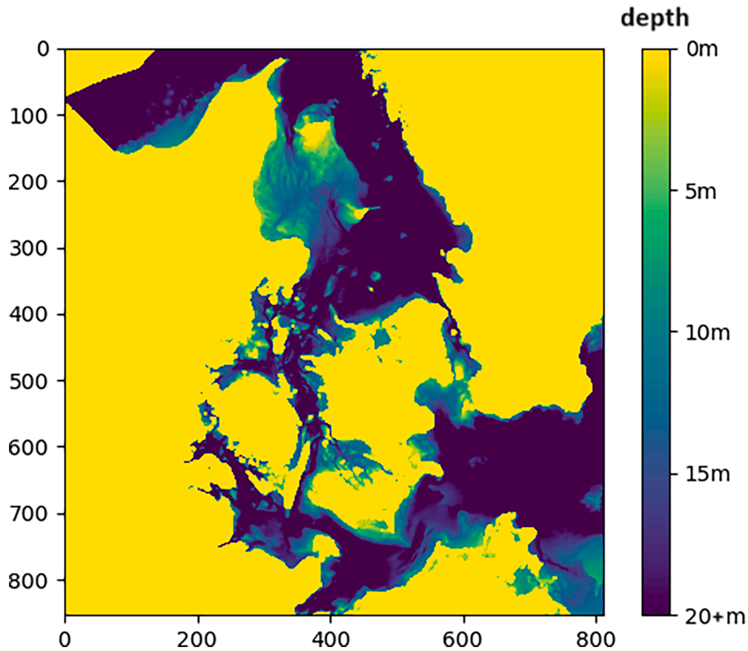

Fig. 8. Visualization of the bathymetric grid data (depth in meters) applied in this study.

- *Rather Safe* (a yellow indicator),
- *Safe* (a green indicator).

The algorithms for determining those levels are as follows.

3.6.1. Assigning a safety level to the own ship's current course

For each of the safety levels from *Barely Safe* (orange) to *Safe* (green), the system checks if the conditions for this level are met. If conditions for a given safety level are not fulfilled, a lower level is returned and the procedure ends. Otherwise, a higher level is verified. The conditions for each level concern:

a) the current distance between the own ship and the target, which is compared with the declarative arena reflecting navigators' preferences from Section 3.2;
b) future distance between both ships, which is compared with domain dimensions from Section 3.3;
c) the time remaining to run aground based on a bathymetric profile of the area.

It is assumed here that the own ship's course and speed do not change, as long as there is no intentional maneuvering involved. Minor course deviations due to wind and waves are assumed to be automatically handled by the ship's course control system, while the associated speed deviations are neglected prior to maneuver execution. The exact conditions for a particular safety level are configurable in the system, with the default values being as follows.

Barely Safe:

- a predefined declarative arena has not yet been violated by a target,
- the predicted DDV is smaller than 0.5 (computed f_{min} is larger than 0.5),
- there are at least 5 min left to violate bathymetric constraints.

Rather Safe:

- a 1.5-sized declarative arena has not been violated,
- a minor domain violation is predicted (computed f_{min} is between 0.75 and 1, DDV is between 0 and 0.25),
- there are at least 10 min left to violate bathymetric constraints.

Safe:

- a double-sized declarative arena has not been violated,
- the domain will not be violated (computed f_{min} is larger than 1, DDV is 0),
- there are at least 15 min left to violate bathymetric constraints.

The default values used in the above conditions were selected in the following way. If a certain ship domain is used, this domain should not be violated, which means DDV = 0 ($f_{min} > 1$) must be fulfilled for the "safe" condition. The particular domain dimensions given in the settings (Table 4, Section 4.1) indicate that DDV = 0.5 is the largest value still resulting in a lack of physical incident (assuming no measurement and modeling errors). For DDV > 0.5 ($f_{min} < 0.5$), depending on the particular relative course and bearing of TS, a physical incident may occur. Therefore DDV = 0.5 is set as a threshold value for *Barely Safe*. Consequently, the threshold for *Rather Safe* is set as a simple arithmetic average of thresholds for *Barely Safe* and *Safe*, which, in this case, is DDV = 0.25 ($f_{min} = 0.75$). As for the declarative comfort arena around the ship's domain, its enlarged size (for *Rather Safe* and *Safe*) was selected as a result of simulations in such a way, that the safety levels presented in the Maneuver Selection Display reflect the distance from CADCA in the right display. Finally, the threshold value of time remaining to violate bathymetric constraints is the most problematic and context-dependent of the above conditions. The exact values should be set based on deck officers' experience in navigating in a particular water region (e.g. having a 15-minute time margin may be simply impossible in some restricted water areas).

3.6.2. Assigning a safety level to a particular maneuver

In the present version of the proposed system, it is possible to turn by 20°, 40°, or 60° to either side by deflecting the rudder by 5°, 10°, 15°, or 35° as presented in Table 2. This gives a total of 24 potential maneuvers all denoted by colorful indicators (dots) in Fig. 2. Similarly to assessing the safety of the current course, for each of the maneuvers safety levels from *Barely Safe* to *Safe* are checked and if the conditions are not fulfilled, a lower level is assigned to the particular maneuver. The conditions for each safety level are similar as in the case of the current course: the same times remain to violate bathymetric constraints and f_{min} larger than 0.5, 0.75, or 1 for *Barely Safe, Rather Safe*, and *Safe*, respectively.

However, the navigators' declarative arena is not taken into account in this case, as it only applies to the pre-maneuvering phase of an encounter. It is also worth noting that checking for potential domain violations is more complex in this case – it involves simulating the own ship's trajectory both during and after the maneuver through the meta-model from Section 3.4.

3.7. Ship critical area – the CADCA concept

CADCA is a type of maritime collision-avoidance indicator rooted in the critical area concept, thus it reflects the zone that a navigator must keep free from other objects to ensure ship safety and avoid a collision. The envelope of the CADCA delimits a distance, at which the last-minute maneuver for a specific ship's operational and environmental parameters should be executed. Therefore, it allows also for determining the time remaining to start a given evasive action. As solving the close-quarters situation is a non-static problem that changes in time, and it is impacted by many factors both operational and environmental, the CADCA shape changes according to prevailing conditions.

It is worth noting here the main difference between the CADCA and the ship domain concepts. The latter is an area, that a navigator prefers to keep free from other vessels for the general comfort and safety of navigation. Therefore, the ship domain does not inform a navigator when exactly and what kind of maneuver should be performed to avoid a collision. It merely defines the desired outcome, which may be interpreted as a passing distance after taking evasive action. In comparison, CADCA is a more informative concept, especially for a decision-maker (OOW). This is due to its direct translation to a given evasive maneuver and the distance/time of its execution, rather than the distance resulting from its execution. Thus, for each of the potential course alterations of the own ship, CADCA specifies the last moment when a single-action kind of evasive maneuver can be effectively executed. If a target is about to violate the CADCA envelope, the navigator must act immediately to take proper action. However, if CADCA has already been violated, an assumed combination of both course alteration and rudder deflection is no longer sufficient to solve a dangerous encounter. In this case, if possible due to the ship's characteristics, the OS action should be firmer to ensure a higher rate of turn by a larger magnitude of rudder deflection, or other actions increasing her efficiency. These can be, for instance, either rapid speed reduction or favorable action taken by the target (joint evasive maneuvers of both ships). Nonetheless, if the CADCA determined for maximum rudder deflection and feasible course alteration is violated by the target and there is no possibility of other supporting means, this will lead to an imminent collision of the vessels.

