



# Predicting a passenger ship's response during evasive maneuvers using Bayesian Learning

Mateusz Gil<sup>a</sup>, Jakub Montewka<sup>b,c,d,\*</sup>, Przemysław Krata<sup>b,d</sup>

<sup>a</sup> Research Group on Maritime Transportation Risk and Safety, Faculty of Navigation, Gdynia Maritime University, Jana Pawła II 3, 81-345 Gdynia, Poland

<sup>b</sup> Institute of Naval Architecture, Gdańsk University of Technology, Faculty of Mechanical Engineering and Ship Technology, Gabriela Narutowicza 11/12, 80-233 Gdańsk, Poland

<sup>c</sup> Tallinn University of Technology, Estonian Maritime Academy, Kopli 101, Tallinn, 11712, Estonia

<sup>d</sup> Waterborne Transport Innovation, Trzy Lipy 3, 80-173, Gdańsk, Poland

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## ABSTRACT

The rapidly advancing automation of the maritime industry – for instance, through onboard Decision Support Systems (DSS) – can facilitate the introduction of advanced solutions supporting the process of collision avoidance at sea. Nevertheless, relevant solutions that aim to correctly predict a ship's behavior in irregular waves are only available to a limited extent by omitting the impact of wave stochastics on resulting evasive maneuvers. This is mainly due to the complexity of the phenomena, the existing couplings therein, and the time inefficacy in resolving the problem through real-time simulations.

Therefore, this paper attempts to fill this knowledge gap by presenting a probabilistic, data-driven meta-model trained using an extensive set of 6DOF numerical simulations of vessel motions in irregular waves. For this purpose, machine learning adopting causal probabilistic modeling with Bayesian Belief Network (BBN) was employed. The latter offers two-way reasoning in the presence of uncertainty and provides insight into the meta-model's outcome.

This, in turn, helps estimate a set of safety-critical parameters for a large passenger ship performing an evasive maneuver. This set comprises a huge quantity of ship turning circle parameters as well as the hull's rotational motions and resulting lateral accelerations, all simulated multiple times to consider the stochastic realization of the waves. The proposed meta-model can be used to assist watchkeeping officers' decisions or raise their awareness concerning the possible consequences of evasive maneuvers performed. The achieved accuracy of the meta-model's prediction lies within a range from 81% to 98%, which makes it suitable for this purpose.

## 1. Introduction

Due to the increasing automation of the maritime industry and the progressive introduction of autonomous systems into service, operational Decision Support Systems (DSSs) are needed today more than ever before. These tools, operating on the basis of mathematical models, are able to directly support the Officer of the Watch (OOW) or an autonomous agent in terms of leading performance indicators for safety (LPIs) [1], in a diverse range of ship operations at sea. Among these, one can

mention, for example, shipboard DSSs used in collision and grounding avoidance [2–5], flooding risk assessment [6–9], weather routing [10–12], or fuel and energy optimization [13–15].

The most popular group are DSSs designed for collision avoidance and ensuring a ship's safe operation [16], while DSSs themselves are the main application of modern ship collision avoidance solutions [17]. However, relatively few tools and models, even those focused on stability-related issues, take into account a vessel's operation in a complex marine environment that is prone to external disturbances.

*Abbreviations:* ANN, Artificial Neural Network; BBN, Bayesian Belief Network; CFD, Computational Fluid Dynamics; CNN, Convolutional Neural Network; CPT, Conditional probability table; CV, Cross-Validation; DOF, Degrees of freedom; DSS, Decision Support System; EM, Expected Maximization; IMO, International Maritime Organization; LOA, Length overall; LPI, Leading performance indicator; LSTM, Long Short-Term Memory; MAP, Maximum a posteriori; MASS, Maritime Autonomous Surface Ships; MSI, Motion Sickness Incidence; OOW, Officer of the Watch; TC, turning circle; VCG, Vertical Center of Gravity.

\* Corresponding author at: Institute of Naval Architecture, Gdańsk University of Technology, Faculty of Mechanical Engineering and Ship Technology, Gabriela Narutowicza 11/12, 80-233 Gdańsk, Poland.

*E-mail address:* [jakub.montewka@pg.edu.pl](mailto:jakub.montewka@pg.edu.pl) (J. Montewka).

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Even fewer studies consider the nature of irregular waves and their stochasticity [18–20]. This can significantly affect the outcome of an evasive maneuver, the accompanying hull motions, and their aftermaths, as recently demonstrated in the authors' earlier works [21,22]. The hull motions are as relevant as the successful evasive maneuver itself, since excessive rolling and pitching of the ship as well as lateral acceleration may directly lead to accidents involving passengers. Such an operation could be hardly recognized as safe or even acceptable. Consequently, setting aside the complexity of the marine environment, the meta-models on which existing systems are based, can only to a limited extent predict the short-term response of a ship during her turn resulting, for instance, from the execution of an evasive maneuver. However, in light of the technological solutions for unmanned and autonomous ships [23–25], it seems reasonable to take into account the ship's safety-critical indicators as accurately as possible, to make the decisions prompted by a DSS more realistic.

A variety of methods can be used to design and develop a meta-model for operational decision-making, in particular for predicting a ship's response in waves. In the literature, one can find solutions based on Artificial Intelligence and deep learning models, such as various types of Artificial Neural Networks (ANNs) [26–28]. This family of methods yields numerous benefits resulting mainly from their efficiency [29] and dynamic adaptability [30], especially when the networks are trained using reliable and real-world input data [31].

Probabilistic approaches include the Bayesian Belief Network (BBN) or Monte Carlo simulations, among others [32–34]. For instance, Carchen et al., 2021 assessed passenger comfort on board a ship using Motion Sickness Incidence (MSI) as the leading indicator, with a case study application based on maritime traffic data [35]. Montewka, et al., 2022, proposed a simplified model featuring BBN for predicting the maximum roll angle for a turning vessel [36]. Zhang et al., 2024, in turn, introduced an integrated framework for investigating the risk of intelligent ship collisions on inland waters. They used the Bayesian Network learned using accident data for analyzing multiple simulation scenarios, including verification of factors related to ship maneuvering [37].

In turn, hybrid data-driven methods combine the advantages of various approaches [38,39]. For instance, Zhang et al., 2023 used a hybrid method consisting of a dataset delivered through Computational Fluid Dynamics (CFD) as well as the application of Long Short-Term Memory (LSTM) and Convolutional Neural Network (CNN) to accurately forecast ship rolling amplitudes in rough seas [40]. Similarly, Marlantes & Maki, 2024 as well as La Ferlita et al [41] used CFD to provide input data for machine learning algorithms [42]. This led to the creation of a practical models used for roll, heave and pitch motion predictions through limited computational and time resources.

Given all of the above, the literature lacks a comprehensive and holistic model for predicting a ship's response when maneuvering in irregular waves. This problem is particularly acute in the case of turning, which is the most typically applied solution in collision situations since such a maneuver is relatively quick [43]. Consequently, the period when the ship is exposed to wave action is limited accordingly. Therefore, a wave-induced motion may vary tremendously as the ship reacts to the wave realization as encountered at that moment, which in the short term may differ from average long-term wave parameters [21,22].

Therefore, this paper presents a data-driven, causal-probabilistic model created using Bayesian Belief Networks that can be either used as the core of the navigational DSS or as a source of LPs for future Maritime Autonomous Surface Ships (MASS). BBN has been selected as a modeling method to adequately reflect the stochastic nature of the investigated process, the associated non-linearities, and the resulting uncertainties. The proposed model encompasses several substantial aspects that contribute directly to the existing literature:

Predicting ship response in the context of the ship's maneuverability and the resulting turning circle parameters. These are of utmost

importance in terms of collision avoidance application and triggering evasive actions.

Predicting ship response in the context of hull motions in irregular waves and accompanying stability-related phenomena. This allows for detecting in advance excessive motions leading to a potentially dangerous situation. Also, for considering the comfort of the ship's passengers on board by predicting total lateral accelerations in two representative locations.

Taking into account the stochastic nature of sea waves by considering their multiple realizations for the same wave spectrum. This, in turn, allows the model to respond in the form of data distribution instead of a potentially misleading single value. This illustrates to the end-user what probability a specific range of outcomes might be expected instead of a deterministic trajectory, roll, pitch, and acceleration.

The rest of the paper is structured as follows: in [Section 2](#) the overall modeling process along with adopted methods, dataset, and modeling tools are presented. The developed probabilistic models are presented in [Section 3](#) and further discussed in [Section 4](#). [Section 5](#) concludes the paper.

## 2. Methods, data, and tools

### 2.1. Overarching modeling framework

The adopted modeling framework aimed to develop a meta-model estimating the expected response of a selected passenger ship prior to initiating evasive maneuvers, and taking into account the stochastic nature of irregular waves. By design, the meta-model should allow for two modes of operation, namely predictive and diagnostic, thus employing forward and backward reasoning. The predictive application could be used to assess the probability of the response variables (e.g. ship motions) for given operational or environmental parameters, such as rudder angle, ship speed, wave height, etc. The diagnostic mode could be, in turn, used to figure out how the ship's operational parameters should be set in the observed environmental conditions to maintain response variables in the desired limits.

The adopted process of the model development can be divided into three distinctive stages, namely *training dataset development*, *data processing & Bayesian Learning*, and *meta-model validation*, which are presented in [Fig. 1](#).

To be informative enough for operational purposes during collision avoidance actions, the model was designed to include the following parameters as its outcome: maximum roll and pitch angles, maximum total lateral acceleration in two distinctive locations on board the ship, as well as accompanying maneuvering characteristics, such as tactical diameter, advance, and transfer distances resulting from ship turning circle (TC) as defined in [44]. The set of governing input data covers the following: the initial ship's speed, the magnitude of rudder angle, significant wave height, and the initial angle of wave attack on the ship's hull. These were selected to reflect as best as possible the parameters which have the greatest impact on a ship's evasive maneuver executed in real conditions.

