# A semi-Markov model of fuel combustion process in a diesel engine

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ABSTRACT

*A four-state model of the combustion process in working spaces (cylinders) of a diesel engine is presented in the form of a semi-Markov process, discrete in states and continuous in time. The values in this process are the states : $s_1$, $s_2$, $s_3$, $s_4$ corresponding to commonly accepted types of combustion in those engines, namely : $s_1$ – the process state which corresponds to total (complete and perfect) combustion, $s_2$ – the process state which corresponds to incomplete combustion, $s_3$ – the process state which corresponds to imperfect combustion, and $s_4$ – the process state which corresponds to both incomplete and imperfect combustion. It is also mentioned that proper use of diesel engines can secure a correct course of the combustion process, if it is properly attended and shaped. During this course the state $s_4$ does not take place. Formulas are given which make it possible to determine the probability of staying of the combustion process in the above named states, along with the interpretation of the probability $P_1 = P(s_1)$ as the probability of correct (reliable) engine operation.*

**Keywords** :

## INTRODUCTION

The combustion process in particular working spaces (cylinders) of each diesel engine significantly affects its reliability and durability. Factors which can most significantly affect the course of the combustion process include [6, 8, 10, 11] :

- ✰ physical and chemical properties of the fuel (diesel oil, heavy oil), in particular its chemical composition, cetane number, ignition temperature, and viscosity
- ✰ design properties of the engine (in particular concerning the injection systems) such as, combustion chamber type, piston material, nozzle type, and main engine dimensions
- ✰ energetic characteristics of the engine (dynamics, inertia, accumulative properties of the functional systems)
- ✰ usage, both unconditioned and conditioned by engine operation, mainly resulting from the load (especially thermal) and technical state of the engine, and represented by the following parameters : rotational speed, injection pressure, injection advance angle, volume of exhaust gases remaining from the previous working cycle, and cooling water temperature.

Both the physical and chemical properties of the fuel, and the constructional characteristics of the engine are structured in such a way that in the initial time of operation (in conditions to which this engine was adapted in the design and manufacturing phase) a correct course of combustion process can be obtained in its working spaces (cylinders).

However, the above listed factors are subject to unfavourable changes during engine operation, which frequently result in incorrect course of the combustion process. The correct course of the combustion process is only possible when in each working cycle total (i.e. complete and perfect) combustion takes place. The course of the combustion process is incorrect when instead of the total combustion, defective (i.e. incomplete and imperfect) or only incomplete or imperfect combustion takes place, and also when the combustion takes place in incorrect time with respect to both the beginning and end of its presence.

Both the incorrect and correct course of combustion in engine cylinders can be initiated at an arbitrary time instant and last certain time durations of engine work (operation). These time durations should be treated as random variables, which results from random properties of the above listed factors affecting the course of the combustion process [1, 2, 3, 7, 12].

The combustion process can be evaluated in the dynamic time $\vartheta$ (time during which the combustion takes place in the cylinders), i.e. as the process $[\{X^*(\vartheta) : \vartheta \in \Theta\}$, and in the quasi-static time t (time of engine operation during which certain task is done by the engine user) i.e. as the process $\{Y^*(t) : t \in T\}$. The evaluation of the course of the combustion process taking place in the engine cylinders (i.e. in the dynamic time) is obtained via relevant tests oriented on assessing values of the combustion parameters [8, 10, 11, 12].

From the obtained values of those parameters the course of the combustion process can also be evaluated for a quasi-static state, such as, for instance, engine operation between two preventive services, or the operation before the first engine failure, etc. However, evaluating the combustion process during long-lasting engine operation requires additional probabilistic measures, which can only be determined after working out a relevant model of combustion process in working spaces (cylinders) of the diesel engine.

