FREE SURFACE FLOW MODELING IN NUMERICAL ESTIMATION OF FLOOD RISK ZONES: A CASE STUDY

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Abstract: A case study of potential inundation of Saska Kępa in Warsaw is presented. The flood is a result of a hypothetical breach of a segment of the Vistula river embankment. The inundation's evolution is simulated numerically using a model of shallow water hydrodynamics. The finite volume method is used to solve the mathematical model of the flow. Digital models of the floodplain's relief and land cover, as well as a visualization of the simulation results, are prepared using the Geographical Information System. The computations may be useful in estimations of Warsaw's flood risk zones.

Keywords: mathematical modeling, inundation, flood zones, the Vistula, Saska Kępa (Warsaw)

1. Introduction

Estimation of the reach of river flood (flood zones) in urban areas has become one of hydrology's most important research subjects. Flood risk maps are usually prepared for spatial planning and insurance purposes. In Poland, the creation of flood risk maps is required by Article 82 of the Water Law of 18 July, 2001, concerning elaboration of flood risk maps for unprotected areas and those protected by embankments along major rivers. Although protected (embanked) areas are outside direct flood zones, they are still regions of potential risk due to the possibility of river embankment failures resulting in sudden inundation of the floodplain.

Polish legislation does not specify models or tools needed to simulate inundation evolution for flood risk mapping. However, numerical simulations of inundation hydrodynamics seem to be the only possible manner of proper assessment of actual flood hazard. Numerical calculations of flood wave propagation provide information about the duration and magnitude of inundation and about extreme water depth and velocity in the flooded area. Mathematical models of flood hydrodynamics used in water engineering vary from one-dimensional steady or unsteady flow equations [1, 2] to two-dimensional horizontal water flow equations [3]. The choice of the model depends on the scale of the analyzed problem and the required detail of information about flow hydrodynamics. One-dimensional models are usually used for long river reaches. When detailed analysis of local flow problems is necessary, two-dimensional models should be applied.

In Warsaw, areas directly endangered by flooding are currently limited to the belts between flood protection dikes, although the scope of the potential flood hazard is much wider. In the first studies [4], the range of potential $Q_{1\%}$ flooding of Warsaw was estimated using the combined digital elevation model (DEM) and results of one-dimensional hydraulic modeling. Conditions of a steady flow were assumed to be $Q_{1\%} = 7214\,\mathrm{m^3/s}$ [5]. The reach of the Vistula valley analyzed in the study ranged from the vicinity of Józefów down to Ławice Kiełpińskie, of the length of 47.05km (*i.e.* between the river's 494.76$^{\mathrm{th}}$ and 541.81$^{\mathrm{th}}$ km). It was found that the elevation of flood protection dikes was generally sufficient for the $Q_{1\%}$ discharge.

Nevertheless, three flood risk zones were selected in the districts of Łomianki, Saska Kępa and Wilanów, where flood is possible due to potential embankment breach. The dike breach locations were selected on the analysis of the river channel's morphology, orientation of the current and dikes, and the pattern of flood-related forms on the floodplain behind the dikes. In this paper, a simulation of the flooding of Saska Kępa is presented.

2. Saska Kępa case study: site description

Saska Kępa is located on a right bank of the Vistula on a younger Holocene floodplain (see Figure 1).Two terraces developed during the Holocene at the Vistula in the Warsaw area. Their appearance was due to the variable level of the Baltic Sea and the location of the erosion base. The higher flood terrace (the Wawer terrace) developed in the early Holocene. It stretches along both banks of the river, forming the widest flood plain, within which younger Holocene terraces developed. When forming this terrace, the Vistula was a meandering river. The traces of meander bends have been preserved in the form of undercuts in the lower over-flood (Praga) terrace in the areas of Wawer-Gocław and Powsin. Old river bed remnants persist in both of these floodplains. Due to the regression of the Baltic shoreline, the flood terrace was cut apart and the Vistula has been accumulating alluvial sediment on the lower flood terrace. This terrace has developed along the Vistula river channel in fragments only, limited by the flood protection dikes. The Vistula in Warsaw remained in its natural state until the catastrophic flood of July 1884 gave an impulse for regulation of its channel. Further regulatory work was carried out in the years 1923–1931 with the purpose of protecting the shores and the newly built dikes. Immediately after the World War II, the Vistula river channel in Warsaw became a debris dumping ground, so that cross-sections of high discharges were narrowed down by ca 50% in comparison with the middle Vistula, forming the so-called Warsaw corset in the 507–517km segment. Upstream of the Śląsko-Dąbrowski Bridge, the Vistula's channel was narrowed down to approximately 350m. So significant narrowing of the channel in the entire middle and lower course of the river was unique. It was due to the geological structure (an extension of the Praga terrace) and man-made formations – the rests of the bridgehead of the former Kierbedź Bridge and flood protection dikes. According to designs, the city should be protected against floods of one thousand years' recurrence ($Q_{0.1\%}$) with protection dikes distanced by 400–600 m. The height and state of flood protection dikes has been reviewed recently, but the floodplain nature of the terrain behind the dikes persists. Traces of the old Vistula river channel are still visible in the topography from the Praga port, through the ponds of the Skaryszewski Park, to the green area of the nearby allotments. This paleo-channel of the river should be considered as a potential flood route. In the model, the location of the hypothetic flood breach has been selected taking into account the recent morphology of the river channel and the existence of the paleo-channel behind the dikes.

