THE USE OF NUMERICAL METHODS FOR THE ANALYSIS OF PHASE PHENOMENA IN MULTI-ELEMENT ANTENNAS

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The article describes an analysis of the effects of mutual impedance on the phase of multi-element antenna signals. The calculations were made using a substitute model of a Mason ultrasonic transducer. The effects of self and mutual impedance on the phase of multi-element antenna signals were considered.

INTRODUCTION

The design of a multi-element antenna must follow clearly defined phases transmitted or received by antenna elements. The phases of signals transmitted (received) by the antenna depend on both electrostriction parameters of the transducer and acoustic load. In order to analyse the parameters the Mason wave model of a transducer was used. It is frequently assumed that the transducer is loaded with acoustic impedance typical of the plane wave. In reality, however, because of the transducer's low apertures, the acoustic load is quite different from those in the plane wave assumption and come as the sum of self and mutual impedance. The article includes the results of numerical calculations of the transducers impedances used in typical multi-element antennas. Next, the effects of the impedance on the phase shift between the electric and acoustic signal were calculated. The results are presented in the broad frequency band.

1. WAVE MODEL OF A PIEZOELECTRIC TRANSDUCER LOADED WITH A HOMOGENOUS UNLIMITED MEDIUM

The wave model of the ultrasonic transducer allows the derivation of complete relations between the transducer's electric and acoustic parameters [1]. The complicated form, however, makes a fast and simple analysis of transducer properties difficult by analytical methods. Various methods simplifying the transducer's wave model were described in a number of publications. The article suggests the use of numerical methods for the analysis of the compelte wave model of the transducer, which are quite common recently.

The subject mater of the analysis is a multi-element antenna consisting of identical ultrasonic transducers. A transducer that has the form of a long thin bar (a wide disc) - Fig.1, is built of piezoelectric material with wave impedance Z_1 loaded on both ends with media of known wave impedances Z_0 and Z_2, and according to wave model is described with the following equations [2]:

$$U(j\omega)=\frac{h^2\left(1-e^{-\alpha d-j\beta d}\right)\left[\left(1+b_{12}\right)\left(1-b_{10}\,e^{-\alpha d-j\beta d}\right)+\left(1+b_{10}\right)\left(1-b_{12}\,e^{-\alpha d-j\beta d}\right)\right]}{2Z_1\omega^2A\left(1-b_{10}b_{12}\,e^{-2\alpha d-j2\beta d}\right)}I(j\omega)+\frac{I(j\omega)}{j\omega C_e}$$

$$V(0,j\omega)=-j\frac{h\left(1+b_{10}\right)\left(1-b_{12}\,e^{-\alpha d-j\beta d}\right)}{2Z_1\omega A\left(1-b_{10}b_{12}\,e^{-2\alpha d-j2\beta d}\right)}I(j\omega) \qquad (1)$$

$$V(d,j\omega)=-j\frac{h\left(1+b_{12}\right)\left(1-b_{10}\,e^{-\alpha d-j\beta d}\right)}{2Z_1\omega A\left(1-b_{10}b_{12}\,e^{-2\alpha d-j2\beta d}\right)}I(j\omega)$$

where:

$U(j\omega)$ – voltage on the transducer, $I(j\omega)$ – current – excitation,
α, β – attenuation and wave constant in piezoelectric material, $\beta=2\pi/\lambda$,
Z_1, Z_0, Z_2 - wave impedances of transducer and propagation media,
$V(0,j\omega)$, $V(d,j\omega)$ - velocity on the left and right - hand side transducer surface,
$p(0,j\omega)$, $p(d,j\omega)$ momentary pressure on the left and right - hand side transducer surface,
$b_{10}=(Z_1-Z_o)/(Z_1+Z_o)$, $b_{12}=(Z_1-Z_2)/(Z_1+Z_2)$ reflection coefficients on both transducer ends.

