ON A THRESHOLD INDUCED BIAS IN FISH TARGET STRENGTH ESTIMATION

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In fisheries acoustics the bias introduced by a system threshold in the integration of echoes from fish and other marine organisms has long been recognized. Some simple methods for evaluating such a bias uses Rayleigh assumption applied to fish backscattering length distribution. More adequate methods require that the target strength distribution of the fish and directivity pattern of the transducer be known.
The method proposed here uses statistical approach for correction of target strength probability density function (PDF). The theory of conditional target strength PDF and conditional beam pattern PDF is derived. It provides correction not only for mean value of target strength but for whole distribution. The data obtained from simulations are presented and compared with those obtained from single-beam inverse processing.

INTRODUCTION

In the acoustic surveying of fish stocks and subsequent estimation of biomass, the sampling volume of the acoustic instruments is of great importance, as for randomly distributed targets the received echo energy is linearly related to this volume. As a signal threshold is applied in order to eliminate contribution from noise the effective sampling volume is always less than the full volume of the acoustic beam. The problem was treated in several papers. Kalikhman *et al.* (1981) [1] combined the beam pattern of the transducer with scattering characteristics of the fish averaged over azimuth. Their conclusion is that, for a single fish, the effective equivalent beam angle depends not only on the threshold but also on the angle of insonification. Foote (1991) [2] developed an expression for the effective equivalent beam angle in terms of the directivity of the transducer, the backscattering cross-section of the fish as function of tilt angle and the signal threshold. Detailed literature survey on the problem can be found in papers of Reynisson (1996) [3] and Fleischman (2000) [4]. In this paper, the problem of bias introduced by threshold will be described by statistical analysis of the target strength and beam pattern distributions.

1. STATISTICAL DESCRIPTION OF THRESHOLD PROBLEM

According to the sonar equation expressed in logarithmic domain, the amplitude of echo envelope E_i may be treated as a sum of fish target strength TS_i and the value of beam pattern function Bi corresponding to its angular position:

$$E_i = TS_i + B_i + TVG + K \tag{1}$$

where TVG is time varied gain function and K is the hydroacoustic system constant. Assuming that processed data are TVG corrected and the system gain is included in this correction we can write Eq.(1) as a statistical function of two random variables $\underline{TS}$ and $\underline{B}$, which describes random process of collecting the fish echo data:

$$\underline{E} = \underline{TS} + \underline{B} \tag{2}$$

In simplified and often used case the random variables $\underline{TS}$ and $\underline{B}$ are treated as the independent random variables, which allows expressing its probability density functions (PDFs) as a integral equation:

$$p_E(E) = \int_{-\infty}^{0} p_{TS}(E-B)p_B(B)dB \tag{3}$$

This equation is typically used in statistical removal of so called beam pattern effect in the case of acquisition of data by a single beam system, where exact angular position of the object can not be determined. In the case of dual beam system Eq. (1) is also used as the two sets of data from narrow and wide beam channels allows for direct calculation of beam pattern value B_i for every fish echo and hence its TS_i calculation. However, although we have exact TS for every detected fish echo, introducing system threshold restricts not only the dynamic range of the data but also introduces dependence on $\underline{TS}$ and $\underline{B}$ random variables. Thus, statistically, we can write the equivalent equation for dual beam case:

$$\underline{E'} = \underline{TS'} + \underline{B'} \tag{4}$$

where primes denotes that we operate on conditinal variables. As the consequence, the PDFs of this variables should be expresses by a conditional PDF as follows:

$$p_{E'}(E') = p_E(E \mid E > E_{\min}) \tag{5}$$

$$p_{TS'}(TS') = p_{TS}(TS \mid E > E_{\min}) \tag{6}$$

$$p_{B'}(B') = p_E(B \mid E > E_{\min}) \tag{7}$$

where E_{min} is a echo level threshold value.

The net effect is that the mean value of backscattering cross-section $\overline{\sigma}'$ evaluated from transformed distribution of conditional random variable $\underline{TS}'$ is biased as compared to truth mean value $\overline{\sigma}$ evaluated from variable $\underline{TS}$. It is noteworthy that in the single beam case the fact of introducing the threshold modifies only the range of integration in convolution-like integral:

$$p_E(E \mid E > E_{\min}) = \int_{E_{\min}-B}^{0} p_{TS}(E-B)p_B(B)dB \tag{8}$$

and the reconstructed $p_{TS}(TS)$ is unconditional if properly assumed $p_B(B)$ is used.

