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Design of IIR digital filters with non-standard characteristics using differential evolution algorithm

A. SŁOWIK* and M. BIAŁKO

Department of Electronics and Computer Science, Koszalin University of Technology,
Sniadeckich 2 Street, 75-453 Koszalin, Poland

Abstract. In the paper an application of differential evolution algorithm to design digital filters with non-standard amplitude characteristics is presented. Three filters with characteristics: linearly growing, linearly falling, and non-linearly growing are designed with the use of the proposed method. The digital filters obtained using this method are stable, and their amplitude characteristics fulfill all design assumptions.

Key words: artificial intelligence, differential evolution algorithm, digital filters, design, non-standard amplitude characteristics.

1. Introduction

During design of digital filters it is important to fulfill accepted design assumptions which can be for example: the width of pass-band, the width of stop-band, the value of pass-band ripple, and the value of stop-band attenuation [1]. When digital filters with standard amplitude characteristics are designed, we can use existing approximations such as: Butterworth, Chebyshev (with pass-band ripple or stop-band ripple) and Cauer [2]. However, the problem becomes complicated when designed digital filter is supposed to possess non-standard amplitude characteristics as for example in amplitude or phase equalizers. Then the standard approximations are useless [1]. Methods based on evolutionary algorithms [3–5] or based on differential evolution algorithm [6, 7] are used to design such digital filters since several years. As examples we can mention following papers [1, 8–10]. In the paper [1] the method based on evolutionary algorithm to design IIR digital filters with non-standard amplitude characteristics is described. In this method it is necessary to determine such parameters as: probability of crossover, probability of mutation, population size, and an initial value of auxiliary parameter Δ_1, which is required to correct operation of mutation operator. Moreover, the evolutionary algorithm (presented in paper [1]) has been initialized with the use of Matlab Signal Processing Toolbox to improve its convergence.

Generally, the transfer function of designed IIR digital filter in z domain is described as follows:

$$H(z) = \frac{b_0 + b_1 \cdot z^{-1} + b_2 \cdot z^{-2} + ... + b_n \cdot z^{-n}}{1 - (a_1 \cdot z^{-1} + a_2 \cdot z^{-2} + ... + a_n \cdot z^{-n})}. \quad (1)$$

This function is a multi-modal function, therefore the algorithms based on gradient methods can easily stuck in the local extremes [11]. In order to avoid this problem it is possible to use the technique of the global optimization like for example the differential evolution algorithm [6], which is one of variations of evolutionary algorithms [3, 4, 5]. Differential evolution algorithm has been introduced recently (in the year 1997), and is a heuristic algorithm for global optimization. Its advantages are as follows: a possibility of finding the global minimum of a multi-modal function regardless of initial values of its parameters, quick convergence and a small number of parameters to set up [9]. Moreover, it can lead to solutions unavailable for standard digital filter design methods, because standard methods are searching only certain subset of potential solution space [12].

2. Differential evolution algorithm

The differential evolution algorithm has been proposed by Price and Storn [6]. Its pseudo-code form is as follows:

a) Create an initial population consisting of *PopSize* individuals
b) While (termination criterion is not satisfied) Do Begin
c) For each i individual in the population Begin
d) Randomly generate three integer numbers $r_1, r_2, r_3 \in [1; PopSize]$, where $r_1 \neq r_2 \neq r_3 \neq i$
e) For each j gene in i individual ($j \in [1; n]$) Begin
f) Randomly generate one real number $rand_j \in [0; 1)$
g) If $rand_j < CR$ then $x'_{i,j} := x_{r1,j} + F \cdot (x_{r2,j} - x_{r3,j})$ Else $x'_{i,j} := x_{i,j}$ End;
h) If x'_i is better than x_i then Replace x_i individual by x'_i child individual End;
End;

*e-mail: aslowik@ie.tu.koszalin.pl

This algorithm optimizes the problem having *n* decision variables. The parameters $F \in (0;2]$, and $CR \in [0;1)$ are determined by the user, the $x_{i,j}$ is the value of $j-th$ decision variable stored in $i-th$ individual in population. The *F* parameter is scaling the values added to the particular decision variables, and the *CR* parameter represents the crossover rate. This algorithm is a heuristic algorithm for global optimization, and it operates on decision variables in real number form. The individuals occurring in this algorithm are represented by real number strings. Its searching space must be continuous [7, 13].