## COMBUSTION PROCESS CHARACTERISTICS IN PROBABILISTIC APPROACH

As already mentioned, the process of fuel combustion in particular working spaces (cylinders) of each diesel engine can be analysed either in the dynamic state $\vartheta \in \Theta$ (short time during which this process is executed in one operation cycle of the examined engine) or in the quasi-static time $t \in T$ (longer time, for long-lasting operation (work) of the examined engine).

That means that the combustion process in the cylinders of each engine can be considered a two-dimensional process :

$$\{W^*(\vartheta, t) : \vartheta \in \Theta, t \in T] = [\{X^*(\vartheta) : \vartheta \in \Theta\} \quad \{Y^*(t) : t \in T\}] \;;\; \Theta, T \subset R_+ \tag{1}$$

the components of which are mutually dependent processes $\{X^*(\vartheta) : \vartheta \in \Theta\}$ and $\{Y^*(t) : t \in T\}$, whose parameters : $\vartheta \in \Theta$ and $t \in T$ , respectively, are not random variables. The random variables are the values of those processes attributed to parameters $\vartheta$, composing the set $\Theta$ and to parameters t, composing the set T . The elements of those two sets belong to a set of real non-negative numbers $R_+$.

The combustion process in the dynamic time ($\vartheta$) exerts direct influence on engine durability and reliability, among other parameters. The course of this process is evaluated using various systems which diagnose, in the first place, the quality of transformation of the chemical energy collected in the fuel into the thermal energy. Various indices (parameters) of engine operation are analysed, in particular those which significantly affect not only the energetic qualities of the engine, but also its reliability and durability. These parameters include : average ($\Delta p/\Delta\alpha$) and instantaneous ($dp/d\alpha$) rate of pressure built-up (p) in cylinders as a function of crankshaft rotation angle ($\alpha$), the highest combustion pressures ($p_{max}$) and the highest combustion temperature ($T_{max}$) in the cylinders, heat emission coefficient ($w_c$), heat consumption coefficient ($w_w$), heat evolution coefficient ($w_{wc}$), heat evolution rate (w), the coefficient being the quotient of the maximum heat evolution rate ($w_{max}$) to the ignition delay time ($\tau_{zz}$), pressure gain factor ($\varphi$), preliminary decompression ratio ($\rho$), the exhaust gas composition at the smoke limit, as well as the contents of such chemical compounds as : carbon monoxide (CO), carbon dioxide ($CO_2$) hydrocarbons ($C_nH_m$), nitric oxides ($NO_x$), solid particles, sulphur compounds ($SiO_2$, $SiO_3$, $H_2SO_3$, $H_2SO_4$), aldehydes, etc. in the exhaust gases. The course of the process $\{X^*(\vartheta) : \vartheta \in \Theta\}$ depends on the conditions of engine operation, for instance on its mechanical and thermal load, crankshaft rotational speed, quality of adjustment of the engine injection system and timing gear, and technical state of the engine. All this suggests that the course of the combustion process during engine operation needs proper attendance and prognosing. What is necessary is analysing the execution of the fuel combustion in the engine cylinders during the entire time of engine operation (working time), in other words, analysing the process $\{Y^*(t) : t \in T\}$.

In this process such adjacent states can be named, from which a four-element set of state classes, simply named the states, can be created, namely :

$$S = \{s_1, s_2, s_3, s_4\} \tag{2}$$

with the following (commonly accepted) interpretation of these states :

- $s_1$ − the process state which corresponds to complete and perfect combustion
- $s_2$ − the process state which corresponds to incomplete combustion
- $s_3$ − the process state which corresponds to imperfect combustion
- $s_4$ − the process state which corresponds to both incomplete and imperfect combustion.

These states take place (appear) randomly during engine operation (work), at an arbitrary operating time. Therefore the events consisting in the appearance of these states can be considered random events. If so, the probability should be evaluated for these states to appear during the operation of the diesel engine of concern. Determining those probabilities will make it possible to perform quantitative assessment of the course of the combustion process in the cylinders of this type of engines.