Statistical removal of the bias introduced by the threshold in dual beam processing requires calculation of $p_{TS}(TS)$ from conditional $p_{TS}(TS|E>E_{min})$. The latter distribution, which is *de facto* observed, can be expressed as:

$$p_{TS}(TS \mid E > E_{\min}) = p_{TS}(TS \mid TS + B > E_{\min}) \tag{9}$$

which may be evaluated by integration of joint distribution of independent random variables <u>*TS*</u> and <u>*B*</u>:

$$p_{TS}(TS \mid E > E_{\min}) = \frac{\int\limits_{E_{\min}-TS}^{\infty} p_{TS,B}(TS,B)dB}{\int\limits_{-\infty}^{+\infty}\int\limits_{E_{\min}-TS}^{\infty} p_{TS,B}(TS,B)dBdTS} \tag{10}$$

As the denominator evaluate to the constant value (normalization constant) and independency of variables <u>*TS*</u> and <u>*B*</u> allows replacing joint PDF by multiplication of its PDFs, it results in:

$$p_{TS}(TS \mid E > E_{\min}) = c_1 p_{TS}(TS) \int\limits_{E_{\min}-TS}^{\infty} p_B(B)dB \tag{11}$$

The integral in above equation can be expressed using cumulative distribution function (CDF) $F_B(\,)$ of beam pattern random variable <u>*B*</u>, which finally gives:

$$p_{TS}(TS \mid E > E_{\min}) = c_1 \; p_{TS}(TS)\,[1 - F_B(E_{\min} - TS)] \tag{12}$$

which describes the connection between conditional distribution of observed target strength and required unconditional distribution. Note that, it requires the knowledge of unconditional distribution of beam pattern CDF.

The same approach applied to conditional distribution of beam pattern function $p_B(B|E>E_{min})$ gives following equation:

$$p_B(B \mid E > E_{\min}) = c_2 \; p_B(B)\,[1 - F_{TS}(E_{\min} - B)] \tag{13}$$

In both cases the expression in brackets represents CDF of the second function scaled to domain of first function. Thus dependence introduced by threshold is observed as a multiplication of unconditional PDF of one function by scaled CDF of the second one. The constants c_1 and c_2 normalizes equivalent distributions. The removal of threshold effect on measured distribution of target strength requires solution of equations (12) and (13), which represents a set of integral equations.

2. SIMULATIONS

To verify the legitimacy of presented equations the numeric simulations were performed. In the simulations, the Rayleigh-distributed echo amplitudes were assumed, equivalent to exponentially-distributed backscattering cross-section σ. Thus 100000 doubly exponentially-distributed *TS* variates with modal value TS_{mod} = -40dB ($\overline{\sigma}$=1cm^2) were generated along with the same number of uniformly-distributed *B* variates. They were sum up to obtain echo level and only those greater than threshold value E_{min} = -70dB were used to imitate actual process of data acquisition and calculation of conditional distributions. The mean value of backscattering cross-section $\overline{\sigma}'$ = 1.16 cm^2 was obtained from transformed conditional distribution of *TS* '. Thus, the mean value is biased by 16%, which is equivalent to 0.7dB shift in logarithmic domain. Fig. 1 presents modeled PDFs of target strength *TS*, beam pattern *B* and echo level *E* of unconditional distributions and conditional distributions induced by threshold. Although the bias in mean value is not meaningful, the number of detected fish at modal value of TS distribution is two times smaller. Comparing the distributions of beam pattern functions it is evident that threshold induced bias becomes more pronounced with distance off-axis. For this case the effect of multiplication by CDF function of target strength is easily observed.

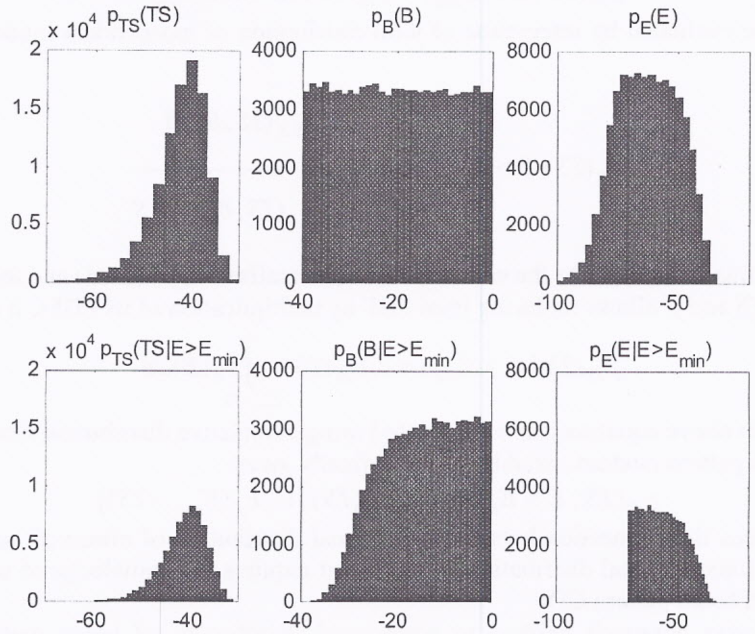

Figure 1. Modeled PDFs of target strength *TS*, beam pattern *B* and echo level *E*. The upper charts represents unconditional distributions while lower one conditional distributions induced by threshold.

3. CONCLUSIONS

In the paper, the statistical analysis of the effects introduced by thresholding the fish echo data is presented. The analysis is performed using probability distribution functions and thus describes not only known effect of bias observed in mean value of fish backscattering cross-section but also the change in the shape of observed target strength and beam pattern histograms.

It also reveals the difference between target strength estimates obtained by indirect processing of single beam data as those corresponding to unconditional distribution and those obtained by direct processing of dual beam data, which in fact are conditionally dependent and generates the bias in mean values of estimates.

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