Figure 1. An aerial photograph of Saska Kępa in Warsaw [6]

3. Inundation: hydrodynamic model

Flood zones can be identified in river valleys using mathematical models of flood wave propagation and digital terrain models (DTM) of the flooded area. One-dimensional hydrodynamic models are usually used for modeling river water flows. Unfortunately, such models are often inconvenient for predicting inundations of complex valley areas due to their rough representation of the floodplain relief. The flow area's definition in a hydrodynamic model with cross-sections of the river channel and the floodplain may produce errors in the predicted inundation range identified by the intersection line between the interpolated water surface and DTM. More accurate flood simulation results can be obtained using two-dimensional flow models [7], providing detailed (though still discrete) representations of floodplain topography in computations and allowing depth and velocity to be computed at each location. Moreover, two-dimensional hydrodynamic modeling is necessary when two-dimensional water flow effects are expected. Following a breach of flood protection dikes, water is suddenly released from the river channel and a rapidly varied flow can be observed near the breach. The flow in this region must be modeled with two-dimensional models solved with specific numerical methods. In this paper, flood wave propagation due to a river embankment failure is investigated using the shallow water equations (SWE) solved with the finite volume method (FVM) [8].

The SWE model can be written in its conservative form as follows [9]:

$$\frac{\partial \boldsymbol{U}}{\partial t}+\frac{\partial \boldsymbol{E}}{\partial x}+\frac{\partial \boldsymbol{G}}{\partial y}+\boldsymbol{S}=0, \tag{1}$$

where

$$\boldsymbol{U}=\begin{pmatrix} h \\ uh \\ vh \end{pmatrix}, \qquad \boldsymbol{S}=\begin{pmatrix} 0 \\ -gh(S_{ox}-S_{fx}) \\ -gh(S_{oy}-S_{fy}) \end{pmatrix} \tag{2a}$$

and

$$\boldsymbol{E}=\begin{pmatrix} uh \\ u^2h+0.5gh^2 \\ uvh \end{pmatrix}, \qquad \boldsymbol{G}=\begin{pmatrix} vh \\ uvh \\ v^2h+0.5gh^2 \end{pmatrix}. \tag{2b}$$

In this system of equations, h represents water depth, u and v are the depth-averaged components of the velocity vector in the x and y directions, respectively, S_{ox} and S_{oy} denote the bed slope terms, S_{fx} and S_{fy} are the bottom friction terms defined by the Manning formula and g is the acceleration due to gravity. Equation (1) can be written in another vector form:

$$\frac{\partial \boldsymbol{U}}{\partial t}+\operatorname{div}\boldsymbol{F}+\boldsymbol{S}=0, \tag{3}$$

where, assuming a unit vector of $\boldsymbol{n}=(n_x,n_y)^T$, vector $\boldsymbol{F}$ is defined as $\boldsymbol{F}n=\boldsymbol{E}n_x+\boldsymbol{G}n_y$. In order to integrate the SWE in space using the finite volume method, the domain of solution is discretized into a set of triangular cells (see Figure 2).

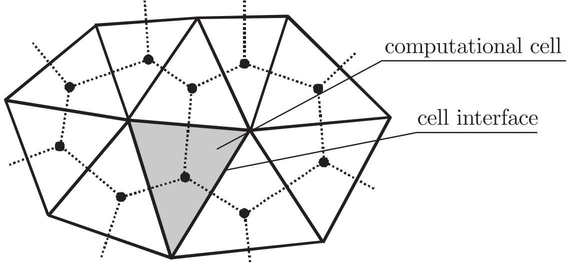

Figure 2. FVM discretization of the calculation domain

After integration and substitution of integrals with the corresponding sums, Equation (3) can be rewritten as:

$$\frac{\partial \boldsymbol{U}_i}{\partial t}\Delta A_i + \sum_{r=1}^{3}(\boldsymbol{F}_r n_r)\Delta L_r + \sum_{r=1}^{3} S_r \Delta A_r = 0, \tag{4}$$

where $\boldsymbol{F}_r$ is the numerical flux (computed at the r^{th} cell interface) and ΔL_r represents the cell-interface length. $\boldsymbol{S}_r$ and ΔA_r are the components of source terms and area of cell i assigned to the r^{th} cell interface.