The dynamics of CADCA arise directly from the method of its determination that employs ship trajectories delivered from the external 6DOF motion model. This, in turn, binds the CADCA's envelope with the vessel's response and maneuverability in irregular waves, impacted by given operational (initial ship speed, magnitude of rudder angle, vertical center of gravity) and environmental (wave spectrum parameters, initial wave direction) conditions.

In general, the CADCAs have been determined on the basis of computer simulations, in which a massive number of geometrical combinations of ship encounters are verified. The virtual ship hulls are sequentially moved backward and forward, respectively. During moving the own ship ahead, the predefined 6DOF trajectory of her turning has been overlaid, in order to verify if this specific maneuver allows for collision avoidance. It is also assumed that the target ship always remains passive during an encounter and she always maintains her course and speed. The entire process is conducted as long as the first opportunity to execute an effective evasive maneuver occurs. Once this specific mutual arrangement of two ships allowing for collision avoidance is determined for a given simulation scenario (described by operational, geometrical, and environmental parameters), its details are calculated

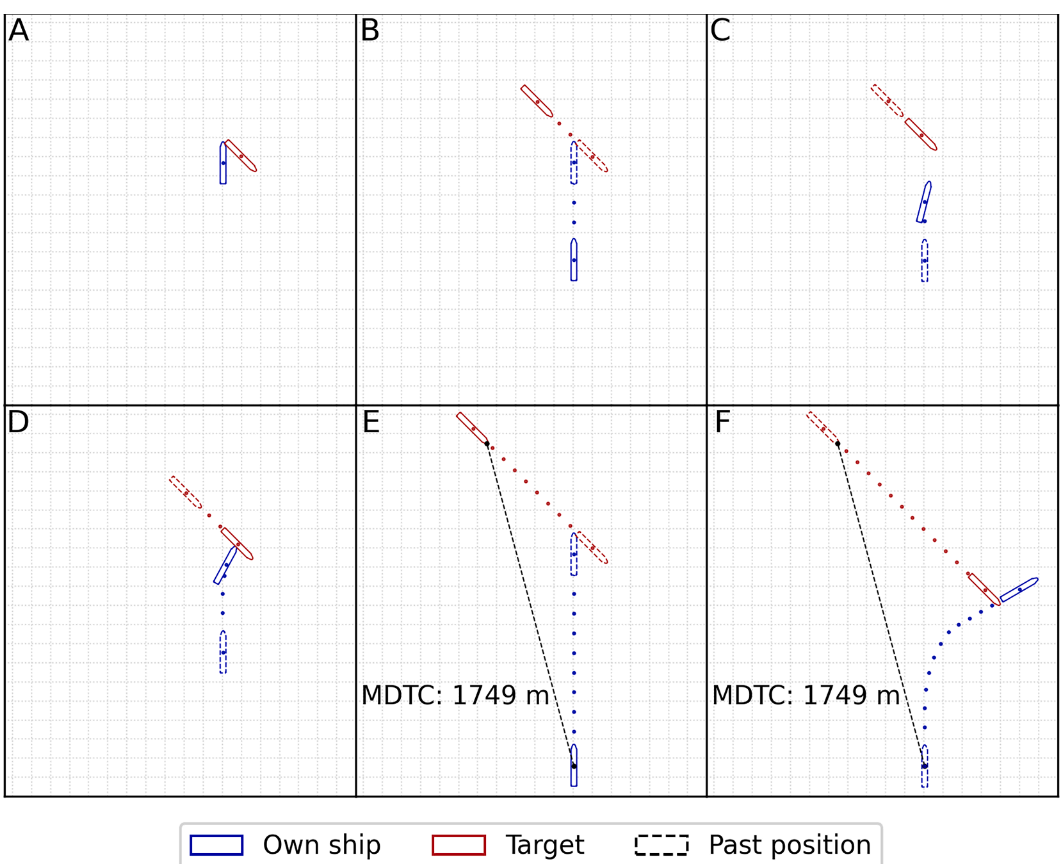

Fig. 9. Visualization of the simulative determination process of a single MDTC.

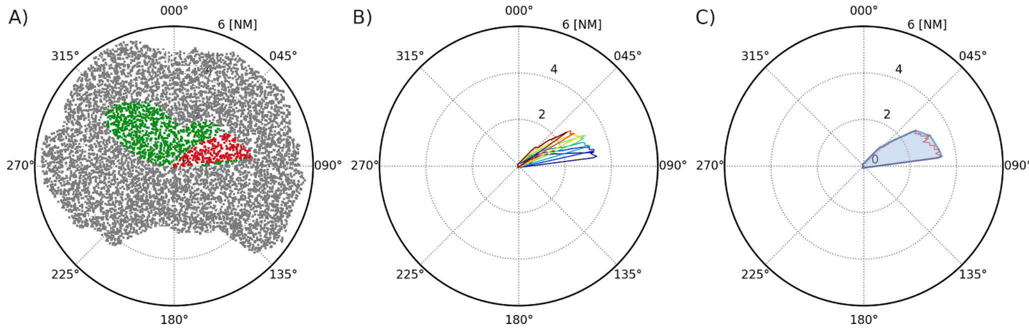

Fig. 10. The illustrative process of creating the CADCA envelope: A) collecting and grouping of single MDTC results; B) construction of the subsectors from maximized MDTCs; C) final envelope through set union and application of convex hull.

and saved. The main ones in this respect are the shortest distance at which the simulated evasive maneuver was effectively executed (the first opportunity to avoid a collision), together with the direction in which the target was located. In literature, this distance is typically called MDTC (Minimum Distance to Collision) [57], while the angle describing the target's location is the relative bearing from own ship to the target. The step-by-step process of the simulative determination of a single MDTC for an exemplary simulation case and specific evasive maneuver is depicted in Fig. 9.

Once the MDTC simulations produced interim results, the process of determining the final CADCA envelope begins, as depicted in a simplified manner in Fig. 10. Firstly, when all pairs of values (MDTC vs. relative bearing) are successfully obtained (all data points in Fig. 10A), these have been afterward collected with respect to each simulation scenario and then grouped (green and red data points in Fig. 10A). Because of the high-fidelity MDTC simulations taking into account dimensions and shapes of virtual ship hulls as well as even minor impact of the waves, an enormous number of single results are eventually obtained, also for the same bearing once rounded to integers. Therefore, for the same bearing always the maximum determined MDTC is taken into consideration in the further steps of the procedure, to assume the worst navigational scenario (as for the largest MDTC within the same simulation case, the same evasive maneuver should be executed at the earliest). Once the decision about the angular resolution of CADCA is made (discrete step of considered own ship headings and the final sector of the area), the maximum MDTC values are processed to construct component sectors for each considered OS heading (Fig. 10B - thin colorful sectors). Afterward, a union of the sectors is made and the final critical area is created (presented as the red, irregular envelope of the CADCA in Fig. 10C). Lastly, to smooth the final envelope and reduce the number of data points consisting of the final polygon, the convex hull is determined and applied.

Summarizing, the presented process of CADCA determination allows for consideration of operationally-accurate results, due to the enormous number of the considered MDTC simulations within the same scenario. This takes into account ship operational parameters, encounter geometry, and wave conditions. The consideration of the worst-case MDTC results as well as their direct translation to a specific evasive maneuver allows for a proactive approach to safety. Additionally, the convex hull increases the applicability of the CADCA concept while maintaining its informative form for future decision-makers. For more details concerning the CADCA concept, an in-depth description of the determination method as well as its application please refer to [51,54,58].

4. Simulation study

To emphasize the practical applications and benefits of using the proposed solution, this section presents the tool applied to three ship encounters. The following subsections cover the settings of the simulations, particular scenarios, and a discussion of the results.