Noteworthy, the first two aforementioned variables, i.e. the ship speed and rudder setting, can be fully controlled by the crew unlike the environmental parameters describing waves, which should always be acknowledged by OOW when considering any collision avoidance action. However, the wave system characteristics are a statistical description by definition, since the irregular wave spectrum remains the nondeterministic description of the process. Simply, the assumed wave spectrum with parameters such as the significant wave height and the peak period produces a countless number of wave realizations. In the long run, they converge to the statistical description provided by that spectrum, although from a short-term perspective, the wave realization may vary significantly. As the ship turning lasts for a limited time, the

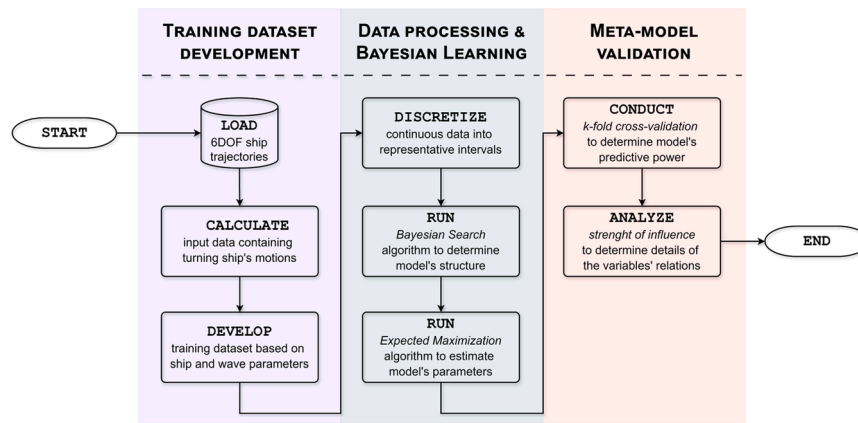


Fig. 1. The process of probabilistic data-driven meta-model development.

corresponding short-time approach is relevant. Therefore, a single simulation or prediction of a ship's response may (with a stroke of luck) or may not be representative of a ship's range of operation, and such an approach would be far from scientific rigor. To prevent such an oversimplified representation of the trajectory and ship motions, hundreds of simulation repetitions were conducted for each single operational scenario. From the practical point of view, the stochastic realization of irregular waves results in a different ship response for the same initial conditions, and therefore a different possible outcome of an evasive maneuver performed. The fact this is dealt with in this paper is one of its major achievements and contributions.

Therefore, to grasp the impact of wave stochastics for each unique set of operational and environmental parameters, the time-domain ship motion simulations repeated 500 times were used, which led to a massive dataset of 96,000 trajectories of the vessel's turn. To deliver necessary input data, a 6DOF (degrees of freedom) ship motion model called *LaiDyn* was selected and used as an exemplary source of numerical data. The foundations of its operation are briefly introduced in Section 2.2.

Each single numerical simulation contained a record of the vessel's turning trajectory along with corresponding ship motion parameters. Based on this input data, parameters related to ship maneuvering as well as resulting lateral accelerations were calculated for each stochastic wave realization and consequently served as a training dataset for the meta-model developed (see Section 2.3 for details). Such a multiple representation of the analyzed phenomena more accurately than deterministic models reflects ship operating conditions and foremost the nature of irregular waves. Therefore, this approach is more suitable for the operational decision support that the model is designed for.

Once the training dataset was established, the process of data discretization began to accurately depict the operational ranges of the parameters considered within the future meta-model. Afterward, *Bayesian Learning* was employed to determine the model's structure through the Bayesian Search algorithm as well as its parameters by the *Expected Maximization* method. This stage of model development is described in detail in Section 2.4.

## 2.2. Simulation of ship motions in irregular waves

Numerical simulations of ship motions were performed with the use of a 6DOF ship motion model, called *LaiDyn*, described in detail in [45].

*LaiDyn* has been developed as a hybrid non-linear model for time domain simulations comprising the ship's response to the external excitation by waves as well as the resistance, propulsion, and steering forces at the same time. From the planar motion point of view, the so-called reference technique [46] was applied consisting of the adoption of the hydrodynamic derivatives and their refinements based on a

comparison to the most similar known ship. However, the propeller thrust is directly modeled with the use of *Kt* curve approximation. The lift and drag forces generated on the rudder fin are directly modeled as well, accounting for the estimated relative velocity of water flow around the rudder [45]. The *LaiDyn* model does not need any speed control option, which is advantageous from the perspective of outcome fidelity. Instantaneous velocity is the result of a calculation performed at each time interval using Newton's equations of motion, with thrust and total resistance as the input values. As the code is based on a panel approach, the wave-added resistance is determined while integrating the forces across all the panels distributed over the hull.

Besides the forces related to maneuvering manifested in the outcome of simulations as the ship trajectory (spatial-temporal location), a vastly important role is played by modeling the forces involved in the ship oscillatory motions including all 6DOF i.e. roll, pitch, yaw, heave, surge, and sway. The Froude-Krylov forces directly result from water pressure integration over the instantaneously submerged hull panels. The nonlinear approach to the Froude-Krylov component refers to the pressure distribution accounting for both the waveform and the current position of every panel due to the ship's motion in the Earth-fixed coordinate system [47].

Furthermore, the radiation and diffraction forces are considered as well and they are calculated by linear approximation using the convolution integral approach. The retardation function implements the memory effect on the radiation forces [45].

Bearing in mind the objectives of this research that involve the stochastic realization of sea wave process, and therefore the nondeterministic response of the ship, it is essential to generate a mathematical model for the waves. Therefore, the assumed wave spectrum (JONSWAP here) is reconstructed by superposing a number of considered components of an irregular wave (here 19 components). Those contain two random parameters for the sake of repetitive generation avoidance, namely the random phase shift and the random frequency increment.

Even though the *LaiDyn* model takes into account the fairly complex phenomena taking place in irregular waves, its output is an approximation of the real-world behavior of the vessel, as every model. In order to confirm a credible reproduction of reality, the code has been involved in several benchmark studies worldwide. Seakeeping-related studies were of most interest [48–50].

Furthermore, some model tests focused on both the nonlinear phenomena like parametric rolling as well as maneuverability: see, for example, [46,51,52]. Therefore, the code has been acknowledged as credible and used in numerous studies related to complex ship motion phenomena in irregular waves: see, for example, [53].

Fig. 2 depicts the exemplary trajectory of the selected passenger ship, along with accompanying motion and turning circle parameters. These include, among others, the elevation of waves encountered by the ship,

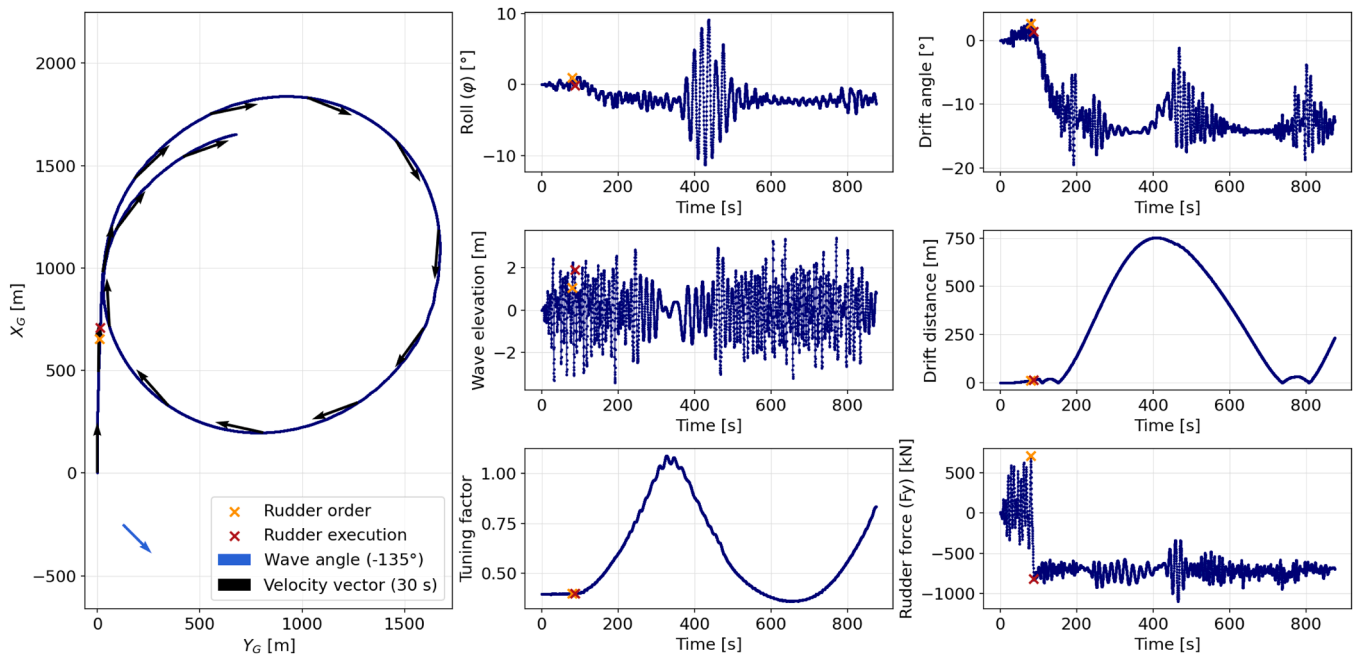


Fig. 2. Visualization of a sample turning trajectory and selected ship motion parameters.

roll angle, or tuning factor ( $T_e/T_r$ ) as a ratio of the encountered wave period ( $T_e$ ) to the natural roll period ( $T_r$ ). Once the tuning factor reaches a value close to 1, synchronous rolling has been noted to occur. This means that *LaiDyn* not only simulates the irregular seas but also mimics the ship's response with respect to accompanying wave-induced phenomena which can occur at certain (possibly unfavorable) initial directions of wave attack on the hull, such as bow-quartering seas, shown as the sample case in Fig. 2.

