Paper published on pages 481-485 in:
Shell Structures: Theory and Applications – Pietraszkiewicz & Szymczak (eds)
 ISBN 0 415 38390 0

Critical examination of benchmark problems for large rotation analysis of laminated shells

I. Kreja
Gdańsk University of Technology, Gdańsk, Poland

ABSTRACT: The paper presents a critical review of benchmark problems for large rotation analysis of laminated shells. Several examples taken from the literature have been re-analysed using the author's own computer programs based on different levels of the geometrical non-linearity. The comparative analysis allows one to drawn some conclusions on suitability of the considered examples for the use as benchmark problems.

1 INTRODUCTION

In last years a relatively large number of publications were dedicated to the geometrically nonlinear FE analysis of laminated shells undergoing large rotations (see e.g. Balah & Al-Ghamedy 2002, Başar et al. 1993, 2000, Brank et al. 1995, Kim & Voyiadjis 1999, Kulikov & Plotnikova 2003, Masud et al. 2000). It is quite natural that every author of a new finite element code for large rotation analysis of laminated shells starts to test his formulation by confronting his results with those published by other authors. However, in contrast to the state of the art in the field of isotropic shells, there is lack of a commonly accepted set of benchmark problems for laminated shells undergoing large rotations. As a consequence, it happens that an advanced formulation accounting for finite rotations in laminated shells is illustrated with numerical examples where rotations stay well within the range of small rotations (see e.g. Brank et al. 1995).

A critical examination of benchmark problems for the large rotation analysis of laminated shells performed in the present report is based on a re-calculation of all considered examples with the author's own computer programs with gradually increased and clearly distinguished levels of geometrical non-linearity. The four FE formulations for laminated shells based on the First Order Shear Deformation hypothesis were considered in the present analysis:

- RVK5 – FE model (Kreja et al. 1997) of anisotropic shells with the von Kármán type non-linearity (Reddy 1982);
- MRT5 – FE formulation (Kreja et al. 1997) based on the Moderate Rotation Theory of anisotropic shells (Schmidt & Reddy 1988);
- LRT5 – FE realization (Ferro et al. 1998) of the Large Rotation Theory for anisotropic shells (Librescu 1987);
- LRT56 – a revised FE implementation of LRT (Librescu 1987) based on the use of Euler angles (Kreja & Schmidt 2002, 2005).

The 8-URI shell element has been applied (Kreja et al. 1997). Each time the calculations have been performed using a finite element mesh of a density that provided a convergent solution. The arc-length control method used in the incremental calculations allowed us for the investigation of snap-through and snap-back problems.

2 SELECTED EXAMPLES

2.1 *Hinged cylindrical panel under point load*

The laminated hinged cylindrical panel under the point load as presented in Figure 1 was proposed by Saigal et al. (1986). It was adopted as a benchmark example by many other researchers (see e.g. Brank et al. 1995, Kim & Voyiadjis 1999, Sze et al. 2004).

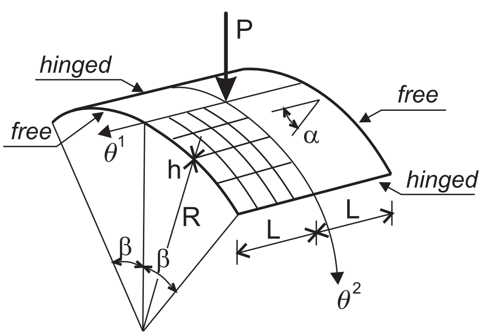

Figure 1. Hinged cylindrical panel under point load

The orthotropic material of the shell is characterized by the following parameters: $E_a = 3.3$ kN/mm^2, $E_b = 1.1$ kN/mm^2, $G_{ab} = G_{ac} = G_{bc} = 0.66$ kN/mm^2 and $\nu_{ab} = 0.25$. Dimensions of the panel are assumed as R = 2540 mm, L = 254 mm and $\beta = 0.1$.

Three different thicknesses of the shell were considered. In the first attempt, the thickness of the shell was equal to 12.6 mm following the original proposal of Saigal et al. (1986). As one can observe in Figure 2, there is almost no difference between results of the LRT56, MRT5 or RVK5 formulations. Therefore, one can assume that the magnitude of deformations of the panel stays within the limits of small rotations.