The Roe scheme [10] is used to calculate the $\boldsymbol{F}_r$ fluxes. A detailed description of the method is available in the literature [7, 11] and has thus been omitted here. The source term vector $\boldsymbol{S}$ contains two kinds of elements, which respectively depend on bottom and friction slopes. Both of them involve numerical integration difficulties. In order to ensure proper bottom-slope approximation, the term is up-winded in the same way as fluxes $\boldsymbol{F}$. The friction source term is inconvenient with a flow of great velocity and small depth, observed during flood wave propagation on a dry floodplain after a levee breach. The splitting technique with respect to the physical processes has been applied in order to avoid the dry bottom problem. The numerical algorithm is completed with a two-step explicit scheme of the finite difference method for SWE integration in time. This scheme is of second-order accuracy in time and its stability is restricted by the Courant number. The stability condition for the 2D SWE can be written as follows [12]:

$$\mathrm{Cr} = \frac{\max\left(\sqrt{u_i^2+v_i^2}+\sqrt{gh_i}\right)}{\min(d_r)/\Delta t} \le \frac{1}{\sqrt{2}}, \tag{5}$$

where the i subscript denotes the given cell and d_r represents the distances between centre points of cell i and its neighboring volumes.

The described numerical technique was prepared and tested for simulations of flood wave propagation resulting from dam-break events in natural river valleys [7] and then adapted for modeling the flow in urban areas [13] and flood inundation in developed areas due to potential river embankment failures [14]. The computational code for numerical simulation of catastrophic floods was prepared at the Hydraulic and Hydrology Department of the Gdansk University of Technology. Herein, the model is used to predict the potential flooding event at Saska Kępa in Warsaw.

4. Data for the inundation model

When estimating the extent of flood zones, a digital model of terrain, including elevation, land cover and land use information, is needed in addition to a mathematical model of flood wave propagation,. In order to simulate a flood event after an embankment failure, hydrological data defining the initial and boundary conditions are also necessary.

The DEM data were prepared at the Hydrology Department of the University of Warsaw and by compilation from various sources. The geometry of the Vistula river channel was first measured in an echo-sounder survey carried out on 24^{th}–30^{th} September 1998 between the 490.76^{th} and 541.80^{th}km of the river [5]. On the basis of the obtained profiles, contour lines were drawn on a map by hand, considering the river bed forms and the more important water engineering structures. The information on the river channel's pattern and depth distribution was used to select the point of possible embankment failure.

The data on elevation of crests of the levees were originally prepared by Warsaw's Hydroprojekt design office. Tabular data were transferred to a system of orthogonal cartographic coordinates by interpolation. The calculations resulted in a set of coordinate values (x,y,z) describing the distribution of dike crest elevations on both river banks. The floodplain relief data were acquired from aerial photographs by photogrametric processing at the District Geodesic and Cartographic Department and the Office of the Surveyor of Mazowieckie Voivodship.

The data describing the course of the embankment, the relief of the floodplain and the river bottom were merged into a single file containing *ca.* 0.77 million points. On the basis of this set, the DEM was interpolated through a procedure of inverse distance and resolution of 20m (see Figure 3). During the flood simulation, the elevation was linearly interpolated inside each computational cell using the DEM data.

The digital model of land cover was prepared using the land-use maps and aerial photographs. The spatial distribution of the Manning roughness coefficient's values was estimated on the basis of the land cover model (see Figure 4). The following values of the roughness coefficient, n $[\mathrm{m}^{-1/3}\mathrm{s}]$, were adopted: 0.02 for the Vistula river channel, 0.04 for meadows, 0.045 for single trees, 0.051 for arable land, 0.055 for orchards, 0.12 for forests, 0.15 for osiers and bushes, 0.021 for roads and boulevards. In numerical simulations of flow in developed areas, buildings were included in the land cover model by applying a roughness coefficient of $0.1\,\mathrm{m}^{-1/3}\mathrm{s}$.

Additional data for numerical simulation of a flood event following a river embankment failure depends on the flow scenario assumed in the calculation. In the Saska Kępa case study, the embankment breach was assumed to be located at the 508^{th}km of the Vistula, on its right bank. The process of levee breaking was not analyzed in the simulation: a sudden and total collapse of the embankment segment was assumed. The crest level of the breach was accepted on the ground level. Finally, a rectangular shape of the breach was used, with the width of 100m and the depth equal to the levee's height. The embankment crest elevation near the breach location equaled 88.3masl.

The constant water level elevation in the river channel at the given $Q_{1\%} = 7210\,\mathrm{m}^3/\mathrm{s}$ was equal to 85.38masl. The flow in the river was neglected: the discharge and water level at the cross-section where the breach was located was based on previous calculations [4] using one-dimensional flow modeling.