### 2.3. Training dataset

The training dataset includes 96,000 simulations of the starboard-side turning of the Floodstand-B passenger ship [54], which were previously used in other studies focusing on the impact of stochastic realization of irregular waves on ship motions [21,22]. Input ship trajectories were created using the aforementioned 6DOF *LaiDyn* motion model [45], for an assumed set of operational and environmental parameters. Their values were selected to suitably reflect a range of conditions that cruise or passenger ships can meet during both routine and demanding operations in open waters [55,56], including the execution of smooth and firm evasive maneuvers [57,58]. The considered set of ship operational parameters consisted of:

- one Vertical Center of Gravity (VCG): 15.19 m;
- two initial forward ship speed values: 12 and 16 kts (6 and 8 m/s, respectively);
- four magnitudes of rudder angles: 10°, 15°, 20°, and 30° (starboard side);

and the following parameters describing the marine environment, particularly irregular waves:

- deep, unrestricted water;
- JONSWAP wave spectrum with the peakedness  $\gamma = 3.3$ ;
- 19 components of irregular waves;
- 500 stochastic wave realizations;
- 3 significant wave heights ( $H_S$ ): 1.5 m, 3.5 m, and 5.5 m;
- 3 accompanying, most likely wave peak periods ( $T_p$ ) that can occur for selected  $H_S$  based on [59]: 6.99 s, 7.92 s, and 8.80 s;

- 8 initial angles of encountered wave ( $\mu$ ): 0°, ±45°, ±90°, ±135°, and 180° In this notation 0° indicates the following seas, positive values indicate starboard while negative ones refer to the port side, and 180° stands for head seas. For instance, +135° should be interpreted as starboard side bow-quartering seas.

The selected model ship is a mid-size concept passenger vessel. Its profile is depicted in Fig. 3 while all details regarding its dimensions are given in Table 1.

Once the input data had been prepared, i.e. the ship motion simulations outcome, further processing was applied, so the components and total lateral accelerations acting on a passenger located at two selected representative locations were calculated for each time step of the ship's motion simulations while turning. For this purpose, the bow-most location on the sun deck (#1 in Fig. 3) was selected, as well as the restaurant situated in the aft part of the ship's 5th deck (#2 in Fig. 3). The total lateral acceleration consists of the sum of lateral components of accelerations acting in relation to the ship's roll, heave, sway, pitch, and yaw, as well as due to gravity. For each trajectory, the final value was maximized in order to identify the largest occurring result in each simulation scenario.

The ship turning trajectories fluctuate even when the initial conditions in the numerical simulations remain unchanged. Thus, these cause a different ship response to encountered irregular waves and consequently, different turning circle parameters and values of lateral accelerations. This is due to the representation of wave stochasticity by *LaiDyn*, which was previously verified in the work [22]. This can be also noted in Fig. 4 where a single overlaying marker represents on the scatterplot matrix [60] a single result from 96,000 simulations used in the prepared training dataset.

Therefore, to organize such an extensive input dataset into a meaningful meta-model, it is necessary to employ an appropriate modeling technique capable of representing the inherent stochastic nature of the analyzed process. To this end, causal-probabilistic modeling using Bayesian Belief Networks (BBNs) was used along with associated learning algorithms.

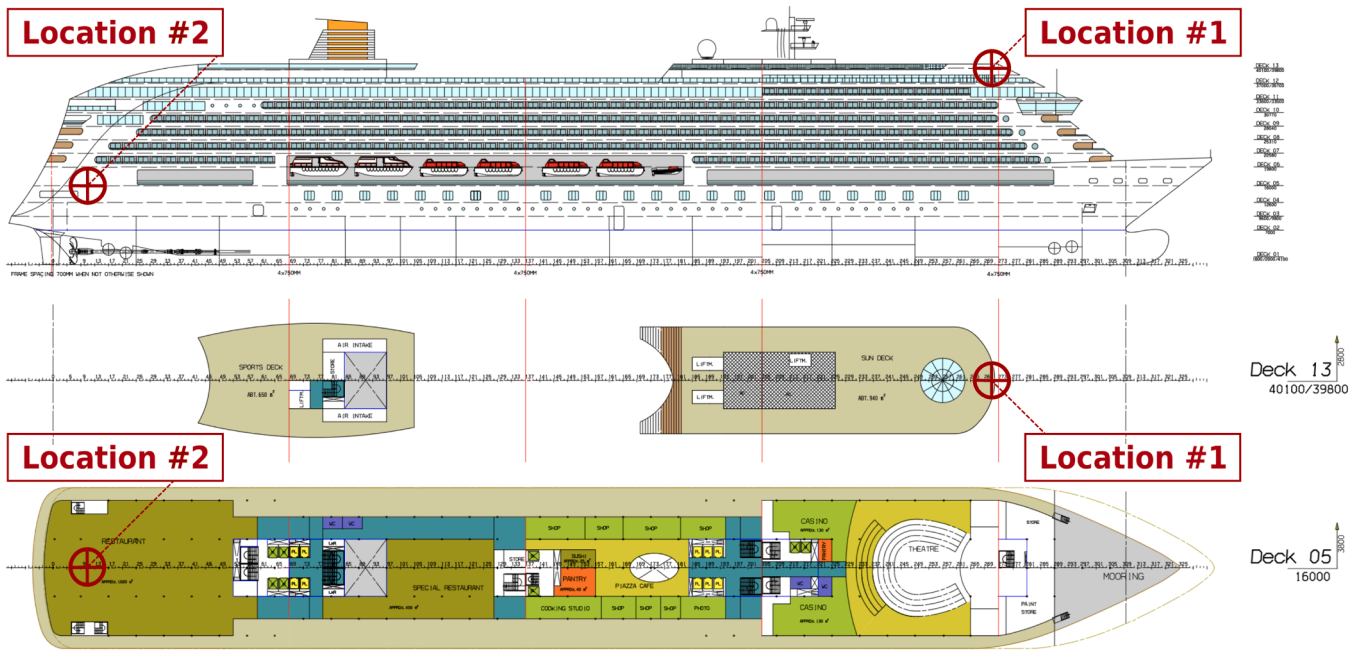


Fig. 3. Profile of the selected passenger vessel with two chosen locations as given in [22], based on [54].

Table 1

Main characteristics of the selected Floodstand-B passenger ship.

Length overall (LOA) [m]	Beam [m]	Draft [m]	Displacement [t]	Gross tonnage (GT)
238.0	32.2	7.2	34,367	63,000

### 2.4. Bayesian learning

The methodology employed in this study to construct a meta-model is referred to as Bayesian Belief Networks and belongs to a family of probabilistic models. This approach can capture the inter-relationships among the attributes of the input dataset and express them in a probabilistic manner. Furthermore, BBNs facilitate a more comprehensible representation of the model's structure and enable fast modifications, such as enhancing predicted accuracy. Furthermore, BBNs can be effortlessly refreshed with new data.

These advantages are significant when compared to other predictive methods, such as regression or neural networks.

Probabilistic graphical models are mathematical representations of random variables and their conditional dependencies, [61]. These models are typically represented using a directed acyclic graph (DAG).

A typical BBN consists of a pair  $N = \{G, P\}$ , where  $G$  is a DAG with a set of variables  $V = \{V_1, \dots, V_i\}$ , and a set of edges  $E$  representing the connections between the variables.  $P$  represents a collection of probability distributions for the variable  $V$ . BBNs are a collection of variables, which have two types of dependencies: a quantitative dependency represented by  $P$ , and a qualitative dependency represented by  $G$ . Therefore, a network  $N = \{G, P\}$  serves as a proficient depiction of a joint probability distribution  $P(V)$  throughout  $V$ , based on the structure of Goutlined below:

$$P(V) = \prod_{X \in V} P(X|parents(X)), \quad (1)$$

The conditional probability table (CPT) represents the probability distribution of a variable given a specific set of parent variables. BBNs can calculate the conditional probability of a variable, based on the values assigned to the other variables. The process of reasoning, known as probability propagation or belief updating, occurs through the

transmission of information within the network, and this transmission is not restricted to the directions of the arcs.

BBNs can be conditioned on any subset of their variables, allowing for reasoning in either direction. So, it is possible to perform forward (predictive) reasoning by using fresh information about causes (explanatory variables) to update ideas about the consequences (response variables), in accordance with the network edges. Alternatively, one might engage in backward (diagnostic) reasoning, which involves determining the most likely causes based on the observed effects. This process involves propagating information in the model against the direction of the edges. For example, a BBN representing the relationships between an expected roll angle (the response variable) and operational as well as environmental conditions (the explanatory variables) can be used to:

- predict the probability for a ship to attain a certain value (or rather interval) of roll angle given the values assigned to explanatory variables,
- diagnose the most probable distribution of explanatory variables for the specific value of roll angle, see for example, [36].

In other words, such a BBN can predict the response of the ship given the maneuver parameters; additionally, it can diagnose the most likely maneuver parameters for the predefined response of the ship. BBN-based models can be created either by human construction using domain knowledge or by discovering them from data using a set of learning algorithms.

In this research, the latter technique by constructing a probabilistic meta-model based on a comprehensive training dataset generated on the basis of the 6DOF ship motion simulations in the time domain was employed. To this end, selected learning algorithms that helped discover and present causality were used.

#### 2.4.1. Learning algorithms

In learning BBNs two major tasks can be distinguished: 1) structure development ( $G$ ) and 2) parameter estimation ( $P$ ).

