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Stability analysis of cylindrical composite shells in MSC/Nastran

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In the paper, the capabilities of the *MSC/NASTRAN* system in the field of stability analysis of composite laminated shells are critically tested. Two selected benchmark examples of laminated cylindrical panels under axial compression are examined. The *MSC/NASTRAN* results obtained either in buckling analysis or in nonlinear incremental calculations are compared with the solutions available in the literature.

Keywords: *composite shells, buckling, FE analysis*

1. Motivation

In the last few years, structural engineers who use commercial computer systems for FE structural analysis have considerably increased in number. Among the main advantages one can recognize the following features of a typical *big commercial FEA system*

- a convenient access to the computational module through a graphical interface of pre- and postprocessors,
- wide range of linear and nonlinear analyses offered,
- a rich library of elements available in the system.

On the other hand, every user trying to perform any non-standard calculations meets also disadvantages of a big system, to mention here just a relatively complex manual and very limited information on a theoretical background. The latter together with a lack of any access to the source code makes the user see the system as a proverbial *black box*, where all one knows is the input and the output but few really know what is happening inside.

The author shares belief that *NASTRAN* can be treated as a very typical member of the family of *big commercial FEA systems.* It happened that the author had to perform a nonlinear analysis of laminated composite shells with the *MSC/NASTRAN* system after a rather short experience with that program. According to promotional materials

1. The *MSC/NASTRAN* system is a powerful tool in the range of linear and nonlinear analyses of structures.
2. The shell element QUAD4 available in system is suitable to model laminated shells.

Trying to verify those promises the author has applied the *MSC/NASTRAN* system to recalculate several well-known benchmark examples of large deformation analysis for composite laminated shells. However, the scope of the present paper is limited to the stability analysis of laminated cylindrical panel under axial compression as shown in Figure 1. It is assumed that the curved edge *BC* is fixed, whereas the boundary conditions at the curved edge *AD* allow only a rigid translation of the whole rim along the generatrix. The boundary conditions at the straight edges *AB* and *CD* vary, depending on the example considered.

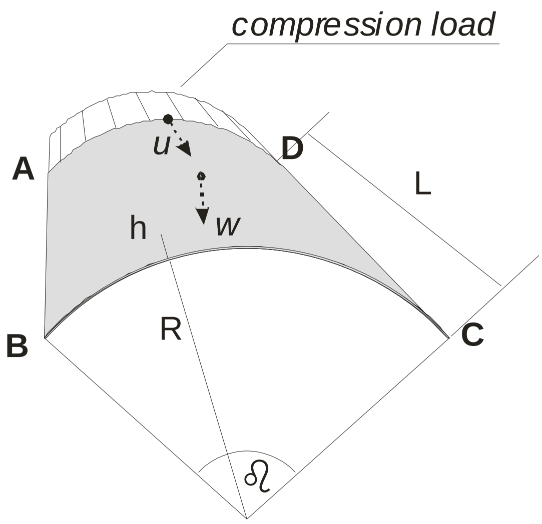

Fig. 1. Composite cylindrical panel under axial compression

One can easily notice an obvious similarity between the problem considered and the stability analysis of the isotropic cylindrical panel under axial compression being the classical illustration of the buckling problem with non-symmetric bifurcation point [1]. The basic difference herein lies in the different boundary conditions and the layered structure of the panel.

2. Computational model

2.1. Literature review

Numerical analysis of laminated plates and shells has been presented in a large number of research papers. Due to a limited space of the present report, it is impossible to list here a comprehensive bibliography of the subject. Let us focus mainly on review papers, each carrying a long list of references. At the beginning of the nineties Noor et al. published a series of articles [2, 3] presenting the state-of-the-art with regard to computational models for laminated shells. Piskunov and Rasskazov [4] surveyed 180 papers to trace an evolution of theoretical models for laminated plates and shells. The list of references of the cross-sectional article by Qatu [5] contains as many as 374 positions. An extensive bibliography of the topic can be found also in the recent papers of Carrera [6, 7]. As the Finite Element Method is the predominant tool in the computational analysis of laminated shells, one may not omit here papers devoted to the review of shell finite elements [8–10].