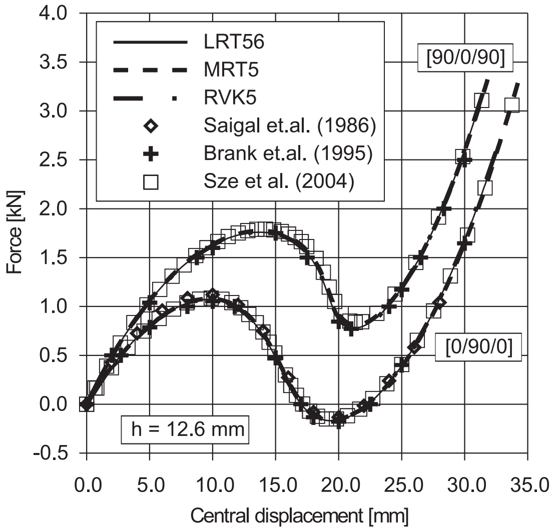

Figure 2. Central deflection for laminated panel 12.6 mm thick.

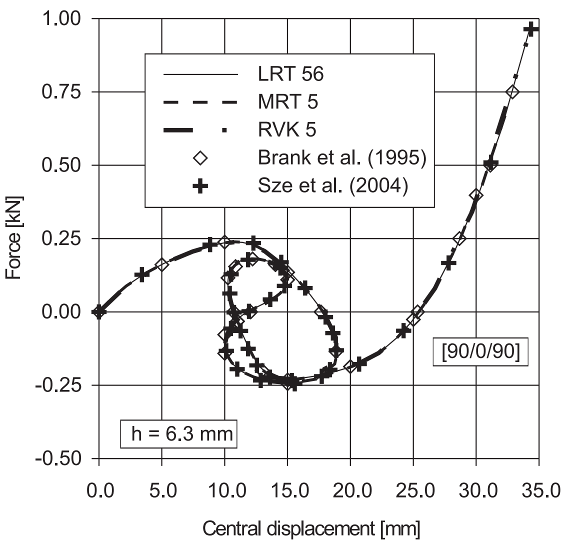

Figure 3. Central deflection for laminated panel 6.3 mm thick.

Brank et al. (1995) introduced a laminated panel with the thickness reduced by half (h = 6.3 mm) hopping probably to gain a more pronounced snapping behavior. Looking at the graphs in the Figure 3, one can observe that the equilibrium path is represented by an evidently more complex curve than for h = 12.6 mm. Nevertheless, here again the responses for LRT56, MRT5 and RVK5 formulations seem to be the same.

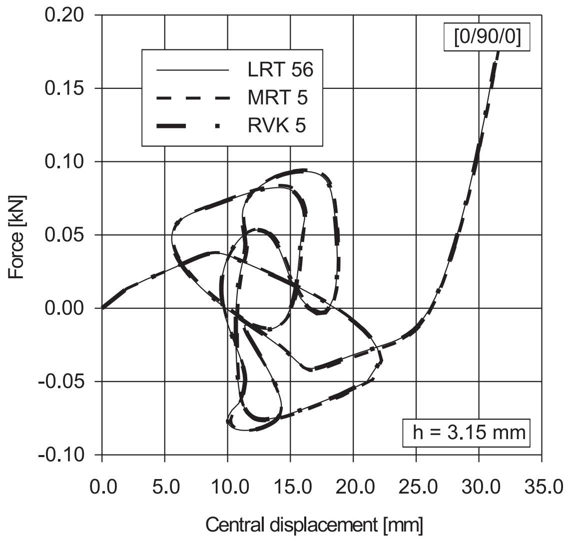

Figure 4. Central deflection for laminated panel 3.15 mm thick.

In the third case of the present analysis even a further reduction of the panel thickness was presumed taking h = 3.15 mm, what resulted in a rather complicated shape of the equilibrium path (see Figure 4). However, even in such a case, it is also difficult to notice any distinction between the curves obtained for LRT56, MRT5 or RVK5 models (Fig. 4).

One can remark that although the decrease of the thickness of the shell has reduced its stiffness, nevertheless, the range of rotations has not exceeded the limits of small rotations. It is obvious that because of that the considered example of the hinged laminated cylindrical panel cannot serve as a proper test problem for the large rotations shell analysis.

2.2 *Clamped cylindrical panel under point load*

In the second example, a deep cylindrical laminated panel is considered (Fig. 5) after Tsai et al. (1991).