The differential evolution algorithm, by computation of difference between two randomly chosen individuals from population, determines a function gradient in a given area, but not in a single point, and therefore prevents sticking the solution in local extremum of optimized function [7, 13]. The other important property of this algorithm is local limitation of selection operator only to the two individuals: parent (x_i) and child (x_i'), and due to this property the selection operator is more effective and faster [7, 13].

3. DE-IIRF method

In this paper a design method of IIR digital filters with non-standard amplitude characteristics based on differential evolution algorithm is presented. In comparison to method described in paper [1], in presented method only one parameter determining the number of individuals in the population is required. Moreover, due to its very good convergence an introduction of initial solution, for example obtained by *Matlab Signal Processing Toolbox* to the population, is not required. Presented method is named *DE-IIRFD (Differential Evolution – Infinite Impulse Response Filter Design)*. The elaborated method is operating as follows.

In the first step the initial population is created randomly. This population consists of *PopSize* individuals. Each individual x_i consists of $2 \cdot n+1$ genes (where *n* represents the order of designed filter), and is coded as follows:

$$x_i = (b_0, b_1, b_2, ..., b_{n-1}, b_n, a_1, a_2, ..., a_{n-1}, a_n) \quad (2)$$

where: a_i and b_i are the transfer function (1) coefficients, which are the genes of x_i individual.

Each $j-th\,(j \in [1; 2 \cdot n+1])$ gene of x_i individual can have value from prearranged range from min_j to max_j. In described method it is assumed that $min_j = -1$ and $max_j = 1$.

In the second step, the mutant individual (vector) v_i is created for each x_i individual from population according to the formula:

$$v_i = x_{r1} + F \cdot (x_{r2} - x_{r3}) \quad (3)$$

where: $F \in (0;2]$, and $r_1, r_2, r_3, i \in [1; PopSize]$ fulfill the constraint:

$$r_1 \neq r_2 \neq r_3 \neq i. \quad (4)$$

In this method it is assumed, that the value of *F* parameter from the range (0; 2] for each mutant individual v_i is separately randomly generated. The index r_1 indicates the individual which has the lowest value of the objective function. The objective function is described by function *COST(.)*. The main task of the proposed method *DE-IIRFD* is minimization of objective function *COST(.)*.

In order to compute the value of function *COST(.)* for any individual, the *FFT* transform of filter coefficients a_i and b_i (see formula 2), is performed. Having the *FFT* results, the amplitude characteristics $H(f)$ [dB] of predesigned digital filter is determined according to formula:

$$H(f) = 20 \cdot log_{10}\left(\sqrt{H_{real}(f)^2 + H_{imag}(f)^2}\right) [dB] \quad (5)$$

The value of the objective function *COST(.)* for each individual becomes higher when the sum of absolute values of not allowed deviations (see Fig. 1) exceeding constraints for particular frequencies f_i, is higher. The value of objective function is increased additionally in the case when a given set of filter coefficients a_i and b_i (coded in individual) leads to an unstable filter. In such a case to the value of the function *COST(.)* the value of penalty is added, which is higher when more poles of the filter transfer function are located outside the unit circle in the z plane.

During design of filter characteristics some restrictions (constraints) posed on their shapes must be fulfilled. In Figure 1 the assumed areas of constraints for amplitude characteristics are shown.