4.1. Simulation overview and settings

Three real-life ship encounter situations have been selected as use cases of the developed 3-stage synthesis-oriented DSS tool (described in Section 3.1). Animation files (MP4) presenting the selected real-life encounter situations are attached to this paper as supplementary materials. The selected encounters cover typical crossings (target on starboard – Scenario 1 and target on port side – Scenario 2) as well as a more debatable encounter – a crossing bordering on head-on (Scenario 3). For convenience, all DSS screenshots for Scenarios 1–3 are provided in Appendices A1-A3, respectively. In each case, the scenario data such as ships' positions, speeds, headings, etc. have been imported from a ship encounter database containing historical AIS data recorded in the southern part of the Baltic Sea around the Danish straits from 2013 to 2018 [59]. The bathymetric data was provided using the S-57 data gathered for the South Baltic region and then prepared as introduced in Section 3.5.

For each of the three selected real-life scenarios, a particular moment in time has been selected (when the ships were 2NM or 3NM apart) from which the simulation scenario starts. Table 3 shows details of the real-life scenarios and their accompanying simulation scenarios (Scenario 1 – Scenario 3), whereas Table 4 presents a configuration of the ship model selected, as well as ship domains and arenas applied to the scenarios. An off-centered elliptic shape introduced by Coldwell [24] has been used here as the ship domain, however, its dimensions have been updated following more recent AIS data-based empirical research [38, 39].

4.2. Scenario 1 – crossing with TS on starboard

In the first scenario (situation overview in Fig. 11, DSS screenshots in Appendix A.1) TS is initially 3NM from OS and approaching from starboard. The latter means that OS is obliged to give way in case of collision risk as both ships are power-driven vessels and visibility conditions are good. As can be seen in Fig. A1-1, the current course indicator is yellow (the dot marked with a blue boundary), which means a minor domain violation. All of the maneuvers to starboard, regardless of the size of rudder deflection and course alteration, are marked in green, meaning they are safe and can be performed successfully in time to avoid the above-mentioned domain violation. As for turns to port, not only they are incompliant with COLREG, but also ineffective, as shown by orange and yellow indicators on the left.

In the following figures in Appendix A.1, we can trace the development of the scenario, assuming OS does not take the desired action. In Fig. A1-2 we can see the CADCA for the turn by 20° to starboard (and rudder deflected by 5°). We can also see how the yellow indicators (representing maneuvers to port) gradually turn to orange and the orange ones – to red in Figs. A1-2, A1-3, A1-4, as the ships get closer to each other. In the case of selecting a red indicator maneuver (Fig. A1-3), we can see that it is indeed unsafe, as TS is already within CADCA

Table 3
Characteristics of the real-life scenarios and accompanying simulation scenarios.

	Parameter name		Parameter value
Scenario 1	Region		Baltic Sea, Fehmarn Belt (Danish straits)
	Encounter start (date & time)		2018–04–16, 10:32:45
	Encounter database ID		Scenario 6
	Own ship (OS) MMSI		304374000
	Target ship (TS) MMSI		211188000
	Navigational status		Power-driven vessel (OS and TS)
	Simulation scenario start (frame no.)		399
	Distance between OS and TS on simulation scenario start		3NM
	Initial wave conditions	wave height	0.9 m
		wave period	4.5 s
		wave angle	0°
Scenario 2	Region		Baltic Sea, approach to the Trelleborg port (Sweden)
	Encounter start (date & time)		2013–05–12, 06:12:21
	Encounter database ID		Scenario 39
	Own ship (OS) MMSI		265186000
	Target ship (TS) MMSI		236490000
	Navigational status		Power-driven vessel (OS and TS)
	Simulation scenario start (frame no.)		364
	Distance between OS and TS on simulation scenario start		2NM
	Initial wave conditions	wave height	4.3 m
		wave period	5.5 s
		wave angle	−90°
Scenario 3	Region		Baltic Sea, Fehmarn Belt (Danish straits)
	Encounter start (date & time)		2017–04–21, 03:04:57
	Encounter database ID		Scenario 140
	Own ship (OS) MMSI		211190000
	Target ship (TS) MMSI		211188000
	Navigational status		Power-driven vessel (OS and TS)
	Simulation scenario start (frame no.)		157
	Distance between OS and TS on simulation scenario start		2NM
	Initial wave conditions	wave height	1.8 m
		wave period	5.5 s
		wave angle	+45°

Table 4
Parameters of the ship domains, and ship arenas applied to the scenarios.

	Parameter name	Parameter value
Ship domain	domain type (shape)	Coldwell's domain (elliptic)
	semi-major axis *a*	4 · LOA = 886 m
	semi-minor axis *b*	2 · LOA = 443 m
	ship's displacement from the ellipse's center towards aft along the semi-major axis *da*	1 · LOA = 221.5 m
	ship's displacement from the ellipse's center towards the port side along the semi-minor axis *db*	0.5 · LOA = 110.8 m
Ship arena	arena type (shape)	Coldwell's
	semi-major axis *a*	4 NM = 7 409.1 m
	semi-minor axis *b*	4 NM = 7 409.1 m
	ship's displacement from the ellipse's center towards aft along the semi-major axis *da*	1 NM = 1 852.3 m
	ship's displacement from the ellipse's center towards the port side along the semi-minor axis *db*	0 NM = 0 m

determined for this particular turn. We can also observe in Fig. A1-4 that TS is directly on the CADCA's envelope for a single yellow indicator maneuver for the largest and fastest turn to port (alteration of 60° and rudder deflected by 35°). Please note that the CADCA display scale was automatically changed from the 3NM range (Fig. A1-3) to the 2NM range (Fig. A1-4) for better visualization.

Further on, despite TS approaching OS, we can see that turns to starboard remain a safe solution. This is particularly true for the largest turn (course alteration of 60° and rudder deflection of 20°) shown in Fig. A1-5. In Fig. A1-6 we can see that when TS crosses in front of OS, nearly all maneuver indicators turn yellow. The reason for this is that the maneuvers take longer than the passing of both ships, so they would have little to no impact on the collision risk now. The exceptions are maneuvers to the port involving a large rudder deflection, which would make the situation even more risky, as OS would turn towards TS. Finally, in Fig. A1-7, all maneuver indicators turn green as soon as TS and OS passed each other. The current course of OS is safe again and no maneuver can lead to collision risk, as TS would be further away by the time such maneuver is completed.

4.3. Scenario 2 – crossing with TS on port board

In Scenario 2 (situation overview in Fig. 12, DSS screenshots in Appendix A.2), the current course of OS is again marked with a yellow indicator, meaning a minor collision risk only. However, unlike Scenario 1, in Scenario 2 TS is on port and thus obliged to give way. Therefore, OS is expected to keep the course until its own maneuver can no longer be postponed. As we can see in Fig. A2-1, all turns to port are safe in terms of avoiding domain violations, while the ones to starboard can only make the situation more risky and increase the predicted domain violation (orange and red indicators). What is more, Fig. A2-1 shows, that even for the smallest turn to port (course alteration of 20° and rudder deflection of 5°), TS will not violate CADCA, but will pass near its boundary.