It should be emphasized that the topic of stability analysis of cylindrical composite shells is just a fraction of the subject matter of the aforementioned papers. A review of the literature dedicated strictly to the buckling analysis of cylindrical laminated shells can be found, for example, in [11].

2.2. Basic equations in stability analysis

The first approximation to a critical load and a corresponding buckling mode can be obtained in a *linearized buckling analysis* [12]. A standard eigenvalue problem to be solved in such a case can be described by the following equation

$$[\mathbf{K}^{(\mathrm{con})} + \lambda \mathbf{K}^{(\sigma)}]\mathbf{v} = \mathbf{0}\,, \tag{1}$$

where $\mathbf{K}^{(\mathrm{con})}$ is the constitutive stiffness matrix, $\mathbf{K}^{(\sigma)}$ stands for the stress (geometrical) stiffness matrix, λ is the critical load multiplier, and $\mathbf{v}$ symbolizes the eigenvector representing the buckling mode of the structure. One should notice that Equation (1) has been obtained with strong linearization assumptions and therefore should not be applied to examine problems with severe nonlinearities.

The second option is to trace the whole equilibrium path of the structure by means of the *nonlinear incremental analysis* [12]. Depending on the algorithm applied, such a strategy enables one to find singular points of different kind – for example adopting the arc-length technique based on the application of extended system of equations one can detect limit points as well as bifurcation points [12]. A governing equation of an incremental approach in its standard form can be written as

$$\begin{aligned} &[\mathbf{K}_T({}^1\mathbf{q})]\Delta\mathbf{q} = \mathbf{R}({}^1\mathbf{q})\,, \\ &{}^2\mathbf{q} = {}^1\mathbf{q} + \Delta\mathbf{q}\,, \end{aligned} \tag{2}$$

where ${}^1\mathbf{q}$ and ${}^2\mathbf{q}$ symbolize the global vector for displacements at the actual and at the searched configuration, respectively, and $\Delta\mathbf{q}$ represents the increment of displacements. The tangential stiffness matrix $\mathbf{K}_T({}^1\mathbf{q})$ and the vector of residual forces $\mathbf{R}({}^1\mathbf{q})$ depend on the actual state of deformation.

2.3. Laminated shells analysis with Nastran

The history of Nastran is almost 40 years long, the first version of the program was designed in the course of a NASA-sponsored project which still founds its reflection in the name of the program (NASTRAN = **NA**SA **STR**uctural **AN**alysis Program). The program is available in several different releases offered simultaneously by various vendors. The most popular version of the program, the *MSC/NASTRAN for Windows* [13, 14] distributed by the MacNeal-Schwendler Corporation, has been selected for use in the present research.

According to promotional materials the MSC/NASTRAN system is *a general purpose, computer-aided engineering tool based on Finite Element Method (FEM)*. Among different finite elements available in the system one can find a 4-node shell element QUAD4 that is applicable in a structural analysis of laminated composite shells. It is quite understandable that details of the FE procedures applied are trade secrets of the MSC. One can guess that the current shell element QUAD4 originates from the shell element proposed by MacNeal in 1978 [15]. QUAD4 appeared to be one of the most effective low-order FE elements in the analysis of isotropic shells of the Mindlin–Reissner type [9, 16]. An extension of the QUAD4 element formulation to the geometrical non-linear analysis is based on the corotational concept [17]. The layered structure of the shell is considered according to the *Classical Lamination Theory*, i.e. it is assumed that the laminas are perfectly bonded together (no slip is allowed between laminas) and each lamina is in a plane stress state. According to the *First Order Shear Deformation Theory* a linear variation of deformations through the laminated thickness is postulated; however, an appropriate shear correction factor is applied to fix the error of constant transverse shear strains in contrast to the more realistic parabolic distribution. Stability analysis in *MSC/Nastran for Windows* is possible either as a *linear buckling* (see Equation (1)) or as a *nonlinear incremental analysis* (Equation (2)). The latter can be performed with application of *arc-length technique* which allows tracing quite complicated equilibrium paths; however, the details of the procedures offered are not accessible which forces users to adopt the choice of default parameters.

