Archives of Hydro-Engineering and Environmental Mechanics
Vol. 53 (2006), No. 1, pp. 7–30

Comprehensive Circumscribing of Non-Linearity Cases of a Water Supply System with Smooth Flow Control

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(Received February 12, 2004; revised December 20, 2005)

Abstract

The descriptions found in the literature, of cases of even relatively strong non-linearity of the water supply systems, as well as their mathematical treatment, are rare and incomplete. They refer mainly to the non-linearity with respect to smooth flow control components (for instance, to the operation of pressure reducing valves (PRV), flow reducing valves (FRV) and pumps with motors equipped with smooth rotational speed control – variable frequency drives (VFD)). The article proposes a formally correct method of a comprehensive solution of the above non-linearity cases based on successive simulations with some dynamic analysis of the sensitivity of the system to the operation of the smooth control components mentioned. An appropriate algorithm controlling the effectiveness of the calculations secures correct description of the behaviour of the system for an arbitrary, in practice, method of control of flows, even if all the proposed smooth control methods are used simultaneously. The effects of the use of the solutions presented in mathematical modelling are illustrated by selected results of simulation carried out for an existing water supply system.

Key words: flow reducing valve, non-linearity of the system, pressure reducing valve, steady flows, variable frequency drives, water supply system

Notations:

$a_n, a_{n-1}, ..., a_1, a_0$	–	coefficients of a polynomial approximating pump characteristic,
A	–	section area corresponding to the diameter of the reducer [m^2],
g	–	gravity acceleration [m/s^2],
h_r	–	frictional head loss on the reducer [m],
H	–	ordinate of pressure head, ordinate of total pressure head [m],
H_c	–	assumed pressure head supervising smooth control of the pump‘s operation [m],

H_s	–	corresponding simulated total pressure head [m],
H_u	–	effective pump delivery head [m],
k	–	dynamically determined coefficient taking into account sensitivity of the system to changes of ϖ or ξ,
Q	–	flow rate, pump delivery [m^3/s],
Q_c	–	required, current discharge of the tank, determined from the continuity equation modelling the execution of the assumed trajectory of water level changes in the tank [m^3/s],
Q_s	–	corresponding simulated discharge of the tank [m^3/s],
ΔH	–	control pressure head simulation error [m],
ΔQ	–	tank discharge simulation error [m^3/s],
$\Delta\xi$	–	correction of resistance coefficient of the flow through the reducer,
$\Delta\omega$	–	correction of pump rotational speed,
ε_1 and ε_2	–	assumed control accuracies [m] and [m^3/s],
ξ	–	resistance coefficient of the flow through the reducer,
$\xi_{\min}, \xi_{\max}$	–	resistance coefficients of the flow through the fully opened and fully closed reducer, respectively,
ϖ	–	pump rotational speed,
$\varpi_{\min}, \varpi_{\max}$	–	assumed limiting values of pump rotational speed.

Superscripts:

n	–	exponent, degree of a polynomial,
p	–	number of simulation.

1. Introduction

1.1. Preliminary Description of the Problem

Cases of non-linearity observed in the water supply systems can be divided into three groups: (1) system non-linearity with respect to the parameters (mainly the resistance of pipes depending on Reynolds number), (2) system non-linearity with respect to random disturbances, having the form of water consumption, and (3) system non-linearity with respect to flow control in the system, mainly with respect to the operation of pressure reducers, flow reducers, and various cases of smooth control of the pump rotational speed. Treatment of cases of non-linearity group (3) is still troublesome for engineers dealing with the problem. Neglecting this type of non-linearity or its too-simplified approach will result in generation of large errors in calculations, or making use of the computer model, inconvenient and requiring additional artificial actions, frequently incorrect from the formal point of view, to be taken by the model user. When studying the manuals of available computer programmes, one can note whether the calculation algorithms, generally kept a secret, are based on formally correct solutions only after checking required data assortment and the preparation method. The above conclusion is often negative. A frequently used incorrect simplification is creating a model which requires determining, among other parameters, the following input data: 1. instantaneous values of rotational speed of smoothly controlled pumps, and 2. instantaneous resistance coefficients of the flow through the pressure and flow reducers. Those quantities, however, are unknown a priori and should not be given as input data. What can well constitute the input data is the parameters which supervise smooth control of the above mentioned devices, i.e. pressures, flow rates, and/or water levels in the tanks, assumed at selected points of the system to be kept by the control system.

The problem of developing a formally correct and efficient mathematical and numerical model of the above cases of non-linearity in water supply systems with respect to smooth flow control is of increasing significance, as this type of control is becoming more and more frequently used in engineering practice, due to various advantages of technical, economical, and reliability nature, widely discussed in, among other publications, ACEE (1991), Lingireddy and Wood (1998), Orłowski (1999, 1999a, 2003), Ula et al (1991). The advantages mainly include: a smaller volume of electric power consumed by the pumps, very efficient flow and pressure control in the water supply system both in average and extreme conditions, minimisation of water losses resulting from system leakages, optimisation of operation of surge tanks, and, finally, reduction of the effects of water hammers.

Descriptions of formally correct models of some cases of non-linearity group (3) can be found in: Jeppson and Davis (1976), Lingireddy et al (1992), Lingireddy and Wood (1995, 1998), Wood (1993).

The above models reveal, however, numerous drawbacks.

1. The proposed solutions require artificial changes in the calculation schemes (graphs describing water supply systems). Tanks are introduced, along with additional, artificial continuity equations. This results in certain difficulties, most of all those connected with modelling the operation of PRV and FRV reducers. The model does not allow smooth switch in calculations between different regimes of operation of those devices, as completely different calculation schemes are used in particular regimes. In case of PRV, for instance, in the basic regime this valve is substituted by a tank in the calculation scheme, while in the regime where it acts as a non-return valve the above tank is replaced in the scheme by two terminal nodes. In the regime when

the valve is fully open, it is represented in the network scheme by a continuous conduit, or an open valve of small resistance, located in the same place.

2. The models are incompatible, in various publications different methods are presented for different smooth control components, which makes it impossible to create a general model fully applicable for any controlled arbitrarily water supply system.
3. They are incomplete and do not cover, altogether, all smooth flow control components, or provide the opportunity for arbitrary localisation of points at which the parameters supervising smooth control are measured, nor for flexibility of the way in which they are assumed (they are assumed constant, and not variable, defined according to selected principles). It is particularly troublesome in cases of smooth control of operation of a pumping station in order to generate a trajectory of water level changes in a surge tank co-operating with the network. The states of operation of the reducers mentioned in 1. above are modelled imprecisely. In particular, PRV-type valves are assumed always to open when the pressure in front of the valve is lower than its setting (pressure assumed behind the valve), without checking whether the conditions exist to force its closure.