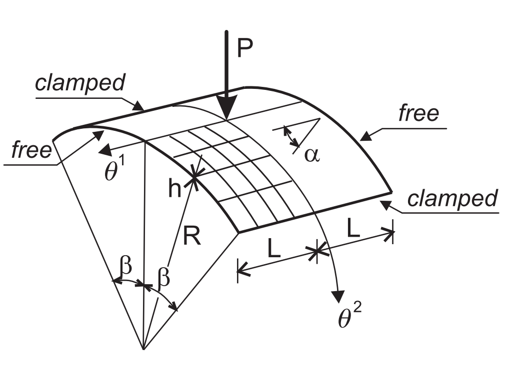

Figure 5. Clamped cylindrical panel under point load

The following parameters are assumed: R = 12 in, L = 5.5 in, β = 0.5, E_{11} = 20.46·10^6 psi, ν_{12} = 0.313, E_{22} = 4.092·10^6 psi, G_{12} = G_{13} = 2.53704·10^6 psi, G_{23} = 1.26852·10^6 psi. The laminate is composed of four layers (0/90/90/0), each of them being 0.01 in thick.

As one can see in Table 1, the buckling load levels predicted with the LRT56, LRT5, MRT5 and RVK5 are very close each other and in a quite good agreement with the solution given by Tsai et al. (1991).

Looking at the graphs of the central deflection of the panel versus the central load as presented in Figure 6, one can notice that distinction between obtained results is evident only in the post-buckling response. The RVK5 solution is surprisingly close to the results of the most advanced non-linear model in this study – the LRT56. None of the present results agrees with the reference solution of Tsai et al. (1991). After some further examinations (for details we refer to Kreja & Schmidt 2005) one can find that the formulation applied by Tsai et al. (1991) is rather closer to the LRT5 model (a lack of a proper updating of rotations) but differs by excluding all non-linear terms in the transverse shear strains. Although the difference among the results of LRT56, LRT5, MRT5 and RVK5 is noticeable for the considered clamped cylindrical panel, nevertheless the lack of a clear distinction between curves of LRT56 and RVK5 could be a problem for the test example.

Table 1. Buckling load for clamped cylindrical panel

Model	Snap-through load
LRT56	25.83 lb
LRT5	26.28 lb
MRT5	26.67 lb
RVK5	25.19 lb
Tsai et al. (1991)	~26 lb

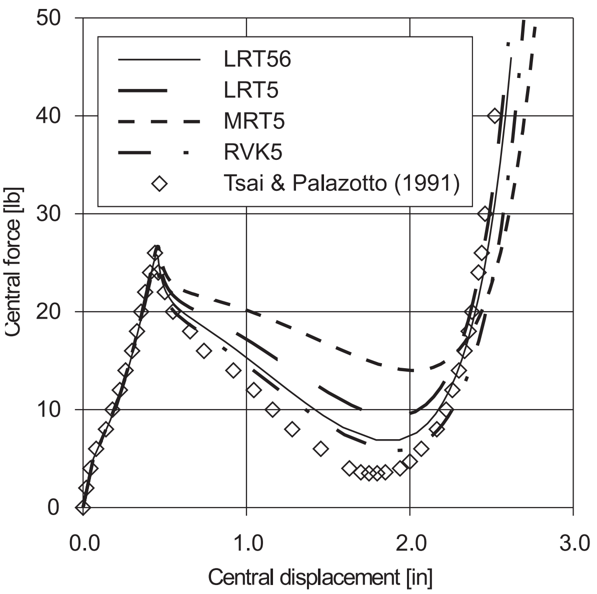

Figure 6. Central deflection for clamped panel.

2.3 *Stretching of open laminated cylinder*

In the next example, stretching of a short laminated cylinder is considered, as shown in Figure 7, with R = 4.953 in, L =5.175 in, h = 0.094 in, and β = 0.5. The open cylinder loaded by two opposite stretching forces in its middle section exemplifies a very popular benchmark test for the nonlinear analysis of isotropic shells (see Sze et al. 2004). Its composite variant was proposed by Masud et al. (2000) who considered the laminated (0/90) shell with the following orthotropic material data: E_{11} = 30500 ksi, E_{22} = 10500 ksi, G_{12} = G_{13} = G_{23} = 4000 ksi and ν_{12} = 0.3125,

The graphs of the outer deflection of the loaded points are presented in Figure 8. It is quite symptomatic that the snap-through behavior appears only in the LRT56 curve. Paradoxically, the MRT5 solution seems to give a better estimation of the panel response than the LRT5 formulation.