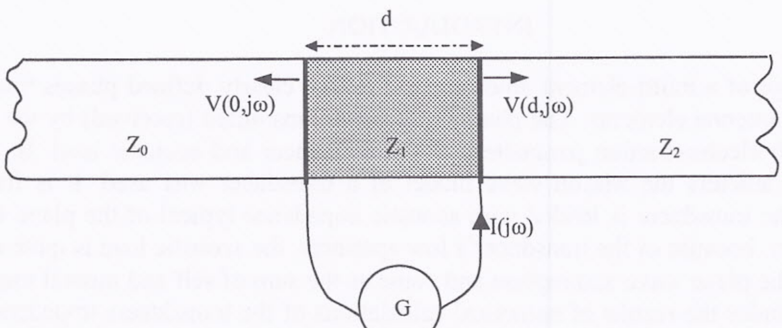

Fig.1 Physical model of a longitudinally polarised transducer with loads on both ends.

The frequent assumption is that a plane wave is propagated in the transducer and in the medium surrounding it. When aperture values are actually comparable to wave length, the transducer is loaded with radiation impedance different from mechanical impedance. For a multi-element antenna the impedance also includes the effects of adjacent elements, leading to the additional impedance of mutual radiation. The impedances were calculated numerically based on papers [3] [4], and then used in the analysis of their effects on the phase properties of the acoustic wave.

2. THE EFFECTS OF SELF AND MUTUAL RADIATION IMPEDANCE

In the paper a simple algorithm to calculate self and mutual impedance [4] is used. It is based on numerical integration of the pressures from the wave radiated by the adjacent transducer S_n to the transducer under analysis S_m. The effects were analysed involving an antenna consisting of two transducers aligned along a single line, as shown in Fig.2

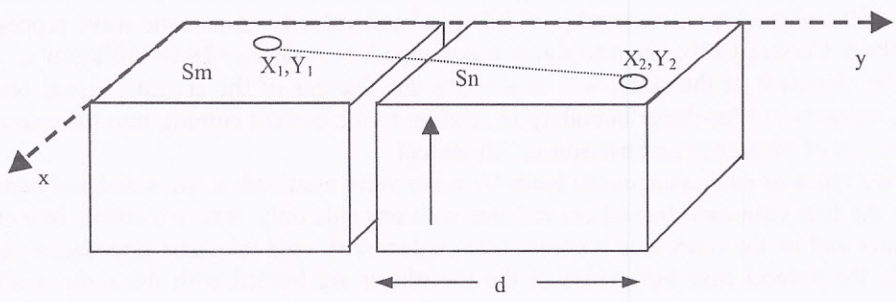

Fig.2 Model of the two-element transducer

For the purpose of the analysis, it was assumed that each element of one transducer (S_n) sends a spherical wave which reaches all elements of the second transducer (S_m). Given this, mutual impedance can be determined using Rayleigh's formula (2).

$$Z_{mn} = \frac{j\rho ck}{2\pi} \int\int_{Sm} dx_1 dy_1 \int\int_{Sn} dx_2 dy_2 \times \frac{e^{-jk\sqrt{(X+x_1-x_2)^2+(Y+y_1-y_2)^2}}}{\sqrt{(X+x_1-x_2)^2+(Y+y_1-y_2)^2}} \qquad (2)$$

For rectangular transducer apertures, Eq. (2) representing 4-dimentional integral can be reduced to 2-dimentional integral [3]:

$$Z_{mn} = \frac{j\rho ck}{2\pi} \int_{-d}^{d} du \int_{-d}^{d} dv (d-|u|)(d-|v|) \times \frac{e^{-jk\sqrt{(X+u)^2+(Y+v)^2}}}{\sqrt{(X+u)^2+(Y+v)^2}} \qquad (3)$$

where X, Y are the coordinates of the transducer separation. Thus, the calculation time of the mutual impedance is highly reduced.