The fundamentals of more general approach to the non-linearity problems of a water supply system are given by Orłowski (1997, 2002).

1.2. Tasks to be Solved by the Author of the Article

The first task is formulating the principles of a general, formally correct, deterministic simulation of flows in a water supply system simultaneously equipped with arbitrarily available PRV, FRV and VFD components of smooth control (with VFD controlling network pressures or tank water level changes).

The calculation methods proposed and described by the author are *the universal methods of solving flow group (3) non-linearity cases, based on successive corrections made to instantaneous characteristics of smooth control components (i.e. tools)* of flow in water pipelines (current rotational speeds of smoothly controlled pumps and resistance coefficients of the reducers are corrected until the required values of smooth control parameters are obtained with an assumed accuracy). These methods can be used in practically all known models, such as for instance, Newton-Raphson, and Cross-Lobaczew models, or the model based on the linearisation of the energy conservation equations, which solve the steady-state equations for the water supply system. Descriptions of these methods, presented from the point of view of creating effective computer programmes, can be found in, among other sources, Epp and Fowler (1970), Walski (1985), and Orłowski (1997).

The proposed simulation methods taking into account the discussed cases of system non-linearity, reveal a number of significant advantages, the most important of which are briefly discussed below.

1. Their formula includes the most general case, i.e. the task having the form of an analysis of a given system. In such a task, system graph arcs, on which the conservation equations to be solved are modelled, includes not only pipes, but also the remaining components, such as particular tanks (wells, and surge and pressure tanks), pumps, gate valves, reducers, filters, water meters, etc.
2. The generality of the formula also consists in providing opportunities for taking into account:

   (a) simultaneous presence of arbitrary water supply system smooth control components, modelled by a uniform system of equations,
   (b) arbitrary localisation, on the water supply network, of points at which the parameters supervising the operation of the smooth control components are measured. Those points can be localised directly "behind" the reducers and pumping stations, or at any distance along the network (the only limitation here is: proper technical execution of such a control, and the necessary presence of the direct hydraulic action of smooth control components on the control parameters generated by them),
   (c) arbitrary method of determining values of control quantities (see item 3 below).

3. The values of controlling quantities can, in practice, be defined constant or varying in time. Therefore special attention was paid in the article to the mathematical context of performing the simulation in the case when the smooth control of pumping station operation is expected to generate certain changes in water level in the network tank (the object of interest is the method of determination of a proper instantaneous discharge of the tank, generated by the pumping station). Let us remark that instantaneous values of pressures and flow rates, generated by relevant reducers, do not need such additional calculations.
4. An additional advantage of high importance of the proposed methods is that the smooth control components are not substituted in the calculation schemes by tanks. In consequence, real rotational speeds and resistance coefficients are calculated for the pumps and reducers during the simulation. This not only results in high clarity and simplicity of the calculation algorithms, but also provides opportunities for proper control of simulated operation of motors driving the pumps (the object of interest here is the

efficiency of motors with changeable rotational speed) and authorities of the reducers (having fundamental impact on their durability). To avoid misunderstandings let us note that the so-called "external authority" of the reduction valve mounted on the water supply network is defined as the quotient of the pressure loss on this valve and the difference between pressure ordinates recorded at the following nodes: the supply node of the water supply network zone at the valve inlet, and the node situated directly behind the valve (the latter pressure corresponding to the valve setting pressure).

The second task of the article is paying special attention in the discussed mathematical and numerical methods to the following aspects:

(a) taking into account the sensitivity of the water supply system to the smooth flow control in the designs of those controls (basic condition) and in the algorithms of the numerical calculations (to secure their proper convergence characteristics),

(b) analysing correctness of the flow control project which takes into account the operation of particular smooth control components (reducers, pumps) in various regimes (ranges), depending on current hydraulic and technical conditions in the simulated operating situations (time instants). The described computer modelling allows, among other advantages, easy and smooth switch between the operating regimes of all smoothly controlled components. The switch may be executed during the calculations (both in successive iterations and in successive simulations, i.e. simulations of different operating situations in the same water supply system) with no need for making any changes in the calculation scheme (graph) of the system, or in the form of the equations applied. This ability is secured by proper calculation control principles, which may also control the correctness of execution of particular flow control projects in the system, as the real operation of the system is simulated under the assumption that not all settings of the smooth control components which were initially adopted in the simulation can be physically executed.

Finally, *the third task* of the article is presenting the effects of formally correct flow simulations, and the sensitivity of the system to the described control, to illustrate the significance of the discussed problem of non-linearity. For this purpose the author presents and discusses selected results of personally carried out simulations and corresponding measurements carried out for a real medium-sized water supply system.

1.3. Principles and Regimes of Operation of Automatically and Smoothly Controlled Pumps and Reducers

Pumps driven by motors with smoothly controlled rotational speed may operate in one of three basic states/regimes: (a) the basic regime, i.e. when the pump works at changing rotational speed, kept within the range $\varpi \in (\varpi_{\min}\,;\,\varpi_{\max})$ permissible from the technical and economical point of view, (b) the pump works at constant, minimum permissible rotational speed, when the generation of a proper value of the smooth control parameter requires reducing the rotational speed of the pump below $\varpi_{\min}$, and (c) the pump works at constant rotational speed equal to $\varpi_{\max}$, when the generation of a proper value of the smooth control parameter requires increasing the rotational speed of the pump beyond the maximum limit.

Formal description of the operating states (regimes) of all *automatically and smoothly controlled reducers* is slightly more complicated. This description will be presented here using the pressure reduction valve – PRV as an example. Such a reducer is given an assumed ("set", "controlling") pressure at a node indicated by the operator. The article deals with the general case, in which the control pressure can be assumed in a node situated at a distance from the reducer. For this case we must properly recognise hydraulic relations in the system to make sure that the action of the valve on the pressure at that node is allowed by a relatively simple programme of the controller supervising the operation of this valve. Generally, however, the reduction valves are direct action automation components and the control pressure is assumed at a node situated directly "behind" the reducer. In those cases the design of the reducer enables it to act as the non-return valve precluding the flow in the opposite direction. According to the above general principles of operation of the pressure reducer, its operating regimes are the following: (a) basic regime, when the reducer is partially closed, i.e. the resistance coefficient during the flow through the reducer $\xi \in (\xi_{\min} ; \xi_{\max})$ corresponds to the situation when the control pressure can be obtained, (b) the reducer is fully open, i.e. the resistance coefficient $\xi = \xi_{\min}$ corresponds to the situation when the pressure at the measuring node drops below the assumed value of the (set) control pressure and the generation of the control pressure is not possible (as the pressure "in front of" the valve is too low), (c) the reducer is fully closed, i.e. the resistance coefficient $\xi = \xi_{\max}$. The limiting value $\xi_{\max}$, assumed for the purpose of computer programming, corresponds to the flow through the practically closed reducer. Let us remark that the reducer closes when the pressure at the node "behind" the reducer is higher than of that the set pressure, or than that "in front of" the reducer, playing in this case the role of a non-return valve. Thus, the operating regime with the fully closed reducer is only possible for a water supply system simultaneously meeting the two following conditions: 1. The reducer is of a direct action type in the system, 2. Independent pressure increase is possible in the zone "behind" the reducer, i.e. the pressure cannot be decreased to an arbitrary level by closing the reducer. Such a situation can take place when the zone supplied through this reducer is in direct hydraulic co-operation with the zone "in front of" the reducer (by-passes on the network), or with the tanks or water system parts including pumping stations, and is not entirely created by a structure of pipes separated by closed gate valves from the rest of the system.

