

SOUND VELOCITY AND PARAMETER OF NONLINEARITY IN THE TWO-COMPONENT RELAXING MIXTURES

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General formulae on small-signal sound velocity and parameter of nonlinearity B/A of a two-component relaxing mixture in the two limiting cases of very low and very high periods of sound (when the thermal equilibrium between components has enough time to establish or not) are derived. Sound parameters are expressed in the terms of partial derivatives of individual equations of state. For the two cases: the mixture of van der Waals gases and the suspension consisting of the ideal gas and tiny solid or liquid inclusions, sound velocity and parameter of nonlinearity B/A are evaluated as functions of mass concentration of one of the parts. The first example concerns to the mixtures consisting of oxygen and helium, and the second one to the suspension of air and graphite and the water fog. General conclusions about acoustic features of the two-component mixtures under very high and low frequencies are drawn out.

Keywords: relaxation processes, irreversible thermodynamics, parameter of nonlinearity.

1. Introduction

Relaxation processes consider a finite temporal delay of deviation of microscopic system from thermodynamic equilibrium, or of returning to it. During sound propagation, a part of energy goes into heat. The beginning of understanding of relaxation processes as transfer of energy from the external into internal degrees of freedom, and *vice versa*, comes to the works by KNESER [1]. Sound propagation in relaxing media is not only dissipative, but disperse as well [2–6], because the behavior of a given internal process depends on ratio of its relaxation time and period of acoustic wave. Links of disperse and damping properties of relaxing medium are expressed by the famous Kramers–Kronig relations. The relaxation theory of gases is most developed [3]. For gases, dependence of sound velocity on frequency may be easily illustrated. If sound is of low frequency and period much greater than the relaxation time: $T \gg \tau$, equilibrium has enough time to follow the sound propagation, and the molar heat capacity under

constant volume is a sum of internal and oscillatory parts, $c_{v,\text{int}} + c_{v,\text{osc}}$. The correspondent infinitely small-signal sound velocity equals:

$$c_0 = \sqrt{\frac{p_0}{\rho_0} \left(1 + \frac{R}{c_{v,\text{int}} + c_{v,\text{osc}}} \right)}, \quad (1)$$

where p_0, ρ_0 are unperturbed pressure and density of the medium, $R = 8.314 \text{ J/mol/K}$ is the universal gas constant. For the high frequencies $T \ll \tau$, $c_{v,\text{int}} \rightarrow 0$, and adiabatic sound velocity equals:

$$c_\infty = \sqrt{\frac{p_0}{\rho_0} \left(1 + \frac{R}{c_{v,\text{osc}}} \right)} > c_0. \quad (2)$$

There are other types of relaxation processes in gases, such as translational relaxation while molecular velocities tend to Maxwell distribution (very fast process), chemical associations and dissociations [2], phase transitions and other ones. In the air, relaxation process takes place mainly by means of vibration of oxygen and nitrogen molecules, and in seawater by means of dissociation of boric acid and magnesium sulfate molecules [6]. In liquids, a structural relaxation may take place [3]. The majority of relaxation processes associate with bulk viscosity depending on frequency.

The most exciting problem of non-equilibrium thermodynamics is “acoustic spectroscopy”. Unfortunately, distinguishing of two or more relaxation processes and their characteristic times, meets many difficulties. Nevertheless, about 1 percent of injections may be experimentally founded by observations of sound velocity and its attenuation. Acoustic measurements sometimes are an only equipment to conclude about relaxation processes. They are fruitful in research of liquids with very small time of structural rebuilding, $\tau \sim 10^{-11}$ seconds [3].

Author develops the non-equilibrium thermodynamics to evaluate parameter of non-linearity of mixture B/A as a function of individual equations of state and mass concentrations of components. Evaluations in the two limiting cases of sound are provided, while frequency tends to zero (AT, 0 limit), and under the very high frequency (AA, ∞ limit). So that, the thermal relaxation between two parts is considered, though the individual relaxations also may be taken into account in the frames of the discussion below. It is known, that parameter B/A is expressed in terms of the second partial derivatives of thermodynamic functions and is more sensitive to the curvature of thermodynamic functions than linear sound velocity [7]. So that, theoretical predictions of B/A may provide more trustable than that of sound velocity information about concentration of every component in the mixture.

