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Driving force of acoustic streaming caused by aperiodic sound beam in unbounded volumes

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Abstract

Instantaneous driving force of acoustic streaming in the thermoviscous medium is the subject of investigation. Dynamic equation of the Eulerian streaming velocity is a result of splitting the hydrodynamic equations into acoustic and non-acoustic parts. The acoustic force represents a sum of three parts, one is the classic one, which being averaged over the sound period coincides with the well-known expression. The second one is connected to the periodicity of the sound, it becomes exact zero after averaging for the strictly periodic sound but is not zero for other acoustic wave. The last term originates from the sound divergence. All terms are nonlinear and proportional to the overall attenuation. The consistent comparative analysis of both formula for quasi-periodic and modulated sound is proceeded.

Keywords: Instantaneous acoustic streaming, Radiation force, Non-linear sound propagation force
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1 Introduction

The term acoustic streaming refers to a bulk movement arising from the transfer of momentum from an acoustic field to a fluid. The well-understood origins of acoustic streaming are nonlinearity and attenuation. Nonlinear losses in momentum of intense sound cause solenoidal mean mass flow, which arises exclusively in the multi-dimensional flows. The theory of streaming deals as a rule with acoustic beam in the role of origin of the driving force. The reducing to the quasi-plane geometry allows to use series in small divergency parameter in seeking of governing equations. The problem of general description is still poorly analyzed even in the unbounded flow which is not affected by external forces, over initially uniform background.

The traditional method for successive separation of different types of motion consists in linear combination and averaging over the sound period of the continuity and momentum equations [1,2]. It presupposes temporal average over the sound period of quantity $\partial\rho/\partial t$ be equal zero, where ρ is total density. In the thermoviscous flows, excess density includes, among acoustic part, the slowly decreasing part originated from isobaric heating, so that the averaged value of $\partial\rho/\partial t$ is not longer zero. The important inconsistency of classical treatment is supposing that the fluid is incompressible [1]: streaming is itself generated by sound which can propagate because of fluid compressibility. The traditional method starts from the continuity and the Navier-Stokes equations in a viscous incompressible fluid. It does not account for energy balance and, therefore, discards thermal conductivity, though it is well-understood that streaming depends on total attenuation involving heat conduction [2,3]. The traditional method needs many intermediate discussions of individual roles of every term arising in the averaged acoustic force [1,2]. We can avoid that by means of instantaneous projecting of initial equations.

The present study continues investigations of acoustic streaming and heating basing on consistent division of conservative equations into specific parts [4,5]. The procedure includes the determination of all modes (or possible types of fluid motion) as links of hydrodynamic variables, perturbations of two thermodynamic quantities and components of velocity. These links originate from roots of dispersion relation of the infinitely small-amplitude flow, they are independent on time. The links, or, in the other words, polarization relations for vorticity, acoustic and entropy motions may be found in the Chapter 10.3 of [6]. The overall flow represents a superposition of these possible motions of a fluid. For correct description of nonlinear thermoviscous phenomena, the links should account for attenuation. The next important step is pointing out the ways to separate dynamic equation for every mode by linear combining of initial equations. This procedure is valid in any time. It may be formally proceeded by instantaneous projecting, which decouples modes in the linear part of equations but yields in nonlinear terms responsible for their interaction, including self-action (in the meaning of nonlinear terms of exclusively one mode) [4,5]. In the studies of acoustic streaming, a sum of nonlinear acoustic terms, which is a reason for vortex flow following intense sound, is called the driving force of acoustic streaming. The correspondence in the leading order of classical acoustic driving force and that obtained by projecting is demonstrated in the case of strictly periodic sound in Sec.2. The role of sound aperiodicity and effects of modulated sound are discussed in Sec.3.