3. Numerical examples

3.1. Cylindrical panel No. 1 – simply supported straight edges

In the first numerical example, an axial compression of a 16-layer composite cylindrical panel is considered assuming that the straight edges *AB* and *CD* are simply supported with the possibility of moving along the generatrix. The lamination scheme can be described as $[45/\text{-}45_2/45/0_4]_S$. Each lamina is made of carbon-epoxy composite XAS-914C with the following parameters: $E_a = 130{\cdot}10^6$ kPa, $E_b = 10{\cdot}10^6$ kPa, $G_{ab} = G_{ac} = G_{bc} = 5{\cdot}10^6$ kPa and $\nu_{ab} = 0.3$. The geometry of the panel is characterized by the height $h = 16\times0.125 = 2$ mm, the radius $R = 250$ mm, the length $L = 540$ mm and the opening angle $\beta = 1.6848$ rad.

The origin of this, one of the most popular buckling problems of laminated shells, is referred to the experimental and numerical study by Snell and Morley [18] which was, however, not available to the author of the present report. Jun and Hong [19] performed a nonlinear buckling analysis using 8-node degenerated shell elements within Updated Lagrangian formulation. Laschet and Jeusette [20] presented results of linear and nonlinear buckling analyses obtained with solid-shell multilayered 16-node finite elements (3 translational DOFs per node). Wagner [21] calculated the linear buckling load of the panel employing different meshes of 4-node shell elements with reduced integration and hourglass control. Brank and Carrera [22] applied 4-node mixed ANS shell elements based on the refined FSDT with finite rotations.

It is quite symptomatic that the descriptions of the analyzed panel given by the authors of the five papers cited above are not quite consistent. There are some differences in the interpretation of boundary conditions on the straight edges which are described as “simply supported” – for instance Jun and Hong [19] and Wagner [21] constrained only radial and circumferential translations at all nodes lying on the straight edges. However, due to the isoparametric formulation of the finite elements applied this approach does not fix the rotations about the normals to the edge. One can expect that the deformation of the panel obtained in this model largely depends on the number of nodes assumed along the straight edges. The details of the boundary conditions applied by Laschet and Jeusette [20] are not clear – just from the figure given in their paper one can expect that they applied an additional row of shell elements on both sides of the panel. Brank and Carrera [22] admitted that they themselves met some problems with the description of boundary conditions.

The MSC/Nastran has been used to compute the critical load for the examined cylindrical panel applying a linear buckling analysis as well as a non-linear incremental analysis. The calculations have been performed using uniform meshes of 20×20, 40×40 and 80×80 QUAD4 elements. The results obtained with the MSC/Nastran are in a good agreement with reference solutions [18–21] as is shown in Table 1. The only exception is the solution of Brank and Carrera [22] which noticeably differs from all the others. The difference with respect to the experimental results is contained within the range of just several per cents. One can observe that an increase in the mesh density results in a decrease in the buckling load estimated.

As the values of the critical load estimated in the linear buckling analysis are very close to those obtained from the non-linear incremental analysis, one can conclude that the pre-buckling deformations do not differ too much from the linear solution. In these circumstances, one could expect a better agreement between the buckling mode corresponding to the first eigenvector (Figure 2) and the deformation form determined in the non-linear analysis (Figure 3).