For a pure relaxing medium, sound velocity and attenuation depend on a circular frequency in the following manner [3]:

$$\begin{aligned} c(\omega) &= c_0 \left(1 + \frac{m}{2} \frac{\omega^2 \tau^2}{1 + \omega^2 \tau^2} \right), \\ \alpha(\omega) &= \frac{m}{2c_0 \tau} \frac{\omega^2 \tau^2}{1 + \omega^2 \tau^2}, \end{aligned} \quad (3)$$

where $m = (c_\infty^2 - c_0^2)/c_0^2 \ll 1$. These formulae give maximal possible value of attenuation:

$$\alpha\lambda_{\max} = \alpha\lambda(\omega\tau \approx 1 + m/4) \approx \frac{\pi m}{2}. \quad (4)$$

Predictions of AA and AT sound velocities in a mixture give maximum value of attenuation in mixture which is not obviously a simple sum of attenuations in every component. In the mixtures, the acoustic properties depend on mass concentrations of the parts. We can conclude about maximal attenuation in mixtures knowing the correspondent value of m in the mixture as a whole, which depends on mass concentration of parts and equilibrium thermodynamic state of the mixture.

2. Infinitely small-signal sound velocity and parameter of nonlinearity B/A in the relaxing mixtures

2.1. Adiabatic-adiabatic processes

When processes occur so fast that heat transfer between components is completely absent, every component individually and the mixture as a whole behave adiabatically. A compressibility of such mixture is called adiabatic-adiabatic (AA, ∞) compressibility. It may be easily derived knowing the compressibility of every component. The adiabatic compressibility of every homogeneous component β_∞ and its linear sound velocity c_∞ may be determined in the following manner:

$$\beta_\infty = -\left(\frac{\partial V}{\partial p}\right)_s = -\frac{C_v}{C_p}\left(\frac{\partial V}{\partial p}\right)_T = -\left(\frac{\partial V}{\partial p}\right)_T - \frac{T}{C_p}\left(\frac{\partial V}{\partial T}\right)_p^2, \quad (5)$$

$$c_\infty = V\beta_\infty^{-1/2},$$

where $V = 1/\rho$ is a specific volume, T is temperature, and C_v , C_p are heat capacity under constant volume and pressure per unit mass, correspondingly. Specific volume of a substance consisting of two homogeneous parts marked by indices 1 and 2 is:

$$V = (1-x)V_1 + xV_2, \quad (6)$$

where x is a constant (over the whole volume) mass concentration of the second component in the mixture. AA compressibility of the mixture and sound velocity therefore equal:

$$\beta_\infty = (1-x)\beta_{1,\infty} + x\beta_{2,\infty}, \quad (7)$$

$$c_\infty = V(\beta_\infty)^{-1/2}.$$



Equivalently, c_∞ is a function of ρ_1 , ρ_2 and $\gamma_1 = \left(\frac{C_p}{C_v}\right)_1$, $\gamma_2 = \left(\frac{C_p}{C_v}\right)_2$:

$$\begin{aligned} c_\infty &= \frac{1}{\rho} \left(\frac{1-x}{c_1^2 \rho_1^2} + \frac{x}{c_2^2 \rho_2^2} \right)^{-1/2} \\ &= \left(\frac{1-x}{\rho_1} + \frac{x}{\rho_2} \right) \left(-\frac{1-x}{\gamma_1} \left(\frac{\partial V_1}{\partial p} \right)_T - \frac{x}{\gamma_2} \left(\frac{\partial V_2}{\partial p} \right)_T \right)^{-1/2}. \end{aligned} \quad (8)$$

Coefficient of nonlinearity ε may be expressed in terms of the second derivative of specific volume:

$$\varepsilon_\infty = 1 + \left(\frac{B}{2A} \right)_\infty = \frac{1}{2} \rho^3 c_\infty^4 \left(\frac{\partial^2 V}{\partial p^2} \right)_{s,\infty}. \quad (9)$$

The second partial derivative in the right-hand side of Eq. (9) should be evaluated in the proper way, like it will be proceeded in the next subsection. For the most cases, partial parameters of nonlinearity $(B/A)_\infty$ are known. Taking into account Eqs. (7), (8), we conclude that the coefficient of nonlinearity depends on partial quantities as follows:

$$\begin{aligned} \varepsilon_\infty &= \left(\frac{1-x}{\rho_1} + \frac{x}{\rho_2} \right) \left(\frac{1-x}{(c_{1,\infty} \rho_1)^2} + \frac{x}{(c_{2,\infty} \rho_2)^2} \right)^{-2} \\ &\quad \times \left(\frac{1-x}{\rho_1^3 c_{1,\infty}^4} \varepsilon_{1,\infty} + \frac{x}{\rho_2^3 c_{2,\infty}^4} \varepsilon_{2,\infty} \right). \end{aligned} \quad (10)$$

Formulae (8)–(10) may be easily extrapolated for any number of components in a mixture.