The course of the combustion process taking place in engine cylinders in time $t \in T$ can be quantitatively assessed based on the mathematical model $\{Y(t) : t \in T\}$ of the real combustion process $\{Y^*(t) : t \in T\}$, the values of which are the states $s_i \in S$ (i = 1, 2, 3, 4). The analysis of chances for the appearance of those states indicates that the above model $Y(t) : t \in T\}$ can be worked out in the form of a semi-Markov process, continuous in time and having a limited (four-element) set of states.

## MODEL OF COMBUSTION PROCESS IN ENGINE CYLINDERS

It results from the theory of semi-Markov processes that working out a model of an arbitrary real process in the form of a semi-Markov process is possible if its initial distribution and functional matrix can be defined [5, 8].

Taking into account the specifics of the combustion process in the working spaces (cylinders) of the diesel engines we can assume that the combustion process $\{Y(t) : t \in T\}$ with the state set $S = \{s_1, s_2, s_3, s_4\}$ has the following initial distribution [2, 4] :

$$P_i = P\{Y(0) = s_i\} = \begin{cases} 1 \text{ for } i = 1 \\ 0 \text{ for } i = 2, 3, 4 \end{cases} \tag{3}$$

and functional matrix :

$$Q(t) = \begin{bmatrix} 0 & Q_{12}(t) & Q_{13}(t) & 0 \\ Q_{21}(t) & 0 & 0 & Q_{24}(t) \\ Q_{31}(t) & 0 & 0 & Q_{34}(t) \\ Q_{41}(t) & 0 & 0 & 0 \end{bmatrix} \tag{4}$$

where :

$$Q_{ij}(t) = P\{Y(\tau_{n+1}) = s_j, \tau_{n+1} - \tau_n < t \,|\, Y(\tau_n) = s_i\}$$

$$s_i, s_j \in S \;;\; i, j = 1,\ 2,\ 3,\ 4 \;;\; i \neq j$$

The above initial distribution (3) bases on the fact that after manufacturing the combustion engine should be fully efficient [2], and its combustion process $\{Y^*(t) : t \in T\}$ (taking place in particular cylinders) should take such a course that the state $s_1$ can be recognised as the value of the process $\{Y(t) : t \in T\}$ being the model of the already mentioned process $\{Y^*(t) : t \in T\}$. The appearance of the remaining states $s_i \in S$ (i = 2, 3, 4) results from worsening engine abilities, in particular incorrect action of injection apparatuses during engine operation.

The change state graph for the process $\{Y(t) : t \in T\}$ as resulting from the functional matrix (4) is given in Fig. 1. This graph, and, as a consequence, the functional matrix (4), result from the rational use of the engine. This use takes place when after detecting the state $s_2$, or $s_3$, by the engine user, relevant technical action is taken to regain full engine abilities, i.e. to return to the state $s_1$. In case the diagnosing system detects the state $s_3$, the engine user should remove causes of its appearance and bring again the engine to the state $s_1$, with total (complete and perfect) combustion in the cylinders. In practice, this action is not always possible, as a consequence of which the state $s_4$ frequently appears. Then the only sensible action of the engine user is that leading to the technical service which will return the process to the state $s_1$.

