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Spain Water  
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## 27<sup>th</sup> IAHR International Symposium on Ice

Gdańsk, Poland, 9–13 June 2024

DOI: 10.1000.10/123456

### Mathematical Modeling of Ice Dynamics in the Area of the Planned Siarzewo Reservoir

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#### Abstract

River regulation is an inseparable aspect of the economic progress of countries, and based on the examples of highly developed countries, it may be concluded that their development could be partially supported by properly conducted water management. The construction of dams in cold regions or areas where ice phenomena are observed in winter, requires taking into account the impact of ice on the structure and the impact of the designed facilities on ice transport. In the context of the impact of ice on structures, the design of hydrotechnical facilities should consider the determination of forces transferred to the planned facilities, the impact of ice on the operation of locks and outer ports, the load of ice on ecological compensations, erosion processes caused by the presence of ice, and increasing losses in electricity production. Large hydrotechnical facilities will also significantly influence the dynamics of ice processes, both in terms of ice cover formation and the method of ice flow during ice-breaking operations, and, above all, changes in the river's congestion potential.

The aim of the work is to determine the impact of the planned investment in the form of the Siarzewo dam on the Vistula River on the ice dynamics in the newly constructed reservoir and in the section of the river directly below the planned dam. To achieve the intended goal, it was necessary to implement a mathematical model and conduct numerical simulations for possible scenarios. The calculations were used to determine the route of ice flow through the reservoir and below the reservoir, the forces with which the ice will affect the banks and bottom of the river, and the forces that the islands designed in the reservoir will have to withstand. The research covered the section of the Vistula between the existing Włocławek barrage (km 674.850) and km 715, i.e. approximately 8 km below the planned Siarzewo barrage. Based on the conducted research, it was found, among other things, that ice dynamics in the reservoir is largely influenced by the strength and direction of the wind.

**Keywords:** ice dynamics; mathematical modeling; wind interaction; Vistula River; Siarzewo Dam

# 1. Introduction

The development of river infrastructure is an integral aspect of the economic growth of countries, and based on examples from highly developed European countries and the United States, it can be concluded that their development may have been partly supported by proper water management (Szymkiewicz 2017). Currently, around 3,700 large dams with a power output of over 1 MW are being planned or constructed worldwide, most of them in countries aspiring to become economic powers or those experiencing dynamic growth (Zarfl et al. 2015). The construction of dams in polar regions or areas where ice phenomena are observed in winter necessitates considering the impact of ice on the structure and the influence of the designed facilities on ice transport (Gebre et al. 2013). In the context of the impact of ice on structures in hydrotechnical projects, it is necessary to account for the forces exerted on the planned structures (Kolerski, Zima, and Szydłowski 2019), the influence of ice on the operation of locks and foreports (Tuthill, Liu, and Shen 2004), the ice load on ecological compensations (Kolerski, Shen, and Knack 2010), erosion processes caused by the presence of ice (Carr and Tuthill 2011; Knack and Shen 2017), and increased losses on grates at hydropower plant intakes (Walczak, Walczak, and Nieć 2020; Daly 1991). Large hydrotechnical structures will also significantly influence the dynamics of ice processes, both in terms of ice cover formation (Kolerski 2015) and ice movement during icebreaking operations (Kolerski 2016a), as well as changing the river's potential for ice jams (Pawłowski 2019).

The aim of this study is to determine the impact of the proposed Siarzewo dam on the Vistula River on ice dynamics in the area of the newly created reservoir and on the river section directly downstream of the planned dam. To achieve this goal, it was necessary to implement a mathematical model and conduct numerical simulations for possible scenarios. The calculations were aimed at determining the path of ice transport through the reservoir and downstream of it, the forces exerted by the ice on the riverbanks and bed, and the forces that the designed islands in the reservoir will have to withstand. The study covered the Vistula River section between the existing Włocławek dam (km 674.850) and km 715, which is about 8 km downstream of the proposed Siarzewo dam. Due to the lack of impact of the section below the dam on the conditions in the planned reservoir, the calculations were divided into two computational areas:

- Siarzewo reservoir,
- Free-flowing river section below the planned dam.

In the first area, an additional region was delineated in the immediate vicinity of the dam for a precise analysis of ice movement at the spillway forefield and in the upper lock foreport area. For each area, current bathymetric data and a digital terrain model provided by the client were adopted. The simulation accounted for changes in river bathymetry resulting from the construction of the dam, which included deepening the river channel at the spillway forefield and below the spillway and hydropower plant. Additionally, dredging work on the left bank of the Vistula, where the hydro-power plant is to be located, was considered. In detailed calculations for ice transport in the upper lock foreport area, the riverbed position according to data from the State Water Holding Polish Waters (PGW WP) was also considered.