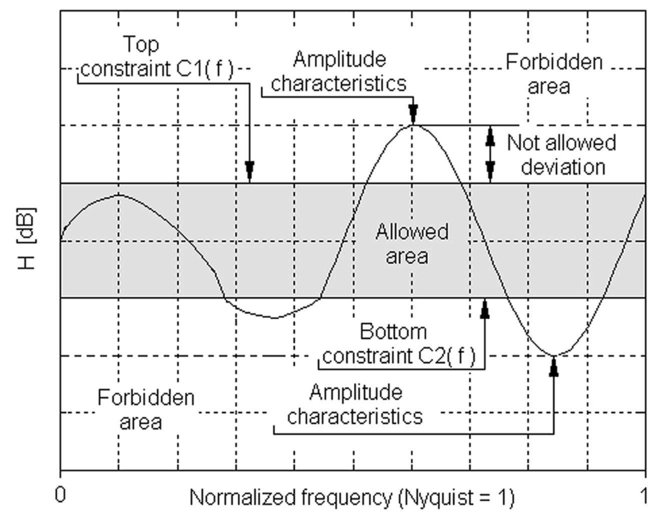

Fig. 1. Amplitude characteristics of designed digital filter with assumed constraints

In Fig. 2 the assumed areas of constraints for poles of designed digital filter are presented.

Taking into account constraints of Figure 1 and Figure 2, the objective function *COST(.)* can be defined as follows:

$$COST(.) = \sum_{i=1}^{k} Er(f_i) + \sum_{i=1}^{m} Stab_i \quad (6)$$

where:

$$Er(f_i) = \begin{cases} |H(f_i) - C_1(f_i)|, & when\ H(f_i) > C_1(f_i) \\ |H(f_i) - C_2(f_i)|, & when\ H(f_i) < C_2(f_i) \\ 0, & when\ H(f_i) \in [C_2(f_i); C_1(f_i)] \end{cases}$$

$$Stab_i = \begin{cases} (|z_i| - 1) \cdot w + w, & when\ |z_i| \geq 1 \\ 0, & when\ |z_i| < 1 \end{cases}$$

where: *m* – number of poles of the transfer function, *k* – number of output samples from *FFT* transform divided by 2 (assumed *k*=256), *w* – the value of penalty (assumed $w = 10^5$), f_i – *i*-th value of normalized frequency, $|z_i|$ – the absolute value of *i*-th pole of the transfer function in z plane.

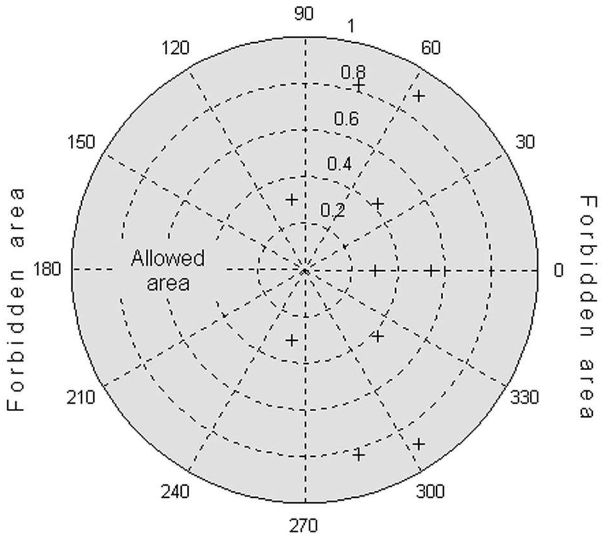

Fig. 2. Poles of designed digital filter with assumed constraints

In the third step, all x_i individuals are crossed-over with v_i individuals corresponding to them. As a result of this crossover an individual u_i is created. The crossover operates as follows:

let individual $x_i = (x_{i1}, x_{i2}, x_{i3}, ..., x_{ij})$, and individual $v_i = (v_{i1}, v_{i2}, v_{i3}, ..., v_{ij})$; for each gene $j \in [1; 2 \cdot n + 1]$ of individual x_i, randomly generate a number $rand_j$ from the range $[0; 1)$, and use the following rule:

$$\texttt{If}\ rand_j < CR\ \texttt{then}\ u_{ij} = v_{ij} \qquad (7)$$
$$\texttt{Else}\ u_{ij} = x_{ij}$$

In this paper it is assumed that the *CR* coefficient value is randomly generated from the range $[0; 1)$ for each individual x_i subject to crossover operation.