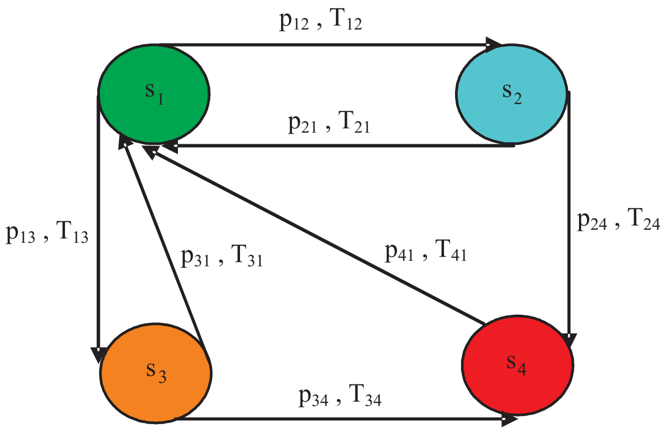


***Fig. 1.*** *State change graph for process $\{Y(t) : t \in T\}$ :*
*$s_1$ – state corresponding to complete and perfect combustion; $s_2$ – state corresponding to incomplete combustion; $s_3$ – state corresponding to imperfect combustion; $s_4$ – state corresponding to incomplete and imperfect combustion; $p_{ij}$ – probability of process change from state $s_i$ to state $s_j$; $T_{ij}$ – time duration of state $s_i$ provided that the next state is $s_j$; ($i, j = 1, 2, 3, 4; i \neq j$).*

An important characteristic of the process $\{Y(t) : t \in T\}$, applicable for evaluating the course of the combustion process $Y^*(t) : t \in T\}$ in diesel engine cylinders, is a set of conditional probabilities of passing to particular states during the time not longer than t, namely :

$$P_{ij}(t) = P\{Y(t) = s_j \mid Y(0) = s_i\} \qquad (i, j = 1, 2, 3, 4 ; i \neq j) \tag{5}$$

bearing the name of probabilities of passing from the state $s_i \in S$ at time $t = 0$, to the state $s_j \in S$ at time $t \neq 0$.

The probabilities $P_{ij}(t)$, formula (5), can be calculated using the Kronecker symbol $\delta_{ij}$ and making use of the equation system [4] :

$$P_{ij}(t) = \delta_{ij}[1 - G_i(t)] + \sum_{k=1}^{4} \int_0^t P_{kj}(t-\zeta) dQ_{ik}(\zeta) \tag{6}$$

which is operationally solved using the Laplace-Stieltjes transformation. In formula (6) : $G_i(t)$ is the distribution function of a random variable $T_j$ representing the time duration of the state $s_i$ ($i = 1, 2, 3, 4$) of the process $\{Y(t) : t \in T\}$, independently of the state to which the process is going to pass. When analysing in longer time intervals t (theoretically $t \to \infty$), these probabilities tend to constant values, and after this time form the limiting distribution [2, 8, 12] :

$$P_1 = \lim_{t\to\infty} P\{Y(t) = s_1\},\ P_2 = \lim_{t\to\infty} P\{Y(t) = s_3\}$$
$$P_3 = \lim_{t\to\infty} P\{Y(t) = s_3\},\ P_4 = \lim_{t\to\infty} P\{Y(t) = s_4\} \tag{7}$$

Detailed solutions, see : formula 6, applicable in operating practice can be obtained provided that the distributions $Q_{ij}(t)$ and $G_i(t)$ are known. These quantities can be determined from the following relations [2, 8, 12] :

$$Q_{ij}(t) = p_{ij} F_{ij}(t) \ ; \ G_i(t) = \sum_{j=1}^{4} Q_{ij}(t) \tag{8}$$

where :

$p_{ij}$ – probability of passing of the Markov chain placed in the process $\{Y(t) : t \in T\}$

$F_{ij}(t)$ – distribution function of a random variable $T_{ij}$, representing the time duration of the state $s_i$ provided that the next state is $s_j$ in the process $\{Y(t) : t \in T\}$, ($i, j = 1, 2, 3, 4; i \neq j$).

Formula (8) reveals that the subsequent characteristics of the process $\{Y(t) : t \in T\}$are quantities $Q_{ij}(t)$ and $G_i(t)$ representing random variables $T_{ij}$ and $T_i$, respectively.