Continuing with Fig. A2-2, we can see that one of the orange indicators representing the ship turning to starboard has changed to red

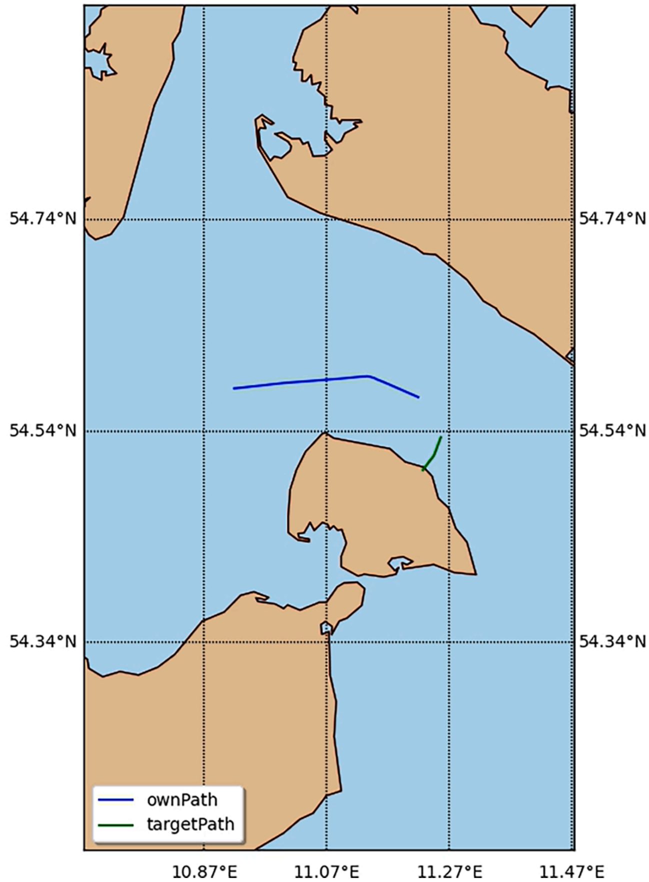

Fig. 11. Real OS & TS encounter situation at the beginning of Scenario 1 (OS path marked in blue and TS path – in green).

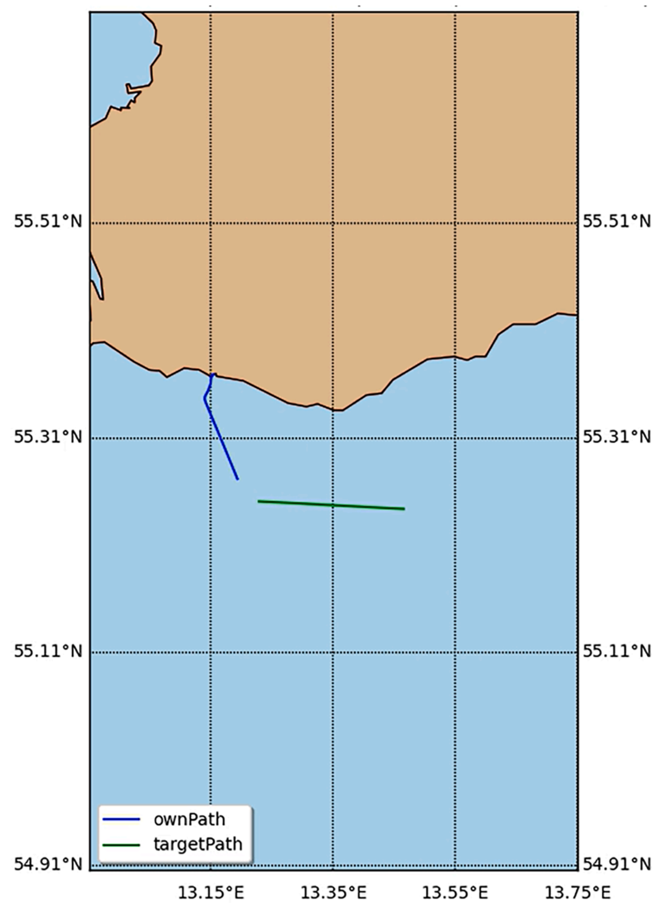

Fig. 12. Real OS & TS encounter situation at the beginning of Scenario 2 (OS path marked in blue and TS path – in green).

(when compared to Fig. A2-1). We can also observe that indeed TS has just violated CADCA for this maneuver and an alarm is generated to inform the navigator that the selected action is dangerous. As for the turns to port, they remain safe until TS is about to cross ahead of OS, which is depicted in Fig. A2-3. We can notice there, that some of the maneuvers to port would not bring the desired effect (green indicators have turned to yellow), as there is not enough time to perform them. For the same reason, some of maneuvers to starboard are now marked as less dangerous than before in Fig. A2-3.

Once TS has crossed ahead of OS bow (Fig. A2-4), most of maneuver indicators turn yellow, which informs an operator that those turns would not affect the situation due to their execution time. This, however, does not include larger and faster turns to starboard, which may still result in increased collision risk. Finally, when OS and TS have passed each other (Fig. A2-5), the current course indicator, as well as all maneuver indicators, are again marked in green meaning they are safe (TS moves away from OS and no OS maneuver can change it).

4.4. Scenario 3 – crossing bordering on head-on

This scenario (situation overview in Fig. 13, DSS screenshots in Appendix A.3) is a crossing situation with TS on port (2 NM from OS) and therefore obliged to give way. However, the encounter is close to head-on and thus may lead to decision problems. As can be seen in Fig. A3-1, the OS current course indicator is orange (*Barely Safe*), unlike in the previous scenarios, indicating that maneuvering would be recommended. The figure shows also that turns to starboard are ineffective (orange and red indicators), while even minor maneuvers to port can result in a safe passage (green indicators). The former is exemplified in Figs. A3-1, A3-2, A3-3 (TS about to enter CADCA or already inside CADCA) and the latter – in Fig. A3-4 (TS outside CADCA and moving along its boundary).

As TS gets closer to OS, slower turns to port (marked as yellow indicators in Fig. A3-5) gradually become less effective. At the same time, smaller or slower turns to starboard are less dangerous than larger ones. This is because, in the initial phase of a turn, the vessel deviates to port before it starts moving to starboard. The larger and faster the turn to starboard, the more likely it is to cause an incident (red indicators in Fig. A3-5). This tendency grows stronger with the diminishing distance between ships and can be observed in Fig. A3-6: no turn to port is now fully effective (green indicators have turned to yellow). Also, the consequences of maneuvering to starboard are less severe, as there is simply not enough time for those turns to seriously affect the already significant risk (orange indicators). Eventually, when TS is crossing ahead of OS bow (Fig. A3-7), some turns to port can no longer be recommended (orange indicators for smaller rudder deflections).

Finally, when TS passes OS (Fig. A3-8), OS maneuvers can no longer be recommended because the effect of own ship dynamics would dominate over the intended results of a turn. And as soon as the ships have passed each other, all indicators will become green again, because TS will be moving away from OS regardless of the latter ship's actions.