2.2. Adiabatic-isothermal processes

The another important type of processes in a mixture are so slow while expanding and compressing, that heat exchange between components follows these processes. Therefore, all components have equal temperature, and the parts of the mixture are in the thermal equilibrium, in contrast to the adiabatic-adiabatic processes considered in the previous subsection. The system is closed, no heat exchange with surrounding occurs, so that the whole mixture behaves adiabatically. The processes of this kind are called adiabatic-isothermal (AT, 0) processes. AT processes take place under frequencies so small that there is enough time for establishing of thermal equilibrium between components. Specific volume of the mixture is a function of p and T , in contrast to the adiabatic-adiabatic processes, where it is a function of p and entropy s . In the role of the starting point for the further evaluations, the formula (5) for partial compressibility and sound velocity may be used. Taking into account (6) and the equality below:

$$C_p = (1-x)C_{p,1} + xC_{p,2}, \quad (11)$$

adiabatic-thermal compressibility takes the form:

$$\beta_0 = -(1-x) \left(\frac{\partial V_1}{\partial p} \right)_T - x \left(\frac{\partial V_2}{\partial p} \right)_T - \frac{T}{(1-x)C_{p,1} + xC_{p,2}} \left[(1-x) \left(\frac{\partial V_1}{\partial T} \right)_p + x \left(\frac{\partial V_2}{\partial T} \right)_p \right]^2. \quad (12)$$

Adiabatic-isothermal sound velocity equals:

$$c_0 = \left(\frac{1-x}{\rho_1} + \frac{x}{\rho_2} \right) \beta_0^{-1/2}. \quad (13)$$

Going to evaluating of coefficient of nonlinearity, one should also take into account that V is now a function of p and T :

$$\left(\frac{\partial V}{\partial p} \right)_s = f(p, T). \quad (14)$$

For a homogeneous substance, it follows from (14):

$$\left(\frac{\partial^2 V}{\partial p^2} \right)_s = \left(\frac{\partial f}{\partial p} \right)_T + \left(\frac{\partial f}{\partial T} \right)_p \left(\frac{\partial T}{\partial p} \right)_s, \quad (15)$$

where

$$\begin{aligned} \left(\frac{\partial f}{\partial p} \right)_T &= \left(\frac{\partial^2 V}{\partial p^2} \right)_T + \frac{2T}{C_p} \left(\frac{\partial V}{\partial T} \right)_p \frac{\partial^2 V}{\partial p \partial T} - \frac{T}{C_p^2} \frac{\partial C_p}{\partial p} \left(\frac{\partial V}{\partial T} \right)_p^2, \\ \left(\frac{\partial f}{\partial T} \right)_p &= \frac{\partial^2 V}{\partial p \partial T} + \frac{1}{C_p} \left[\left(\frac{\partial V}{\partial T} \right)_p^2 + 2T \left(\frac{\partial V}{\partial T} \right)_p \left(\frac{\partial^2 V}{\partial T^2} \right)_p \right] - \frac{T}{C_p^2} \frac{\partial C_p}{\partial T} \left(\frac{\partial V}{\partial T} \right)_p^2. \end{aligned} \quad (16)$$

Using the thermodynamic equality

$$\left(\frac{\partial T}{\partial p} \right)_s = \frac{T}{C_p} \left(\frac{\partial V}{\partial T} \right)_p, \quad (17)$$

Equations (15), (16) for a homogeneous medium give finally:

$$\begin{aligned} \left(\frac{\partial^2 V}{\partial p^2} \right)_s &= \left(\frac{\partial^2 V}{\partial p^2} \right)_T + \frac{3T}{C_p} \left(\frac{\partial V}{\partial T} \right)_p \left(\frac{\partial^2 V}{\partial p \partial T} \right) + \frac{T}{C_p^2} \left(\frac{\partial V}{\partial T} \right)_p^2 \left[\left(\frac{\partial V}{\partial T} \right)_p + 2T \left(\frac{\partial^2 V}{\partial T^2} \right)_p - \frac{\partial C_p}{\partial p} - \frac{T}{C_p} \frac{\partial C_p}{\partial T} \left(\frac{\partial V}{\partial T} \right)_p \right]. \end{aligned} \quad (18)$$