To discover the network structure, several approaches can be adopted, each reflecting a distinctive way of looking at a BBN:

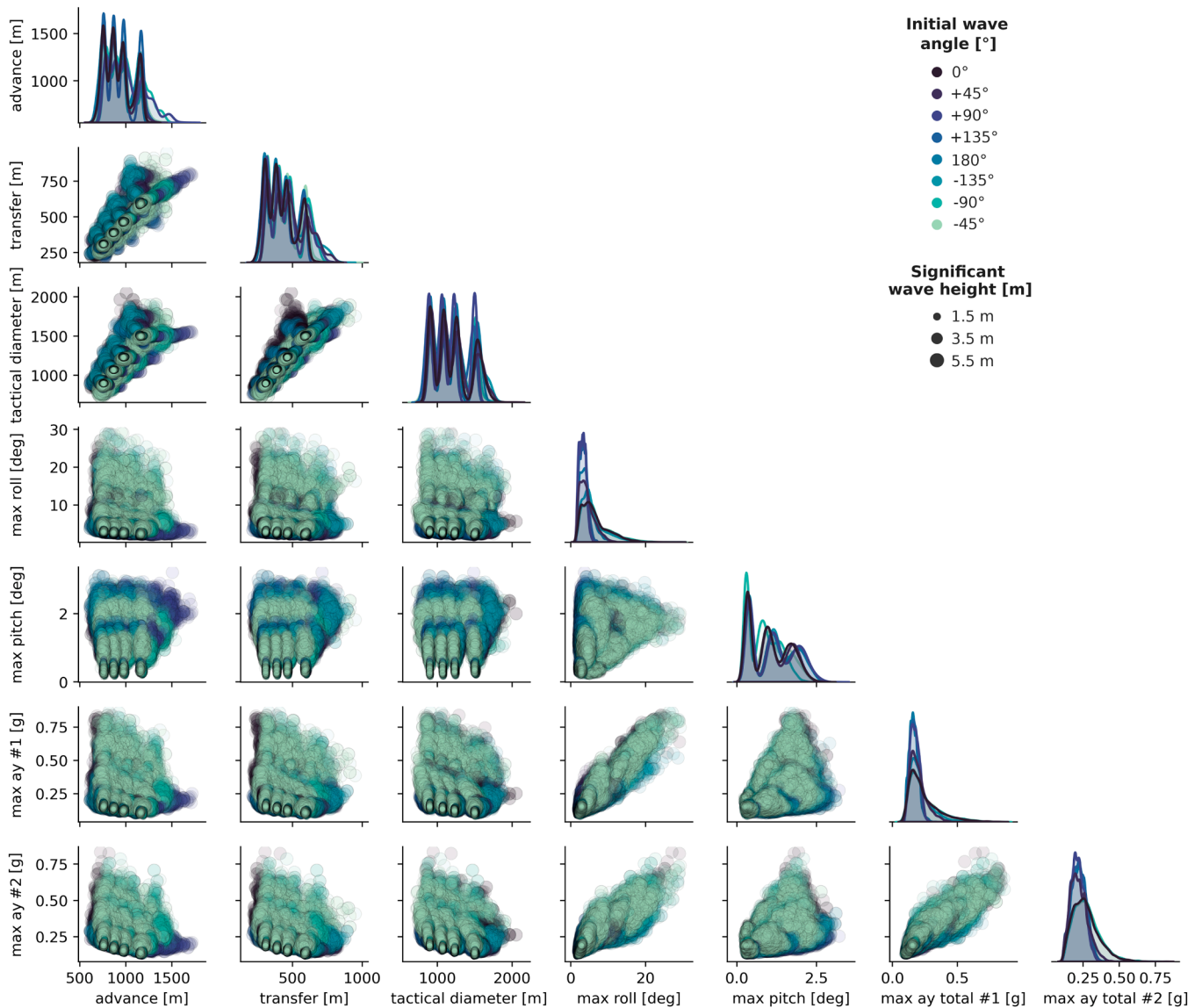


Fig. 4. Exemplary parameters from the training dataset considering wave stochastic realizations.

*Scoring-based learning algorithms*, where a BBN is seen as a structure that encodes the joint probability distribution of the variables, and the best-suited BBN is the one that best fits the training data. This structures the BBN according to scoring-based learning algorithms, where prior ordering of the variables can speed up the learning process and ensure proper representation of the dataset and the analyzed process it intends to model. An exception here is an algorithm called *Naive Bayes*, which assumes full independence of the explanatory variables, and so no prior ordering is needed.

*Constraint-based learning algorithms*, where a BBN structure codes a group of conditional independence relations among the variables via the concept of *d-separation* [62,63]. BBN learning is based on identifying the conditional independence relationships, using relevant statistical tests. Then, the results of the tests are used as constraints to construct a BBN.

To acquire knowledge about the structure and parameters of the meta-model, in this paper, GeNIe software was used, as described in [64]. In order to achieve this objective, an algorithm known as *Bayesian Search* was employed. It is highly renowned, it was introduced by [65] and has been widely adopted since its early inception.

The approach employed is a hill-climbing algorithm, led by a scoring

heuristic specifically the log-likelihood function, and incorporates random restarts. Before learning, the background knowledge of the variables' order is provided, by organizing variables in groups of temporal tiers without specifying any interrelations in order to reflect the existing causality of the analyzed process and indicate what variable belongs to what group (cause or effect).

Subsequently, the algorithm is initiated, which assumes that a node lacks any parents, and thereafter begins to insert the parents that maximally enhance the probability of the eventual structure incrementally. Once the inclusion of any individual parent no longer has the power to raise the likelihood, the process of adding parents to the node is ceased.

The algorithm produces a directed acyclic graph, which attains the maximum score. The score is directly proportional to the likelihood of the data for the given structure reflecting its learning data, providing that we assign equal prior probabilities to all structures. The procedure governing the *Bayesian Search* algorithm is described in detail in [65].

Learning parameters become quite simple once the structure has been established for the provided dataset. One way to accomplish this is by using the conditional frequencies obtained from the data, as demonstrated in the study [62]. Another approach is to employ the expected maximization (EM) method, which allows for the estimation of

maximum likelihood or maximum a posteriori (MAP) parameters in the models, as explained by [63].

The EM method proceeds by iteratively alternating between an expectation (*E*) step and a maximization (*M*) step. The former creates a function that computes the expected value of the log-likelihood by using the current parameter estimate. The latter computes parameters that optimize the expected log-likelihood acquired during the *E* step. Afterward, these estimated parameters are used to determine the distribution of the variables in the following *E* step.

#### 2.4.2. Input data processing

Since only a few learning algorithms support continuous and mixed data, and Bayesian Search is not among those, the input data needs to be properly processed before applying the learning algorithms. This process covers splitting a continuous variable into several intervals by adopting one of the methods such as equal interval; equal quantile; hierarchical; and manual, for more information: see, for example, [66].

The importance of this stage of model preparation lies in two key factors: firstly, it ensures an accurate representation of the analyzed process, and secondly, it enhances the computational efficiency of the model and the accessibility of data for model learning.

Increasing the number of intervals can provide a more accurate representation of continuous data. Nevertheless, the magnitude of the conditional probability table (CPT) for a variable grows exponentially as the number of the variable's parents and the number of intervals for each parent rise. In a three-node network, even with just one outcome variable and two parents, the number of intervals for each variable greatly affects the calculation of conditional probabilities. Specifically, if each variable has two intervals, there would be 8 ( $2^3$ ) conditional probabilities to calculate. However, if each variable has three intervals, there would be 27 ( $3^3$ ) conditional probabilities to calculate for the outcome variable; and in the case of four variables and three established intervals, each will result in a CPT of 81 ( $3^4$ ) entries.

Therefore, in order to warrant the use of multiple intervals, a substantial and diverse dataset is necessary and should contain a sufficient number of data to accurately calculate significant probability values for each combination of variable intervals. Alternatively, certain conditional probabilities may rely on a limited number of data, or perhaps lack the necessary observations, resulting in inconclusive entries in the CPT. A situation may arise where the model's granularity increases with the number of intervals, but its accuracy remains unchanged, as stated in the comparison analysis by [66].

### 2.5. Meta-model validation

To establish trust in the developed meta-model two distinctive analyses are carried out namely: *cross-validation* and *strength of influence*. The former allows determining the meta-model accuracy while the latter assesses the model behavior and the relations among variables in the model.

#### 2.5.1. Cross-validation

To assess the accuracy of the developed meta-model *k-fold Cross-Validation* (CV) is used, in which a portion of the data is used to build the model and a separate portion is used to assess the model's prediction ability. This is how the K-fold algorithm operates:

1. Randomly divide the data set into  $K$  subsets,
2. For each subset  $S$ :
  - a. train a model on the data but not on the subset  $S$ ,
  - b. test model on the subset  $S$ ,
3. Return the average error over the  $K$  subsets.

The CV tests provide accuracy, thus the prediction power of the model with respect to each response variable and its instances defined by the intervals.

#### 2.5.2. Strength of influence analysis

This analysis aims to determine the most influential variables in the model and to indicate the direction of this influence. It helps to determine the direction of information flow within the model and its extent, which in turn enables the specification of the most influential variables in the model. In addition, such an analysis combined with the assessment of the uncertainty of the variables can increase the reliability of the model and ultimately strengthen confidence in it.

The magnitude of the influence between two variables is determined based on the difference between two discrete probability distributions that are used in BBNs to describe those variables. By comparing differences, representing various sets of variables, one may draw valid conclusions about which change is more significant than the other in a particular model.

To express the difference several *distance measures* can be used; however, the *J-divergence* measure is the most appropriate for our purposes, [67]. The concept of *J-distance* uses the cross-entropy notion, which measures the variation between two probability distributions. It is based on the fundamental concept of entropy, which quantifies a distribution's uncertainty or randomness. The distance  $J$  between two probability distributions  $P$  and  $Q$ , which represents the magnitude of the influence of one variable onto the other, is calculated as follows, see [67, 68]:

$$J(P, Q) = \frac{K(P, Q) + K(Q, P)}{2}, \quad (2)$$

$$K(P, Q) = - \sum_{i=1}^n p_i \log_2(q_i) + \sum_{i=1}^n p_i \log_2(p_i) = H(P, Q) - H(P), \quad (3)$$

where  $H(P, Q)$  is the cross-entropy of  $P$  and  $Q$ , and  $H(P)$  is the entropy of  $P$ .

While the direction of influence is expressed qualitatively, the conditional probability tables of a BBN are used for this purpose. The following four types of influence can be distinguished: *positive*, *negative*, *null*, or *ambiguous* marked with the corresponding colors: *green*, *red*, *black*, and *violet*.

The type of influence (*positive*, *negative*, *null*) for a particular arc is indicated by a series of equations. If the influence is neither *positive*, *negative*, or *null* it is considered *ambiguous*. The sign of influence can be determined in both directions (from the parent to the child and from the child to the parent). If these two are not coherent then the sign will also be regarded as being ambiguous. For a detailed mathematical description of this concept, the reader is referred to [67].