Of highest applicability for evaluating correctness of the combustion process course in the diesel engine cylinders over a relatively long time is the limiting distribution. The theory of semi-Markov processes says that this distribution, with the interpretation given by the formula :

$$P_j = \lim_{t\to\infty} P\{y(t) = s_j\} \ ; \ s_j \in S, j = \overline{1,4} \tag{9}$$

can be determined from the following relation [2, 8, 12] :

$$P_j = \frac{\pi_j E(T_j)}{\sum_{k=1}^{4} \pi_k E(T_k)} \ ; \ j = 1, 2, 3, 4 \tag{10}$$

where the limiting distribution $\pi_j$, $j = 1, 2, 3, 4$ of the placed Markov chain $\{Y(\pi_n : n = 0, 1, 2, ...\}$ fulfils the equation system [4, 9, 12] :

$$[\pi_1, \pi_2, \pi_3, \pi_4]\ [redacted]\ = [\pi_1, \pi_2, \pi_3, \pi_4] \tag{11}$$
$$\pi_1 + \pi_2 + \pi_3 + \pi_4 = 1$$

From equations (10) and (11) the following form of the limiting distribution for the process $\{Y(t) : t \in T\}$can be obtained :

$$P_1 = \frac{E(T_1)}{M} \ ; \ P_2 = \frac{p_{12}E(T_2)}{M}$$
$$P_3 = \frac{p_{13}E(T_3)}{M} \ ; \ P_4 = \frac{p_{12}p_{24}E(T_4)}{M} \tag{12}$$

taking into account that :

$$M = E(T_1) + p_{12}E(T_2) + p_{13}E(T_3) + p_{12}p_{24}E(T_4)$$

where :

$P_1, P_2, P_3, P_4$ – probabilities that the process $\{Y(t) : t \in T\}$ is in one of the states : $s_1, s_2, s_3, s_4$, respectively

$E(T_1), E(T_2), E(T_3), E(T_4)$ – expected values of time durations of the states : $s_1, s_2, s_3, s_4$, respectively

$p_{ij}$ – probabilities of process passing from the state $s_i$ to the state $s_j$ : $i, j = 1, 2, 3, 4; i \neq j$.

The probability $P_1$ can be considered an indicator of correct fuel combustion in engine cylinders and, consequently, correct engine operation (work), while the probabilities $P_j$ ($j = 2, 3, 4$) can be considered indicators of incorrect fuel combustion in the cylinders and incorrect engine operation.

Rational action of the engine user should be oriented on avoiding the state $s_4$ (then $P_4 = 0$) and should lead to the situation in which time durations of the states $s_2$ and $s_3$ of the fuel combustion processes were as short as possible. This requires proper control of the combustion process during engine operation, for instance by making use of concepts presented in [2, 8, 10, 11]. Then a combustion process model $\{Z(t) : t \in T\}$ can be obtained, the state change graph of which is shown in Fig. 2.

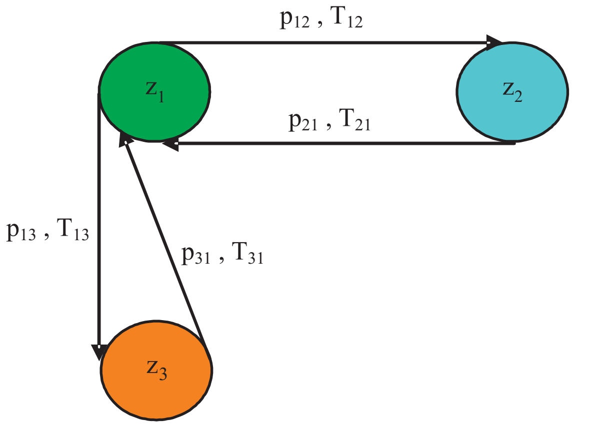


***Fig. 2.*** *State change graph for process $\{Z(t) : t \in T\}$ : $z_1$ – state corresponding to complete and perfect combustion; $z_2$ – state corresponding to incomplete combustion; $z_3$ – state corresponding to imperfect combustion; $p_{ij}$ – probability of process change from state $z_i$ to state $z_j$, $T_{ij}$ – time duration of state $z_i$ provided that the next state is $z_j$; $(i, j = 1, 2, 3 ; i \neq j)$.*

Like for the process $\{Y(t) : t \in T\}$ we can assume that the values of the combustion process $\{ZY(t) : t \in T\}$ are the elements of the state set $Z = \{z_1, z_2, z_3\}$ having the following interpretation :

$z_1$ – the state corresponding to complete and perfect combustion

$z_2$ – the state corresponding to incomplete combustion

$z_3$ – the state corresponding to imperfect combustion.