1.4. Discussed Cases of System Non-Linearity vs. EPS

From the point of view of calculations discussed in the article it is unimportant whether the flow is described: (a) in a set of arbitrary individual (historical, hypothetical, typical, average, extreme) operating situations in the system, or (b) an extended period simulation (EPS) is carried out, consisting in successive simulations of a sequence of so-called "instantaneously steady states" (which should not be confused with unsteady states, i.e. water hammers). In these two cases (a) and (b) we have the same conservation equations based on the assumption of the presence of steady states. Only particular technical situations simulated in the examined system must be defined in a sufficiently precise way. The difference between simulating flow in a set of individual, separated situations and EPS consists in the methods of defining the situations, which may slightly differ, leaving the instantaneous flow simulations themselves identical in both cases.

2. Methods Based on Correcting Instantaneous Characteristics of Pumps and Reducers

Independent of the model applied to finding the solution to the system of conservation equations modelling the water supply system (Subsection 1.2) the calculations are performed in such a manner that the flows which are present in the modelled time instant (i.e. the operating situation in the system) are simulated repeatedly many – in practice several – times, with correcting the characteristics before each simulation until the required values of the smooth control parameters are obtained with satisfactory accuracy.

2.1. Correcting Characteristics of Smoothly Controlled Pumps

In the first simulation we assume an initial hydraulic characteristic of the smoothly controlled pump corresponding to the initial rotational speed $\varpi^{(1)}$. This characteristic is usually taken from a pump catalogue and $\varpi^{(1)}$ is the rotational speed defined in the catalogue for this characteristic. Before each successive simulation, the pump characteristic is corrected by changing ϖ according to a proper algorithm. This operation is carried out repeatedly until the variable (pressure in the selected node, or water level in the selected tank) corresponding to the smooth control parameter of the pump nears its required value with an assumed accuracy.

Therefore the dependence of the pump characteristic on its rotational speed should be determined to describe speed correction algorithms.

The pump characteristic is approximated by a polynomial of the 3rd to 5th order:

$$H_u = a_n \cdot Q^n + a_{n-1} \cdot Q^{n-1} + ... + a_1 \cdot Q + a_0 \tag{1}$$

(H_u – effective pump delivery head, Q – pump delivery).

The initial characteristic of the pump is assumed as a series of points (Q_i, H_{ui}), $i = 1, \ldots 3$ or 5, introduced by the programme operator. Each point corresponds to some different efficiency of the pump. The ordinates of those points change in successive simulations according to the relations:

$$\frac{Q_i^{(p+1)}}{Q_i^{(p)}} = \frac{\varpi^{(p+1)}}{\varpi^{(p)}} \quad \text{and} \quad \frac{H_{u_i}^{(p+1)}}{H_{u_i}^{(p)}} = \left(\frac{\varpi^{(p+1)}}{\varpi^{(p)}}\right)^2, \tag{2}$$

where p denotes the number of simulation. New ordinates of the characteristic points correspond to new coefficients of polynomial (1), determined in each successive simulation.

Two methods of correcting the pump rotational speed turned out to be most effective.

2.1.1. Direct Method Taking into Account Sensitivity of the System

Before each successive iteration, the rotational speed of the pump is corrected taking into account the error of the simulated smooth control parameter. To improve the convergence of the calculations, the formulas determining corrections $\Delta\varpi$ have been complemented by additional correcting coefficients k, which take into account dynamically determined sensitivity of the system to changes introduced to the pump characteristic. After correcting the rotational speed ϖ and determining the new characteristic of the pump, we start the next simulation as the continuation of the previous one, i.e. the result of the previous simulation is assumed to be the first approximation of the next simulation, which considerably speeds up the calculations.

The form of the formula used for determining the correction $\Delta\varpi^{(p+1)}$, depends on whether the control parameter is: (a) pressure assumed at a selected node, or (b) assumed ordinate of the water level in a selected tank (i.e. the tank connected to the selected node). In both cases the selected node, indicated by the operator, can be any node in which the control parameter generated by the smooth control of the pump is sensitive to this control.

In case (a) the correction of the pump rotational speed should generate the correction of the pump delivery head. The absolute value of the correction should not be less than the error of the simulated control pressure assumed by the system operator at the node selected by him, and the sign of the correction value should be opposite that represented by the error. Having applied Eq. 2 we arrive at:

$$\Delta\omega^{(p+1)} = \omega^{(p)}\left(1 - \sqrt{1 + k^{(p+1)} \cdot \frac{\Delta H^{(p)}}{H_u^{(p)}}}\right), \tag{3}$$

where: p – number of simulation, $\Delta H^{(p)} = H_s^{(p)} - H_c$ – control pressure head simulation error at the node (H_s – simulated total pressure head, H_c – assumed control pressure head), $H_u^{(p)}$ – simulated delivery head of the smoothly controlled pump, $k^{(p+1)}$ – dynamically determined increasing coefficient which takes into account sensitivity of the system to the control discussed; for the first time, when $p+1=2$ we assume $k^{(2)} \cong 1.1$, and in successive iterations it is determined from the formula:

$$k^{(p+1)} = \frac{H_u^{(p)} - H_u^{(p-1)}}{H_s^{(p)} - H_s^{(p-1)}}. \tag{4}$$

In case (b) the correction of the pump rotational speed aims at arriving at the situation in which the simulated discharge Q_s of the tank (in which the assumed water level is generated by changing the pump characteristic) is equal to its currently required discharge Q_c, determined from the continuity equation modelling the assumed (no matter how) trajectory of water level changes in the tank. As mentioned in the Introduction, determining this discharge is an independent action, to be taken in case of performing EPS simulation. Therefore it is not included in the procedure of modelling of steady flows taking place in any of the successive technical situations in the system discussed here. In special cases, when the constant water level in the tank is assumed, the discharge must equal zero. Using Eq. 2 a formula has been derived for the correction of the pump rotational speed. The formula takes into account the effect of this speed on the pump delivery, and, indirectly, on the tank discharge:

$$\Delta\omega^{(p+1)} = -k^{(p+1)} \cdot \omega^{(p)} \cdot \frac{\Delta Q^{(p)}}{Q^{(p)}}, \tag{5}$$

where: $\Delta Q^{(p)} = Q_s^{(p)} - Q_c$ – tank discharge simulation error, i.e. the difference between the simulated and required tank discharge (if a constant water level is assumed, then $\Delta Q^{(p)}$ is equal to its discharge simulated in the most recent p-th simulation), $Q^{(p)}$ – simulated delivery of the smoothly controlled pump, $k^{(p+1)}$ – increasing coefficient, taking into account sensitivity of the system to this control; for the first time it is assumed $k^{(2)} \cong 1.1$, and for the successive simulation it is dynamically determined from the formula:

$$k^{(p+1)} = \frac{Q^{(p)} - Q^{(p-1)}}{Q_s^{(p)} - Q_s^{(p-1)}}. \tag{6}$$

2.1.2. Method Making Use of Properly Defined Characteristics of the Water Supply System

The system characteristics are selected relations characterizing the system. Any real characteristic of the system exists for a real, simulated, well-defined operating situation, i.e. a determined water consumption in the entire system and for all currently assumed settings of the fittings and devices which control flows and pressures in the system (including current water levels in the tanks).

In case (a), i.e. when the parameter smoothly controlling the pump operation is the pressure at a selected node, we apply the system characteristics having the form of a dependence of the pressure recorded at this node on the rotational speed of the pump. The non-linearity of the system with respect to smooth control of the pump characteristics is solved numerically using a chord method. The remaining required points of the system characteristics, which correspond to successive (approaching the final solution) values of the rotational speed of the pump rotor, are evaluated in successive p simulations of flows taking place in the simulated situation in the system. Usually, several steps (simulations) return the value securing correct description of flows and pressures in the system. A controlled condition has the form:

$$\left|H_s^{(p)} - H_c\right| < \varepsilon_1, \tag{7}$$

where: $H_s^{(p)}$ – total pressure head simulated at the node, at which the sensor is installed, i.e. at which the pressure controlling the pump operation is assumed, H_c – control pressure head, p – number of simulation, ε_1 – assumed accuracy of control.

In case (b), when the smooth control of pump operation aims at securing the execution of the assumed trajectory of water level changes in the tank by generating proper tank discharge, the system characteristic is defined as the dependence of the tank discharge on the rotational speed of the pump. The calculations are performed here in a way similar to that reported in case (a). A controlled condition has the form:

$$\left|Q_s^{(p)} - Q_c\right| < \varepsilon_2, \tag{8}$$

where: $Q_s^{(p)}$ – simulated tank discharge in p-th simulation of flows in the examined technical situation in the water supply system, Q_c – tank discharge required for executing the assumed trajectory of water level changes in the tank, ε_2 – assumed accuracy. It is noteworthy that when constant water level is assumed in the tank, the condition ending the calculations takes the form:

$$\left|Q_s^{(p)}\right| < \varepsilon_2. \tag{9}$$

2.2. Correcting Characteristics of Automatic Pressure and Flow Reducers

Let us remember that the non-linearity of the water supply system with respect to the pressure and flow reducers results from the fact that during the flow simulation current resistance coefficients of those reducers are to be known to obtain the pressures and flows assumed at the system points indicated by the operator. Those coefficients are unknown a priori in the model.

The principles (regimes) of operation of the pressure reducers, defined in the Introduction, and similar principles concerning the flow reducers are reflected in the universal algorithms developed by the author, used for correcting, in successive iteration, the resistance coefficients ξ during the flow through those reducers. The algorithms make use of the results of comparison of the pressures, simulated and assumed at the control nodes (for direct action reducers, also pressures "in front of" and "behind" the reducers) with corresponding sensitivity of the system.

The author proposes two alternative methods for correcting the resistance coefficients ξ (and characteristics of the reducers in consequence). They allow an effective solution of the discussed non-linearity connected with the operation of the reducers in the water supply system.

2.2.1. Direct Method Taking into Account Sensitivity of the System

The formula for frictional head loss h_r on the reducer has the form:

$$h_r = \xi \cdot \frac{1}{2 \cdot g \cdot A^2} \cdot Q \cdot |Q|, \tag{10}$$

where: A – section area corresponding to the reducer diameter, g – gravity acceleration.

Optional methods of defining the first approximation of the resistance coefficient ξ, include, for instance, a constant value, assumed by the computer programme, or the value determined with the aid of formula (10) on the basis of an expected value of the reducer frictional head loss, suggested by the system operator. In the case of EPS simulation the first approximation of ξ can be taken from the previous time step.

Correction $\Delta\xi$ is equal to:

(a) for the pressure reducer

$$\Delta\xi^{(p+1)} = k^{(p+1)} \cdot \frac{2 \cdot g \cdot A^2}{Q^{(p)}\left|Q^{(p)}\right|} \cdot \Delta H^{(p)}; \tag{11}$$

in this formula $Q^{(p)}$ stands for flow rate through the reducer, while $\Delta H^{(p)}$ and $k^{(p+1)}$ have the same meaning as in Eq. 3. Another relation is used for calculating coefficient k which dynamically takes into account during the calculations the sensitivity of the system (and controlled pressure) to the operation of the applied reducer. We have:

$$k^{(p+1)} = \frac{h_r^{(p)} - h_r^{(p-1)}}{H_s^{(p-1)} - H_s^{(p)}}; \tag{12}$$

notations as in Eqs. 3, 4 and 10;

(b) for the flow reducer

$$\Delta\xi^{(p+1)} = -k^{(p+1)} \cdot 2 \cdot h_r^{(p)} \cdot g \cdot A^2 \cdot \left(\frac{1}{\left(Q_s^{(p)}\right)^2} - \frac{1}{Q_c^2} \right), \tag{13}$$

where: Q_c – flow rate, assumed by the system operator, at a point where the measuring device with the sensor is mounted, $Q_s^{(p)}$ – currently simulated flow rate at the same point, $k^{(p+1)}$ – coefficient taking into account sensitivity of the system to this regulation:

$$k^{(p+1)} = \frac{Q^{(p)} - Q^{(p-1)}}{Q_s^{(p)} - Q_s^{(p-1)}}; \tag{14}$$

notations in the formula the same as above.