2 Dynamic instantaneous equation of acoustic streaming in the thermoviscous unbounded flow

The continuity, momentum and energy equations for a thermoviscous fluid flow in an unbounded space without external forces read:

$$\frac{\partial\rho}{\partial t}+\overrightarrow{\nabla}(\rho\overrightarrow{v})=0,$$

$$\frac{\partial\overrightarrow{v}}{\partial t}+(\overrightarrow{v}\overrightarrow{\nabla})\overrightarrow{v}=\frac{1}{\rho}\left[-\overrightarrow{\nabla}p+\mu\Delta\overrightarrow{v}+\left(\mu_B+\frac{\mu}{3}\right)\overrightarrow{\nabla}(\overrightarrow{\nabla}\overrightarrow{v})\right], \tag{1}$$

$$\frac{\partial e}{\partial t}+(\overrightarrow{v}\overrightarrow{\nabla})e=\frac{1}{\rho}\left[-p\overrightarrow{\nabla}\overrightarrow{v}+\chi\Delta T+\mu_B\left(\overrightarrow{\nabla}\overrightarrow{v}\right)^2+\frac{\mu}{2}\left(\frac{\partial v_i}{\partial x_k}+\frac{\partial v_k}{\partial x_i}-\frac{2}{3}\delta_{ik}\frac{\partial v_l}{\partial x_l}\right)^2\right].$$

Here, $\vec{v}$ denotes Eulerian velocity of fluid, ρ, p are density and pressure, e, T mark internal energy per unit mass and temperature, μ_B, μ, χ are bulk, shear viscosities and thermal conductivity (all supposed to be constants), x_i, t - spacial coordinates and time. Two thermodynamic functions $e(p,\rho), T(p,\rho)$ should complete the system (1). Thermodynamics of ideal gases gives:

$$e(p,\rho)=\frac{p}{(\gamma-1)\rho}=C_vT(p,\rho), \tag{2}$$

where $\gamma=C_p/C_v$ is the ratio of the specific heats at constant pressure (C_p) and constant volume (C_v). Basing on the linearized version of Eqs. (1), the dispersion relations can be obtained for three independent "modes" of small-signal disturbances in an unbounded fluid, called the acoustic (two branches), vortex flow (two branches), and thermal (or entropy) modes.(Taking into account of boundaries and/or external forces would result to the more complex definition of modes.) In general, each of the field variables contains contributions from each of three modes, for example, $\overrightarrow{v}=\overrightarrow{v}_{ac}+\overrightarrow{v}_{ent}+\overrightarrow{v}_{vort}$. The method developed by the author gives possibility of consequent decoupling of the initial system (1) into specific dynamic equations for every mode basing on the specific properties of each mode in weakly nonlinear, thermoviscous and diffracting flow [4,5].

Our limited aim is an equation for acoustic streaming valid within accuracy up to the second order of a number of small parameters. The first is acoustic Mach number $M=v_0/c_0$, where v_0 is a typical particle velocity magnitude, c_0 is the infinitely-small amplitude sound speed. The next small parameters are dimensionless viscosities and thermal conductivity, $\beta=\frac{\mu}{\rho_0c_0\lambda}$, $\beta_B=\frac{\mu_B}{\rho_0c_0\lambda}$, $\delta=\frac{\chi}{\rho_0c_0\lambda}(\frac{1}{C_v}-\frac{1}{C_p})$, λ is characteristic sound scale, ρ_0 is static density. Weak diffraction presupposes smallness of $\epsilon=(\lambda/R_t)^2$, where R_t is a transversal scale of a flow, for example, a radius of a transducer. At last, sound is dominative, so that the ratio of particle velocities correspondent to sound and vortex flow, should keep small. All formulae everywhere below in the text, including links of modes and governing equation, are written on in the leading order. Following Lighthill [7], we choose to treat total attenuation $b=4\beta/3+\beta_B+\delta$ and M of comparable smallness, and we shall discard $O(b^2M)$ and $O(M^3)$ terms in all expansions. The resulting model accounts for the combined effects of nonlinearity, dissipation, and diffraction on three-dimensional sound waves and vortex flow.