Going to the two-component mixture, we account for Eqs. (6), (11), (18) and, importantly, that the mixture is thermally uniform. As a result, one gets the expressions:

$$\varepsilon_0 = 1 + \left(\frac{B}{2A} \right)_0 = \frac{1}{2} \rho^3 c_0^4 \left(\frac{\partial^2 V}{\partial p^2} \right)_{s,0},$$

$$\begin{aligned}
\left(\frac{\partial^2 V}{\partial p^2}\right)_{s,0} &= (1-x) \left(\frac{\partial^2 V_1}{\partial p^2}\right)_T + x \left(\frac{\partial^2 V_2}{\partial p^2}\right)_T \\
&+ \frac{3T}{(1-x)C_{p,1} + xC_{p,2}} \left[(1-x) \left(\frac{\partial V_1}{\partial T}\right)_p + x \left(\frac{\partial V_2}{\partial T}\right)_p \right] \\
&\times \left[(1-x) \left(\frac{\partial^2 V_1}{\partial p \partial T}\right) + x \left(\frac{\partial^2 V_2}{\partial p \partial T}\right) \right] \\
&+ \frac{T}{((1-x)C_{p,1} + xC_{p,2})^2} \left[(1-x) \left(\frac{\partial V_1}{\partial T}\right)_p + x \left(\frac{\partial V_2}{\partial T}\right)_p \right]^2 \\
&\times \left\{ (1-x) \left(\frac{\partial V_1}{\partial T}\right)_p + x \left(\frac{\partial V_2}{\partial T}\right)_p + 2T(1-x) \left(\frac{\partial^2 V_1}{\partial T^2}\right)_p \right. \\
&+ 2Tx \left(\frac{\partial^2 V_2}{\partial T^2}\right)_p - \frac{\partial((1-x)C_{p,1} + xC_{p,2})}{\partial p} - \frac{T}{(1-x)C_{p,1} + xC_{p,2}} \\
&\left. \times \frac{\partial((1-x)C_{p,1} + xC_{p,2})}{\partial T} \left((1-x) \left(\frac{\partial V_1}{\partial T}\right)_p + x \left(\frac{\partial V_2}{\partial T}\right)_p \right) \right\}. \quad (19)
\end{aligned}$$

3. Examples of evaluation of sound velocity and parameter of nonlinearity

3.1. Two van der Waals gases

The linear sound velocity as well as parameter of nonlinearity for two and more van der Waals gases behaving adiabatically (AA) may be found in the papers by DENISOV [8, 9]. The general conclusions are that both sound velocity and parameter of nonlinearity tends to that in the pure component while mass concentration of the other component tends to zero. In contrast to the case of ideal gases, evaluations should take into account the equation of state for the van der Waals gas:

$$\left(p + \frac{a}{\mu^2 V^2}\right) (\mu V - b) = RT, \quad (20)$$

where μ is the molar mass. (The case when the specific excess pressure caused by particles interaction tends to zero, $a \rightarrow 0$, as well as decrease in volume accounting for molecular sizes, $b \rightarrow 0$, corresponds to the ideal gas.) The heat capacity under constant pressure is the same for ideal and van der Waals gases, but the specific volume and heat capacity under constant pressure differ in the leading order from that of ideal gas:

$$\begin{aligned}
V_{vdW} &= \frac{RT}{\mu p} + \frac{b}{\mu} - \frac{a}{\mu RT} = V_{id} + \frac{b}{\mu} - \frac{a}{\mu RT}, \\
C_{v,vdW} &= C_{v,id}, \quad C_{p,vdW} = C_{v,id} + \frac{R}{\mu} + \frac{2pa}{\mu RT^2} = C_{p,id} + \frac{2pa}{\mu RT^2}, \quad (21)
\end{aligned}$$

as well as $\gamma = C_p/C_v$ which influences on the final values of c_∞ and B/A_∞ . The heat capacity under constant pressure is now a function of temperature and pressure which should be taken into account while evaluating of B/A in accordance to Eq. (19).