The effects of self and mutual impedance on transducer parameters – Eq. (1) – is contained in reflection coefficients b_{10} and b_{12}. Because of the self and mutual impedance the coefficients are now complex values and it finally leads to a change of the velocity phase on transducer boundary. Section 3 presents the results of calculations of phase relations between the velocity at transducer aperture and electrical excitation. The phases were calculated using the formula:

$$\varphi = \mathrm{atan}\frac{Im\,\mathrm{V}(\mathrm{d}, \mathrm{j}\omega)}{Re\,\mathrm{V}(\mathrm{d}, \mathrm{j}\omega)} \tag{4}$$

3. RESULTS OF CALCULATIONS

The study involved a number of numerical calculations analysing an antenna model consisting of two linearly aligned ultrasonic transducers (as in Fig.2) with a square shaped aperture. In the numerical calculations the following values were used: aperture side a=0.75cm, transducer height (the condition of resonance frequency 100kHz) 1.8cm. It was assumed that the transducers were made of piezoelectric material with typical parameters, i.e. density of the piezoelectric material ρ_l=7800kg/m^2 and velocity of acoustic wave propagation c_l=3600m/s. Consequently, the transducer wave impedance was Z_l =28.08*10^{6}kg/m^{2}s.

The objective of the study was to identify the changes of the acoustic signal phase or actually velocity at transducer boundary in relation to the current coming into the transducer, as a function of frequency and transducer alignment.

Two types of antenna acoustic loads by water were assumed: a one-sided and two-sided load. In the first case each transducer radiates with one side only, where it comes into contact with water and on the other side it comes into contact with void (acoustic impedance equal to zero). In the second case both sides of the transducer are loaded with the same medium – water.

The first stage involved the calculation of the effects of aperture size and element spacing on self and mutual impedance – Fig.3a and 3b. The thick line represents the real part of the impedance and the thin line the imaginary part. The X-axis was scaled to normalised distances in relation to wave length.

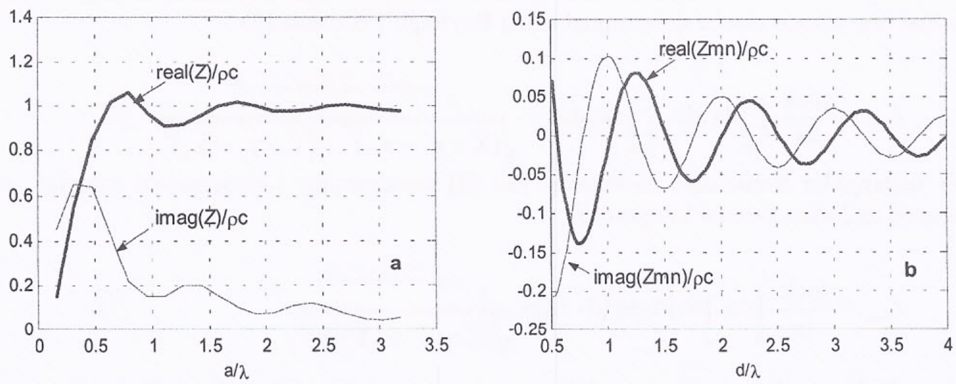

Fig.3 a) Self impedance of a rectangular prism transducer with square aperture,
b) mutual impedance of two transducers as above and in the alignment function.

The values of self and mutual impedance were then substituted to formulas (1) describing the velocities on the boundaries of both transducers of the antenna under analysis.

To compare, analogue calculations were made for the same antenna assuming that the transducers are loaded with mechanical impedance only (as for the plane wave).

The results are shown in Fig. 4 and 5. Figures 4a and 4b show the effects of the frequencies on phase shift between the current and velocity. Figures 5a and 5b show diagrams of phase deviations in the frequency function.