Let y designates the nominal axis of the beam pointing in the propagation direction, and let (x,z) be the coordinates perpendicular to this axis. It is convenient to rearrange formulae in the dimensionless quantities as follows:

$$p'=\frac{p-p_0}{c_0^2\cdot\rho_0},\rho'=\frac{\rho-\rho_0}{\rho_0},\vec{v'}=\frac{\vec{v}}{c_0},x'=\frac{\sqrt{\epsilon}x}{\lambda},y'=\frac{y}{\lambda},z'=\frac{\sqrt{\epsilon}z}{\lambda}t'=\frac{c_0}{\lambda}t, \tag{3}$$

where p_0 is static pressure, c_0 is infinitely small amplitude sound speed.

Everywhere below in the text, primes by dimensionless quantities are dropped. There are five roots of dispersion relation and correspondent eigenvectors of the linearized system (1), including the entropy, or thermal mode. The details of establishing of links connecting the thermodynamic perturbations and velocity components may be found in the paper [4]. Every root of dispersion relation determines the links of Fourier-transforms of excess thermodynamic quantities and velocity components, which specify correspondent links in the $(\overrightarrow{x}, t)$ space. These links account for weak diffraction and attenuation. The acoustic field is represented by two branches, progressive in the positive and negative directions of y, marked by indices 1 and 2, correspondingly. The two branches of vortex motion in perpendicular planes $z = 0$ and $x = 0$, are also indexed by 1 and 2. The acoustic and vortex modes are as follows:

$$\psi_a = \psi_{a,1} + \psi_{a,2} = \begin{pmatrix} v_{x,a} \\ v_{y,a} \\ v_{z,a} \\ p_a \\ \rho_a \end{pmatrix} = \begin{pmatrix} \sqrt{\epsilon}\frac{\partial}{\partial x}\int dy \\ 1 - \frac{\epsilon}{2}\Delta_\perp \int dy \int dy - \frac{b}{2}\partial/\partial y \\ \sqrt{\epsilon}\frac{\partial}{\partial z}\int dy \\ 1 - \delta\partial/\partial y \\ 1 \end{pmatrix} \rho_{a,1} +$$

$$\begin{pmatrix} -\sqrt{\epsilon}\frac{\partial}{\partial x}\int dy \\ -1 + \frac{\epsilon}{2}\Delta_\perp \int dy \int dy - \frac{b}{2}\partial/\partial y \\ -\sqrt{\epsilon}\frac{\partial}{\partial z}\int dy \\ 1 + \delta\partial/\partial y \\ 1 \end{pmatrix} \rho_{a,2}, \tag{4}$$

$$\psi_{vort} = \psi_{vort,1} + \psi_{vort,2} = \begin{pmatrix} v_{x,vort} \\ v_{y,vort} \\ v_{z,vort} \\ p_{vort} \\ \rho_{vort} \end{pmatrix} = \begin{pmatrix} -\frac{\partial}{\partial y} \\ \sqrt{\epsilon}\frac{\partial}{\partial x} \\ 0 \\ 0 \\ 0 \end{pmatrix} \phi_{vort,1} + \begin{pmatrix} 0 \\ \sqrt{\epsilon}\frac{\partial}{\partial z} \\ -\frac{\partial}{\partial y} \\ 0 \\ 0 \end{pmatrix} \phi_{vort,2}.$$

where $\epsilon\Delta_\perp = \epsilon\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)$ is a Laplacian that operates in the plane perpendicular to the axis of beam. In evaluations of modes and correspondent projectors, the series of square root of Laplacian $\Delta = \partial^2/\partial y^2 + \epsilon\Delta_\perp$ is used: $\sqrt{\Delta} \approx \partial/\partial y + 0.5\epsilon\Delta_\perp \int dy$. The links of hydrodynamic perturbations inside every mode reflect the well-known properties of sound and vortices [6]:

$$\overrightarrow{\nabla} \times \overrightarrow{v}_a = \vec{0}, \quad \overrightarrow{\nabla}\overrightarrow{v}_{vort} = 0, \tag{5}$$

where $\overrightarrow{v} = (v_x \quad v_y \quad v_z)^T$, $\overrightarrow{\nabla} = (\sqrt{\epsilon}\partial/\partial x, \quad \partial/\partial y \quad \sqrt{\epsilon}\partial/\partial z)$ is the dimensionless divergency. The method of combining initial equations in order to derive governing equations for each type of hydrodynamic motion by means of projecting is discussed in details and employed to a single acoustic pulse in the paper by the author [4]. All five projectors which sum is the unix matrix, may be found there.