An additional element was ecological compensations in the form of artificial sandy islands, designed in the reservoir (15 islands) and one island at the bypass channel mouth. Since the



details of the islands are unknown, it was assumed that all rise to an elevation of 47 m above sea level, and their banks gently slope to an elevation of 46.0 m above sea level.

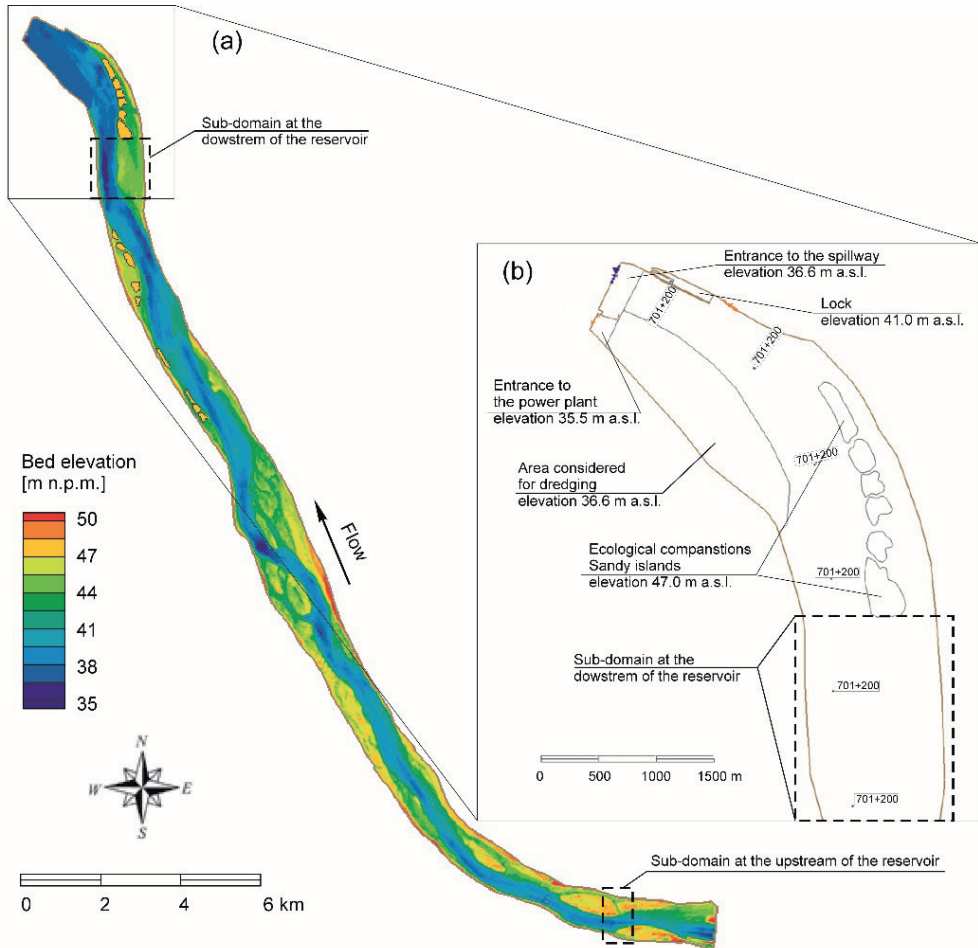
The numerical simulations were performed using the DynaRICE mathematical model, which allows for ice dynamics modeling based on the balance of internal (internal stresses) and external forces (gravitational force, water motion force, and wind force). The modeling results are presented in the form of charts and analyzed concerning the initially set tasks. The model used in the study describes water flow in a two-dimensional system with vertical averaging of water velocity, while ice dynamics are simulated using the Lagrangian Smooth Particle Hydrodynamics (SPH) method (Shen 2010; Kolarski 2016b).

## **2. Mathematical modeling of ice dynamics in the proposed Siarzewo reservoir**

Upon completion of the Siarzewo dam, which is to be located on the Vistula River at approximately kilometer 706+500, a reservoir over 31 kilometers long will be created (Wrzosek et al. 2020). By maintaining water levels at 46 meters above sea level, the planned dam will ensure the creation of an inland waterway of at least class IV, which is crucial for conducting icebreaking operations and protecting against winter floods in this river section. The Siarzewo dam will be the second dam on the Lower Vistula section, after the Włocławek dam, forming part of the so-called Lower Vistula Cascade (Woś et al. 2022).

The purpose of implementing the model for the planned Siarzewo reservoir area is to determine the ice channel path in the Siarzewo reservoir and identify dredging locations to ensure the minimum waterway depth for icebreakers, assumed to be 1.80 meters. Additionally, the load on the islands (ecological compensations) designed in the reservoir area was determined due to the static ice pressure caused by water current and wind action. The load is provided in  $\text{kN/m}^2$  at locations where ice contacts the shore and the reservoir bed, particularly on the island beaches.