### 3. Models and results

#### 3.1. Variables intervals development

The initial aim of the meta-model developed herein is to inform the OOW whether the expected parameters, such as roll and pitch angles, accelerations as well as expected turning circle dimensions that the ship may experience when turning, will remain within the operational limits or will exceed those. Therefore, the variables in the model shall reflect the realm of the analyzed process.

Therefore, in this paper, the hierarchical and manual discretization methods have been adopted to determine the intervals for variables employed in the model. For variables featuring rather simple distributions hierarchical method was enough, for more complex distributions only the manual method was able to reflect the operational aspects and requirements of the analyzed process.

The resulting intervals of the input data discretization are presented in Table 2 where the hierarchical method was applied to the following variables:  $v$ ,  $r$ ,  $H_S$ ,  $\mu$ . However, the remaining variables *acc*, *roll*, *pitch*, *diam*, *adv*, *trsf* required more in-depth analysis to provide justified and interpretable outcomes. Please note that the notation of the wave angle

**Table 2**  
Variables intervals – the result of the conducted discretization process.

Intrv	$v$ [m/s]	$r$ [°]	$H_s$ [m]	$\mu$ [°]	diam [LOA]	adv [LOA]	trsf [LOA]	acc [g]	roll [°]	pitch [°]
1	6	10	1.5	0	<4	<3.5	<1.5	<0.2	<3	<0.58
2	8	15	3.5	45	4–5	3.5–4.5	1.5–2.0	0.2–0.3	3–5	0.58–1.47
3	–	20	4.5	90	5–6	>4.5	2.0–2.5	>0.3	>5	>1.47
4	–	30	–	135	>6	–	>2.5	–	–	–
5	–	–	–	180	–	–	–	–	–	–
6	–	–	–	225	–	–	–	–	–	–
7	–	–	–	270	–	–	–	–	–	–
8	–	–	–	315	–	–	–	–	–	–

Abbreviations: *Intrv* – interval number,  $v$  – ship speed,  $r$  – rudder angle,  $H_s$  – significant wave height,  $\mu$  – wave angle of attack, *diam* – tactical diameter of TC, *adv* – advance distance of TC, *trsf* – transfer distance of TC, *acc* – total lateral acceleration, *roll* and *pitch* – rotational motions of the ship.

of attack has been changed during BBN structure development compared to this introduced in Section 2.3. The previously introduced positive angles are now labeled 45°–135° (starboard side), while negative ones (port side) are 225°–315°

**Lateral accelerations** – variables describing this parameter measured in specific locations on board the ship were divided into three intervals reflecting the normal, moderate, and severe operational conditions correspondingly. A detailed explanation for those conditions is given in the international rules as follows, [69–71]:

- normal conditions* –  $acc \approx 0.15g$  – an average person will keep balance when holding;
- moderate conditions* –  $acc \approx 0.25g$  – maximum load for the mean person keeping balance when holding;
- severe conditions* –  $acc \approx 0.45g$  – an average person will fall out of the seat if not wearing a seat belt.

**Roll and pitch angles** – these are rotational, side-wise (roll) and longitudinal (pitch) motions of the ship, if combined with moderate, and severe accelerations may lead to panic and injury to passengers or crew on a cruise ship, see [72].

The adopted boundaries for the intervals tend to reflect the normal, moderate, and severe operational conditions of this particular ship with a focus on passenger comfort and safety. To this end, the following intervals were adopted with the corresponding limits allowing for the interpretation of roll motions, as given in [73]:

- normal conditions* – roll angle below 3.0° corresponding to the limits of 2.5° for transit passengers and 2.0° for cruise liner;
- moderate conditions* – roll angle between 3.0° and 5.0°, which corresponds to the limit of 4.0° for heavy manual work and 3.0° for intellectual work;
- severe conditions* – roll angle above 5.0°, which corresponds to the limit of 6.0° for light manual work.

For pitch angle of 1.8° denotes a situation where green water enters the bow, which is an unwanted situation from the perspective of passengers. On the other hand, the operational limit designed for a naval ship, and accounting the human performance, is 1.5° [74]. Angles less than 0.5° can be considered normal operations, while those in the range of 0.5°–1.5° are considered moderate.

**Turning circle parameters** – International Maritime Organization (IMO) requires that the tactical diameter shall be less than 5 ship LOA and the advance shall be less than 4.5 LOA [44].

These are considered a reference point to develop the intervals in the meta-model, even though environmental and operational conditions differ from those covered by the mentioned guidelines. Therefore, the adopted intervals tend to reflect:

- the expected dimension of a turning circle if  $diam \in [4, 5]LOA$  and  $adv \in [3.5, 4.5]LOA$ ;*

- smaller than expected:  $diam < 4$  and  $adv < 3.5LOA$ ;*
- larger than expected:  $diam > 5$  and  $adv > 4.5LOA$ .*

### 3.2. BBN-based meta-model

In this section, the developed meta-model is presented with the use of the Bayesian Search algorithm, which pertains to a group of constraint-based algorithms, based on a dataset and temporal ordering of the node performed at the stage of background knowledge incorporation. The following temporal ordering was applied here:

- tier 1: *speed, rudder, significant wave height, wave angle;*
- tier 2: *advance, transfer, tactical diameter, abs max roll, abs max pitch, abs max ay1, abs max ay2.*

The ordering reflects the physics of the analyzed phenomena, where the variables from *tier 1* are considered explanatory and *tier 2* contains predictive variables. The cross-relations among predictive variables that are observed in the dataset are expected to be found by the learning algorithms.

The temporal ordering and further causal discoveries performed by the Bayesian Search algorithm result in the network structure depicted in Fig. 5.

### 3.3. Results of cross-validation

The results of the cross-validity tests (CV) are shown in Table 3. Therein, the overall accuracy of the meta-model and the accuracy for individual predictive variables are presented. The former yields 91%, with the latter ranging from 81% to 97%. The variable intervals are numbered according to the convention previously introduced in Table 2.

In Table 4 comprehensive information about CV tests is given in the form of a confusion matrix. Therein, the distribution of the model predictions across the variable intervals is given as well as information on whether the model tends to overestimate or underestimate the individual intervals of the class variables. For example, a response variable *abs max ay1* has three intervals *Int1–Int3*, and the CV test returns a  $3 \times 3$  confusion matrix showing the distribution of predictions for each interval compared to the actual values recorded in the dataset. It can be seen that the model predicts the variable interval *Int1* with the highest accuracy (91%), while *Int2* and *Int3* have an accuracy of 62% and 72%, respectively.

### 3.4. Strength of influence

The results of the analysis of the strength of influence, are shown graphically in previously introduced Fig. 5 and numerically in Table 5. The higher the value of the tabulated *J-divergence* parameter the stronger the influence among variables, which is also reflected by the arc thickness in the figure. From there the set of variables that have the strongest influence on the model (the thick arrows) and a set of less influential



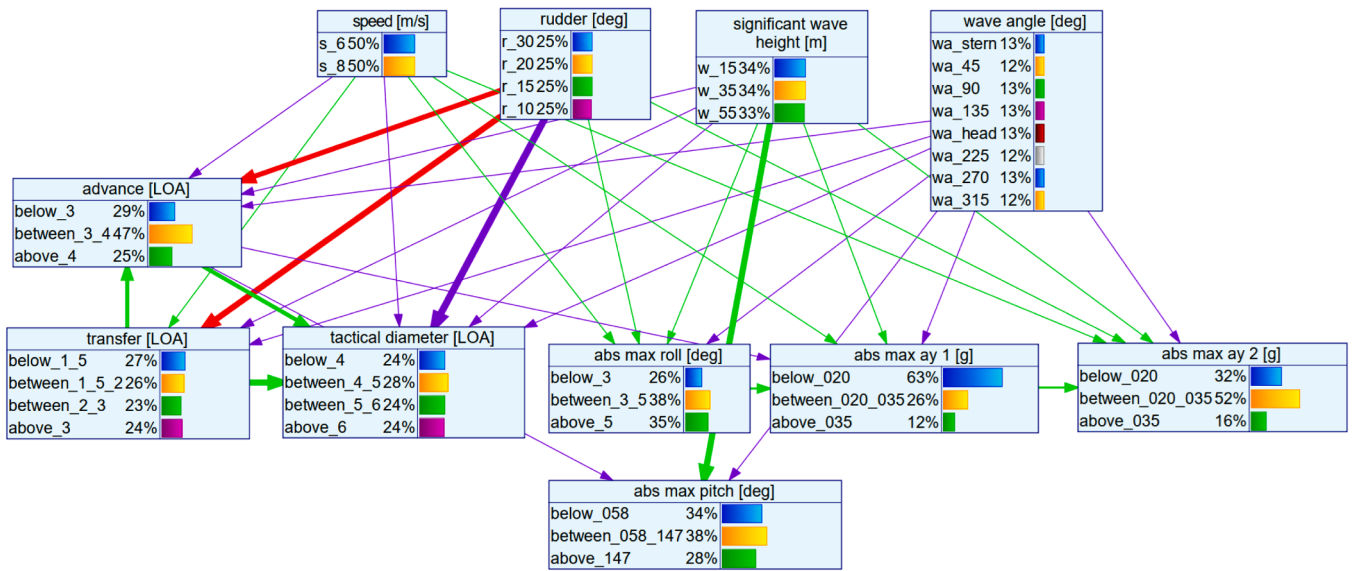


Fig. 5. Meta-model developed with the use of the Bayesian Search learning algorithm.

Table 3

Meta-model validation tests for all the predictive variables and their intervals (Int).

Variable	Accuracy [%]				
	Int1	Int2	Int3	Int4	Average
abs max ay1	91	62	72	-	81
abs max ay2	89	86	78	-	86
abs max pitch	99	93	92	-	95
abs max roll	92	81	91	-	87
advance	94	97	96	-	96
tactical diameter	98	97	97	99	98
transfer	98	94	91	98	95
<b>Overall</b>					<b>91</b>

variables (the thin arrows) can be found.