The projector on vortex motion is a matrix operator consisting of five rows and five columns:

$$P_{vort} = \begin{pmatrix} 1 - \epsilon\frac{\partial^2}{\partial x^2}\int dy \int dy & -\sqrt{\epsilon}\frac{\partial}{\partial x}\int dy & -\epsilon\frac{\partial^2}{\partial x \partial z}\int dy \int dy & 0 & 0 \\ -\sqrt{\epsilon}\frac{\partial}{\partial x}\int dy & \epsilon\Delta_\perp \int dy \int dy & -\sqrt{\epsilon}\frac{\partial}{\partial z}\int dy & 0 & 0 \\ -\epsilon\frac{\partial^2}{\partial x \partial z}\int dy \int dy & -\sqrt{\epsilon}\frac{\partial}{\partial z}\int dy & 1 - \epsilon\frac{\partial^2}{\partial z^2}\int dy \int dy & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{6}$$

It grants the requirements below in the leading order:

$$P_{vort}\psi_a = 0, \quad P_{vort}\psi_{vort} = \psi_{vort}. \tag{7}$$

In order of careful comparing with the classic results in the case of periodic sound, note, that the precise equality sounds:

$$P_{vort}\psi_{vort} = (v_{x,vort} \qquad (1+\epsilon\Delta_\perp \int dy \int dy)v_{y,vort} \qquad v_{z,vort} \qquad 0 \qquad 0)^T. \tag{8}$$

Collecting of $O(M)$ terms on the left, $O(M^2)$ on the right in the system (1), and acting P_{vort} on both sides, decouple perturbations in the linear part and yield in nonlinear "forces" reflecting modes interaction. Hence, the operator P_{vort} successfully extracts dynamic equation governing the vortex velocity. The vortex operator actually operates on momentum equation. The right-side "force" is automatically solenoidal.

In the context of acoustic streaming, a ratio of magnitudes of vortex and acoustic velocities is expected to be small, so that the largest, quadratic acoustic terms are kept in the right-hand side. They form the driving force of acoustic streaming. In any thermoviscous nonlinear flow, acoustic energy losses induces heating, which may input noticeably in the background density and temperature. The correspondent acoustic "source" of heating is proportional to the total attenuation, analogously to the acoustic driving force of streaming. Heating does not induce bulk movement of a fluid (though there exists weak movement with velocity proportional to the thermal conductivity [5]). This type of slow process is left of account in the present study. That means, that conclusions are true over temporal and spacial domains, where acoustic perturbations are dominant comparatively to both other slow modes, solenoidal and entropy.

Acting of the second row of P_{vort} at the momentum equation results in the dynamic equation for the longitudinal component of vortex flow velocity $v_{vort,y}$:

$$(1+\epsilon\Delta_\perp \int dy \int dy)\left(\frac{\partial v_{vort,y}}{\partial t} - \beta\Delta v_{vort,y}\right) = \begin{pmatrix} -\sqrt{\epsilon}\frac{\partial}{\partial x}\int dy \\ \epsilon\Delta_\perp \int dy \int dy \\ -\sqrt{\epsilon}\frac{\partial}{\partial z}\int dy \end{pmatrix}^T \cdot T, \tag{9}$$

where T is a vector consisting of three right-hand sides of momentum equation of order M^2 [4]. Nonlinear terms standing by attenuation originate from series of density $(1+\rho_a)^{-1}$ and thermoviscous links connecting $v_{a,y}$, p_a and ρ_a (4). The second-order term, originated from $(\overrightarrow{v}_{vort}\overrightarrow{\nabla})\overrightarrow{v}_{vort}$, may be removed to the left-hand side of the resulting equation, in order to remind the hydrodynamic nonlinearity. Though it is small compared with other terms, the important role of hydrodynamic nonlinearity in establishing of streaming was underlined in many studies [8-10].