Table 1. Buckling load for cylindrical panel with simply supported straight edges

Model	Mesh	Critical load [kN]	
		Linear buckling	Incremental analysis
8-node elements Jun & Hong [19]	8×10	–	143.2
16-node elements Laschet & Jeusette [20]	8×10 12×18	143.9 140.3	137.8 –
4-node elements Wagner [21]	4×12 4×16 4×20 4×40 4×80	145.6 142.2 140.8 140.0 139.6	– – – – –
4-node elements Brank & Carrera [22]	32×32	–	150
QUAD4 MSC/Nastran	**20×20** **40×40** **80×80**	**144.56** **141.56** **140.34**	**144.35** **142.34** **140.38**
[18]	Experiment	134	

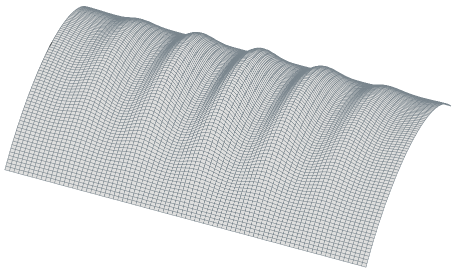

Fig. 2. The first buckling mode, $P_{\mathrm{crit}} = 140.34$ kN

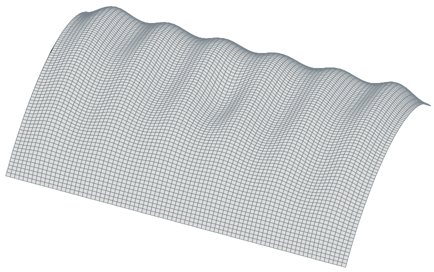

Fig. 3. Deformation at $P_{\max t} = 140.38$ kN

3.2. Cylindrical panel No. 2 – free straight edges

A 16-layer composite cylindrical panel analysed in the second example is very similar to that considered above. The main difference lays in the boundary conditions at the straight edges *AB* and *CD*, which now remain free of any support. A buckling of such a panel made of graphite-epoxy composite AS4/3501-6 had been examined by Chaplin and Palazotto in [23]. The material parameters taken after [23] are: $E_a = 135.8 \cdot 10^6$ kPa, $E_b = 10.9 \cdot 10^6$ kPa, $G_{ab} = G_{ac} = 6.4 \cdot 10^6$ kPa, $G_{bc} = 3.2 \cdot 10^6$ kPa and $\nu_{ab} = 0.276$. A geometry of the panel is described by the following data: $h = 16 \times 0.127 = 2.032$ mm, $R = 304.8$ mm, $L = 508$ mm and $\beta = 1$ rad. The assumed layer stacking sequence is $[0/45/{-45}/90]_{2S}$.

Two different meshes of finite elements have been used in the computations:

- model A – 24×40 QUAD4 elements,
- model B – 48×80 QUAD4 elements.

The equilibrium paths in the geometrically non-linear analysis traced with the arc-length control technique are presented in Figure 4.