The mathematical model area of the planned Siarzewo reservoir covered over a 31-kilometer section of the Vistula River from km 674+850 (axis of the Włocławek dam) to km 706+380 (axis of the planned Siarzewo dam). The model area is shown in the figure below. The bathymetry of the area was determined based on cross-sections and a digital terrain model, including the planned islands. In the region of the planned dam, changes in the river bathymetry were adopted, resulting from the assumed elevation of the stilling basin slab (36.6 meters above sea level) and the inlet concrete basin of the hydropower plant (elevation 35.5 meters above sea level). The effect of dredging operations conducted on the left bank of the reservoir, directly above the inlet to the hydropower plant, was also considered, assuming a uniform bed elevation of 36.6 meters above sea level in this area (see Figure 1a).



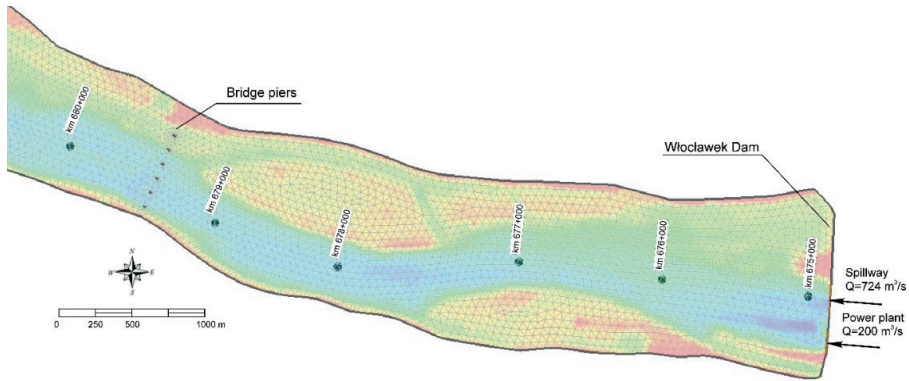
**Figure 1.** Bathymetry of the Siarzewo reservoir area incorporated into the mathematical model, showing islands and dredged area; (a) Lower section of the reservoir with marked changes in river bathymetry due to the construction of Siarzewo dam

In the upper section of the reservoir, approximately 4.5 km from the Włocławek dam, there is the Rydz-Śmigły bridge, whose main supports significantly narrow the river channel. Due to the considerable impact of these supports on ice dynamics in the immediate vicinity of the Włocławek dam (see Figure 2), they were included in the model as a closed bank – preventing the flow of ice and water. The numerical grid considering the bridge piers is shown in Figure 3.





**Figure 2.** Rydz-Śmigły Bridge in Włocławek during ice flow, photo courtesy of M. Grześ



**Figure 3.** The fragment of the numerical grid adapted for the upper part of the Siarzewo reservoir area, with the bridge piers of Rydz-Śmigły bridge in Włocławek visible on the figure

Ecological compensations in the form of sand islands were adopted according to available information, with the island shorelines having an elevation of 46.0 meters above sea level, corresponding to the normal water level in the reservoir. The central part of each island was elevated to 47.0 meters above sea level, with the shores gently sloping down to the shoreline, forming beaches. The outline of all the islands along their shorelines and the bathymetry of the area, including the islands, are shown in Figure 1.

The model did not include details regarding the shape of the spillway and hydropower plant pillars or the lock forebay, due to the high computation time for the extensive reservoir area. Therefore, it was decided that a detailed analysis of ice dynamics in the forebay would be conducted using a separate mathematical model, encompassing only the lower 5-kilometer section of the reservoir. This model will complement the entire reservoir model, with boundary condition data transferred from the overall model.

Mathematical modeling was conducted for boundary conditions as shown in Figure 3, assuming that the upper boundary (Włocławek dam) specified the flow (separately through the spillway and hydropower plant), and the lower boundary specified the water level. Additionally, it



was assumed that in the area of the inlet structure to the bypass channel, the flow was simulated according to guidelines from the spillway project. Simulations were carried out for six different flow conditions, under which icebreaking operations and ice transport through the spillway are possible. Calculations were conducted for six flows ranging from the mean flow  $Q = 924 \text{ m}^3/\text{s}$  to the high flow  $Q = 6104 \text{ m}^3/\text{s}$ .

All numerical simulations were conducted without considering wind and with considering wind blowing from the west, north, east, and south. Wind speed was assumed to be 3 to 5 m/s, as for lower flows (924 and 1308  $\text{m}^3/\text{s}$ ), wind at 5 m/s caused complete stoppage of ice flow in the lower part of the reservoir.

In all simulations, an initial ice thickness of 0.3 meters was assumed. Other ice parameters were adopted according to literature guidelines. Ice was released through the Włocławek dam and then moved down the Siarzewo reservoir according to the dynamic balance of forces. An equal concentration of incoming ice was assumed at the upper boundary condition,  $N = 0.4$ . To protect the hydropower plant inlet from incoming ice, a constant ice cover was assumed on the right bank of the reservoir for about 2 km above the dam in all simulations.