For simplicity, only rightwards progressive sound is considered in the role of the driving force source ($\rho_a = \rho_{a,1}$:

$$(1+\epsilon\Delta_\perp \int dy \int dy)(\frac{\partial v_{vort,y}}{\partial t} - \beta\frac{\partial^2 v_{vort,y}}{\partial y^2}) + (\overrightarrow{v}_{vort}\overrightarrow{\nabla})v_{vort,y} =$$

$$\frac{\epsilon b}{2}\int dy \left(3\frac{\partial}{\partial x}(\rho_a\frac{\partial^2\rho_a}{\partial x\partial y}) + 3\frac{\partial}{\partial z}(\rho_a\frac{\partial^2\rho_a}{\partial z\partial y}) - 2\Delta_\perp \int dy(\rho_a\frac{\partial^2\rho_a}{\partial y^2})\right). \tag{10}$$

Links for acoustic mode (4) was used to express all perturbations through excess density. Considering the axial symmetry relatively to the axis y of beam propagation: $\rho_a(x,y,z) = \rho_a(r=\sqrt{x^2+z^2}, y)$, $\Delta_\perp = 1/r\partial/\partial r + \partial^2/\partial r^2$, it is easy to rearrange (10) into the following equation:

$$(1+\epsilon\Delta_\perp \int dy \int dy)(\frac{\partial v_{y,vort}}{\partial t} - \beta\frac{\partial^2 v_{y,vort}}{\partial y^2}) + (\overrightarrow{v}_{vort}\overrightarrow{\nabla})v_{vort,y} = \tag{11}$$

$$F_y = \frac{\epsilon b}{2}\left[\frac{3}{2}\left(\frac{\partial \rho_a}{\partial r}\right)^2 + \int dy \left(3\rho_a\Delta_\perp\frac{\partial\rho_a}{\partial y} - 2\Delta_\perp\int dy\rho_a\frac{\partial^2\rho_a}{\partial y^2}\right)\right]$$

In its form (11), the governing equation exhibits that absorption, nonlinearity and sound divergence are the origins of streaming. It is useful to establish the equivalence of the acoustic force from formula (11) and the well-known one for periodic acoustic wave coming from [1]. It should be taken into account, that excess acoustic density (and other quantities: pressure and velocity) of the rightwards progressive beam satisfy the famous Khokhlov-Zabolotskaya-Kuznetsov equation (the well-known version using the dimensionless retarded time $\tau = t - y$, which is convenient in the boundary regime problems, follows the first one in the brackets):

$$\frac{\partial\rho_a}{\partial t} + \frac{\partial\rho_a}{\partial y} + \frac{\epsilon}{2}\int\Delta_\perp\rho_a dy + \frac{\gamma+1}{2}\rho_a\frac{\partial\rho_a}{\partial y} - \frac{b}{2}\frac{\partial^2\rho_a}{\partial y^2} = 0, \tag{12}$$

$$\left(\frac{\partial}{\partial\tau}\left(\frac{\partial\rho_a}{\partial y} - \frac{\gamma+1}{4}\frac{\partial\rho_a^2}{\partial\tau} - \frac{b}{2}\frac{\partial^2\rho_a}{\partial\tau^2}\right) - \frac{\epsilon}{2}\Delta_\perp\rho_a = 0\right)$$

which may be derived consistently on the base of projecting [4]. Note the difference between the both forms of (12): the first one needs establishment of the integration constant accordingly to the physical meaning of the problem (generally, it is any smooth function of r and t). That applies as well to the terms forming the driving force below. For flows over media different from an ideal gas, the term $(\gamma+1)/2$ should be replaced by $1+B/2A$, where $B/A = (\rho_0/c_0^2)(\partial^2 p/\partial\rho^2)_s$ is the parameter of fluid nonlinearity, evaluated at the unperturbed state. Equation (12) allows to replace the Laplacian $\Delta_\perp$ acting on acoustic excess density in the following way:

$$\epsilon\Delta_\perp = \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial y^2} + O(M,b), \quad \epsilon\Delta_\perp = -2\frac{\partial^2}{\partial t\partial y} - 2\frac{\partial^2}{\partial y^2} + O(M,b). \tag{13}$$

