Paper

Theory of scattering by an array of lossy dielectric, ferrite and conducting cylinders

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Abstract — Theory of scattering by lossy dielectric, ferrite and/or conducting cylinders is investigated using a combination of an iterative scattering procedure and the orthogonal expansion method. The addition theorems for vector cylindrical harmonics, which transform harmonics from one coordinate system to another, are presented.

Keywords — iterative scattering, waveguide junctions, conducting and dielectric cylinders.

1. Introduction

Considering the electromagnetic wave scattering from two-dimensional arbitrary obstacles we can observe a two areas of active research. The first approach concerns open problems – obstacles in free space, where the far scattered field patterns can be investigated [1, 2], while the second – closed problems – presents the frequency responses of described structure in a rectangular waveguide [3, 4].

In the last decade, a recursive algorithm has been developed for the scattering by arbitrarily shaped obstacles [1]. Elsherbeni *et al.* [2] proposed an iterative solution for the scattering by M different parallel circular cylinders. Recently, Valero and Ferrando [4] presented the method, which segments the problem into regions that are characterized by their generalized admittance matrices.

In this paper we apply modified iterative scattering procedure, which has been used for open problems [2] and the orthogonal expansion method to describe an equivalent scattered field by lossy dielectric, ferrite and/or conducting cylinders on the surface of a separated interaction region, which then can be used both for open and closed structures. The main advantage of this method is that we can obtain a total scattered field from all cylinders and match it with other incident fields to define scattering matrix of investigating structure. This technique can be applied to analyze a waveguide structures where incident fields are the TE_{m0} mode and open structures to define the far scattered field patterns for E_z-wave excitation.

2. Basic formulation

Consider harmonic E_z-wave excitation in global coordinates as infinite series of Bessel functions of the first kind with unknown coefficients a_n, where the electric field has

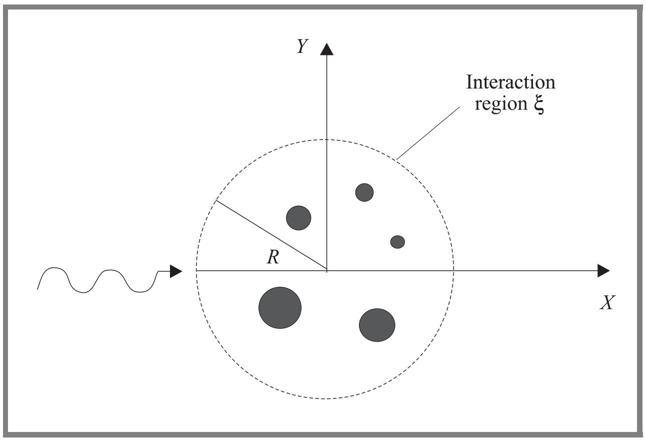

Fig. 1. Cylindrical obstacles in the interaction region excited by E_z-wave.

a z component only with all vectors independent of z of the cylindrical coordinates (ρ, ϕ, z):

$$E_z^{inc(0)} = \sum_{n=-\infty}^{\infty} a_n J_n(k_0\rho) e^{jn\phi}, \tag{1}$$

where k_0 is the wave number in free space.

Now we assume that field (1) excites all of the M homogeneous, lossy dielectric, ferrite or perfectly conducting cylinders (see Fig. 1) and has to be defined in their local coordinates. For the ith cylinder using an addition theorem for Bessel functions [5] we have

$$E_{zi}^{inc(0)} = \sum_{n=-\infty}^{\infty} a_n \sum_{m=-\infty}^{\infty} J_m(k_0 r_i) e^{jm\phi_i} J_{m-n}(k_0 d_{io}) e^{j(n-m)\phi_{io}}, \tag{2}$$

where d_{io}, ϕ_{io} are defined in Fig. 2.

In response to our excitation, a zero order scattered field is created from each of M cylinders by forcing the tangential components of both the electric and magnetic fields, on the surface of each cylinder, to be continuous:

$$E_{zi}^{inc(0)}(r_i, \phi_i) + E_{zi}^{s(0)}(r_i, \phi_i) = E_{zi}^{d(0)}(r_i, \phi_i), \tag{3}$$

$$H_{\phi i}^{inc(0)}(r_i, \phi_i) + H_{\phi i}^{s(0)}(r_i, \phi_i) = H_{\phi i}^{d(0)}(r_i, \phi_i), \tag{4}$$

where r_i is the radius of the ith cylinder.