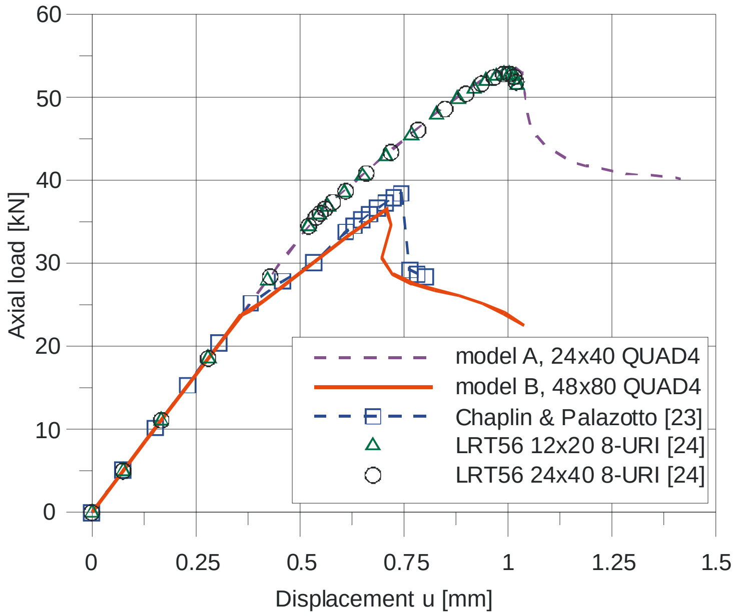

Fig. 4. Equilibrium paths for cylindrical panel No. 2

It can be observed in Figure 4 that the graph obtained for model B is very close to that given in [23]. At the first glimpse, the difference between the graphs for models A and B seems to result from the variation of the mesh density in those two models. To verify those findings additional computations have been performed with the own author's program for analysis of laminated shells SHL04 [24]. The results obtained with SHL04 for two discretizations: 12×20 and 24×40 8-node elements are almost identical with those of model A. Since on one hand the formulation incorporated in program SHL04 provides a very exact description of the geometry, and on the other hand all calculations in SHL04 are performed with a double precision, one can suppose that the different answer for model B can result from the jump between the fundamental and the post-bifurcation paths. To verify this deduction a linear buckling problem has been solved in MSC/NASTRAN for the cylindrical shell under consideration. The five lowest eigenvalues calculated for models A and B are gathered in Table 2.

Numbers presented in Table 2 show that in a case of a linear buckling analysis there are very little differences between the results for the models A and B. It is also quite characteristic that, on the contrary to the previous case of the panel No. 1, the lowest eigenvalue computed for the panel No. 2 in the linear buckling analysis (24.4 kN) is significantly smaller than the critical load estimated in the incremental analysis (52.8 kN for model A and 36.5 kN for the model B). Looking again at the curves in Figure 4, one can observe that the distinction between the paths obtained for models A and B starts at the load level near the lowest eigenvalue determined in the linear buckling analysis (24.4 kN). This observation seems to support the opinion that the graph for model B does not represent the (fundamental) equilibrium path for an ideal structure.

Table 2. Linear buckling solution for panel No. 2

N	Eigenvalues of the buckling load [kN]	
	Model A	Model B
1	24.4326	24.3988
2	27.5549	27.4879
3	28.4660	28.4560
4	29.3149	29.3056
5	36.9095	36.8081