In the leading order, these operators apply not only to the rightwards acoustic values V , but also to a product VW, if W also satisfies the wave equation for the rightwards progressive sound (12). The consequent replacing of operators in Eq.(11), and acting on the both sides by $(1-\epsilon\Delta_\perp\int dy\int dy)$ yield finally:

$$\frac{\partial v_{vort,y}}{\partial t} - \beta\frac{\partial^2 v_{vort,y}}{\partial y^2} + (\overrightarrow{v}_{vort}\overrightarrow{\nabla})v_{vort,y} = F_y = F_{y,class} + F_{y,1} + F_{y,2},$$

$$F_{y,class} = -b\rho_a\frac{\partial^2\rho_a}{\partial t^2},$$

$$F_{y,1} = b\frac{\partial^2}{\partial t^2}\int dy\int dy\left(\frac{1}{2}\rho_a\frac{\partial^2\rho_a}{\partial y^2} + \frac{3\epsilon}{4}\left(\frac{\partial\rho_a}{\partial r}\right)^2\right) + \tag{14}$$

$$b\left(2 - \frac{\partial^2}{\partial t^2}\int dy\int dy\right)\left(\frac{3}{4}\frac{\partial^2\rho_a^2}{\partial t^2} - \frac{\partial^2}{\partial t^2}\int dy\int dy\cdot\rho_a\frac{\partial^2\rho_a}{\partial y^2} + \frac{3}{2}\frac{\partial}{\partial t}\int dy\left(\frac{\partial\rho_a}{\partial y}\right)^2\right),$$

$$F_{y,2} = b\epsilon\left(\rho_a\Delta_\perp\rho_a - \frac{3}{2}\left(\frac{\partial\rho_a}{\partial r}\right)^2\right).$$

For the strictly periodic sound, averaged acoustic driving force may be rewritten in the form as follows (overbaring denotes temporal average over sound period, 2π in dimensionless quantities, $\overline{F_y} = \frac{1}{2\pi}\int_t^{t+2\pi} F_y dt$):

$$\overline{F_{y,periodic}} = \overline{F_{y,class}} + \overline{F_{y,2}} = b\left(-\overline{\rho_a \frac{\partial^2 \rho_a}{\partial t^2}} + \overline{\epsilon \rho_a \Delta_\perp \rho_a} - \frac{3\epsilon}{2}\overline{\left(\frac{\partial \rho_a}{\partial r}\right)^2}\right) \tag{15}$$

Formula (15) differs from the classic result $\overline{F_{y,class}}$ by the last two terms in brackets, which sum is a small value and equals $b\epsilon\overline{\rho_a \partial^2\rho_a/\partial r^2}$ at the axis of beam propagation. Since the celebrated formula describes the driving force in the leading order, both expressions on averaged force produced by periodic sound, are equivalent. Estimations predicted by the classic theory somewhat overestimate experimental data of streaming velocity in the cross section of acoustic beam [11,12]. The last two terms in brackets in Eq. (15) might correct this discrepancy. Simple estimation of additional terms for the Gaussian profile $\rho_a \sim \exp(-r^2)$ results is a quantity negative for every r.

Advance of projecting is instantaneous dynamic vectorial equation for acoustic streaming (Eq.(11) and its equivalent form Eq.(14) govern the longitudinal component) which applies to every type of sound (both periodic and aperiodic). An excess dimensionless acoustic density in the right-hand side (the driving force) may be replaced by acoustic pressure or axial velocity in view of links (4).