These states have the same interpretation as the states $s_1$, $s_2$, $s_3$ in the process $\{Y(t) : t \in T\}$. The above mentioned process $\{ZY(t) : t \in T\}$ has the following initial distribution [2] :

$Q(t) =$ [redacted] (13)

and functional matrix :

$$P_i = P\{Z(0) = s_i\} = \begin{cases} 1 \text{ for } i = 1 \\ 0 \text{ for } i = 2,\ 3 \end{cases} \quad (14)$$

where :

$$Q_{ij}(t) = P\{Z(\tau_{n+1}) = s_j, \tau_{n+1} - \tau_n < t | Z(\tau_n) = s_i\}$$

$$s_i, s_j \in S\ ;\ i, j = 1,\ 2,\ 3\ ;\ i \neq j$$

Like for the process $\{Y(t) : t \in T\}$, when discussing the model of the real combustion process in the form of the process $\{Z(t) : t \in T\}$, we can define its limiting distribution as :

$$P_1 = \frac{E(T_1)}{N}\ ;\ P_2 = \frac{p_{12}E(T_2)}{N}\ ;\ P_3 = \frac{p_{13}E(T_3)}{N} \quad (15)$$

taking into account that :

$$N = E(T_1) + p_{12}E(T_2) + p_{13}E(T_3)$$

where :

$P_1$, $P_2$, $P_3$, $P_4$ – probabilities that the process $\{Z(t) : t \in T\}$ is in one of the states : $z_1$, $z_2$, $z_3$, respectively

$E(T_1)$, $E(T_2)$, $E(T_3)$ – expected values of time durations of the states : $s_1$, $s_2$, $s_3$, respectively

$p_{ij}$ – probabilities of process passing from the state $z_i$ to the state $z_j$ : $i, j = 1, 2, 3; i \neq j$.

The probability $P_1$ can be considered an indicator of correct fuel combustion in engine cylinders, while probabilities $P_j$ ($j = 2, 3$) are to be considered indicators of incorrect fuel combustion in the cylinders and, consequently, incorrect engine operation (work). Rational action of the engine user should lead to the situation in which time durations of the states $z_2$ and $z_3$ of the fuel combustion process were as short as possible.

## FINAL REMARKS AND CONCLUSIONS

- The presented probabilistic models of the combustion process in diesel engine cylinders can be used for evaluating adaptation of certain types of engines for complete and perfect fuel combustion during a relatively long time of their operation (work), and for assessing correctness of use of these engines in certain operating conditions [3, 5, 6, 11]. It can be assumed in general that lower probabilities $P_1$ indicate more incorrect use of the engines, and opposite, when the remaining probabilities are low, the engine is better used.
- The probabilities defined by relations (12) and (15) can be considered indices of diesel engine reliability. Determining values for these probabilities requires relevant operating examination to assess the number of state changes for the presented models of fuel combustion processes in engine cylinders. The entire procedure consists in determining numbers $n_{ij}$, for a sufficiently long time interval $[0, t]$ and obtaining the realisation $t_{ij}^{m}$ , $m = 1, 2, ..., n_{ij}$ of random variables $T_{ij}$. All this will make it possible to calculate the distribution functions $F_{ij}(t)$, probabilities $p_{ij}$ and expected values $E(T_j)$, and, finally, probabilities of staying of the combustion processes in particular states $s_i \in S$ ($i = 1, 2, 3, 4$) or $z_i \in Z$ ($i = 1, 2, 3$).

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