2.2.2. Method Making Use of Properly Defined Characteristics of the Water Supply System

As mentioned in the case of pump characteristics, the system characteristics refer to a certain time instant characterized by a well-defined instantaneous water consumption at particular network nodes and complete, clearly defined operating situation in the system (current settings of the fittings and devices and water levels in the tanks). For the reducers, system characteristics defined to find a solution to the non-linearity problem are the following:

(a) when the non-linearity is caused by the operation of the pressure reducer, it is the dependence of the pressure at the controlled node on the resistance coefficient ξ of this reducer,

(b) when the non-linearity is caused by the operation of the flow reducer, it is the dependence of the flow recorded at a point controlled by the reducer on the resistance coefficient ξ of this reducer.

The searched value of the coefficient ξ corresponding to the sufficiently accurate value of the control parameter (a or b) is found using a successive approximation method: for successive ξ values the corresponding values of (a): controlled pressure or (b): controlled flow rate are determined from successive flow simulations repeated for the same examined situation in the system. High convergence was obtained here using the chord method, analogous to that described above for the smooth pump control; in a medium-size water supply system 4 to 5 simulations were enough to obtain two successive values of the control parameter differing by less than (a) 0.04 m and (b) 0.00001 m^3/s.

2.3. Remarks on Steady Flow Simulations Performed with the Aid of the Method Correcting Characteristics of the Pumps and Reducers

As already mentioned, a correct simulation of steady flows in the system takes into account all significant cases of non-linearity existing in the system. They include the non-linearity system of groups (1), (2) and (3) mentioned in the Introduction. A method of controlling the calculations is given in the diagram shown in Fig. 1. The non-linearity groups (1) and (2) are solved using iterative methods together with the system of non-linear equations of conservation which model the flows in the system, while the non-linearity group (3), is solved, using the methods presented in the article, by multiple repetition of the simulation of the same examined situation, with subsequent simultaneous correction of hydraulic characteristics of all elements composing the smooth control tool.

It is worth reminding here that the methods of correcting ϖ and ξ presented in previous Sections take into account the sensitivity of the system and the ability to use in a p-th simulation results obtained in a $p-1$-th simulation, or make use of properly defined and dynamically determined characteristics of the system. All this secures very good convergence of calculations due to fast reaction during the calculations, to current hydraulic conditions forcing the operation of the examined pumps and reducers (relatively high correction values provide the opportunity for the devices to obtain current states of their operation quickly).

The experience gained by the author says that the computer model acts optimally when in the initial few simulations (i.e. approximations) the corrections $\Delta\omega$ and $\Delta\xi$ are determined using the direct method, while in final simulations, when the convergence characteristics become monotonic for all approximated control parameters, ϖ and ξ values are corrected with the aid of the method making use of properly defined system characteristics.

Let us notice that not each parameter which we want to generate using smooth control method can be generated by this method. This refers to both the value of the parameter and the localization of a node at which we want to assume it. The above observations may lead to the following strategy of the approach to mathematical modelling of flows in a designed or existing system with smooth control:

1. Study thoroughly the hydraulic scheme of the water supply system to eliminate, at the beginning, control projects which for sure cannot be executed,

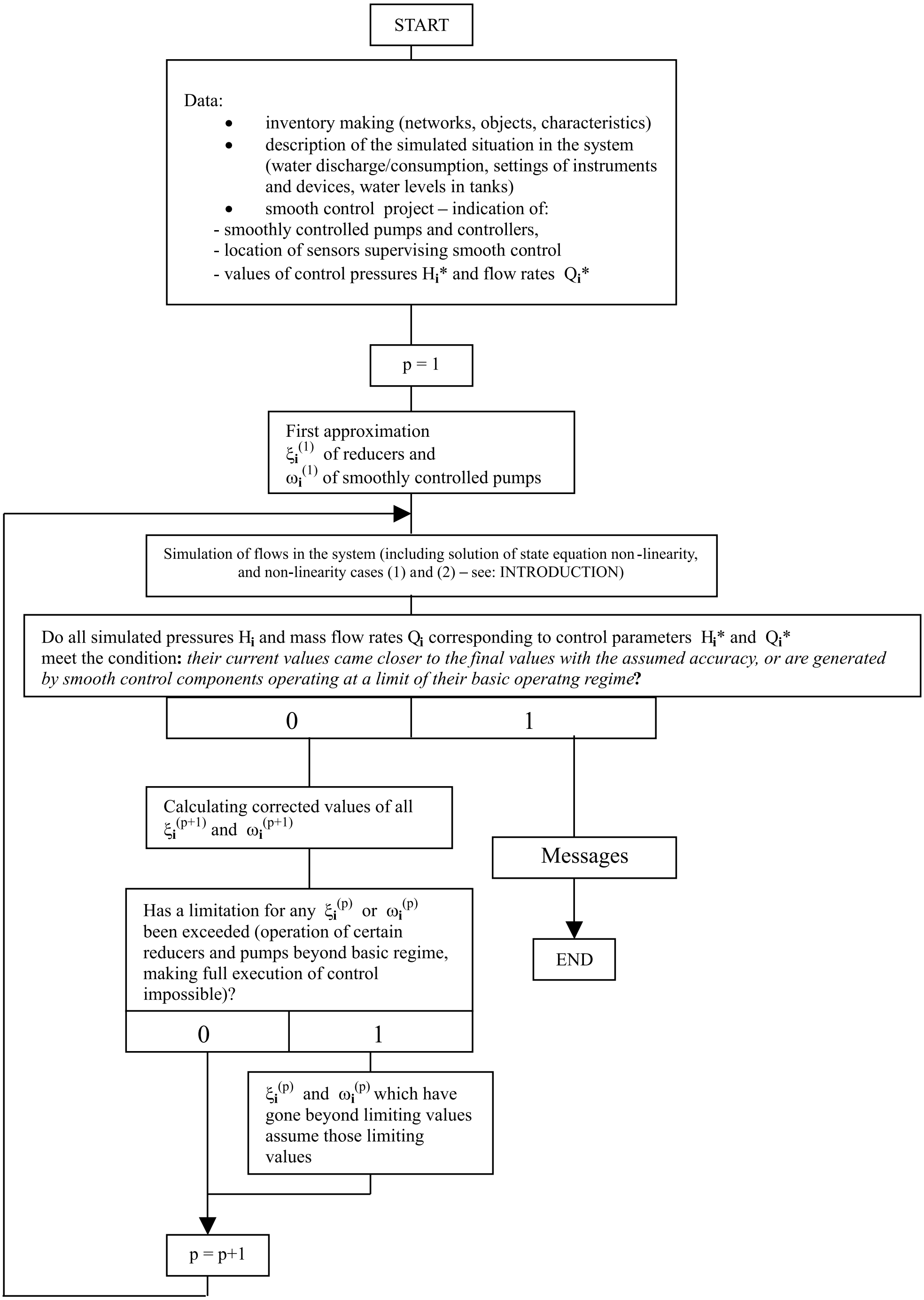

Fig. 1. General principle of performing steady flow simulation in the system (cases of non-linearity with respect to smooth control are solved using the method of multiple simulation with corrections made to the characteristics of pumps and reducers)

i.e. when the control parameter is not sensitive to changes of characteristics of the control element. This situation most often takes place when the duct from a smoothly controlled pump or reducer to the node at which we want to control the pressure or flow is linked by a single pipe (without other parallel pipes), to which a tank is connected. In this situation there is no direct hydraulic interaction between the node and the control element.