It is important, that acoustic force should be solenoidal in the leading order: $\overrightarrow{\nabla}\overrightarrow{F} = 0$, because $\overrightarrow{F}$ equals in the leading order $\partial \overrightarrow{v}_{vort}/\partial t$, and $\overrightarrow{v}_{vort}$ is solenoidal. The projecting supports this property. The vector force proportional to $-\overline{\rho_a \Delta \overrightarrow{v_a}}$ available in [8] is not obviously solenoidal. The majority of experiments deals with measurement of axial velocity of streaming which dynamics agrees with the classic theory predictions. As far as author knows, the transversal velocity is simply calculated basing on the axial one [3,8,10-12]. That is probably the reason for less attention to correct description of perpendicular to beam axis components of $\overrightarrow{F}$.

Consideration of this chapter was restricted by the acoustic field represented by the rightwards progressive beam, thought it may be easily expanded on leftwards one or any superposition of two acoustic branches, by means of collecting nonlinear terms correspondent to different acoustic branches and using links (4).

3 Examples of quasi-periodic diffracting beam

The complexity of mutual solution of (12), (14) is obvious. The limited aim of the present study, among deriving of governing equation (14), is to give simple illustrations on the role of aperiodicity in the acoustic driving force. In the role of sound, the two solutions, exact and approximate, of the linear wave equation without attenuation

$$\frac{\partial \rho_a}{\partial t} + \frac{\partial \rho_a}{\partial y} + \frac{\epsilon}{2}\int \Delta_\perp \rho_a dy = 0, \tag{16}$$

will be considered. The classic formula for strictly periodic sound will be compared with the total driving force, averaged over the approximate sound period, 2π in dimensionless quantities. Accordingly to (14), it is a sum of three parts:

$$\overline{F_y} = \overline{F_{y,class}} + \overline{F_{y,1}} + \overline{F_{y,2}}, \tag{17}$$

one being the classic force, the second which equals exactly zero for strictly periodic sound, and the last small part proportional to the beam divergence.

3.1 Quasi-periodic sound

The first example relates to the waveform:

$$\rho_a = \frac{\rho_{a,0}}{2}\left(-\frac{i}{1-i\cdot 2\epsilon t}\exp\left(-\frac{r^2}{1-i\cdot 2\epsilon t}+i(t-y)\right)\right)+cc. \tag{18}$$

In view of computational difficulty, the expansions in series in powers of ϵ are used in both source and driving force, that is valid inside the temporal domain ($0 \le t \le \epsilon^{-1}$). Simple evaluations yield in the series in diffraction ϵ as follows:

$$\begin{aligned}\overline{F_{y,class}} &= \rho_{a,0}^2(0.5+\epsilon(r^2-1)\exp(-2r^2)(\cos(2(t-y))-2)),\\ \overline{F_{y,1}} &= -\rho_{a,0}^2\epsilon\exp(-2r^2)(r^2-1)\cos(2(t-y)),\\ \overline{F_{y,2}} &= -\rho_{a,0}^2\epsilon(r^2+2)\exp(-2r^2).\end{aligned} \tag{19}$$

Hence, for approximately periodic sound (18), both additional terms are small, but the second one would result in constant negative value for all distances from the axis of beam propagation r, while the first one is periodic.

3.2 Periodic waveform with envelope

The second example is the periodic waveform multiplied by a slowly varying envelope:

$$\rho_a = \frac{\rho_{a,0}}{2}(1-\exp(-\Omega(t-y)))\left(-\frac{i}{1-i\cdot 2\epsilon y}\exp\left(-\frac{r^2}{1-i\cdot 2\epsilon y}+i(t-y)\right)\right)+cc. \tag{20}$$

The waveform (20) is a solution of the linear wave equation (16) with accuracy of the order $\mathrm{Max}(\epsilon\Omega,\epsilon^2)$. The values of illustrative parameters correspond to those often used in experiments. The carrying frequency of sound is $5MHz$, and the radius of transducer is $R_t = 0.01m$, that jointly with sound velocity in water $c = 1560m/s$ yield the small parameter $\epsilon = 0.001$. The dimensionless parameter Ω equals to reverse number of periods before establishing of waveform. In calculations of averaged parts of the driving force, the spacial derivatives of envelope are discarded. The calculations are trustable for $0 \le y \le \epsilon^{-1}$. Temporal behavior of parts of acoustic driving force at different distances from the transducer is shown by Fig. 1(a,b).