Potential ice jam risk was observed in the upper part of the reservoir, around km 677. This is an approximately 650-meter section of the reservoir located a short distance downstream from the Włocławek dam and directly upstream of the Rydz-Śmigły bridge. Ice outflow in this area is hindered due to the bridge piers in the main current, significantly narrowing the cross-section. This is evident in the numerical calculation results under average flow conditions and the simulated flow of 1308  $\text{m}^3/\text{s}$ . For higher flows, the force from the moving water is strong enough to prevent ice outflow blockage on the bridge piers. Considering the ice release by the Włocławek dam at average flow, it is noteworthy that despite ice outflow blockage on the bridge piers, accumulation does not build upstream of the reservoir or increase in thickness. This is because the ice continues to flow out of the problematic area, and a stable ice jam does not form, only localized ice accumulation. When transporting ice under average flow conditions, special attention should be given to this area, maintaining a continuously working line icebreaker.

## 2.1. Discussion of the numerical model results

In the lower section of the reservoir, the results of mathematical modeling indicate a high risk of ice inflow into the upper forebay area of the lock. This occurs under almost all flow conditions, with westerly and northerly winds exacerbating the process by causing ice to drift toward the left bank and into the lock forebay. To counteract this process, a fixed or floating barrier could be used to direct the ice towards the upper spillway area (Kolerski 2017). Another, significantly cheaper option is to maintain an ice cover in the forebay area, which would prevent ice from drifting.

A potential ice jam was also observed, caused by the concentration of flow near the planned island (the sixth from the axis of the Siarzewo dam). Ice accumulated on the southern shore of the island, building up and forming a jam upstream. The thickness of this accumulation is not significant, reaching only 0.6 meters after 24 hours, but the problematic aspect is that it obstructs the ice outflow towards the weir. Such a situation could cause the jam to build



up upstream in the reservoir, hindering the ice transport process. Significant forces, exceeding  $2 \text{ kN/m}^2$ , were also observed acting on the southern shore of the island.

The route of the ice channel was adopted according to the Conceptual Program and Spatial Plan (KPP). Verification of the route involved checking if the ice could flow freely and if there were any areas along the route where water depths would be insufficient for icebreakers. A critical depth of 1.8 meters was adopted as the safe depth for icebreaker operations.

In the model, the route was simulated by assuming a uniform ice cover over the entire surface of the reservoir, except for the area of the route. Then, ice was released through the Włocławek dam, moving down the channel in the reservoir. The results of the numerical simulation show that the route of the ice channel is correctly designated, except for the section near the Rydz-Śmigły bridge. In this area, the channel was unfortunately routed through two bridge piers. It is suggested to slightly shift the route of the channel towards the right bank so that two bridge spans are ice-free, and only one pier is within the channel. With the KPP-compliant route, ice flow is obstructed at the bridge section, but it does not completely stop. Regardless of the chosen variant of the ice channel route, constant monitoring of this section of the reservoir will be necessary. It seems that the constant presence of a line icebreaker near the Rydz-Śmigły bridge should be planned, which would remove the accumulated ice as needed.

### 3. Analysis of ice flow downstream the Siarzewo dam

The objective of the modeling work was to determine the ice flow path below the planned Siarzewo dam under various flow conditions, wind directions, and gate openings. Additionally, the maximum forces exerted by the ice on the Vistula River's bank, the bank of the ecological area Zielona Kępa, and the riverbed in case of ice accumulation on sandbars were assessed.

Numerical calculations were conducted on an eight-kilometer section of the Vistula River downstream of the planned Siarzewo dam, with bathymetry described by 36 cross-sections and supplemented with topographic data derived from a digital terrain model. The model also included the elevation of the weir's outflow slab (35.2 m a.s.l.), the hydropower plant's outflow slab (32.2 m a.s.l.), and the planned dredging work in the area directly downstream of the proposed hydropower plant. It was assumed that the riverbed in this area would be dredged to an elevation of 36.6 m a.s.l., consistent with the design documentation. The bathymetric data used to construct the mathematical model are shown in the figure below. All simulations considered the existence of an artificial island, which will be created at the mouth of the bypass channel as part of ecological compensation measures. This is a sandy island with a maximum elevation of 47 m a.s.l., gently sloping down to the water with beaches.

For high flow conditions ( $Q = 6104 \text{ m}^3/\text{s}$ ), it was anticipated that the ecological area Zielona Kępa would be flooded due to high water levels. Consequently, additional topographic data from the digital terrain model were incorporated and adapted to the modeled area, extending it to include the right-bank area up to the levee.