General formulae for the two van der Waals gases are too long to be presented in the text. For example, the acoustic parameters for mixture of comparatively heavy and light gases, oxygen (1) and helium (2), are evaluated. Data for the both gases under the normal conditions are taken from [10]: $\mu_1 = 32 \cdot 10^{-3}$ kg/mol, $a_1 = 0.1378$ m⁶Pa/mol², $b_1 = 3.183 \cdot 10^{-5}$ m³/mol, $\mu_2 = 4.003 \cdot 10^{-3}$ kg/mol, $a_2 = 0.3457$ m⁶Pa/mol², $b_2 = 2.37 \cdot 10^{-5}$ m³/mol. As it was expected, the AA sound velocity is somewhat greater than AT velocity except of the pure phase, $x = 0$ or $x = 1$. Nonlinear distortions in the AA mixture are stronger as well comparatively to the AT mixture.

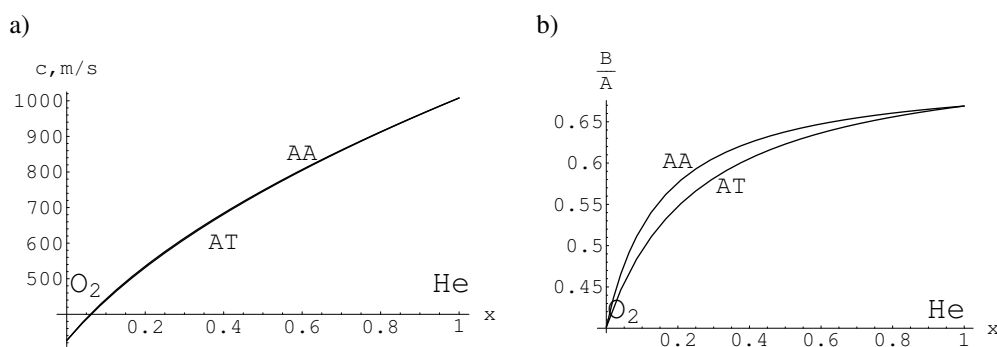


Fig. 1. a) AA and AT sound velocities as functions of mass concentration x of helium in the mixture consisting of helium and oxygen, almost undistinguishable; b) AA and AT parameters of nonlinearity B/A as functions of mass concentration of helium in the mixture helium-oxygen. Both series relate to $p_0 = 101325$ Pa, $T_0 = 293$ K.

It was already mentioned in the Introduction, that the parameter of nonlinearity is highly sensible to the curvature of state equation, while the van der Waals equation of state is itself approximate and may lead to an error. So that curves of the parameter of nonlinearity are rather qualitative. Remarks on the role of equation of state in estimations of the sound velocity and nonlinear parameter may be found in the paper by the author [7].

3.2. Mixtures consisting of ideal gas and tiny inclusions: air-graphite and air-water suspensions

The basic formulae on small-signal sound velocity and parameter of nonlinearity B/A in the both limiting cases in the Sec. 2, are written on in terms of quantities following from both equations of state and include partial second order derivatives of the individual specific volumes with respect to pressure and temperature.

A particular important case of mixtures are those consisting of gas and small tiny inclusions, solid or liquid. A mixture as a whole is further considered as the homogeneous



continuum. Let mark a gas fraction by index "1", and incompressible inclusions (comparatively to the gas) by index "2". Relative incompressibility of the second fraction yields simplifying equalities:

$$\left(\frac{\partial V_2}{\partial p}\right)_T = \left(\frac{\partial V_2}{\partial T}\right)_p = \left(\frac{\partial^2 V_2}{\partial p^2}\right)_T = \left(\frac{\partial^2 V_2}{\partial T^2}\right)_p = \frac{\partial^2 V_2}{\partial p \partial T} = 0. \quad (22)$$