The qualitative and quantitative distinction of the LRT56 solution from the results of the other models (LRT5, MRT5 and RVK5) makes this example suitable for the application as a test problem.

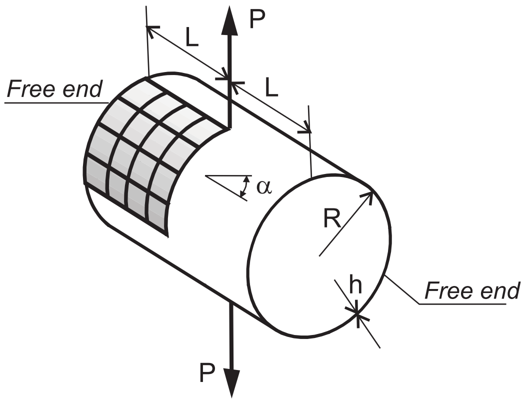

Figure 7. Stretching of open laminated cylinder

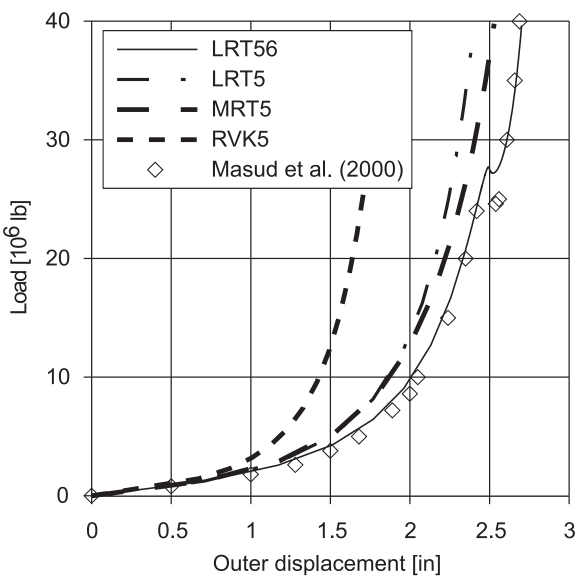

Figure 8. Outer deflection for short cylinder

2.4 *Simply supported plate strip*

In the last example a simply supported asymmetric laminated (0/90) panel as shown in Figure 9 was analyzed assuming a = 9.0 in, b = 1.5 in, h = 0.04 in, with $E_1 = 2.0 \times 10^7$ lb/in^2, $E_2 = 1.4 \times 10^6$ lb/in^2, $\nu_{12} = 0.30$, and $G_{12} = G_{23} = G_{13} = 0.7 \times 10^6$ lb/in^2. The graphs in Figure 7 show that the LRT56 results agree very well with the reference solution of Başar et al. (1993) who solved the problem using a fully non-linear formulation accounting for finite rotations. On the other hand, the LRT56 solution is evidently separated from the curves obtained for the models LRT5, MRT5 and RVK5. Therefore, this example can also be considered as a proper benchmark problem.

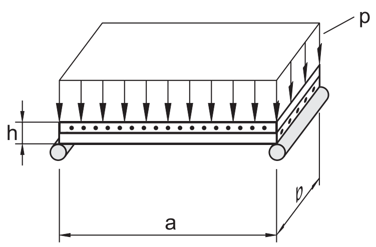

Figure 9. Simply supported plate strip

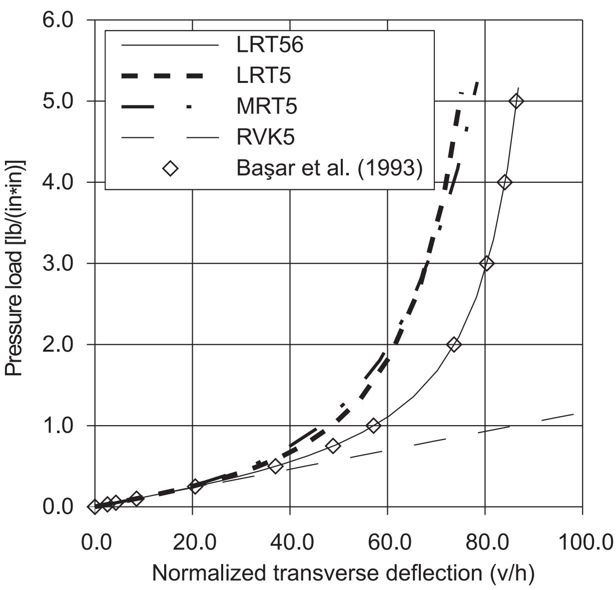

Figure 10. Central deflection of plate strip

3 CONCLUSIONS

The issue of a correct choice of benchmark problems for testing the large rotation formulations of laminated shells was discussed in the paper. Four test examples selected from the literature were critically examined. This investigation indicated that only examples no. 3 and no. 4 could be accepted as proper benchmark tests for the large-rotation analysis of laminated shells.