The upper boundary condition was applied at nodes located in each spillway span, at the outlet of each turbine, and at the mouth of the bypass channel in the area of the planned island. An exception was made for the high flow simulation ( $Q = 6104 \text{ m}^3/\text{s}$ ), where the inflow from the

bypass channel was assumed at a different location. Six different flows and corresponding water levels at the lower, open boundary of the area were used for the numerical analysis. The computational variants from the hydrodynamic conditions perspective are presented for the mean flow ( $Q = 924 \text{ m}^3/\text{s}$ ) in Figure 4.

All numerical simulations were conducted both without considering wind and with considering wind blowing from the west, north, east, and south at a speed of 5 m/s. This resulted in a total of 45 computational variants. Ice parameters were assumed to be the same as in the simulations conducted for the Siarzewo reservoir, with an initial ice thickness (single floe) of 0.3 m. Ice was released through the weir with an initial concentration of 0.4 for all analyzed cases. This means that at the upper boundary condition, the inflow of ice with a thickness of 0.3 m and a surface concentration of 0.4 is assumed. It is worth noting that under conditions of maximum ice cover on the water surface, its concentration is assumed to be 0.6. Further increasing the ice concentration on the surface leads to an increase in its thickness and accumulation.

### 3.1. Discussion of the numerical model results

The results are shown using an example of a single simulation for average flow and wind blowing from the south. Two graphs are presented, showing the ice thickness at the 12<sup>th</sup> hour of the simulation and the maximum load from the ice transferred to the riverbanks and riverbed. The force is given in kN per  $\text{m}^2$ , while the ice thickness is given in meters. Based on the obtained results, it can be seen that the ice, after being discharged through the spillway site, flows without major problems; however, its impact on the riverbanks is significant. The southern edge of the Zielona Kępa ecological area is particularly vulnerable. Significant forces are also observed to be transferred to the riverbed in the area of the shallow water zone located below the dam's tailwater. This is a place where ice flow may be obstructed, which is particularly likely in the case of low water flows and high ice concentration.

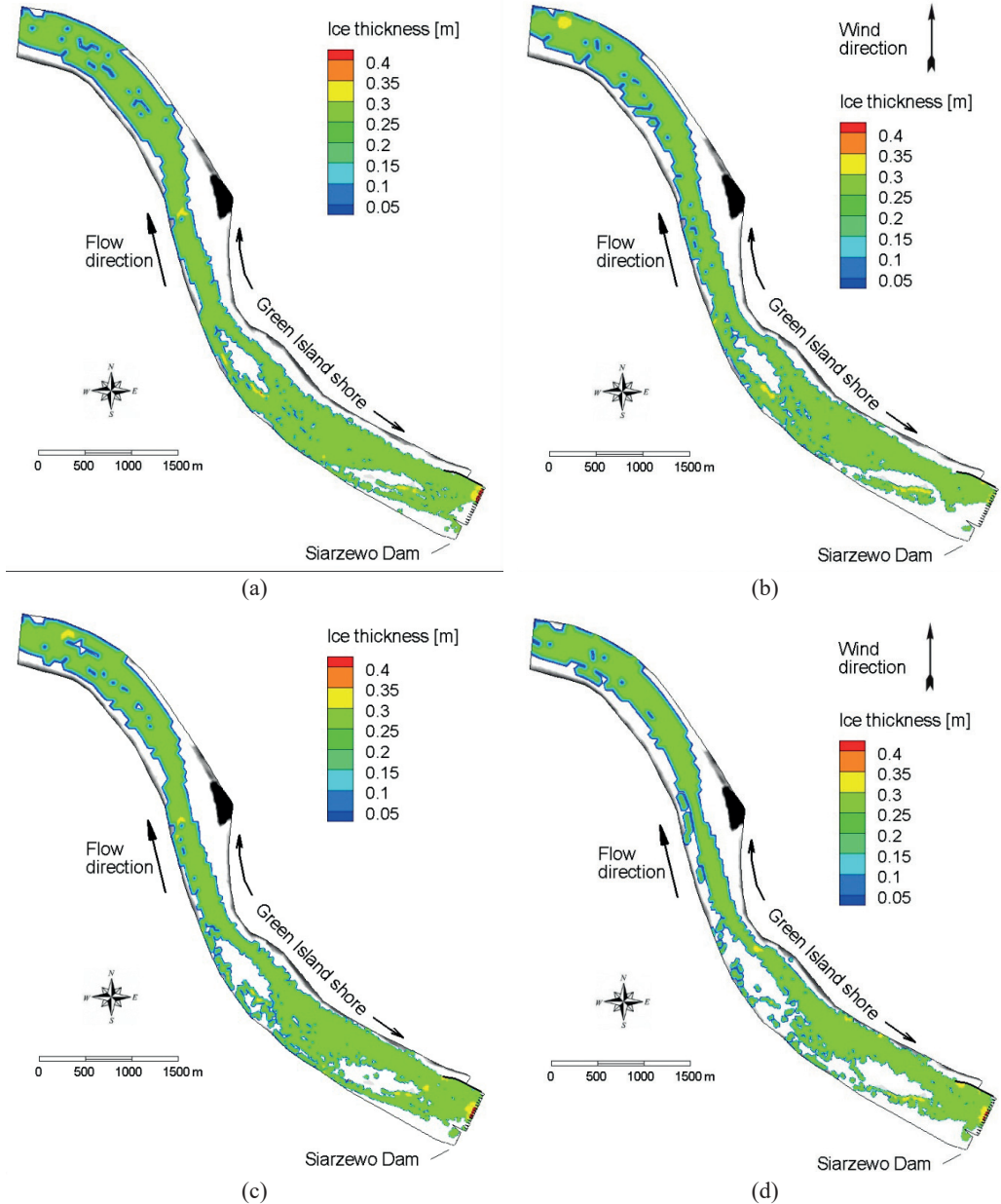
The greatest risk of ice jam formation is observed during ice discharge under average flow conditions ( $SSQ = 924 \text{ m}^3/\text{s}$ ). Under these conditions, the ice generates the highest forces on the Vistula riverbanks and accumulates on the shoal in the central part of the river, directly downstream the spillway. Local forces exceeding  $3 \text{ kN}/\text{m}^2$  are observed, primarily exerted on the shoal near km 707. For average flow, ice discharge was tested through five spans in various configurations, showing that the least issues and smallest forces are observed when the middle spans are open. However, the choice of spans for ice sluicing will depend on how the ice acts against the spillway from the upstream side.