2. The analysis of the hydraulic scheme should also lead to the elimination of possible errors in the smooth control design, such as: (i) localization in series of elements controlling the same variable (the only pressure cascades are permissible in pressure zoning for instance), (ii) conflicting action of control components on the same controlled variable.
3. The remaining controls, along with possible erroneous controls, mentioned under Items 1 and 2 above, which have been overlooked by the designer (operator), can be analysed during the flow simulation in the system. If the sensitivity of the controlled variable to the designed control is too low, or the designed value of this variable is not within the obtainable range, it will always manifest itself by the controlled device going beyond the range of operating parameters, and the corresponding value of ϖ (for a pump) or ξ (for a reducer) is determined at the level of the limiting value which has been exceeded (Subsection 1.3).

Since, in some cases, there is a possibility of changing the sign of the correction $\Delta\varpi$ (or $\Delta\xi$) in successive approximations, its value is determined to the very end of the calculations, i.e. until the generation is completed of all smooth control parameters, which have been adopted in the system and can be generated. This correction, however, will only be taken into account, when its sign changes and the device returns to the basic regime of operation in the successive iteration (simulation). Such a situation can only take place in rare cases of complicated control systems. The final effect of the simulation is the presentation, how the system works (or would work) at the assumed settings of the smooth flow control components (including their location, assortment, and assumed values of control parameters) at the defined operating time instant/situation. In the examined system some controls can be executed, while others cannot (due to the lack or too low sensitivity of the control parameter on the operation of the control element). In those latter cases, some control elements usually work near the limit of their basis regime of operation, and the obtained values of the control parameters are the best which can possibly be obtained in the given situation.

At the end of calculations, messages are displayed for the operator to decide as to possible changes in the control project.

3. Selected Results of Simulation for an Existing Water Supply System

3.1. The Examined Water Supply System

In the article, an example of a real water supply system in operation in a medium-size town located near Gdańsk is presented. For this system a simulation of flows forced was performed, among other agents, by smooth control of operation of pumps and pressure controllers. Below is a brief characteristic of the water supply system, valid for the time when the simulations were performed i.e. for the year 1999.

The scheme of the water supply system is shown in Fig. 2. The system delivers water to a town of administrative and industrial characteristics. An average 24-hour water consumption in the town is equal to $Q_d = 3400$ m^3/d, which gives an average 1-hour water consumption in an average day equal to $Q_h = 141.7$ m^3/h. Such a water consumption level is observed during the time period between 9.30 a.m. and 10.30 a.m. The extreme values of the 1-hour water consumption on an average day are equal to $Q_{h\,\min} = 59.0$ m^3/h (between 4.00 a.m. and 5.00 a.m.) and $Q_{h\max}$=222.0 m^3/h (between 6.00 p.m. and 7.00 p.m.).

Due to considerable differences in ground level, the pressure in the system is zoned with the aid of a pressure reducer, mounted on a pipeline ϕ250 mm supplying the district constituting the north-east part of the town, where ground ordinates are within the limits from about 1.0 to about 7.0 meters above sea level. The remaining part of the town is situated higher. The ground ordinates in western parts reach as much as about 34.0 meters above sea level. The water supply system includes, among other components, three water intakes. Two automatically controlled intakes "U1" and "U2" are in permanent operation. They are both characterized by high discharge margin and in case of failure in one intake the entire town can be, in practice, supplied from the other intake. The intake "U3" is an extra, emergency intake, not operated in general.

Below are given basic data characterizing those intakes and flow control in the system:

- intake "U1"
  The average 24-hour discharge of this intake covers about 53% of the average water consumption in the town. The pumping station at the intake pumps the water from the treated water tank directly to the water supply network. Three working pumps and one emergency pump are installed in this pumping station. The operation of the pumps is controlled by a controller programmed in such a way that two pumps are in operation in an "on-off" (i.e. two-stage) mode, and one is smoothly controlled and keeps the pressure trajectory assumed for the node linking the pumping station with the remaining part of the system.