A more general, though somewhat obvious direct conclusion from the above case studies is that, being provided appropriate tools, navigators may be able to apply (within limits dictated by COLREG) their own (or company's) policies. Depending on their preferences and particular ship's maneuvering characteristics, the evasive action may be either earlier and milder or later and more rapid. The preferred action's extent and execution time may be configured in terms of assumed course alteration and how fast it is achieved by a given magnitude of rudder

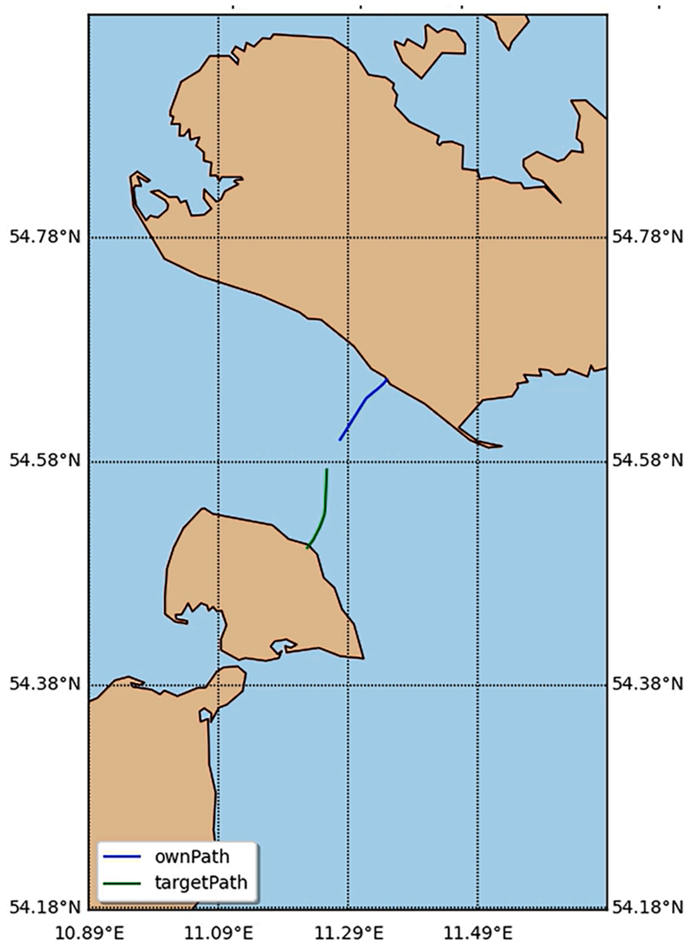

Fig. 13. Real OS & TS encounter situation at the beginning of Scenario 3 (OS path marked in blue and TS path – in green).

deflection. Likewise, the predefined ship's domain (which defines a relative bearing-dependent safe passing distance) and outer declarative arena reflecting navigators' preferences are employed in this regard.

5. System's limitations and future development

Use cases of the DSS presented in Sections 4.2 – 4.4 show that the proposed tool could be a valuable situation awareness monitor as well as a handful navigation aid for OOW, especially in a close-quarters situation. However, as for now, the proposed solution is based on the following assumptions imposing certain limitations to its possible on-board implementation.

The currently identified shortcomings of the proposed DSS may be broken down into two types, namely conceptual and applicational ones. The former refers to the development of the presented system's framework and the scope of considered phenomena within it. The latter consists of DSS deployment and more technical improvements allowing for its implementation and effective work in real conditions.

Regarding the conceptual ones, the most important seems to be the lack of model generalization, as all currently used components of the DSS, i.e. the navigators' declarative arena as well as ship maneuverability-related CADCA are delivered and tailored for the case study Ro-Pax ship. When it comes to applied methods, only a single target ship (TS) can be handled in an encounter scenario, as CADCA cannot be easily extended to a group of targets to date. Furthermore, in the pre-maneuvering phase, OS and TS are assumed to keep their speeds and courses unchanged, while minor deviations are neglected and assumed to be handled by ships' control systems.

Concerning more technical constraints, only discrete values of rudder deflections and course alterations of the own ship can be simulated so far by the system. Also, real-time modeling of CADCAs is not yet supported as the areas are generated offline for the provided ship model. The process of the CADCAs preparation is directly related to the utilized ship motion model and simulation of a given evasive maneuver. Therefore, another limitation is that only course alteration is considered a potential action to resolve a dangerous encounter to date. Despite this being claimed to be the most effective way to avoid a collision, it could be additionally expanded within DSS by preparation of similar trajectories reflecting other ship's behavior than only her turning, for instance, speed reduction.

The main future work should focus on bringing the DSS from the Proof of Concept phase into a fully operational system. To this end, it will be necessary to expand the underlying ship motion data tailored from the specific case-study vessel, into the generalized one, covering a variety of types and dimensions. This could be achieved in two ways: i) by the preparation of many modules tailored for a given kind of ship, ii) or the preparation and utilization of a simplified but generalized motion model, using which the conflict detection and collision resolution through recomputed CADCAs will be conducted.

In an effort to implement the tool onboard, all of the above limitations should be overcome. Therefore, the following method's extensions and future development are planned. The most important of those is that an extended CADCA-based scheme will be developed, taking into account multiple target ships. Also, other types of collision resolution could be considered by applying speed reduction maneuvers or combined ones, including both course and speed change.

Following this, extensive tests with real end-users will be performed, aiming at both functional validation and gathering feedback regarding the customization of system features. As for quantitative progress, the authors intend to apply big data solutions to handle a vast set of CADCA envelopes as well as parallel computation techniques. All of this aims to improve the tool's performance and thus allow real-time modeling and data monitoring.

6. Conclusions

This paper presents a Proof of Concept (PoC) of a shipborne Decision Support System (DSS) for watchkeeping officers. The proposed solution supports three phases of operations when handling an encounter situation, namely conflict detection, maneuver selection, and maneuver execution. In its holistic approach, the solution comprises notions of the survey-based declarative comfort arena, ship safety domain, and critical area indicating maneuvering zone - CADCA (Collision Avoidance Dynamic Critical Area). This kind of operation of the proposed DSS allows for capturing the complexity of the collision avoidance process at sea. The presented tool rooted in the CD&R (Conflict Detection & Resolution) concept, takes into account the navigators' preferences in detecting a dangerous target, monitors the development of the encounter situation by employing ship domain-based indicators, as well as allows to plan the execution of the effective last-chance maneuver through the critical area reflecting physics motion of the objects.

The paper's contribution can be seen twofold – namely, it distinguished its potential impact on the field of application – marine DSS systems – and collision avoidance-oriented research. As for the former, it has been depicted in the case studies based on real-life historical data (Section 4.2 – Section 4.4) that the tool may be a valuable support for the OOW. When using it, the navigator gets a clear picture of an ongoing situation, possible threats, as well as potential solutions to dangerous encounter situations. The applied coloring scheme together with displayed alerts and CADCA envelope are efficient ways to keep the OOW updated and well-informed throughout the encounter. Moreover, it allows for instant and intuitive conflict resolution by selecting a safe maneuver and informing OOW of the time window when the planned evasive action will remain effective (by displaying the CADCA envelope and time to CADCA). Depending on the navigator's attitude (which may be dictated by multiple factors), either a moderate earlier maneuver can be selected or a later, but more substantial one. Summing up, the proposed 3-stage synthesis-oriented DSS tool can significantly improve the safety of the ship, crew, and cargo in close-quarter encounters, at same time still considering the navigator's individual preferences and leaving room for context-dependent decisions.

As for the contribution to the published research, we can conclude what follows. While the current paper does not bring any single major innovation, it manages to integrate several concepts and techniques that have not been combined before. It evidences that they can supplement each other and accentuate the differences between various domain-related concepts and their application. A traditionally understood ship domain (a geometric generalization of a safe passing distance) can be successfully used to assess safety levels of an array of evasive maneuvers without affecting the system's computational performance, due to applied DDV (Degree of Domain Violation) measure and associated analytical solution. Also, the concept of ship arena is still valid (as indicated by the introduced preference-based navigators' declarative arena), and can be utilized to determine the time when most navigators would take an action. Finally, we see how this relates to the ship dynamics-based time window when a particular evasive maneuver needs to be performed to avoid a collision. The latter is made with respect to ship maneuverability and the impact of environmental conditions. All of the above is integrated within a single holistic system, which takes into account also the bathymetric constraints and assists in making a collision avoidance decision, without limiting OOW choice options (other than abandoning risky maneuvers).