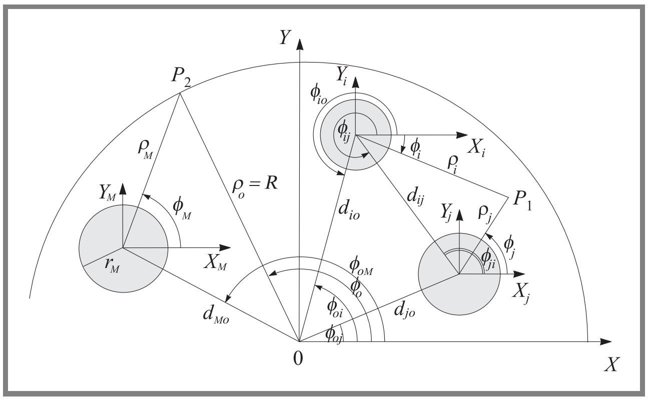

Fig. 2. Notation used for a change of coordinate system for Bessel functions.

The scattered electric field component for the ith cylinder can be expressed as

$$E_{zi}^{s(0)}(\rho_i,\phi_i) = \sum_{n=-\infty}^{\infty} c_{in}^0 H_n^{(2)}(k_0\rho_i)e^{jn\phi_i} \tag{5}$$

while transmitted field component inside the dielectric material of the ith cylinder is given by

$$E_{zi}^{d(0)}(\rho_i,\phi_i) = \sum_{n=-\infty}^{\infty} b_{in}^0 J_n(k_i\rho_i)e^{jn\phi_i}, \tag{6}$$

where c_{in}^0 and b_{in}^0 are the unknown coefficients, $J_n(k_i\rho_i)$, $H_n^{(2)}(k_0\rho_i)$ denotes Bessel and Hankel functions, respectively and $k_i = \omega\sqrt{\varepsilon_{f(i)}\mu_{eff(i)}}$, $\mu_{eff(i)} = (\mu_i^2 - \mu_{a(i)}^2)/\mu_i$, denotes the effective ferrite permeability where μ_i, $\mu_{a(i)}$ are tensor elements. The corresponding magnetic field along ϕ direction can be established from

$$H_\phi = -\frac{1}{j\omega\mu_0\,\mu_{eff(i)}}\left(\frac{\partial E_z}{\partial\rho} + j\,\frac{\mu_{a(i)}}{\mu_i\rho}\,\frac{\partial E_z}{\partial\phi}\right). \tag{7}$$

Applying (2) into Eqs. (3) and (4) and orthogonalizing by $e^{-jm\phi_i}$, the solution is obtained from the point of view of the unknown coefficients c_{in}^0 of the ith cylinder

$$[c_i^0] = [G_i]\cdot[T_{io}]\cdot[a], \tag{8}$$

where $[G_i]$ is shown in (8a) at the top of the following page.

Here the prime symbol denotes the derivative with respect to argument. For dielectric structures we assume that $\mu_{a(i)} = 0$ and $\mu_{eff(i)} = \mu_i = 1$. Transformation of Bessel functions from global coordinates to the local coordinates of the ith cylinder is expressed by matrix

$$[T_{io}] = \Big[J_{m-n}(k_0 d_{io})\,e^{j(n-m)\phi_{io}}\Big]_{m,n=-\infty}^{\infty} \tag{9}$$

and m, n are rows and columns indexes respectively, while $[a]$ defines a vector

$$[a] = \big[\ldots\ a_{-m}\ \ldots\ a_0\ a_1\ \ldots\ a_m\ \ldots\big]^T. \tag{10}$$

In the next interaction, we use scattered fields from $M-1$ cylinders from the previous interaction as a new incident field on the ith remaining cylinder

$$E_{zi}^{inc(1)} = E_0 \sum_{\substack{j=1\\ j\neq i}}^{M}\ \sum_{n=-\infty}^{\infty} c_{jn}^0 H_n^{(2)}(k_0\rho_j)\,e^{jn\phi_j}. \tag{11}$$

To transfer the scattered fields from $M-1$ cylinders to the local coordinate of the ith cylinder the Graf's addition theorem for Bessel functions is used [5] (see Eq. (12) at the top of the following page).