It is obvious that the variables describing the dimensions of the ship's turning circle, i.e. the variables *advanced*, *transfer*, and *tactical diameter*, are explained to a large extent by the rudder setting represented by the variable *rudder*.

The extent of the angular movement of the vessel *abs max pitch*, is mainly explained by the environmental factor of the *significant wave height*.

The variability of *abs max ay 1*, *abs max ay 2*, and *abs max roll*, on the other hand, can only be explained by a combination of more than just one variable. For example, the variation of *abs max ay 1* can only be explained by a set of input variables, such as max *abs roll*, *significant wave height*, *wave angle*, *speed*, and *advance*.

The signs of influence color-coded in Fig. 5 suggest the expected and proper behavior of the meta-model since:

- the increase of the *rudder* decreases the value of *advance* and *transfer* (marked red);
- the increase of *significant wave height* increases *abs max pitch*; (marked green)
- the increase in *ship speed*, *rudder angle*, and *wave height* lead to an increase in the values of variables describing the accelerations and angular motions of the ship (marked green).

The ambiguous sign for the strength of influence (marked violet) means that the calculations performed are inconclusive. To understand the cause of this type of influence, one needs to look at the conditional probability tables (CPTs) that describe the relationships between the

Table 4

Results of confusion matrix concerning the response variables and their intervals (Int). The values obtained from the model (*model*) are shown together with these observed ones (*actual*).

	Int1 <sub>model</sub>	Int2 <sub>model</sub>	Int3 <sub>model</sub>	Int4 <sub>model</sub>
<b>abs max ay1</b>				
Int1 <sub>actual</sub>	54,339 (91%)	4653	605	-
Int2 <sub>actual</sub>	6067	14,940 (62%)	3228	-
Int3 <sub>actual</sub>	54	3061	7849 (72%)	-
<b>abs max ay2</b>				
Int1 <sub>actual</sub>	27,390	3146	1	-
Int2 <sub>actual</sub>	3309	42,425	3295	-
Int3 <sub>actual</sub>	1	3398	11,831	-
<b>abs max pitch</b>				
Int1 <sub>actual</sub>	31,813	200	0	-
Int2 <sub>actual</sub>	123	33,538	2169	-
Int3 <sub>actual</sub>	0	2130	24,823	-
<b>abs max roll</b>				
Int1 <sub>actual</sub>	22,697	2064	0	-
Int2 <sub>actual</sub>	2232	29,392	4739	-
Int3 <sub>actual</sub>	3	3030	30,639	-
<b>advance</b>				
Int1 <sub>actual</sub>	25,543	1548	0	-
Int2 <sub>actual</sub>	644	43,178	565	-
Int3 <sub>actual</sub>	0	895	22,423	-
<b>tactical diam</b>				
Int1 <sub>actual</sub>	22,477	493	0	0
Int2 <sub>actual</sub>	303	25,447	586	0
Int3 <sub>actual</sub>	0	328	22,178	456
Int4 <sub>actual</sub>	0	0	47	22,482
<b>transfer</b>				
Int1 <sub>actual</sub>	24,902	529	0	0
Int2 <sub>actual</sub>	838	23,312	734	13
Int3 <sub>actual</sub>	0	963	19,727	993
Int4 <sub>actual</sub>	0	10	414	22,361

analyzed variables. Often the cause lies in limited data to fill the CPTs, and some of the CPT columns contain uniform values for the given combinations of input variables, which makes the calculation of the influence inconclusive. Apparently, that is the case here.

The results demonstrate the high predictive power of the meta-model which makes it suitable for any further analysis, focusing on the estimation of the motion parameters of the large passenger ship for the given operational scenarios, as shown in selected case studies in the following section.

**Table 5**  
Results of the strength of influence test.

Parent node	Child node	J-divergence weighted
rudder [deg]	tactical diameter [LOA]	0.7018
significant wave height [m]	abs max pitch [deg]	0.6603
rudder [deg]	transfer [LOA]	0.6371
transfer [LOA]	tactical diameter [LOA]	0.6119
rudder [deg]	advance [LOA]	0.5062
advance [LOA]	tactical diameter [LOA]	0.4523
transfer [LOA]	advance [LOA]	0.4435
abs max ay 1 [g]	abs max ay 2 [g]	0.1924
abs max roll [deg]	abs max ay 1 [g]	0.1803
speed [m/s]	abs max roll [deg]	0.1087
significant wave height [m]	abs max ay 2 [g]	0.1086
significant wave height [m]	abs max ay 1 [g]	0.0935
wave angle [deg]	abs max roll [deg]	0.0780
significant wave height [m]	abs max roll [deg]	0.0712
wave angle [deg]	abs max ay 1 [g]	0.0690
speed [m/s]	abs max ay 2 [g]	0.0618
wave angle [deg]	abs max ay 2 [g]	0.0317
speed [m/s]	abs max ay 1 [g]	0.0307
rudder [deg]	abs max ay 2 [g]	0.0271
advance [LOA]	abs max ay 1 [g]	0.0149
wave angle [deg]	abs max pitch [deg]	0.0090
rudder [deg]	abs max roll [deg]	0.0055
wave angle [deg]	transfer [LOA]	0.0046
wave angle [deg]	advance [LOA]	0.0041
wave angle [deg]	tactical diameter [LOA]	0.0015
significant wave height [m]	tactical diameter [LOA]	0.0011
significant wave height [m]	advance [LOA]	0.0011
advance [LOA]	abs max pitch [deg]	0.0010
significant wave height [m]	transfer [LOA]	0.0001
speed [m/s]	advance [LOA]	0.0001
speed [m/s]	transfer [LOA]	0.0001
speed [m/s]	tactical diameter [LOA]	0.0001

3.5. Case studies

A set of studies is presented here demonstrating the usefulness of the developed meta-model using the inherent features of the BBNs namely forward and backward propagation. The summary of all conducted case studies is presented in Table 6 while their results are discussed in the following subsections of the paper.

3.5.1. Predictive mode of application

First, the model is applied in a forward reasoning mode, where for a set of observed operating conditions expressed by the explanatory variables, the probability of the response variables is estimated, as shown in Figs. 6, 7, 8, and 9.

In case #1, depicted in Fig. 6, the ship is proceeding at 6 m/s (12 kts) in the stern quartering seas (wave approaching at 45° from the ship's stern on the starboard side), with a wave height of 1.5 m and a maneuver is performed by ordering rudder angle of 10°. The ship will experience accelerations and angular motions that are among the lowest values (the safe ones), with large dimensions of a turning circle. This corresponds to a smooth and wide turn under these environmental and operating conditions.

In case #2, the significant wave height is 5.5 m, while the other explanatory variables remain the same as in case #1. However, the results of the model change significantly. The distributions of lateral

**Table 6**  
Summary of the set meta-model parameters for the case studies performed.

Case study	Reasoning mode	Figure number	Preset parameters of the meta-model				
			speed [m/s]	rudder [deg]	wave height [m]	wave angle [deg]	abs max ay loc. #2 [g]
#1	Predictive (forward)	Fig. 6	6	10	1.5	45	–
#2	Predictive (forward)	Fig. 7	6	10	5.5	45	–
#3	Predictive (forward)	Fig. 8	8	10	1.5	45	–
#4	Predictive (forward)	Fig. 9	8	10	5.5	45	–
#5	Diagnostic (backward)	Fig. 10	–	–	5.5	45	< 0.2

accelerations show that the probability of experiencing moderate values is quite high, while the ship is expected to develop a serious pitching motion and moderate rolling. However, the diameter of the turning circle remains comparable to that of case #1. The change in wave height resulted in a turn of similar size to calm sea conditions, but one should expect behavior of the vessel that is uncomfortable for the passengers.

Case #3 is similar to case #1, but the vessel is proceeding at 8 m/s (16 kts) and the only difference in her response is the 20% chance of developing a moderate rolling motion (in a range of 3–5°).

In case #4, the significant wave height is 5.5 m and the ship proceeds at 8 m/s (16 kts). This leads to serious pitching and rolling motions and accelerations that can be uncomfortable or even unsafe for passengers.

3.5.2. Diagnostic mode of application

The second type of reasoning using BBN is a diagnostic mode (backward reasoning), in which the desired probability of an instance of the response variable is specified along with the other observable variables, and the most probable instances of the explanatory variables are sought. The predictive mode can be used to evaluate the most likely maneuvers (speed and rudder angle) given the current environmental conditions (wave height and direction) and the desired state of the response variable (e.g. acceleration or angular movements).

In this case, the most likely ship speed and rudder angle that can be applied under the given observable environmental conditions, so as to keep the lateral accelerations at the lowest level that is completely safe for the passengers were sought.

Therefore, the most likely speed should be low (12 kts), and small rudder angles should be applied (10°–15°) to achieve the target values as shown in Fig. 10.

4. Discussion

4.1. Findings

Due to the stochastic character of the analyzed process, appropriate modeling methods and tools need to be applied. Therefore, employing probabilistic models and supervised machine learning algorithms to organize the results obtained in the course of massive numerical simulations of ship motions in waves is a sound solution.

The presented meta-model and modeling framework seem sufficient for the given purpose, namely for the operational prediction of ship response in irregular waves during the ship turning. In the case of the decision support tool suitable for daily navigation, extreme accuracy or decimal precision is neither required nor expected. Instead, a reliable model indicating whether a given maneuver will be safe or not would be helpful – especially, when bearing in mind its potential application in modern collision-avoidance solutions, where the effect of stochastic realizations of the waves on the resulting ship motions has not been accounted for so far.

There, the meta-model can serve OOW or an autonomous agent by providing a new, holistic perspective on evasive maneuver execution. Offering information about the possible scope of ship responses due to operational and environmental conditions, as well as the nature of the sea waves, seems to be of utmost importance for the sake of navigational safety.