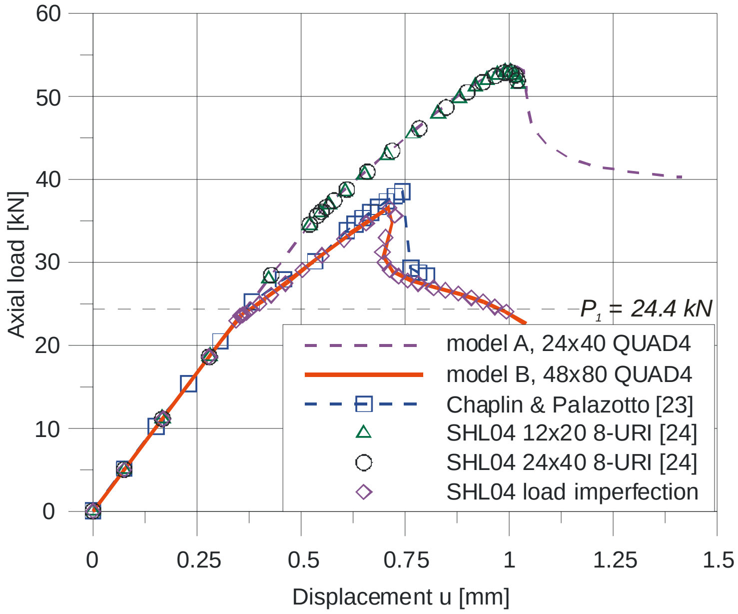

Fig. 5. Influence of imperfection in analysis of cylindrical panel No. 2

To decisively verify this suggestion additional computations have been performed with the program SHL04, where, additionally, to the axial load a very small load imperfection has been introduced taken as a transverse force acting in the middle of the panel and equal to 0.0001 fraction of the axial load. The curve representing the imperfection case (see Figure 5) almost accurately matches the curve corresponding to model B, which, in author's opinion, entirely confirms the hypothesis that numerical round-off errors appearing in the large MSC/Nastran model B (inaccurate mapping of geometry and single precision computations) acted as a kind of imperfection which can direct a solution into the post-bifurcation path. However, on the other hand, it is important to remark that in a case that is as strongly sensitive to imperfections as the panel analysed, the results obtained for the ideal structure on no account should be used to determine the load capacity.

3. Conclusions

Capabilities of the *MSC/NASTRAN system for Windows* in the field of stability analysis of composite structures were critically tested. Two selected examples of 16-layer composite cylindrical panels under axial compression were examined with varied boundary conditions. The results obtained with *MSC/NASTRAN for Windows* were compared with the solutions available in the literature. The comparative study presented in the paper confirmed in full the power of the system to perform an advanced stability analysis of composite shells either as buckling analysis or as nonlinear incremental calculations. Additionally, it was observed that boundary conditions along the longitudinal edges significantly affect the bifurcation sensitivity of the panel. It was also shown that some numerical round-off errors can cause that the solution obtained in the *MSC/NASTRAN* jumps from a primary equilibrium path to a post-bifurcation branch.

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Analiza stateczności kompozytowych paneli cylindrycznych w MSC/NASTRAN

Zanalizowano przydatność systemu *MSC/NASTRAN for Windows* w zakresie analizy stateczności kompozytowych powłok cylindrycznych. Przedstawiono krótki przegląd literatury dotyczącej numerycznej analizy powłok warstwowych. Omówiono zasadnicze równania opi-

sujące problem stateczności konstrukcji w zakresie uogólnionego zagadnienia własnego stateczności początkowej oraz na drodze wyznaczenia pełnej ścieżki równowagi układu z zastosowaniem podejścia przyrostowego. Zaprezentowano podstawowe informacje o systemie *MSC/NASTRAN for Windows* ze szczególnym uwzględnieniem elementu powłokowego QUAD4. Obliczenia przeprowadzono dla dwóch wybranych przykładów paneli cylindrycznych poddanych równomiernemu ściskaniu w kierunku tworzącej, dokonując analizy zarówno stateczności początkowej, jak i problemu geometrycznie nieliniowego w procesie przyrostowym. Podstawowa różnica między analizowanymi przykładami polegała na przyjęciu innych warunków podparcia na prostych krawędziach: w przypadku pierwszego badanego panelu przyjęto swobodne podparcie prostych brzegów, podczas gdy w drugiej rozpatrywanej powłoce proste krawędzie były swobodne. Dla obu rozważanych wariantów przyjęto, że zakrzywione krawędzie są utwierdzone, z tym że jedna z nich ma możliwość sztywnej translacji na kierunku tworzącej. Otrzymane wyniki zestawiono z rozwiązaniami dostępnymi w literaturze oraz z rozwiązaniami uzyskanymi za pomocą własnego programu SHL04. Przeprowadzone badania porównawcze w pełni potwierdziły bogate możliwości systemu *MSC/NASTRAN for Windows*. Zaobserwowano ponadto, że zmiana warunków podparcia na wzdłużnych krawędziach paneli ma decydujący wpływ na zmianę jej podatności na imperfekcje. Jak wykazano w drugim przykładzie, numeryczne niedokładności modelu MSC/Nastran w przypadku konstrukcji wrażliwej na imperfekcje mogą prowadzić do przeskoku rozwiązania na ścieżkę pobifurkacyjną.