In response to our new excitation, the first order ($p = 1$) scattered and transmitted field is created from each of M cylinders like in Eqs. (5) and (6) but with new unknown coefficients c_{in}^1 and b_{in}^1. Using Eqs. (3) and (4) with the first order fields the following solution is obtained:

$$\big[c_i^1\big] = \big[G_i\big]\sum_{\substack{j=1\\ j\neq i}}^{M}\big[T_{ij}^H\big]\cdot\big[c_j^0\big], \tag{13}$$

where $\big[T_{ij}^H\big] = \big[H_{m-n}^{(2)}(k_0 d_{ij})e^{j(n-m)\phi_{ij}}\big]_{m,n=-\infty}^{\infty}$ and m, n are rows and columns indexes, respectively. The matrix $\big[T_{ij}^H\big]$ provides transformation of Hankel functions of the second kind located in the coordinates of the jth cylinder to the ones located in the coordinates of the ith cylinder.

This approach gives us a next order scattered field and repeated for each individual cylinder leads us to an iterative scattering procedure where the coefficients of the pth interaction depend only on the coefficients of the $(p-1)$th interaction

$$\big[C^p\big] = \big[T^{ij}\big]\cdot\big[C^{p-1}\big]\cdot[a], \tag{14}$$

where

$$\big[C^p\big] = \begin{bmatrix}\big[c_1^p\big]\\ \cdot\\ \big[c_i^p\big]\\ \cdot\\ \big[c_M^p\big]\end{bmatrix},\quad \big[T_{ij}\big] = \begin{bmatrix}[0] & \cdot & [T_{1,j}] & \cdot & [T_{1,M}]\\ \cdot & [0] & \cdot & \cdot & \cdot\\ [T_{j,1}] & \cdot & \cdot & \cdot & [T_{j,M}]\\ \cdot & \cdot & [T_{i,j}] & [0] & \cdot\\ [T_{M,1}] & \cdot & [T_{M,j}] & \cdot & [0]\end{bmatrix}$$

for $p = 2, 3, \ldots$ and $\big[c_i^p\big]$, $[0]$, $\big[T_{i,j}\big]$ are square sub-matrices where

$$\big[T_{i,j}\big] = \big[G_i\big]\cdot\big[T_{ij}^H\big]. \tag{15}$$

Iterative procedure gives us the scattered field from the ith cylinder in its local coordinates as follows

$$\big[E_{zi}^S\big] = \big[H_i^\rho\big]\cdot\big[C_i\big]\cdot[a], \tag{16}$$

where $\big[C_i\big] = \sum_{p=0}^{N}\big[c_i^p\big]$, $\big[H_i^\rho\big] = diag\big(H_m^{(2)}(k_0\rho_i)e^{jm\phi_i}\big)_{m=-\infty}^{\infty}$ and N is the number of interactions.

$$[G_i] = diag\left(\frac{k_0 J_m(k_i r_i) J_m'(k_0 r_i) - J_m(k_0 r_i)\left[\frac{k_i}{\mu_{eff(i)}} J_m'(k_i r_i) - m\,\frac{\mu_{a(i)}}{\mu_{eff(i)}\mu_i r_i} J_m(k_i r_i)\right]}{H_m^{(2)}(k_o r_i)\left[\frac{k_i}{\mu_{eff(i)}} J_m'(k_i r_i) - m\frac{\mu_{a(i)}}{\mu_{eff(i)}\mu_i r_i} J_m(k_i r_i)\right] - k_0 J_m(k_i r_i)\, H_m^{(2)\prime}(k_0 r_i)}\right)_{m=-\infty}^{\infty}. \tag{8a}$$

$$H_n^{(2)}(k_0\rho_j)e^{jn\phi_j} = \left\{\begin{array}{ll} \sum\limits_{m=-\infty}^{\infty} H_{m-n}^{(2)}(k_0 d_{ij})e^{j(n-m)\phi_{ij}} J_m(k_0\rho_i)e^{jm\phi_i} & \text{for} \quad d_{ij} \geq \rho_i \\ \sum\limits_{m=-\infty}^{\infty} J_{m-n}(k_0 d_{ij})e^{j(n-m)\phi_{ij}} H_m^{(2)}(k_0\rho_i)e^{jm\phi_i} & \text{for} \quad d_{ij} < \rho_i \end{array}\right\}. \tag{12}$$

Using transformation (12) for $d_{ij} < \rho_o$ the scattered field from each cylinder is transferred to global coordinate system. Therefore the scattered electric field from the ith cylinder on the surface of the interaction region (see Fig. 2) is given as

$$\left[E_{zi}^{SG}\right] = \left[H_i^R\right]\cdot\left[T_{oi}^G\right]\cdot\left[C_i\right]\cdot[a], \tag{17}$$

where

$$\left[H_i^R\right] = diag\left(H_m^{(2)}(k_0 R)e^{jm\phi}\right)_{m=-\infty}^{\infty}$$

$$\left[T_{oi}^G\right] = \left[J_{m-n}(k_0 d_{oi})e^{j(n-m)\phi_{oi}}\right]_{m,n=-\infty}^{\infty}$$

and m, n are rows and columns indexes, respectively.