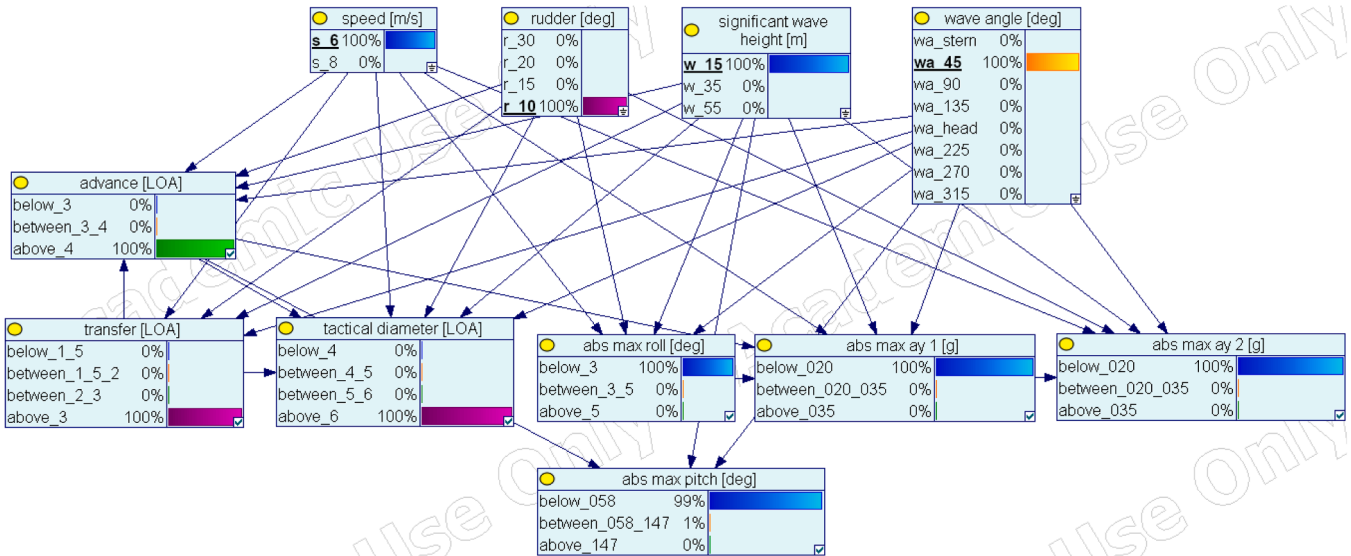


Fig. 6. Case study #1 – a sample application of meta-model in predictive mode – forward reasoning, low speed, small wave, small rudder angle.

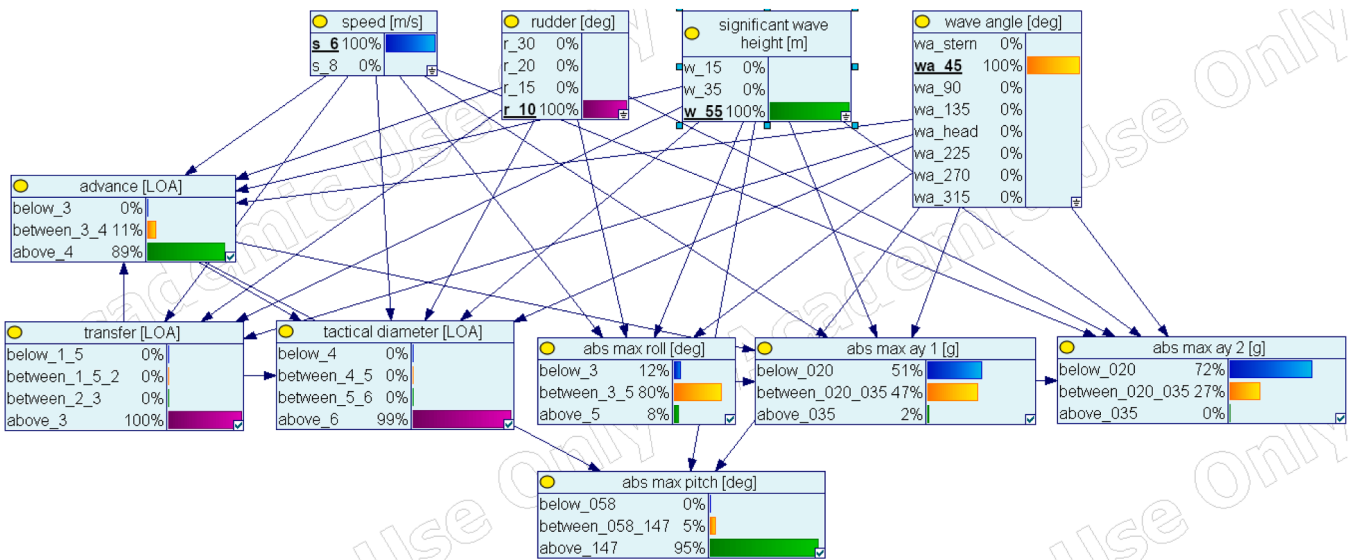


Fig. 7. Case study #2 – a sample application of meta-model in predictive mode – forward reasoning, low speed, high wave, small rudder.

The credibility of the results is also supported by the good qualitative agreement of the obtained predictions clearly showing that the ship turning circle parameters are highly dependent on the rudder setting, as shown in Fig. 5. This may appear straightforward and obvious for deck officers. However, in-depth analysis of the model’s results also revealed some non-trivial observations related to the expected ship motions in irregular waves. The sensitivity graph presented in Fig. 5 unambiguously shows that neither the lateral acceleration nor the amplitudes of pitch and roll motions can be intuitively predicted, as they are highly complex. There are no obvious predictors for such safety-critical responses of a ship when it comes to passengers’ health or comfort. Thus, OOW as well as systems using deterministic models are not able to comprehensively estimate motion-related threats. This, in turn, vastly justifies the application of the probabilistic models, as the one developed within this study.

In general, the accuracy of the developed meta-model is very high (91% on average), with the highest value for predicting the parameters of a turning circle (95%–98%), pitch angle (95%), roll angle (87%), and slightly lower for accelerations in two distinctive locations on board the

ship (81% and 86%). The meta-model features the lowest prediction accuracy with respect to the acceleration in location #1 (the sun deck of the ship) since two out of three intervals are predicted with an accuracy of 62 and 72%. For other response variables and their instances, the prediction power is always above 78% with a geometric mean of 92%.

The variables have been divided into meaningful and interpretative intervals, assuring the high predictive power of the meta-model. Nevertheless, some other division lines for the variables are feasible, especially if the end-users’ needs and preferences significantly differ from the views expressed here. This manipulation obviously will affect both the granularity and accuracy of the model, which needs to be recalculated each time the intervals are modified.

#### 4.2. Limitations

Massive numerical simulations of ship motions are performed with the use of a numerical, high-fidelity ship motion model called *LaiDyn*, mimicking the behavior of the ship in 6DOF in irregular waves. For the given purpose, namely for the analyzed ship type, hydro-meteorological

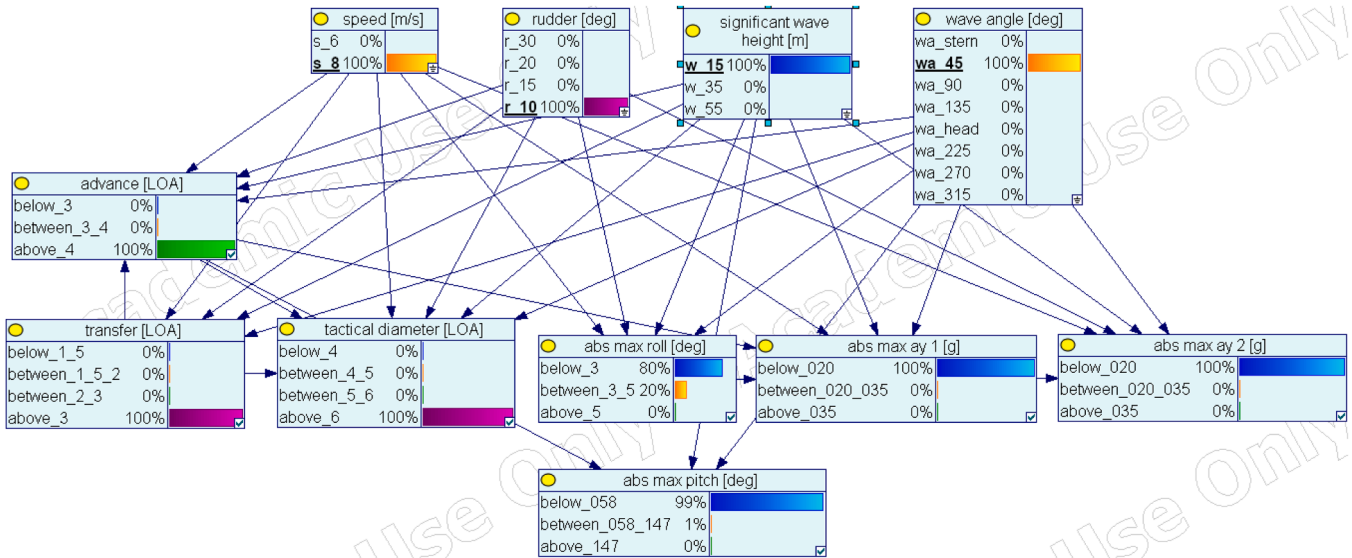


Fig. 8. Case study #3 – a sample application of meta-model in predictive mode – forward reasoning, high speed, small wave, small rudder angle.