In the fourth step, the selection of individuals to the new population is performed according to following rule:

$$\texttt{If}\ COST\,(u_i) < COST\,(x_i)\ \texttt{then} \qquad (8)$$
$$\texttt{replace}\ x_i\ \texttt{by}\ u_i\ \texttt{in the new population}$$
$$\texttt{Else leave}\ x_i\ \texttt{in the new population}$$

The rule (8) refers to the minimization task of the objective function *COST(.)*. The minimization of objective function value is the main goal of the presented method.

In the fifth step, the value of function $COST(x_{r1})$ is checked. If it is equal to zero then the algorithm is stopped, and the result represents by x_{r1} individual is returned (r_1 – the index pointing to the best individual with the lowest value of objective function *COST(.)* in the population). If it is not equal to zero, then the algorithm jumps to step two.

4. Description of experiments

Proposed method has been tested by a design of the 10-th order (*n* = 10) IIR digital filters with: linearly falling amplitude characteristics (for normalized frequency *f*=0 the value of gain *H* is equal to 0 [dB], and for normalized frequency *f*=1 the value of gain is equal to -40 [dB]), linearly growing amplitude characteristics (for normalized frequency *f*=0 the value of gain is equal to -40 [dB], and for normalized frequency *f*=1 the value of gain is equal to 0 [dB]), and non-linearly growing amplitude characteristics (the attenuation is represented by quadratic characteristics; for normalized frequency *f*=0 the value of gain is equal to -40 [dB], and for normalized frequency *f*=1 the value of gain is equal to 0 [dB]). It is assumed, that the deviation values of the gain characteristics from ideal case can not be higher than ± 0.2 [dB]. The shapes of assumed amplitude characteristics are presented graphically in Fig. 3.

In Fig. 4 the amplitude characteristics obtained using the *DE-IIRFD* method (for *PopSize*=100) are presented. The linearly falling amplitude characteristics fulfilling design constraints has been obtained after 403 generations, the linearly growing amplitude characteristics has been obtained after 443 generations, and the non-linearly growing amplitude characteristics has been obtained after 398 generations.

In Fig. 5 the differences (deviations) between ideal characteristics $H(f)$ (Fig. 3), and characteristics of the designed filter $H'(f)$ (Fig. 4) obtained using the *DE-IIRFD* method are presented.

In Fig. 6 the phase characteristics corresponding to the designed 10-order IIR digital filters are presented. Additionally, in Fig. 7 the pole locations of the transfer function of designed filters with linearly falling amplitude characteristics, linearly growing amplitude characteristics, and non-linearly growing amplitude characteristics are shown.

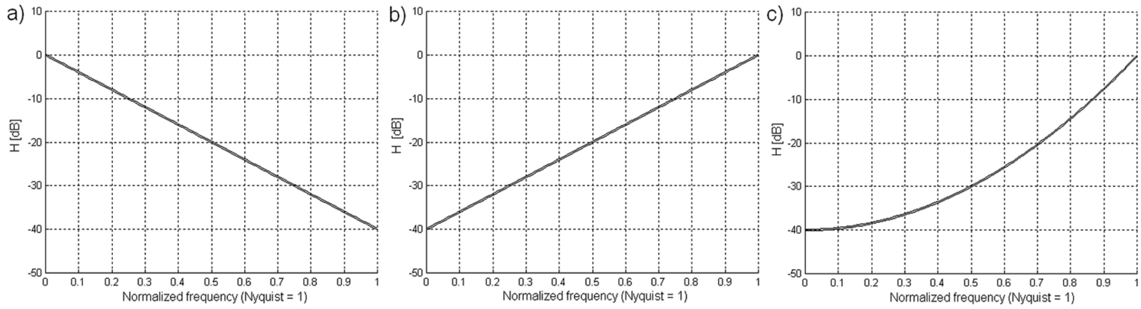

Fig. 3. Graphical presentation of assumed parameters for designed 10-order IIR digital filters with gain amplitude characteristics H: linearly falling (a), linearly growing (b), non-linearly growing (c); allowed deviations: $\pm$ 0.2 [dB]