Despite the above-mentioned advantages, the system is not free of limitations, which have been discussed in Section 5. Therefore, future development of the solution is planned and it will head in three directions. First, own ship's maneuvering will be extended to include speed changes, which can in some cases supplement or substitute course alterations. Second, the CADCA concept will be extended to handle encounters involving multiple targets. Finally, as described in detail in Section 5, future developments will be focused on making a transition from the PoC to a fully functional on-board prototype, ready to be implemented for real ship bridge operations. It should be mentioned that such work toward systems test implementation has already been started.

CRediT authorship contribution statement

Rafał Szłapczyński: Conceptualization, Methodology, Formal analysis, Investigation, Software, Validation, Data curation, Writing – original draft, Writing – review & editing, Supervision, Project administration, Funding acquisition. **Joanna Szłapczyńska:** Methodology, Formal analysis, Investigation, Software, Validation, Data curation, Writing – original draft, Writing – review & editing, Visualization, Supervision, Project administration, Funding acquisition. **Mateusz Gil:** Methodology, Investigation, Resources, Data curation, Visualization, Writing – original draft, Writing – review & editing. **Marcin Życzkowski:** Data curation, Visualization, Software, Writing – review & editing, Writing – original draft. **Jakub Montewka:** Conceptualization, Project administration, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.ress.2024.110232.

Appendix A.1. screenshots for Scenario 1

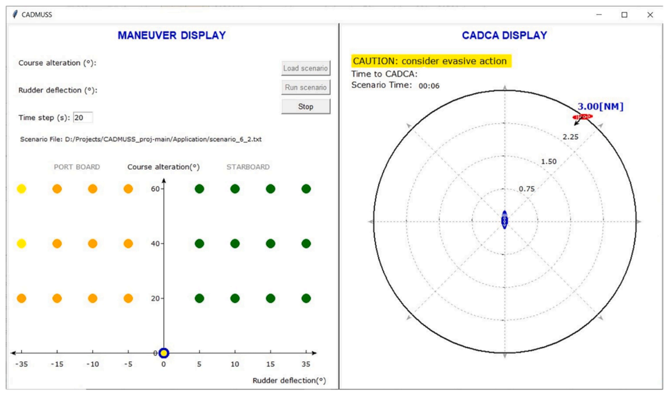

Fig. A1–1. The start of Scenario 1: maneuvers to starboard are safe as opposed to minor or major risks associated with staying on course or turning to port.

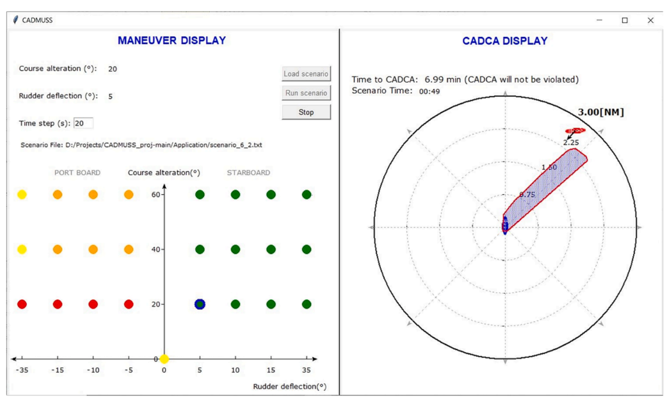

Fig. A1–2. Scenario 1: even the smallest turn to starboard is still safe, as TS is outside of the CADCA. At the same time, maneuvers to port become riskier.

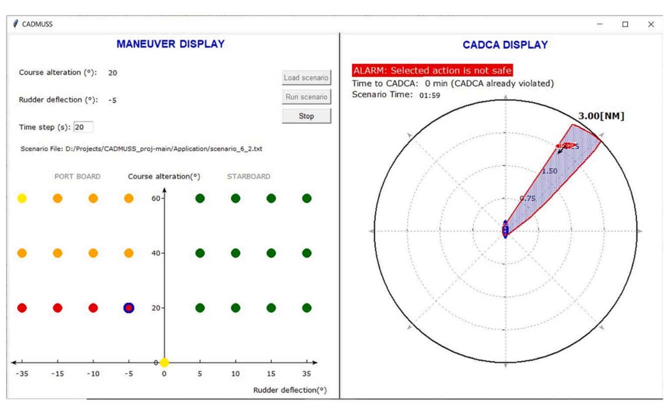

Fig. A1–3. Scenario 1: a turn to port is not a safe solution, as evidenced by TS being inside CADCA determined for this particular maneuver.

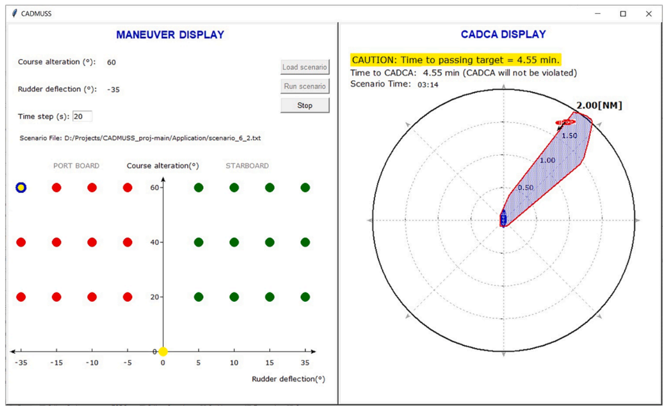

Fig. A1–4. Scenario 1: even the largest turn to port is not safe, as evidenced by TS being on the boundary of the CADCA determined for this particular maneuver. Please note that the CADCA display has just been rescaled (from 3NM to 2NM range).

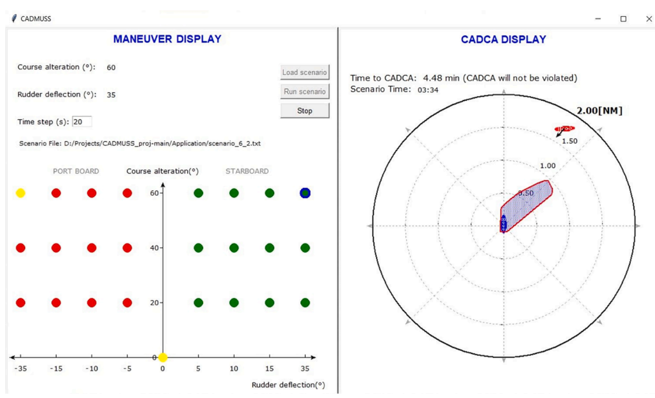

Fig. A1–5. Scenario 1: the larger the turn to starboard, the more time we have to perform it, as OS is still nearly 1 NM from CADCA's boundary and time to passing OS is nearly 5 min.

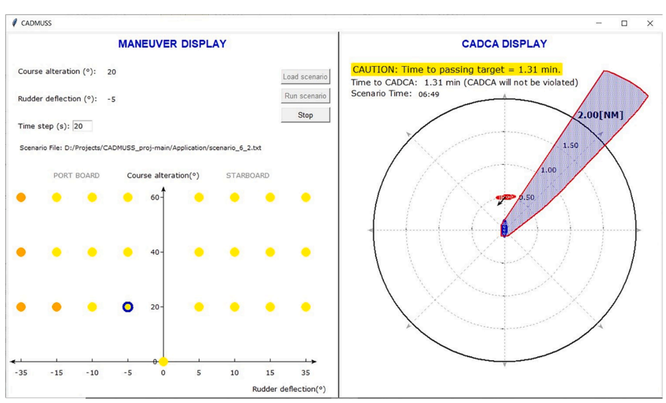

Fig. A1–6. Scenario 1: TS crosses directly in front of OS. No maneuver is safe now, as the time necessary for performing it is too large.