REFERENCES

Balah, M. & Al-Ghamedy, H. N. 2002. Finite element formulation of a third order laminated finite rotation shell element. *Computers & Structures* 80: 1975-1990.

Başar, Y. Ding, Y. & Schultz, R. 1993. Refined shear-deformation models for composite laminates with finite rotations. *International Journal of Solids & Structures* 30: 2611-2638.

Başar, Y., Itskov, M. & Eckstein, A. 2000. Composite laminates: nonlinear interlaminar stress analysis by multi-layer shell elements. *Computer Methods in Applied Mechanics & Engineering* 185: 367-397.

Brank, B., Peric, D. & Damjanic, F. B. 1995. On implementation of a nonlinear four node shell finite element for thin multilayered elastic shells. *Computational Mechanics* 16: 341-359.

Ferro, V., Kreja, I. & Weichert, D. 1998. Geometrically nonlinear analysis of laminated shells. In J. Chróścielewski & W. Pietraszkiewicz (eds), *Shell Structures: Theory and Applications, Proc. the 6th Conf. SSTA 98, Gdańsk - Jurata, 11-13 October 1998*: 113-114. Gdańsk: TUG Press.

Kim, K. & Voyiadjis, G. Y. 1999. Non-linear finite element analysis of composite panels. *Composites: Part B* 30: 365-381.

Kreja, I., Schmidt, R. & Reddy, J. N. 1997. Finite elements based on the first-order shear deformation moderate rotation shell theory with applications to the analysis of composite structures. *International Journal of Non-Linear Mechanics* 32: 1123-1142.

Kreja, I. & Schmidt, R. 2002. On the FEM implementation of the large rotation shell theory for elastic anisotropic shells. In P. Kłosowski & W. Pietraszkiewicz (eds), *Shell Structures: Theory and Applications, Proc. the 7th Conf. SSTA, Gdańsk - Jurata, 9-11 October 2002*: 139-140. Gdańsk: GUT Press.

Kreja, I., & Schmidt, R. 2005. Large rotations in first-order shear deformation FE analysis of laminated shells. *International Journal of Non-Linear Mechanics* (in print).

Kulikov, G. M. & Plotnikova, S. V. 2003. Non-linear strain-displacement equations exactly representing large rigid-body motions. Part I Timoshenko-Mindlin shell theory. *Computer Methods in Applied Mechanics & Engineering* 192: 851-875.

Librescu, L. 1987. Refined geometrically nonlinear theories of anisotropic laminated shells. *Quarterly of Applied Mathematics* XLV: 1-22.

Masud, A., Tham, C. L. & Liu, W. K. 2000. A stabilized 3-D co-rotational formulation for geometrically nonlinear analysis of multi-layered shells. *Computational Mechanics* 26: 1-12.

Reddy J. N. 1982. Bending of laminated anisotropic shells by a shear deformable Finite Element. *Fibre Science & Technology* 17: 9-24.

Saigal, S., Kapania, R. K. & Yang, T. Y. 1986. Geometrically nonlinear finite element analysis of imperfect laminated shells. *Journal of Composite Materials* 20: 197-214.

Schmidt, R. & Reddy, J. N. 1988. A refined small strain and moderate rotation theory of elastic anisotropic shells. *Journal of Applied Mechanics, Transactions of ASME* 55: 611-617.

Sze, K. Y., Liu, X. H. & Lo, S. H. 2004. Popular benchmark problems for geometric nonlinear analysis of shells. *Finite Elements in Analysis and Design* 40: 1551-1569.

Tsai, C. T., Palazotto, A. N. & Dennis, S. T. 1991. Large-rotation snap-through buckling in laminated cylindrical panels. *Finite Elements in Analysis and Design* 9: 65-75.