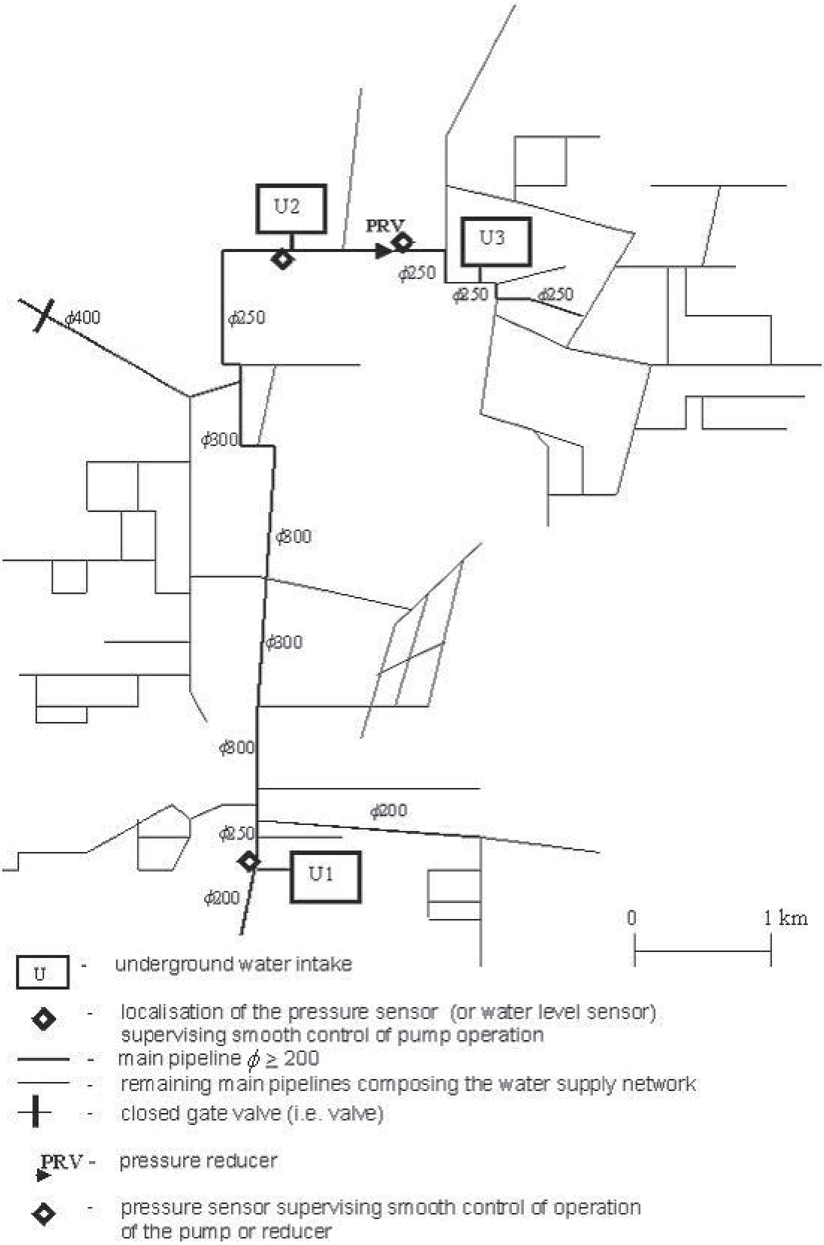

Fig. 2. Scheme of the water supply system at work for which the presented simulation was performed

- intake "U2"

  Here, the pumping station also pumps water from the treated water tank directly to the water supply network. The operation of this pumping station is controlled in a way similar to the case of the intake "U1". Four, out of five working pumps, are controlled in the "on-off" mode and one is smoothly controlled.

  During the time period examined, the assumed trajectories of ordinates of the pressure heads controlling the operation of the pumps at both water intakes were identical: from 59.0 meters above sea level at night to 61 meters above sea level in hours of maximum water consumption. Borderlines between the areas supplied with water from each intake change slightly in time due to water consumption changes in time and space. It can generally be stated that the "U2" intake delivers water to the district situated behind the pressure reducer and part of the town at the main pipeline between the two intakes. The remaining part of the town, including western districts, is supplied with water from the "U1" intake.

  At the intakes, the treated water tanks have a relatively high capacity (800 m^3) and are considered, at the same time, to be tanks collecting water for emergency cases (fire, failure of the well or the treatment station).

- pressure reducer "PRV"

  The pressure reducer is set for a pre-selected constant pressure, which secures the pressure at the ends of the network in this district to be close to the minimum required limit in case of maximum water consumption recorded in the district situated behind the reducer.

3.2. Information on Selected Performed Simulations and Their Results

The computer model of the examined sample water supply system was tuned using a direct method, i.e. with the aid of the following measurements: (1) pressures at selected nodes of the water supply network, (2) pressures and flow rates at all intakes and other pumping stations, and (3) water levels and discharges of surge tanks. Using a relatively high number of complete measurements (i.e. operating situations with complete measurement data in the system), the network pipe roughness coefficients *k*, used in the Colebroock-White formula, were calculated, and spatial distributions of water consumptions in particular hours in the system were more precisely determined. The procedures used for the above mentioned model tuning can be found in, among other sources, Orłowski (1997), and Siwoń (1998). They go beyond the scope of the present article and are not discussed here in detail. Then, the detailed descriptions and results of tuning of the computer model of the water supply system under discussion can be found in Romiński (1999).

The author's intention was to make use of simulations, to which he is going to refer, concerning the smooth control of flows in a real system. Such an approach aimed at: (1.) providing opportunities for verification of correctness of calculation methods used in the descriptions of smooth control systems by comparing the results of pressure head simulation and measurements (a) in the vicinity of nodes linking the smoothly controlled pumping stations with the water supply network, and (b) in front of the pressure reducer, (2.) illustrating the sensitivity of the system (and in consequence, models in which formally correct methods of smooth flow control are applied) to those controls. This sensitivity is illustrated by ranges of recorded pressure head fluctuations mentioned in (1b.), and changes in ranges of the instantaneous characteristics of smoothly controlled pumps presented.

3.3. Results of the Simulations

In the article we refer to the selected results of the extended period simulation (EPS) in the system, covering historical 24 hours. We present the results directly relating to the smooth control of the pump operation at intakes "U1" and "U2" and to the operation of the pressure reducer "PRV". They are shown in Figs. 3, 4 and 5.

Figs. 3 and 4 present the comparison of three instantaneous characteristics of smoothly controlled pumps (recorded during hours of minimum, medium, and maximum consumption) with their catalogue equivalent. These diagrams are presented to illustrate the sensitivity of variable characteristics of smoothly controlled pumps installed at intakes "U1" and "U2", respectively, to the operating situation in the system. The main aspects of this situation are: the changing in time of volume of water consumed by the receivers, and the assumed pressure trajectories at nodes linking the above intakes with the rest of the system to control the operation of the pump stations at those intakes. These pump stations react to the above disturbances by: (1.) changing, in a way shown in the diagrams, the characteristics of the smoothly controlled pumps, (2.) changing the numbers of switched-on pumps operated in an "on-off" mode (normally, all "on-off" pumps are switched off, due to considerable overdimensioning of the intakes with respect to the real water consumption) and finally (3.) changing in time of water levels in the treated water tanks at both intakes.

Finally, Fig. 5 presents diagrams of the simulated and measured pressure heads in front of the pressure reducer "PRV" for the examined 24 hours. The similarity of these two diagrams confirms correctness of the simulation of the reducer's operation, while the scale of pressure fluctuations make the measure of system's sensitivity to the operation of the reducer, which reacts to changing operating situation in the system. Let us notice, however, that in the examined case, the range of pressure fluctuations in front of the reducer is limited by the fact that the intake "U2" with the sensor controlling the operation of the pumping station