The flow region's geometry is presented in Figure 5. It is composed of an area located outside the Vistula river's right levee and the 100m long breach in the embankment. The river channel and floodplain between embankments were excluded from the computational domain. The line along the river embankment (except for the breach) was assumed as the closed boundary of the computational domain. The other boundaries, including the breach, were assumed to be open.

The entire surface of the flood area was assumed to have been initially dry. The calculations were carried out with the time step $\Delta t = 1\,\mathrm{s}$ and the total simulation time was 12h.

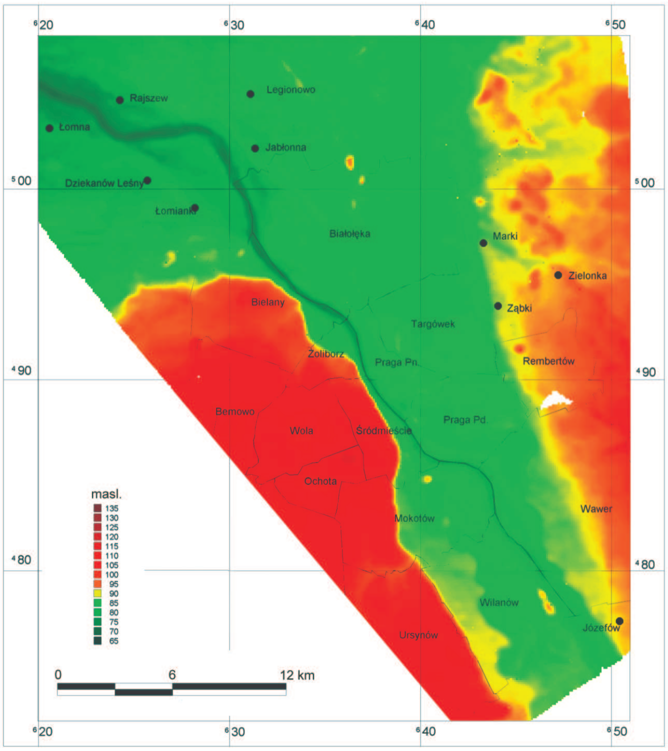

Figure 3. DEM of the Vistula river and Warsaw [15]

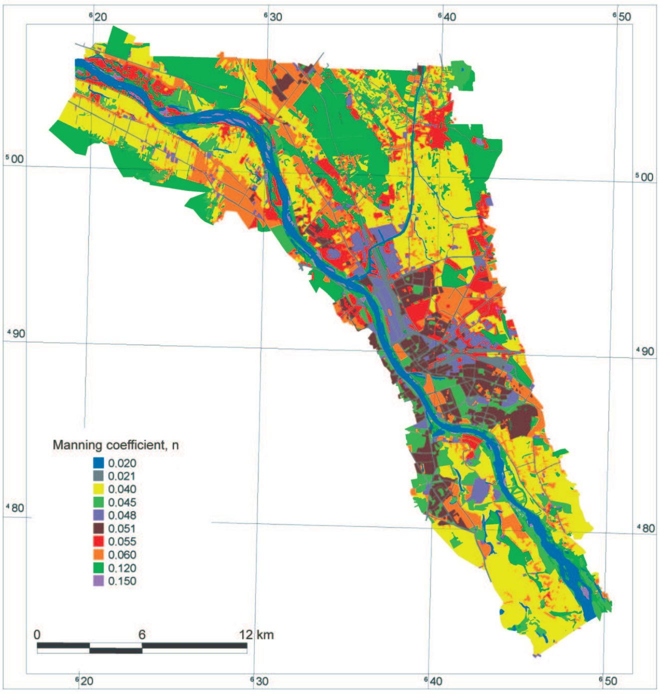

Figure 4. Distribution of the Manning roughness coefficient [15]

5. Numerical simulation

In order to simulate a flood event due to river embankment collapse in the vicinity of Saska Kępa, the domain of solution was covered with an unstructured triangular mesh composed of 15 278 computational cells (see Figure 6). The mesh was locally refined along the embankment to better represent the relief of the floodplain and the complex structure of the flow near the breach. In this region, the length of elements' sides equaled 20m. It increased to 250m with the distance from the levee.

The flood simulation results are presented in Figures 7 till 12, created as the background for flood risk mapping. Such graphical results and computed flow parameters can be directly used for identification of hazard zones. In the first two Figures (7 and 8), two time steps of the flood range evolution are presented: contours of water depth are shown against the area's topography background for two moments ($t = 3$h, $t = 6$h) after the river embankment's collapse. After levee breaking, water is released from the river and the flood wave propagates in many directions on the horizontal plane. The flood expands in accordance with the location of the embankment breach and the relief of the floodplain.

Information about arrival time of the flood wave front propagating across the inundated area is presented in Figure 9. The isochrones allow us to identify the zones of potential inundation in order to manage the risk in the floodplain. The colored areas represent zones which can be flooded in less than 15, 30, 45 minutes and 1, 2, 6, 12 hours. Such data is useful for developing emergency plans or maps and increasing public safety.