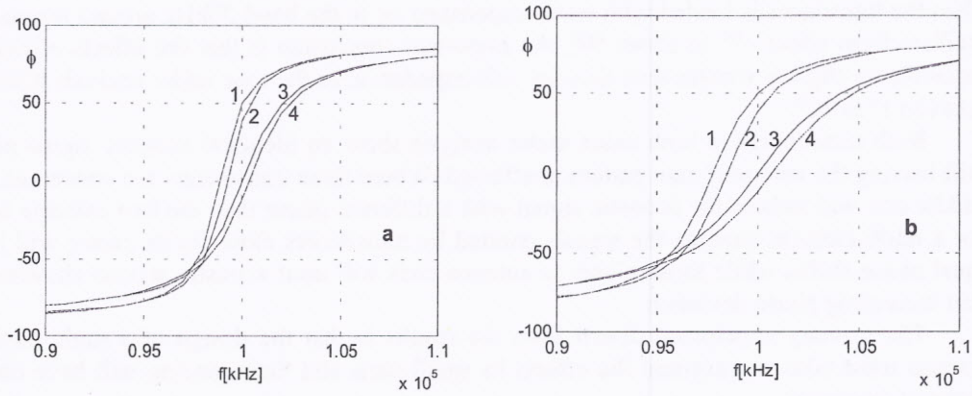

Fig.4 The effects of self and mutual impedance on phase shift of the acoustic signal
a) one-sided load of the transducer, b) two-sided load of the transducer,
line: 1 - for self impedance load excluding mutual radiation,
2 - for loads that are the result of self and mutual impedance,
3 - for acoustic impedance loads (plane wave),
4 - for acoustic impedance load (plane wave including mutual impedance).

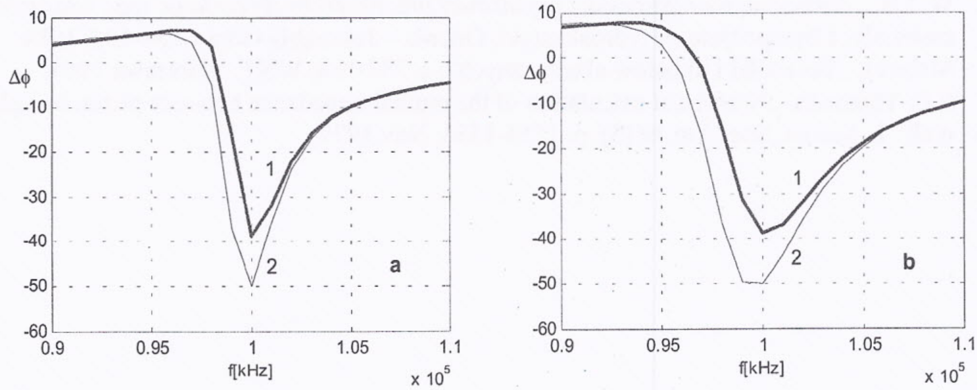

Fig.5 Phase deviations caused by self and mutual impedance: a) transducer with one-sided load, b) transducer with two-sided load. Line 1 - deviation between the phase shift for a transducer loaded with acoustic impedance assuming a plane wave and with self and mutual impedance. Line 2 – deviation between the phase shift for a transducer loaded with acoustic impedance assuming a plane wave and with the self impedance (excluding the mutual impedance).

4. CONCLUSION

The results of the calculations show that in the case of transducers which are half the size of the wave length, the effects of self and mutual impedance on phase shift of the acoustic signal are very strong compared to electric excitation. How strong the effects are depends on frequency; they are highest when approaching resonance of the antenna. In both considered cases (one-sided load and double-sided load), phase deviations from phase shift when the transducer is loaded with wave impedance ρc in the band 20kHz around resonance, oscillate from about –7° to about 50°. An important conclusion is that the effects of mutual radiation are slightly weaker than those of self impedance. In the case under analysis it ranges between 1° to 20°.

Both transducers in both cases under analysis show an identical acoustic signal phase shift leaving the antenna beam pattern unaffected. Where three transducers are concerned, the middle one will radiate the acoustic signal with a different phase than the two extreme ones. For a multi-element antenna the signals emitted by transducers closer to its centre will have equal phase shifts, while those closer to antenna ends will emit acoustic signals showing an ever increasing phase deviation.

The primary conclusion drawn from the results is that the design of a multi-element antenna must take into account the effects its small parts and their spacing will have on the phase of the signal.

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