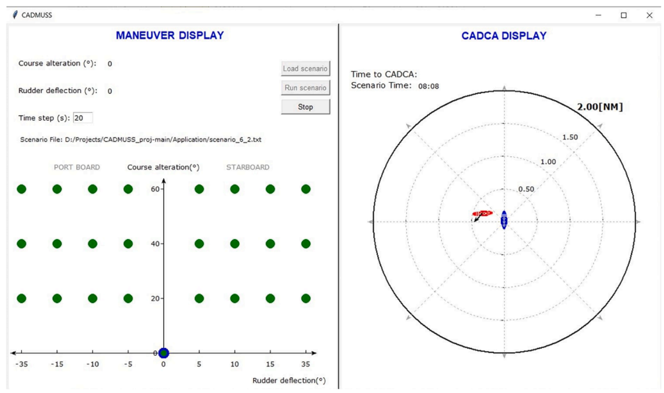

Fig. A1–7. Scenario 1: TS and OS have already passed each other and no OS maneuver can seriously affect the situation now as TS is already moving away from OS (green indicators for all turns).

Appendix A.2. screenshots for Scenario 2

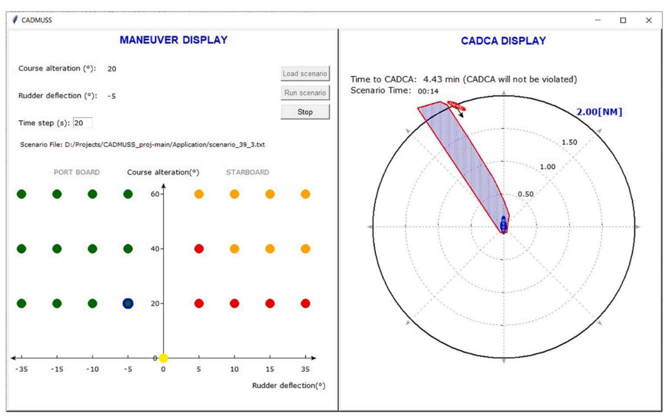

Fig. A2–1. Scenario 2: The smallest turn to port is still safe, as evidenced by TS remaining outside of CADCA determined for this maneuver.

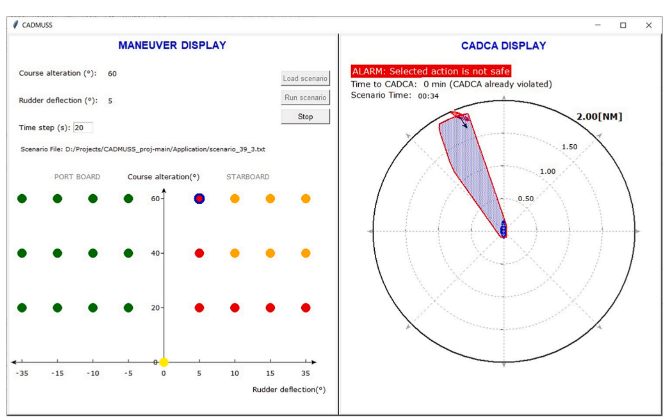

Fig. A2–2. Scenario 2: Selected turn is highly dangerous, as evidenced by TS within CADCA.

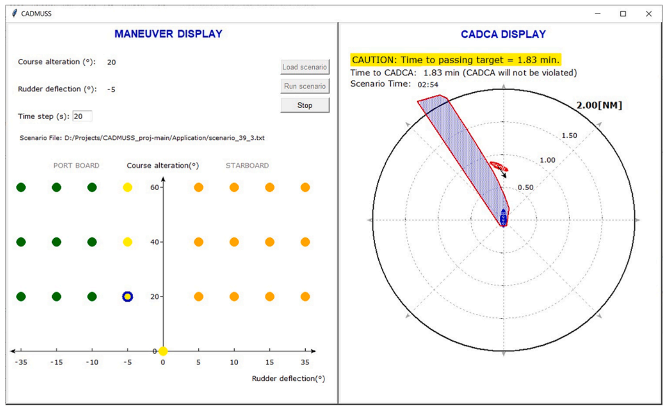

Fig. A2–3. Scenario 2: As TS is about to cross ahead of OS bow, slow turns to port are no longer effective (yellow indicators).

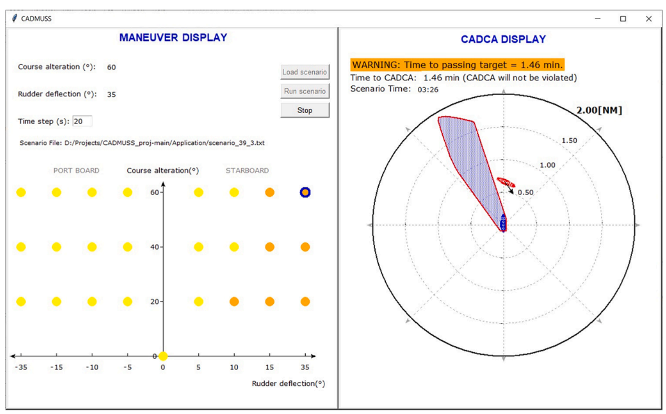

Fig. A2–4. Scenario 2: When TS crosses ahead of OS bow, most of the OS maneuvers cannot bring significant effect, apart from faster turns to starboard, which could further increase collision risk.

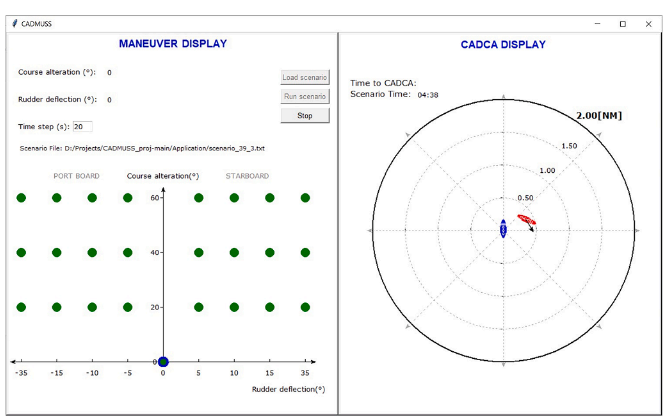

Fig. A2–5. Scenario 2: OS and TS passed each other and both the current course of OS and all turns are now safe, as they would not result in domain violations.

Appendix A.3. screenshots for Scenario 3

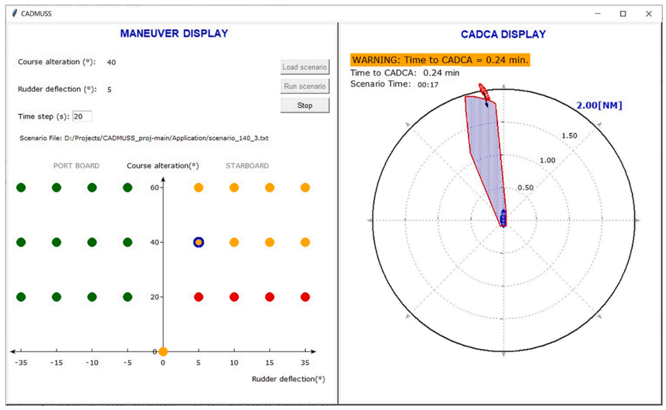

Fig. A3–1. Scenario 3: Turns to starboard are not effective (orange and red indicators). TS is about to enter CADCA determined for a selected maneuver.