Writing (17) for electric and magnetic field for each of M cylinders we obtain the following matrix equations:

$$\left[E_z^{SG}\right] = \left[H^R\right]\cdot\left[T^G\right]\cdot[C]\cdot[a], \tag{18}$$

$$\left[H_\phi^{SG}\right] = \frac{1}{j\omega\mu_0}\left[H'^R\right]\cdot\left[T^G\right]\cdot[C]\cdot[a], \tag{19}$$

where

$$\left[E_z^{SG}\right] = \left[\left[E_{z1}^{SG}\right]\ ..\ \left[E_{zi}^{SG}\right]\ ..\ \left[E_{zM}^{SG}\right]\right]^T,$$

$$\left[H_\phi^{SG}\right] = \left[\left[H_{\phi 1}^{SG}\right]\ ..\ \left[H_{\phi i}^{SG}\right]\ ..\ \left[H_{\phi M}^{SG}\right]\right]^T,$$

and

$$\left[H^R\right] = \begin{bmatrix} [H_1^R] & . & [0] & . & [0] \\ . & . & . & . & . \\ [0] & . & [H_i^R] & . & [0] \\ . & . & . & . & . \\ [0] & . & [0] & . & [H_M^R] \end{bmatrix},$$

$$\left[T^G\right] = \begin{bmatrix} [T_{o1}^G] & . & [0] & . & [0] \\ . & . & . & . & . \\ [0] & . & [T_{oi}^G] & . & [0] \\ . & . & . & . & . \\ [0] & . & [0] & . & [T_{oM}^G] \end{bmatrix}, \quad [C] = \begin{bmatrix} [C_1] \\ . \\ [C_i] \\ . \\ [C_M] \end{bmatrix}.$$

Matrices $\left[H_i^R\right]$, $\left[T_{oi}^G\right]$, $\left[C_i\right]$ and $[0]$ are square sub-matrices.

The total scattered electric and magnetic field from all cylinders, can be easy obtained from

$$\left[E_z^{SGT}\right] = [I]\cdot\left[E_z^{SG}\right], \tag{20}$$

$$\left[H_\phi^{SGT}\right] = [I]\cdot\left[H_\phi^{SG}\right], \tag{21}$$

where matrix $[I]$ consists of diagonal sub-matrices $[I_i] = = diag(1)_{m=-\infty}^{\infty}$ as shown $[I] = \left[[I_1]\ \ldots\ [I_i]\ \ldots\ [I_M]\right]$.

Now the total field on the surface of the interaction region can be defined as

$$\left[E_z^T\right] = \left[E_z^{inc(0)}\right] + \left[E_z^{SGT}\right], \tag{22}$$

$$\left[H_\phi^T\right] = \left[H_\phi^{inc(0)}\right] + \left[H_\phi^{SGT}\right], \tag{23}$$

where $\left[E_z^{inc(0)}\right]$ and $\left[H_\phi^{inc(0)}\right]$ are diagonal matrices based on (1). To eliminate unknown coefficients (10), a relation between electric and magnetic field on the surface of the interaction region is defined:

$$\left[E_z^T\right] = [Z]\cdot\left[H_\phi^T\right]. \tag{24}$$

Hence, the matrix $[Z]$ is given as

$$[Z] = \left(\left[E_z^{inc(0)}\right] + \left[E_z^{SGT}\right]\right)\cdot\left(\left[H_\phi^{inc(0)}\right] + \left[H_\phi^{SGT}\right]\right)^{-1}. \tag{25}$$

The formulation of the problem in form of $[Z]$ allows to consider both waveguide and open problems assuming the proper excitations.

3. Conclusions

The analysis for scattering by an array of lossy dielectric, ferrite and/or conducting cylinders has been developed using a combination of modified iterative scattering procedure and the orthogonal expansion method. This approach is convenient for investigations of the open and waveguide problems.

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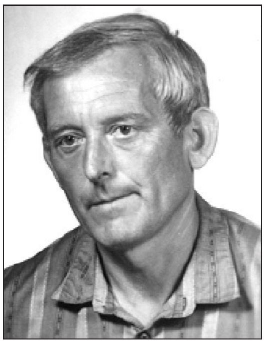

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