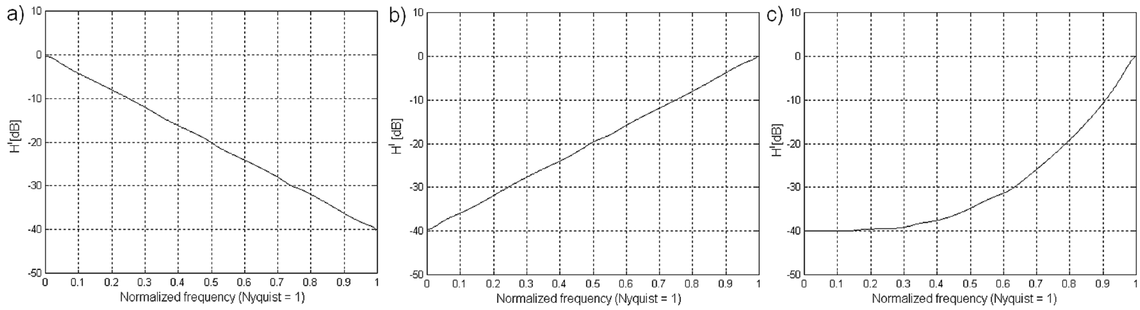

Fig. 4. Amplitude characteristics obtained by *DE-IIRFD* method for 10-order IIR digital filters with characteristics: linearly falling (a), linearly growing (b), non-linearly growing (c)

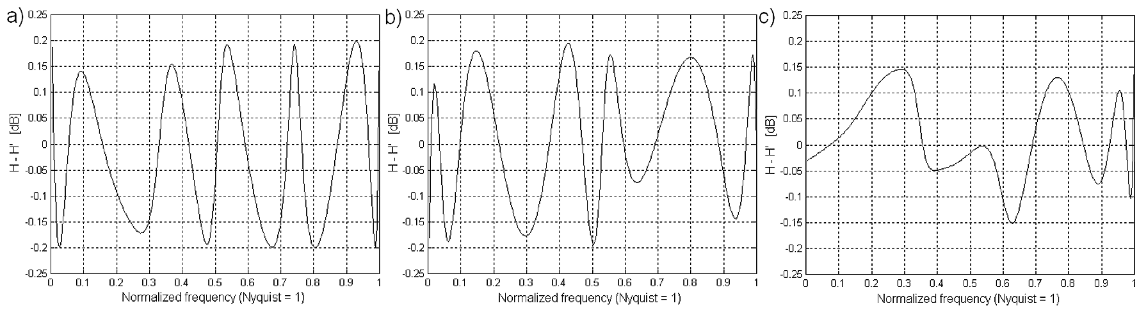

Fig. 5. Deviation between ideal characteristics $H(f)$, and characteristics $H'(f)$ obtained with the use of *DE-IIRFD* method for 10-order IIR digital filters with characteristics: linearly falling (a), linearly growing (b), non-linearly growing (c)