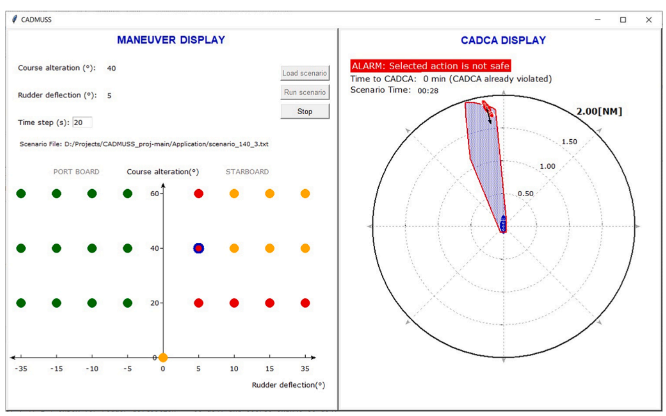

Fig. A3–2. Scenario 3: As TS gets closer, maneuvers to starboard are less and less effective. TS is already within CADCA determined for the considered maneuver.

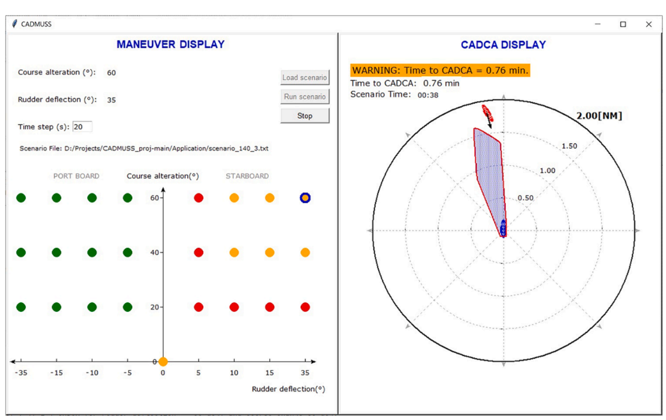

Fig. A3–3. Scenario 3: Even the largest (by 60°) and fastest (rudder deflected by 35°) turn to starboard is still not safe, as TS would soon enter CADCA determined for this maneuver.

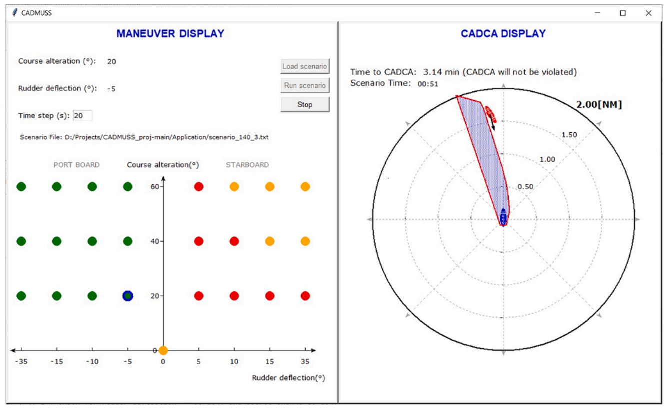

Fig. A3–4. Scenario 3: As opposed to maneuvers to starboard, turns to port are more effective as indicated by green indicators and evidenced by TS remaining outside of the CADCA for the selected maneuver.

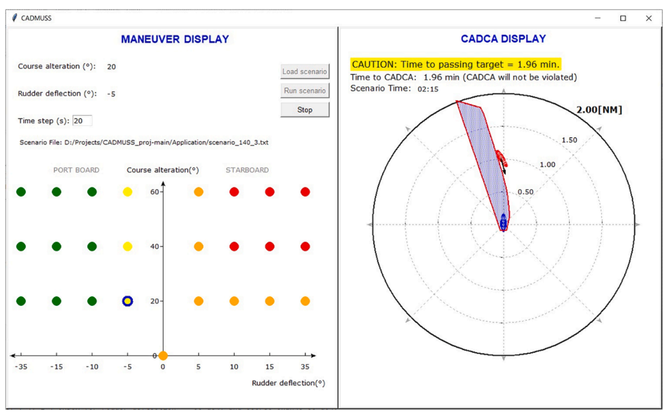

Fig. A3–5. Scenario 3: As TS gets closer to OS, slower turns to port (involving smaller rudder deflections) are less effective. Also, smaller or slower turns to starboard do not affect collision risk, while the larger and faster may actually increase it.

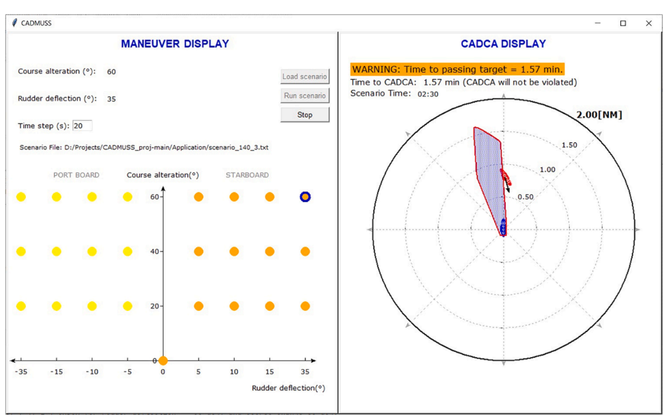

Fig. A3–6. Scenario 3: TS crosses ahead of OS bow. There are less than 2 min left to the passing of the two ships. Due to insufficient time, turns to port would now have limited effect, while turns to starboard would be completely ineffective.

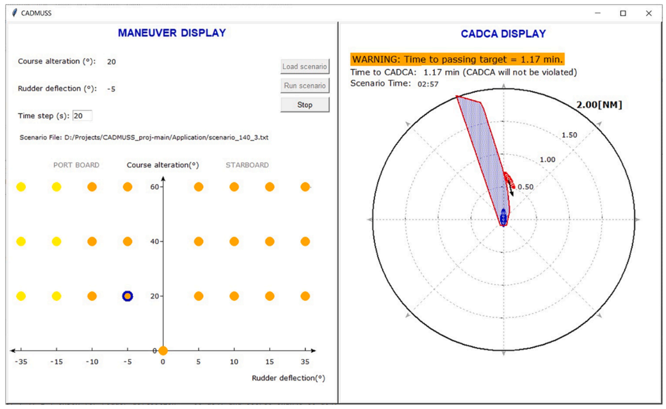

Fig. A3–7. Scenario 3: TS is so close to OS, that only fast turns to port (larger rudder deflections) may be considered and even those maneuvers are not entirely effective.

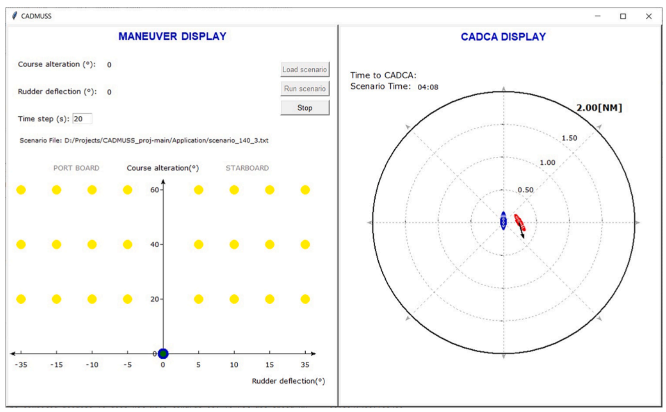

Fig. A3–8. Scenario 3: TS is passing OS. At this point, no action from OS can diminish the collision risk.

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