The flood's hydrodynamic parameters are presented in Figures 10 and 11. The distribution of maximal water depth after 12 hours of simulation is shown in the former. The predicted inundation can be compared with the flood range computed using a one-dimensional hydrodynamic model (marked as blue line in Figure 10) [4]. The flood zones fit each other quite well, following the floodplain's relief. However, the one-dimensional steady-flow simulation does not allow representation of the inundation zone's evolution. Clearly, Saska Kępa, located between the old and present river channels, and other areas in the Vistula valley are in a flood hazard zone. Finally, the absolute values of maximum flow velocities are presented in Figure 11. Extreme velocities – over 1m/s – can be observed near the breach. Significant values of flow velocity also occur along the old Vistula river channel, now separated from the main channel with an embankment. The predicted information allows drawing inundation maps for the floodplain and controlling development in the potential flood zone of the Vistula valley near Saska Kępa.

The computed parameters of a potential flood may also be useful in assessing the impact of flood on buildings' construction [16] and economic losses in urban areas [17]. In the Saska Kępa case study, the information about inundation of individual buildings was obtained by crossing the map of land cover with the flood inundation depth at time 12h. The accuracy of buildings' geometry corresponded to the newest edition topographic maps of Warsaw in the 1:10 000 scale, dated 1992. The most frequent depth of buildings' inundation in the studied area has been found to range from 0.5m to 1m (see Figure 12). The area of buildings inundated to the depth of 0.5–1m