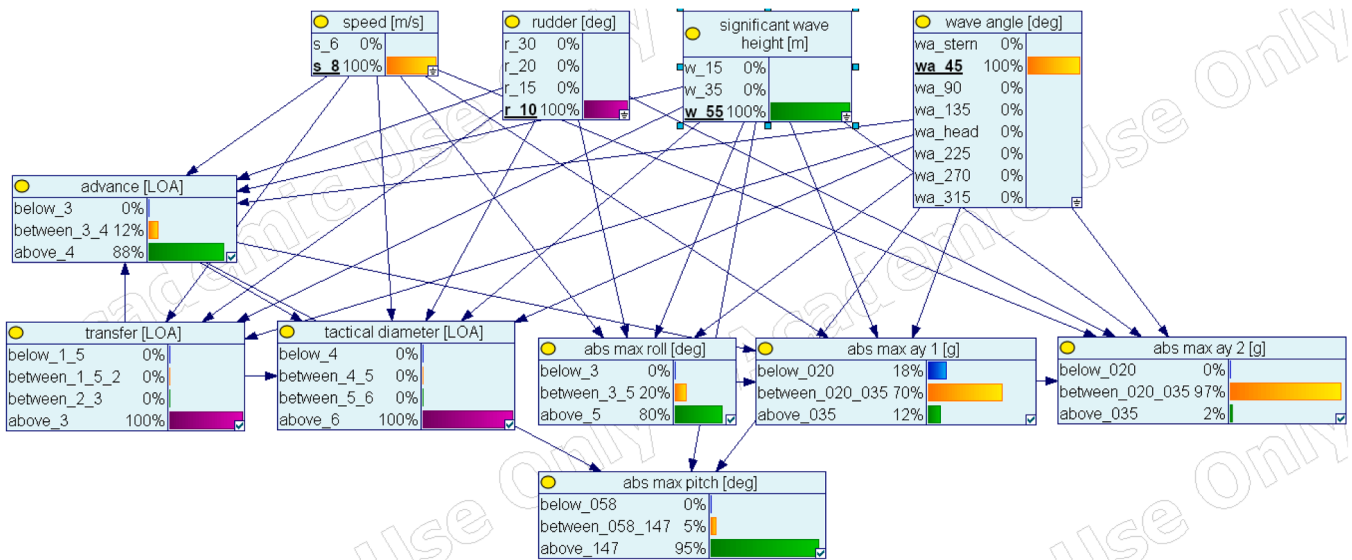


Fig. 9. Case study #4 – a sample application of meta-model in predictive mode – forward reasoning, high speed, high wave, small rudder angle.

conditions, and operational scenarios the *LaiDyn* model is deemed sufficiently accurate. However, several limitations are inherently related to the use of this code. The results may vary to some degree depending on the ship characteristics while a limited number of ships were used for the validation and benchmarking. The long-crested wave is considered with no directional spread of wave energy supply as well as the engine load control is simplified, which introduces some uncertainty to the propeller revolutions estimation. Furthermore, only a screw propeller was modeled whereas POD-type propulsion has been gradually gaining popularity on the market. Even though the approach adopted in this study remains the same regardless of the propulsion details, and generally regardless of considered ship particulars, the quantitative results may vary. The ship adopted in this study has never been built, therefore no data recorded during real operation are available, which involves some unavoidable uncertainties.

Another issue that might have affected the results obtained is the conducted data discretization process. Although the established intervals are reasonable and supported by existing literature or industry standards as presented in Section 3.1., they may require further

investigation and adjustment – especially since some of the maneuvering standards are provided for near-ideal conditions, i.e. to very limited configurations of rudder-speed combinations, which here constitute a minority. Furthermore, too few intervals may insufficiently accurately represent continuous data. On the other hand, too many intervals significantly increase the computational complexity of the established network. Therefore, it was necessary to find a compromise between the two. An inaccurate selection of values that makes an interval too large can, in turn, lead to under-diversification of the results, which may insufficiently represent the actual conditions of ship operation or the complexity of the marine environment.

It should be also acknowledged that the proposed meta-model is tailored for a particular vessel as it is data-driven based on numerical simulations of its maneuvers. Therefore, despite being highly predictive and applicable, this specific meta-model may serve as a decision-supporting aid only for ships that are similar in type and comparable in size. Nevertheless, the approach used herein with the example model ship demonstrates the practical usefulness of such tools and provides room for further work on their development and generalization.

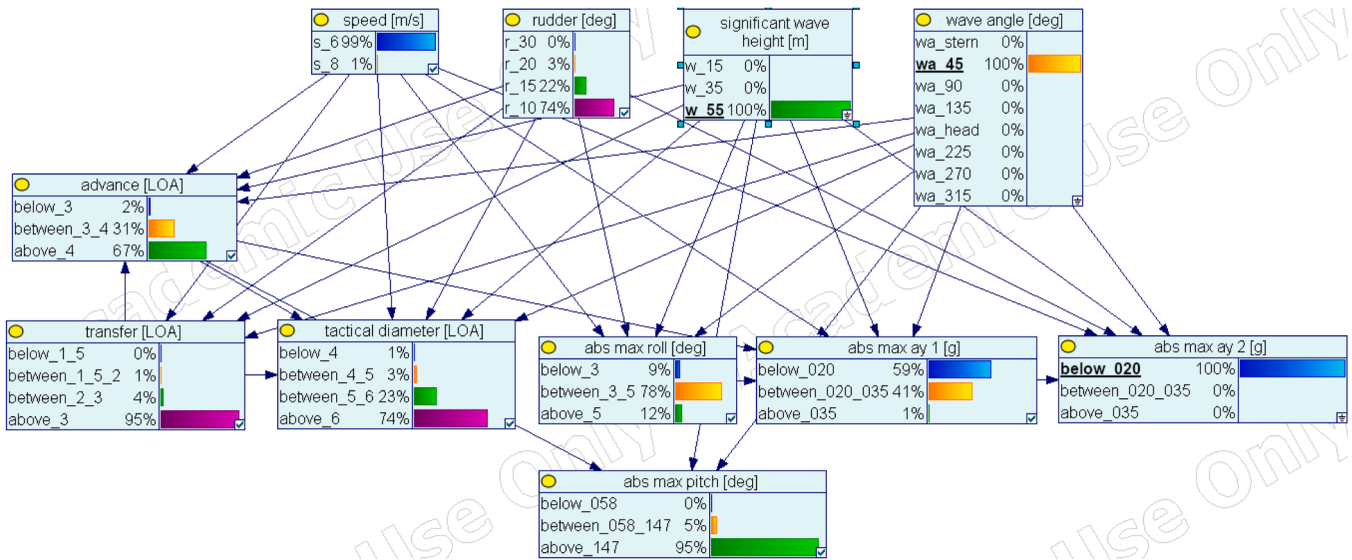


Fig. 10. Case study #5 – a sample application of the meta-model in the diagnostic mode – backward reasoning. For the desired low acceleration level and observable high wave height coming from the bow-quartering seas, the most probable rudder angle and speed of the ship are sought.

4.3. Future work

The training dataset used to develop the unique meta-model presented here is based on the outcome of 96,000 ship-turning simulations. It covers a suitable range of environmental and operational conditions, so it would appear to relevantly reflect the actual operational conditions of a large passenger ship, especially in some selected areas [56]. This helped achieve the wide applicability of the proposed meta-model. However, there are still some combinations of operational and environmental parameters that are poorly represented in the training dataset, such as limited values of considered ship speeds that reach also 20 kts or more [55,75,76]. This may require further simulations with a particular focus on the missing combinations to achieve a sufficient amount of data. To define those combinations a detailed analysis of CPTs is needed.

The presented meta-model operates with the variables represented as probability distributions, however, if the end-user would be interested in single-tone output variables some other machine learning methods, such as ANNs widely used in the marine industry shall be investigated [77]. This could be interesting as the developed training dataset is unique, rich, and extensive, which allows for various areas of meta-model application ranging from operational (collision evasive maneuver planning) to tactical (route planning by excluding the environmental conditions leading to an undesired level of ship motions) purposes. Moreover, the use of physics-guided ANN may help generalize the results obtained [78,79], which could also be the case here and at least partially solve one of the identified limitations of this study.

5. Conclusion

This paper presents a data-driven meta-model estimating a set of safety-critical parameters describing the behavior of a passenger ship while executing an evasive maneuver. This was achieved via 6DOF numerical simulations of ship response in irregular waves with respect to their stochastic realizations and consequently by adopting suitable machine-learning techniques to develop a probabilistic model using Bayesian Belief Networks (BBNs).

The set of response variables comprises the maximum lateral accelerations calculated for two distinct locations on board a ship (sun deck and restaurant), the maximum roll and pitch angles, as well as the maneuvering parameters describing the turning circle of a ship (tactical diameter, advance, and transfer). These are estimated for a predefined

set of explanatory variables pertaining to the environment (significant wave height and initial angle of wave attack) and ship operations (initial ship speed and magnitude of rudder angle).

Massive numerical simulations of ship motion in irregular waves are performed via state-of-the-art code called *LaiDyn*, serving in this study as a sample source of ship motion data. This was conducted by considering the stochastic realization of the waves and repeating each simulation scenario 500 times which resulted in the training dataset describing a total of 96,000 ship turning maneuvers in irregular waves. This dataset, after relevant processing, was then used to develop a meta-model adopting BBNs and the learning algorithm called *Bayesian Search*. Eventually, all three objectives of the study were addressed.

The case studies have proven the instantaneous predictive and diagnostic reasoning ability of the meta-model with high accuracy, making it already possible to serve as an onboard navigational decision support tool. It is worth noting that the proposed meta-model demonstrates a holistic approach for operational risk mitigation through encompassing ship response predictions with respect to the stochastic realization of irregular waves. This was done for both the parameters reflecting hull motions of the vessel as well as the resulting maneuvering parameters describing the ship’s turning circle. As this phenomenon is probabilistic, the outcome obtained using the meta-model is probabilistic as well. This draws the watchkeeping officers’ attention to the fact that they cannot expect the ship to respond in a deterministically predictable manner at all times. Such a conclusion, in turn, increases the officer’s situational awareness, which then improves the safety of navigation.

Future work might also focus on applying various machine learning methods and algorithms, such as Artificial Neural Networks (ANNs), perhaps to compare and investigate modeling approaches that reflect the physics of the analyzed phenomena and may prove to be strongly predictive at the same time.

CRediT authorship contribution statement

**Mateusz Gil:** Writing – original draft, Software, Resources, Investigation, Data curation. **Jakub Montewka:** Writing – original draft, Validation, Supervision, Project administration, Methodology, Funding acquisition, Formal analysis, Conceptualization. **Przemysław Krata:** Writing – original draft, Supervision, Investigation, Data curation, Conceptualization.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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The views expressed are solely those of the authors.

## Data availability

Data will be made available on request.

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