From the calculations conducted for average flow, it is clear that the shoal near km 707 must be removed without delay. This location is prone to ice accumulation and potential jamming, especially during low flow conditions. Another critical location is the shoal below km 709, where ice blockage can also occur.

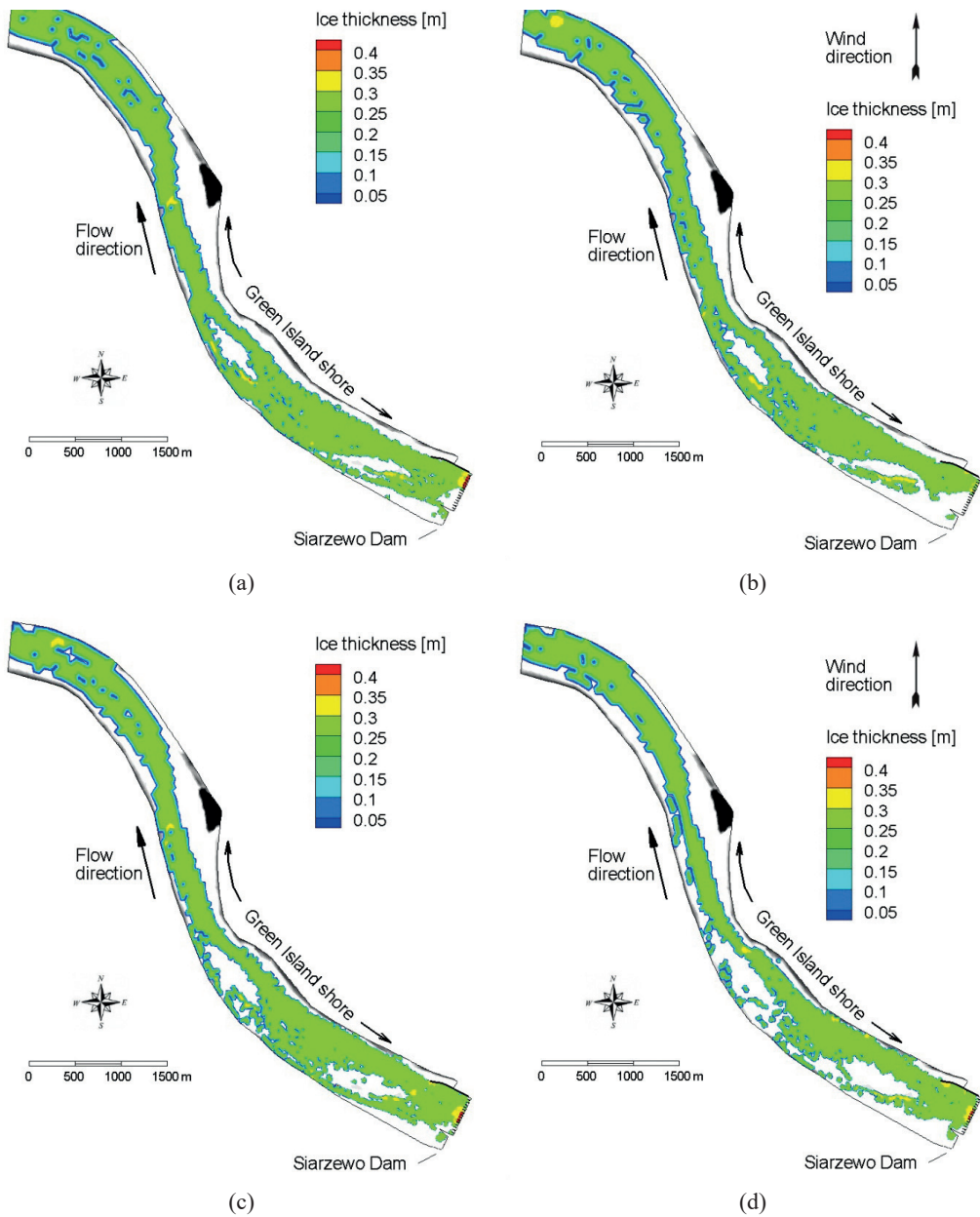
For higher flows and increased water levels, ice flows freely and does not accumulate. Ice contact with the bank is limited, and in several cases, there is no interaction between the ice and the bank. Under maximum flow conditions, with west or south winds, small amounts of ice may intrude into the flooded area of Zielona Kępa. However, the ice exerts minimal force on the island (maximum  $0.6 \text{ kN}/\text{m}^2$ ).