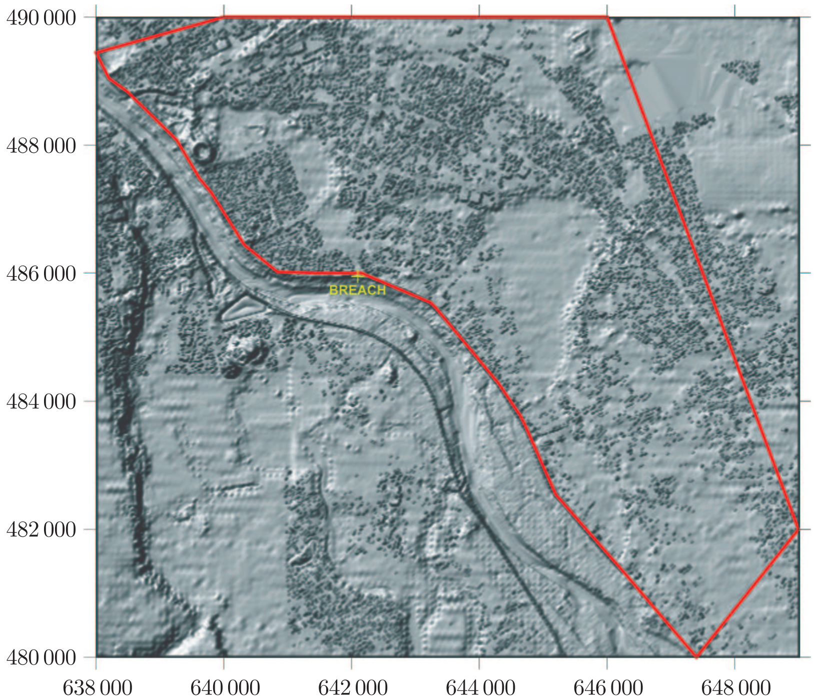

Figure 5. The geometry of computational domain for the Saska Kępa case study

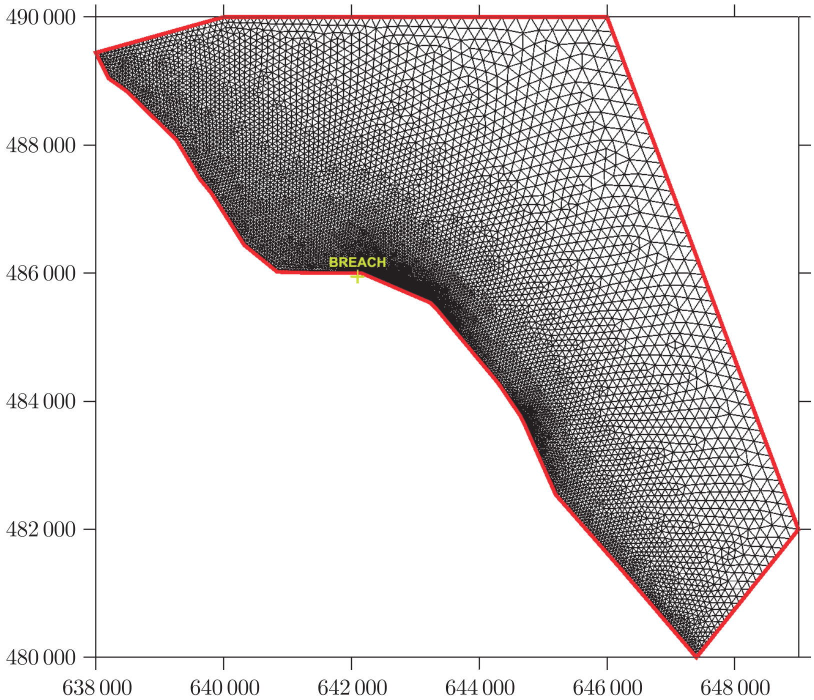

Figure 6. Numerical mesh

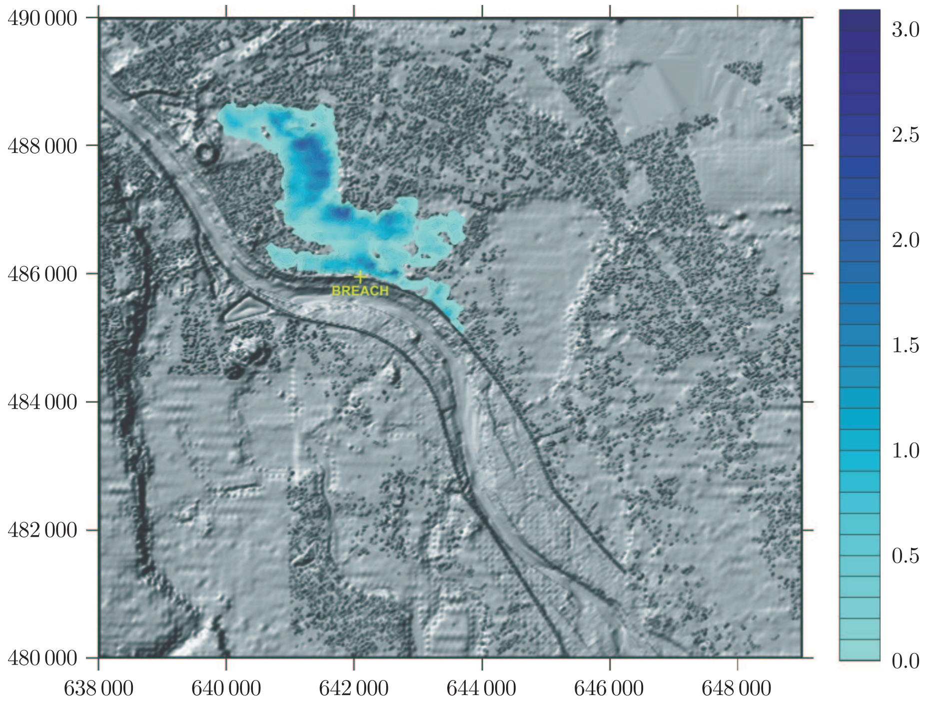

Figure 7. Computed contours of water depth [m] after $t = 3$h

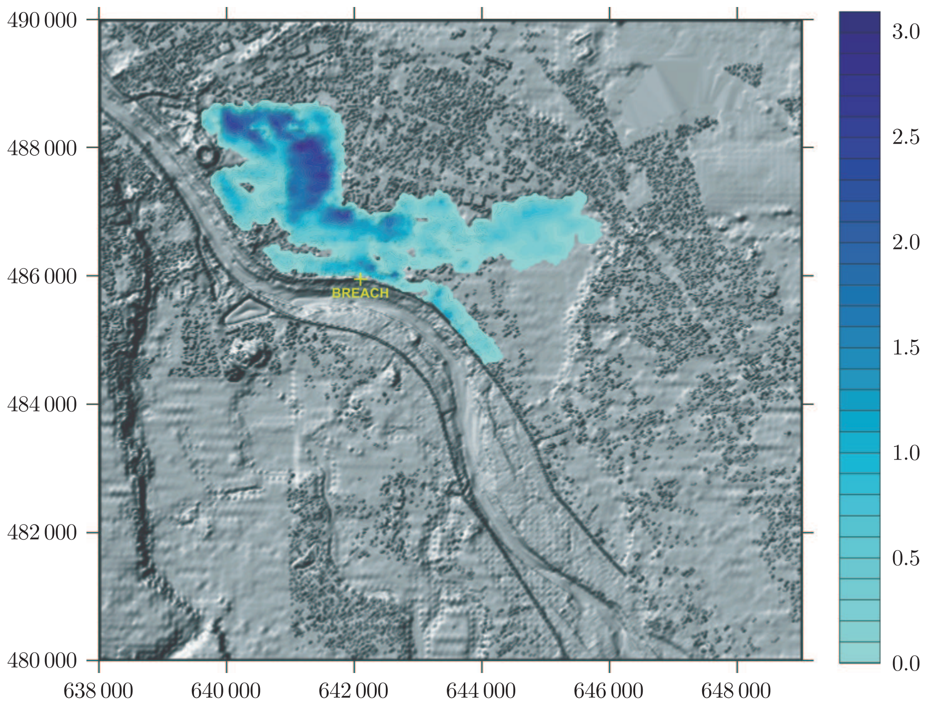

Figure 8. Computed contours of water depth [m] after $t = 6$h

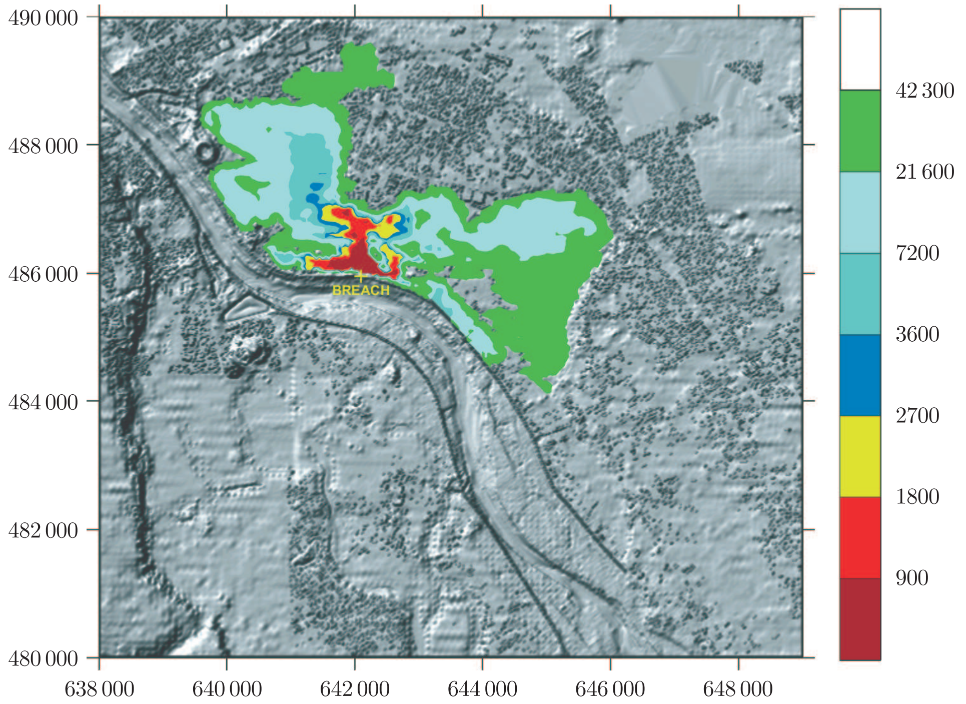

Figure 9. Time [s] zones of potential inundation

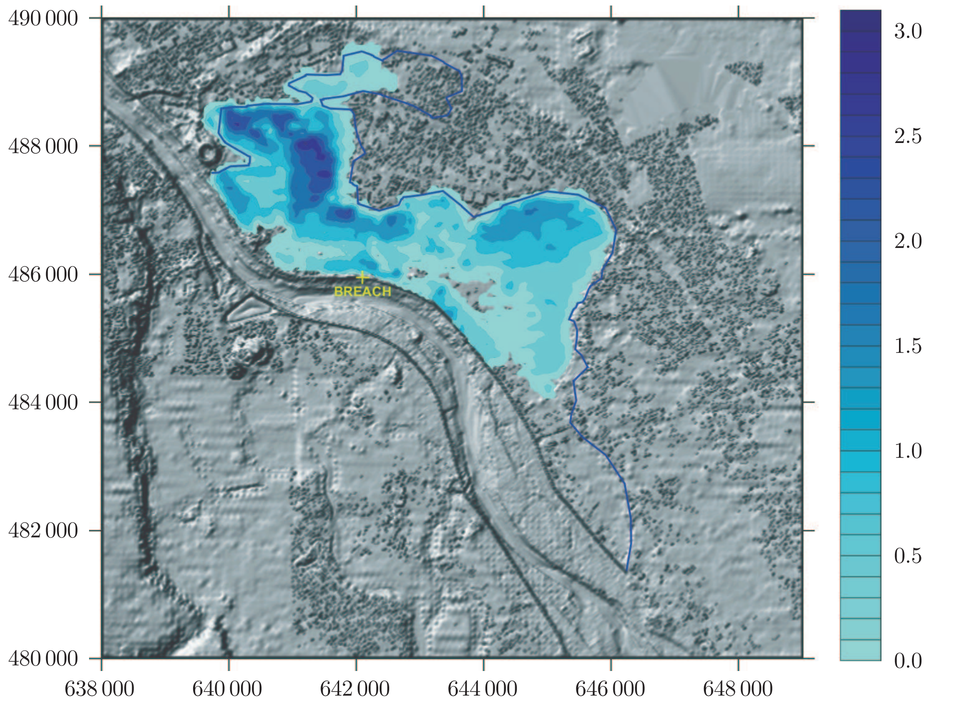

Figure 10. Maximum depth [m] and potential inundation range

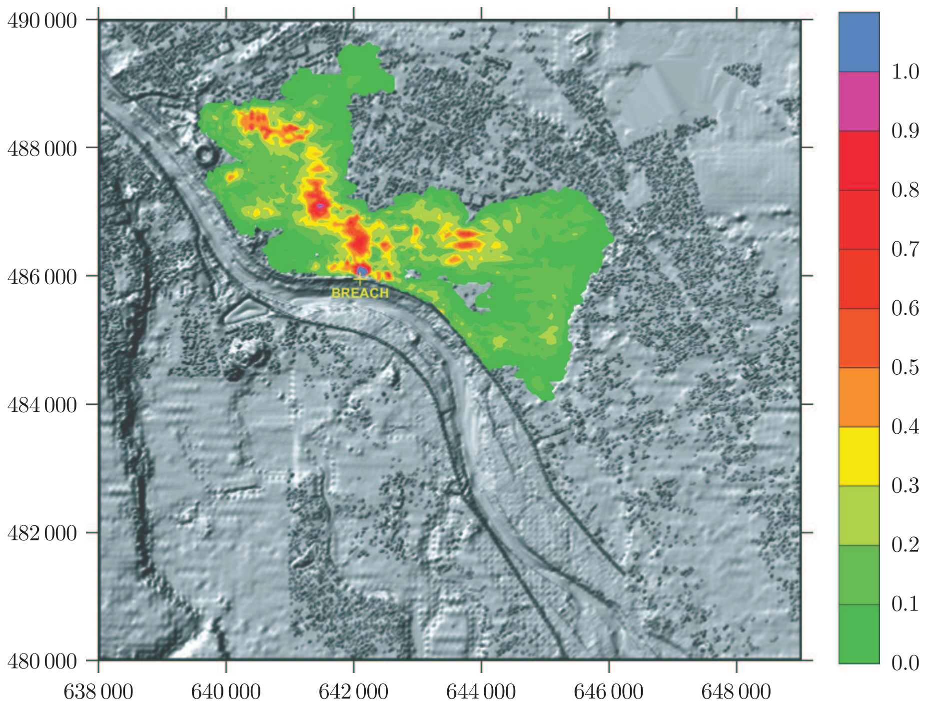

Figure 11. Maximum velocities [m/s] of the flood flow

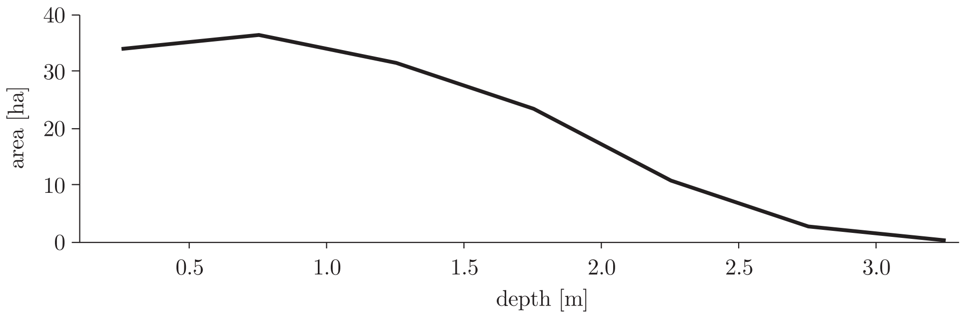

Figure 12. Area of buildings of various inundation depth at Saska Kępa

measured 36 ha. Spatial analysis of the model results enables indicating the area of the greatest flood losses.

6. Conclusions

A numerical simulation of flood inundation of the Saska Kępa floodplain in Warsaw resulting from a river embankment failure has been presented. A two-dimensional model depicts the involved dynamic flooding process accurately.

Flood zone maps are necessary for planning concentration and redistribution of population in the face of growing urbanization. A two-dimensional hydraulic model of flooding is a proper solution to analyze the spatial patterns of the risk of flooding and the resulting losses. The model includes the depth and duration of inundation, as well as velocities and directions of water flow. These parameters, combined with the information on buildings' height and value, enable approximation of flood losses in various scenarios.

Flood modeling is also very important in the context of EU legislation on preventing and limiting floods and their effects on human heath and the environment. A proposal for new directive on the assessment and management of flood risks of 25 April 2007 requires EU Member States to identify river basins and costal areas subject to flooding and take measures to reduce the risks involved. Under the proposed new rules, EU Member States will be required to complete preliminary flood assessments by 2011. The deadlines for compiling flood risk maps and devising management plans will be 2013 and 2015, respectively.

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