As usual, $V_2 \ll V_1$, so that $V \approx (1-x)V_1$. That makes AA and AT sound velocity and coefficient of nonlinearity remarkably simpler:

$$\begin{aligned} c_\infty &= V_1 \left(-\frac{1}{\gamma_1(1-x)} \left(\frac{\partial V_1}{\partial p}\right)_T \right)^{-1/2}, \\ \varepsilon_\infty &= \varepsilon_1, \\ c_0 &= V_1 \left(-\frac{1}{(1-x)} \left(\frac{\partial V_1}{\partial p}\right)_T - \frac{T}{(1-x)C_{p,1} + xC_{p,2}} \left(\frac{\partial V_1}{\partial T}\right)_p \right)^{-1/2}, \\ \varepsilon_0 &= \frac{c_0^4}{2(1-x)^3 V_1^3} \left(\frac{\partial^2 V}{\partial p^2}\right)_{s,0}, \\ \left(\frac{\partial^2 V}{\partial p^2}\right)_{s,0} &= (1-x) \left(\frac{\partial^2 V_1}{\partial p^2}\right)_T + \frac{3T(1-x)^2}{(1-x)C_{p,1} + xC_{p,2}} \left(\frac{\partial V_1}{\partial T}\right)_p \left(\frac{\partial^2 V_1}{\partial p \partial T}\right) \\ &\quad + \frac{T(1-x)^3}{((1-x)C_{p,1} + xC_{p,2})^2} \left(\frac{\partial V_1}{\partial T}\right)_p^2 \\ &\quad \times \left\{ \left(\frac{\partial V_1}{\partial T}\right)_p + 2T \left(\frac{\partial^2 V_1}{\partial T^2}\right)_p - \frac{\partial C_{p,1}}{\partial p} \right. \\ &\quad \left. - \frac{T}{(1-x)C_{p,1} + xC_{p,2}} \left(\frac{\partial(1-x)C_{p,1} + xC_{p,2}}{\partial T}\right) \left(\frac{\partial V_1}{\partial T}\right)_p \right\}. \end{aligned} \quad (23)$$

Note that incompressibility of the second phase means that $c_{0,2} = c_{\infty,2} = \infty$. That is obviously does not agree with experimental data. Sound velocity may be set infinite only comparatively to the sound velocity in gas.

Let consider a mixture including an ideal gas with constant heat capacity $C_{p,1}$. Dependence of $C_{p,2}$ on pressure is known to be weak for solids and liquids. An ideal gas obeys the equation of state as follows:

$$pV = \frac{RT}{\mu}, \quad (24)$$

where R is the universal gas constant and μ a molar mass of gas. The heat capacity $C_{p,1}$ and correspondent partial derivatives for ideal gas take the form:

$$C_{p,1} = \frac{\gamma R}{(\gamma - 1)\mu_1} = \frac{R}{2\mu_1}(f + 2), \quad (25)$$

$$\left(\frac{\partial V_1}{\partial T}\right)_p = \frac{R}{\mu_1 p}, \quad \left(\frac{\partial^2 V_1}{\partial T^2}\right)_p = 0, \quad \left(\frac{\partial V_1}{\partial p}\right)_T = -\frac{RT}{\mu_1 p^2},$$

$$\left(\frac{\partial^2 V_1}{\partial p^2}\right)_T = \frac{2RT}{\mu_1 p^3}, \quad \frac{\partial^2 V_1}{\partial T \partial p} = -\frac{R}{\mu_1 p^2},$$

where f denotes a number of freedom of a molecule. Inserted in the basic formulae (23), these equalities considerably simplify their final form.

For example, let evaluate sound parameters for the mixture consisting of air and graphite $_6\text{C}$, and for the mixture consisting of air and water. Under temperature $T = 293 \text{ K}$ and pressure $p = 101325 \text{ MPa}$, graphite has density $\rho_2 = 2 \cdot 10^3 \text{ kg/m}^3$, its heat capacity is $C_{p,2} = 6.3 \text{ J/kg/K}$, and its temperature gradient in this point is $\partial C_{p,2}/\partial T = 1.08 \text{ J/kg/K}^2$ [11]. Water quantities under the same conditions are as follows: $\rho_2 = 10^3 \text{ kg/m}^3$, $C_{p,2} = 4182 \text{ J/kg/K}$, $\partial C_{p,2}/\partial T = -0.34 \text{ J/kg/K}^2$ [12]. The molar mass of air is $\mu_1 = 28.96 \cdot 10^{-3} \text{ kg/mol}$, $\gamma_1 = 1.4$, so that its computed density and heat capacity equals $\rho_1 = 1.19 \text{ kg/m}^3$, $C_{p,1} = 1004.8 \text{ J/kg/K}$.

Research of sound in solids needs considering not only quasi-longitudinal mode, but quasi-transversal ones (slow and fast) as well. The error caused by limit of incompressible solid grows with mass concentration of solid particles in the mixture. As for the mixture including water, the effects connected with finite drop dimension would lead to strong dispersion. The conclusion is that for large mass concentrations of graphite or water, x , results are not trustable. There is one more reason to conclude that the mixture is not longer suspension for the large x .