It is noteworthy that in practically all simulated scenarios, the planned island has no contact with the ice discharged by the spillway. This means its location is safe and will not be damaged due to ice discharge. The only scenario where ice contacts the island occurs during ice discharge at maximum flow ( $Q = 6104 \text{ m}^3/\text{s}$ ) with a south wind. Even then, the ice contact with the island does not exert significant force on it.



**Figure 4.** Spatial distribution of ice thickness resulting from water discharge  $Q = 924 \text{ m}^3/\text{s}$  through the left-bank spans of the Siarzewo dam under calm conditions (a), with a southern wind (b), and through the central spans of the spillway under calm conditions (c), and with a southern wind (d)



**Figure 4.** Spatial distribution of ice load resulting from water discharge  $Q = 924 \text{ m}^3/\text{s}$  through the left-bank spans of the Siarzewo dam under calm conditions (a), with a southern wind (b), and through the central spans of the spillway under calm conditions (c), and with a southern wind (d)

## 4. Conclusions

Based on the conducted simulations, the following conclusions can be drawn:

1. Along the river section below the planned Siarzewo hydraulic structure, ice discharge flows freely without stopping or forming jams. Only under average flow conditions ( $Q = 924 \text{ m}^3/\text{s}$ ),

ice sluicing may pose certain problems related to shoaling of the riverbed around existing sandy shoals.

2. For SSQ flow, from the perspective of ice transport along the Vistula river section downstream of Siarzewo dam, it is recommended to open the middle spans. For higher flows, this does not significantly affect operations, and spans should be opened where ice exerts the greatest pressure from the upstream side of the Siarzewo reservoir.
3. No significant impact of ice on the island downstream of Siarzewo dam was observed; the island is located on the right bank away from the main ice flow path. Only during high flow conditions, when the entire area of Zielona Kępa is submerged, the beginning of ice interaction with the island is noticed.
4. Wind strength and direction have a significant impact on ice flow on the reservoir.
5. For average flow,  $Q = 924 \text{ m}^3/\text{s}$  ice movement in the lower part of the reservoir is hindered by wind speeds above 3 m/s. With a west wind blowing at 3 m/s, ice drifts slowly downstream, taking over 3 days to reach the dam. The situation is similar with a north wind blowing at 3 m/s, where ice arrival at the dam also exceeds 3 days. In the lower part of the reservoir, a north or west wind at 3 m/s causes ice accumulation around the planned island. In this case, the force transmitted by the ice to the island is significant, exceeding  $2 \text{ kN/m}^2$ . Consideration should be given to changing the shape of the southern edge of the island to direct ice towards the weir.
6. For flow rates around  $1308 \text{ m}^3/\text{s}$ , ice movement in front of the weir is halted with winds stronger than 4 m/s. The least favorable are west and north winds, which not only stop ice discharge through the spillway but also push ice towards the upper sluice bay.
7. For higher flows, ice sluicing through the Siarzewo structure is possible with winds below 5 m/s.
8. The ice sluicing channel route was determined based on the Programmatic-Spatial Concept, tested under ice transport conditions with a water flow of  $Q = 1308 \text{ m}^3/\text{s}$ . No depth deficits were observed along the ice sluicing channel route. The only jam-prone section on the ice sluicing channel route is the bridge section (Rydz-Śmigły bridge in Włocławek). To prevent ice accumulation, which would be halted by bridge piers, one icebreaker should be designated for continuous monitoring of this section during ice sluicing through the Siarzewo structure.
9. Observed ice inflow into the upper sluice bay area may contribute to difficulties in ship sluicing. It is recommended to implement a technical solution in the form of a fixed or floating barrier that directs ice towards the weir. Barrier parameters should be defined during further project and modeling work. Another, significantly cheaper solution is maintaining an ice cover in front of the upper sluice bay.