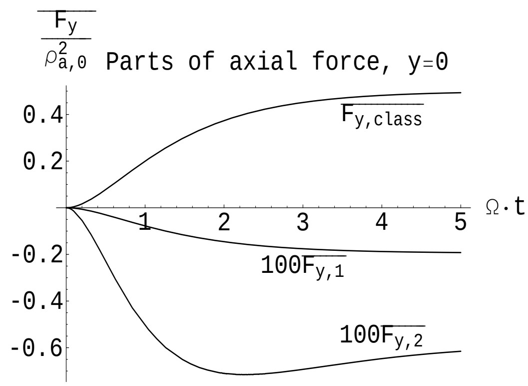

Fig.1a

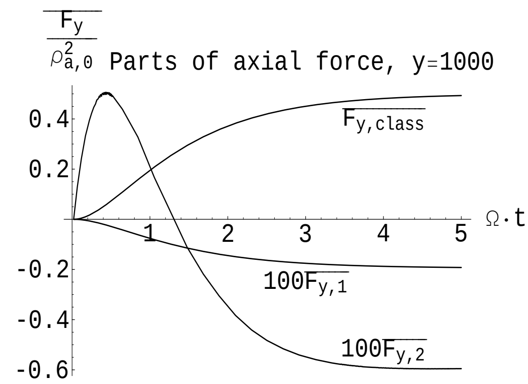

Fig.1b

Parts of the driving force of acoustic streaming, averaged over period of carrying sound at different distances from transducer in accordance to Eqs (14),(20).

4 Conclusions

The main theoretical result is the dynamic equation for acoustic streaming velocity (14) and representation of the driving force in the form convenient for comparison to classical result relating to strictly periodic sound. Approximate equations are thought as series in powers of small parameters. The first is the Mach number M, in order for the differential Navier-Stockes system to be valid. The second is characteristic ratio of amplitudes of acoustic and nonacoustic motions, the value varying with time. The last ones are total attenuation and diffraction.

We do not consider dynamics of acoustic streaming velocity, but only its driving force in the right-hand side of Eq.(14). The general solution of (12), and, therefore, joint solution of Eq.(12) and (14) are not available, and hardly expected to be found in the nearest future. The basic difficulty is nonlinearity in both equations. Tjotta and Naze Tjotta [9] pointed out that the hydrodynamic nonlinearity term (convective term) in the Navier-Stokes equation has the crucial effect on the streaming generation, particularly, in the focal and postfocal region. The role of nonlinear distortions of sound itself, originated from nonlinearity of conservation equations and equation of state, is also well-understood.

The rough illustrations discovering the role of aperiodicity in this study exploit the solution of linear wave parabolic equation for sound beam, though effects of nonlinearity and attenuation should be necessarily considered while deriving the driving force of acoustic streaming. The conclusion is that the difference of the instantaneous (after averaging over sound period) and classic formula of the driving force consists of two parts, one being a negative value proportional to the divergence ϵ, and the other is exact zero for periodic sound and different from zero for any other waveforms. In this last part, the weakly different from periodic sound is hardly expected to produce a noticeable difference comparatively to the classic formula. In the last decades, the attention to the aperiodic sound and caused by it phenomena grows. Many experiments appeared (including medical applications) dealing with aperiodic sources: series of pulses or modulated with slow function sound [13,14].

The possibilities of analytical methods in the study of such multidimensional dependencies are superior over that of experimental as well as numerical investigations. Analytic approach provides usually more flexibility, is less time consuming, and unlike other methods, is not constrained by fixed and limited set of values or varied parameters.

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