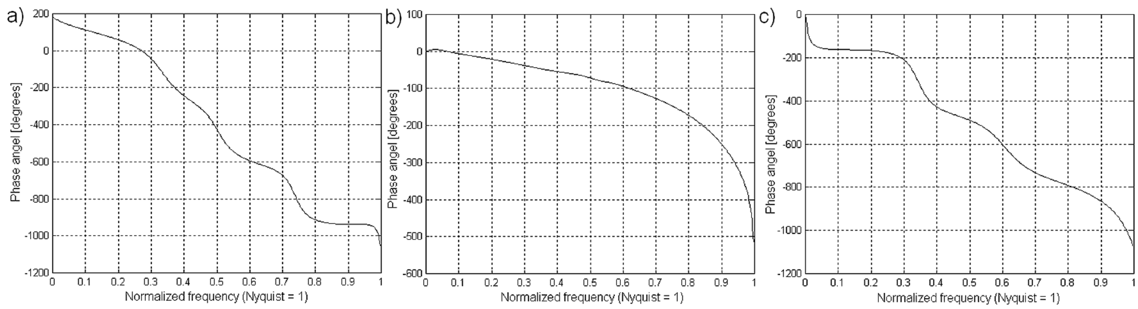

Fig. 6. Phase characteristics obtained by *DE-IIRFD* method for 10-order IIR digital filters with amplitude characteristics: linearly falling (a), linearly growing (b), non-linearly growing (c)

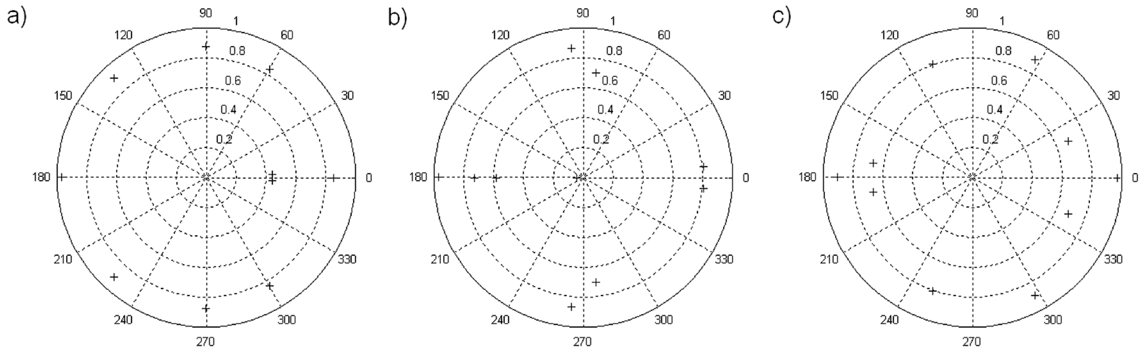

Fig. 7. Pole locations of transfer function of 10-order designed digital filter with amplitude characteristics: linearly falling (a), linearly growing (b), non-linearly growing (c)

It can be seen from Fig. 4, and Fig. 5, that designed filters fulfill prescribed design assumptions (see Fig. 3). The deviation between ideal characteristics $H(f)$, and characteristics $H'(f)$ obtained with the use of *DE-IIRFD* method do not exceed $\pm$ 0.2 [dB] for each value of normalized frequency (see Fig. 5). All designed filters are stable, that is all poles of these transfer function are located inside unit circle in the z plane (see Fig. 7).

5. Conclusions

It has been shown that it is possible to design digital filters with non-standard amplitude characteristics using the differential evolution algorithm. The main advantage of proposed method is setting up only one parameter, which determines a number of individuals in population; elaborated algorithm possesses very good convergence. Moreover, compared to previously elaborated method [1], it is not required to introduce an initial solutions (which can be obtained by *Matlab Signal Processing Toolbox*) to the start population. Due to this, presented method is more simple in use because any knowledge regarding to algorithm settings like the probability of crossover, and the probability of mutation is not required from the user. Also any knowledge regarding to the design theory of digital filters is not required from the user too. Because the objective function *COST(.)* is assumed as a sum of deviations between obtained amplitude characteristics and accepted constraints, the proposed method permits to determine the coefficients of digital filters with amplitude characteristics considerably different from standard approximation (for example Butterworth, Chebyshev or Cauer). Proposed method allows for full automation of the design process of digital filters.

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