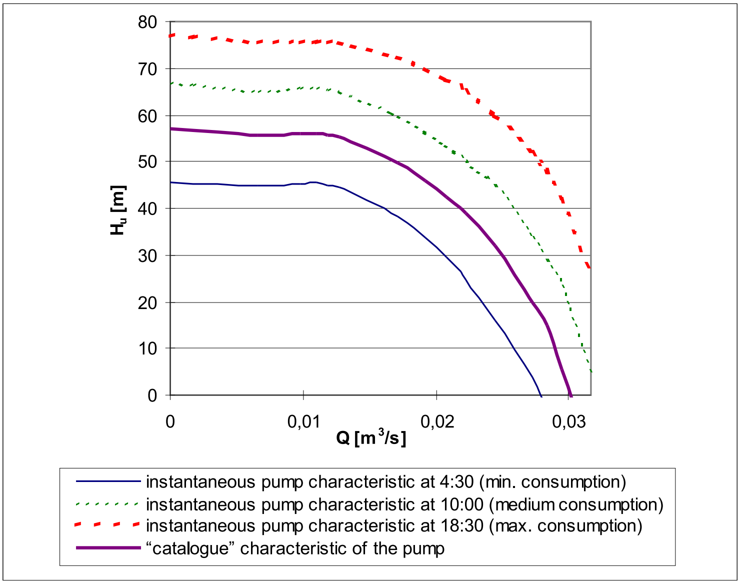

Fig. 3. Instantaneous and "catalogue" characteristics of the smoothly controlled pump at intake "U1"; see the text

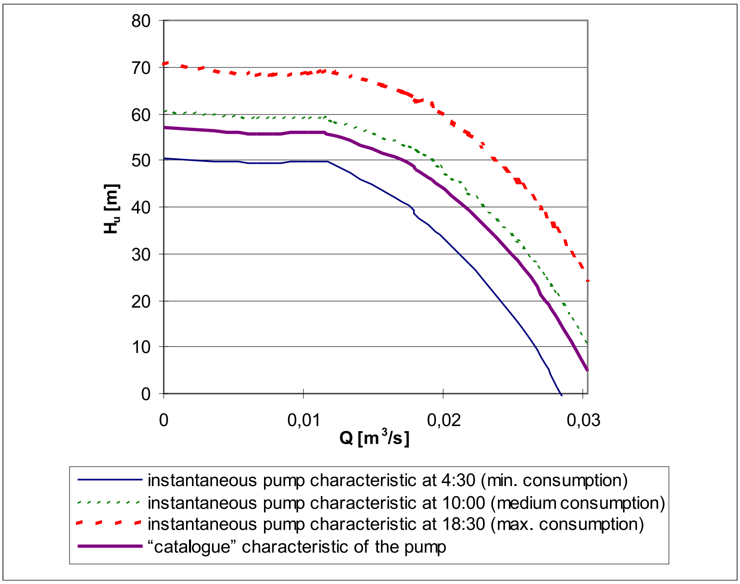

Fig. 4. Instantaneous and "catalogue" characteristics of the smoothly controlled pump at intake "U2"; see the text

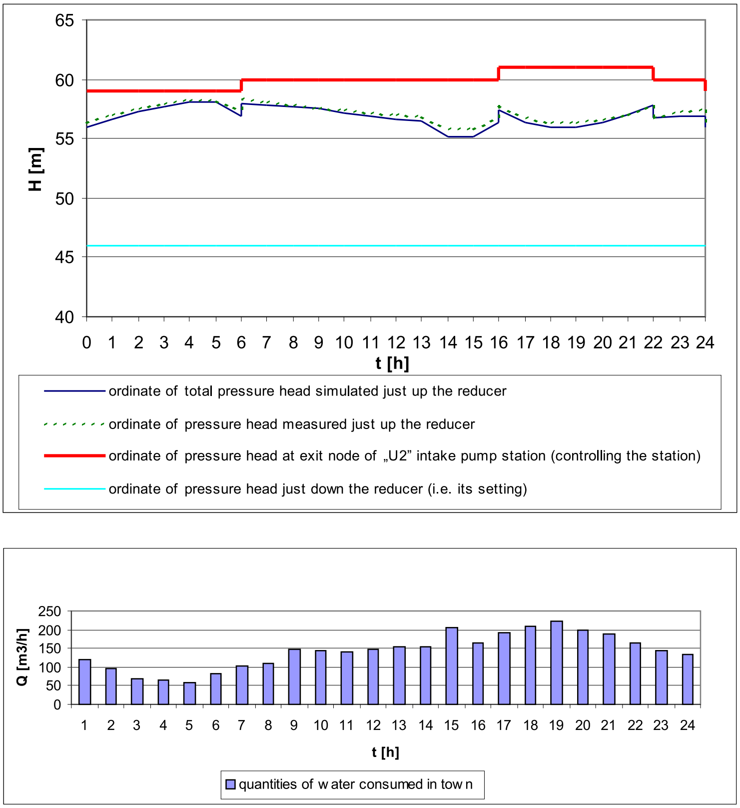

Fig. 5. Ordinates of simulated and measured pressure heads in front of the pressure reducer PRV and curves characterizing the conditions which force the operation of the reducer; see the text

is localized at a relatively small distance in front of the reducer. In this situation, irraspective of the flow rate in the duct between the sensor and the reducer, frictional head losses along this duct are small and the pressure head ordinate in front of the reducer is constantly kept at a slightly lower level than the ordinate of the pressure head controlling the operation of the pumping station. To better illustrate those relations, in the same figure are marked: the reducer operation control pressure head, monitored directly behind the reducer, the trajectory of the pumping station operation control pressure head at the intake "U2", and, finally, the time-history of water consumptions in the town in the examined time period.

4. Conclusions

The non-linearity of the water supply system with respect to smooth regulation of flows inside the system is very strong. Therefore, of high importance is that all system components used for smooth regulation in the mathematical model describing those flows be formally correct. Instantaneous characteristics of those components (i.e. parameters used in their mathematical descriptions) are unknown in the model and should be determined during the flow simulation in the system. Neglecting this fact is the source of large errors in the calculations, which make tuning computer model and its further use pointless. The author of the article has not found in the literature on the subject satisfying models of the problem, i.e. those which would be, at the same time, general (to take into account all possible methods of smooth control and all possible technical situations), formally correct, and convenient in use. Instead, the presented simplified models frequently reveal formal errors.

A formally correct mathematical and numerical model (method of solution) of the above cases of non-linearity was presented. The model bases on correcting characteristics of the system components generating smooth changes of flows in the system. Various effective algorithms for correcting those characteristics taking into account sensitivity of the system were discussed. The description of the method also includes a proper calculation control algorithm to allow: (a) control of correctness (feasibility) of the examined projects of flow control, (b) smooth switch (during calculations) of all smooth control components between various operating states (regimes) provoked by changing hydraulic conditions, and, finally, (c) performing simulation calculations which model real behaviour of the system even in cases when not all controls are correct, i.e. when the execution of corresponding controlling variables is not possible.

The article also presents selected examples of flow simulation in a medium size water supply system, in which the flows are smoothly controlled. These examples aim at illustrating the sensitivity of the system to such control methods executed by smoothly controlled pumps and automatic controllers, and proving the correctness of simulations carried out with the aid of formally correct mathematical and numerical smooth control models.

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