## Acknowledgements

This research was supported by National Water Management Authority 'Wody Polskie' Project no KZGW/KS/306/2019.

## References

- Carr, Meredith L., Andrew M. Tuthill. 2011. „Modeling of scour-inducing ice effects at Melvin Price Lock and Dam”. *Journal of Hydraulic Engineering* 138 (1): 85–92.
- Daly, Steven F. 1991. „Frazil ice blockage of intake trash racks”.
- Gebre, Solomon, Knut Alfredsen, Leif Lia, Morten Stickler, i Einar Tesaker. 2013. „Review of ice effects on hydropower systems”. *Journal of Cold Regions Engineering* 27 (4): 196–222.



- Knack, Ian M., Hung Tao Shen. 2017. „Numerical modeling of ice transport in channels with river restoration structures”. *Canadian Journal of Civil Engineering* 44 (10): 813–19.
- Kolerski, Tomasz. 2015. „Ice cover progression due to flow regulation at the Wloclawek dam”. *Acta Scientiarum Polonorum. Formatio Circumiectus* 14 (1): 229–240.
- Kolerski Tomasz 2016a. „Modeling of Ice Passage Through Reservoirs System on the Vistula River”. W *Hydrodynamic and Mass Transport at Freshwater Aquatic Interfaces: 34<sup>th</sup> International School of Hydraulics*, zredagowane przez Paweł Rowiński i Andrea Marion, 35–47. GeoPlanet: Earth and Planetary Sciences. Cham: Springer International Publishing. [https://doi.org/10.1007/978-3-319-27750-9\\_4](https://doi.org/10.1007/978-3-319-27750-9_4).
- Kolerski Tomasz. 2016b. Modelowanie matematyczne zjawisk lodowych na wodach śródlądowych. Wydawnictwo Politechniki Gdańskiej.
- Kolerski Tomasz. 2017. „Mathematical Modelinig of Ice Booms”. *Acta Scientiarum Polonorum. Formatio Circumiectus* 16 (1): 65.
- Kolerski, Tomasz, Hung Tao Shen, I. M. Knack. 2010. „A nested model for river Ice dynamics”. W *Proceedings, 20<sup>th</sup> IAHR Ice Symposium*.
- Kolerski, T. and Radan, P., 2022. The Application of the Thermal Stabilization Prompted by the Ice Cover Expansion Considering the Energy Production Optimization in the Dam-Reservoir Coupled Systems on the Vistula River. *Energies*, 15(3), p. 823.
- Kolerski, Tomasz, Piotr Zima, Michał Szydłowski. 2019. „Mathematical Modeling of Ice Thrusting on the Shore of the Vistula Lagoon (Baltic Sea) and the Proposed Artificial Island”. *Water* 11 (11): 2297. <https://doi.org/10.3390/w11112297>.
- Pawłowski, Bogusław. 2019. „Ice Jams: Causes and Effects”. W *Encyclopedia of Water*, 1–9. American Cancer Society. <https://doi.org/10.1002/9781119300762.wsts0035>.
- Shen, Hung Tao. 2010. „Mathematical Modeling of River Ice Processes”. *Cold Regions Science and Technology* 62 (1): 3–13. h
- Szymkiewicz, Romuald. 2017. Dolna Wisła – rzeka niewykorzystanych możliwości. Wydawnictwo Politechniki Gdańskiej.
- Tuthill, Andrew, Lianwu Liu, Hung Tao Shen. 2004. „Modeling ice passage at navigation locks”. *Journal of cold regions engineering* 18 (3): 89–109.
- Walczak, Natalia, Zbigniew Walczak, Jakub Nieć. 2020. „Assessment of the Resistance Value of Trash Racks at a Small Hydropower Plant Operating at Low Temperature”. *Energies* 13 (7): 1775.
- Woś, K., Wrzosek, K. and Kolerski, T., 2022. The energy potential of the lower vistula river in the context of the adaptation of polish inland waterways to the standards of routes of international importance. *Energies*, 15(5), p. 1711.
- Wrzosek, Krzysztof, Kacper Jurek, Natalia Górniak-Ziemkowska, Przemysław Sobiesak, Marcin Puchała. 2020. „Stopień Wodny Siarzewo – lokalizacja i rozwiązania techniczne”. *Gospodarka Wodna* 2020 (6).
- Zarfl, Christiane, Alexander E. Lumsdon, Jürgen Berlekamp, Laura Tydecks, Klement Tockner. 2015. „A global boom in hydropower dam construction”. *Aquatic Sciences* 77 (1): 161–170.