AA and AT infinitely small-signal sound velocities differ very weakly for graphite over all the domain x , but AA and AT parameters of nonlinearity B/A differ essentially. $(B/A)_\infty$ approximately equals to that in the pure air 0.4 because of equality $V_2 \ll V_1$. It equals at $x = 0$ to $(B/A)_0$, since in the absence of the second phase, sound propagates over the pure air.

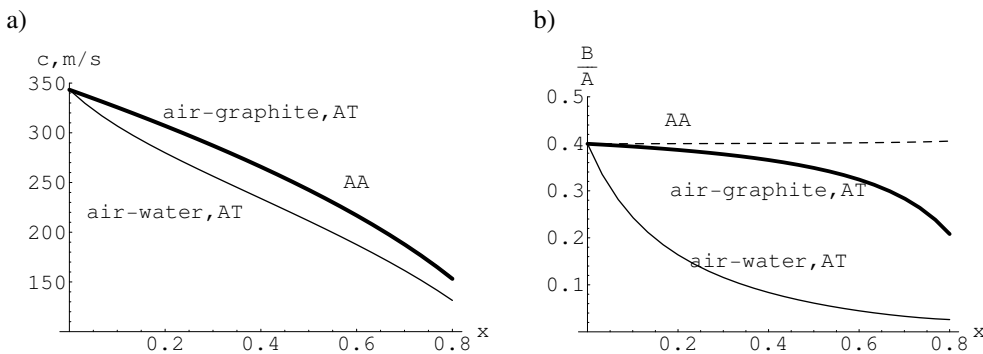


Fig. 2. a) AA (dashing) and AT (solid) sound velocity as function of mass concentration x of graphite or water in the mixture. AA sound velocity in the both mixtures is indistinguishable from AT velocity in the air-graphite mixture; b) AA (dashing) and AT (solid, bold: graphite, thin: water) parameter of nonlinearity B/A as function on mass concentration of graphite or water in the mixture. Curves are trustable for the small concentrations x . Both series relate to $p_0 = 101325 \text{ Pa}$, $T_0 = 293 \text{ K}$.

For the air-water mixture, AA and AT sound velocities differ essentially. The reason is in the large heat capacity of water compared to that of graphite and in its smaller density. As it is expected, AT parameters of nonlinearity B/A considerably deviate from AA quantities. AA processes provide larger nonlinearity. The rough estimations of the sound velocity and parameter of nonlinearity in the air-water and air-graphite mixtures prove that measurements of parameter of nonlinearity rather than sound velocity may be helpful in determination of mass concentration of inclusions.

4. Conclusions

It should be pointed out, that the theory of this research applies in the previous sections, among mixtures of the van der Waals gases, to the air-bearing suspensions including tiny particles of solid (graphite) or liquid (water), that are distributed uniformly over all the mixture volume ($x = \text{const}$). We do not investigate here effects connected with finite volume of inclusions, or inhomogeneous mass concentration. In the reality, other important physical phenomena being out of interest of the present research, may take place. There are, among other: phase transfer (before water and its vapor), effects of surface tension, sound dispersion caused by finite volume of inclusions, effects of gravity force, which makes a mixture disperse and inhomogeneous. In many cases, a mixture could not be treated as a homogeneous continuity.

In spite of the simplifying conditions, the evaluations of sound velocity and parameter of nonlinearity reveal the possibility of determination of mass concentration of one of the parts. Especially it concerns to predictions basing on the parameter of nonlinearity, since it is more sensitive to variations of concentration of inclusions. Expressions for low-frequency (AT) and high-frequency (AA) sound velocity and the parameter of nonlinearity (Eqs. (8), (10), (13), (19)) are exact, the two last being novel. They consider compressibility of every part and dependence of heat capacity on pressure and temperature. The difficulty is in establishing of correspondent experimental data for all components of a concrete suspension. Unfortunately, there are absent data about values of $\partial^2 V / \partial p^2$ and other second derivatives participating in Eq. (19) for the majority of substances. The necessary quantities may be extracted from approximate equations of state like van der Waals equation like it was proceeded in the Sec. 3.1. Analysis reveals that AA processes in the mixture are characterized by the larger both sound speed and nonlinearity comparatively to the AT processes.

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