METHOD FOR IMPROVING MULTIBEAM SONAR BEARING ACCURACY

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The paper presents a simple method for improving multibeam sonar bearing accuracy. The principle proposed here is similar to the monopulse method, a solution commonly used in radars and sonars. With no manual or automatic beam rotation, the method offers a substantial reduction in the demand for sonar computational effort. It significantly reduces bearing error for a relatively high signal to noise ratio. The paper gives a boundary value of the output signal to noise ratio which when exceeded satisfactorily improves bearing accuracy.

INTRODUCTION

It is generally assumed that the bearing accuracy of multibeam sonars with delay-sum beamformers is equal to beam width. To improve accuracy, we can use high resolution methods for estimating spatial spectrum or the monopulse method. Because they are sensitive to noise and interference, spatial spectrum estimation methods are hardly ever used. Originally designed to generate two deflected beams for comparing their echo signal amplitudes, the monopulse method is more common. By rotating both beams echo signal amplitudes are eventually equalised and the angle at which the beams intersect determines the bearing. While today's digital signal processing technology has replaced manual rotation with automatic rotation, obtaining equal signals in both beams is still very much an iterative procedure with a lot of computational effort. The article presents a simpler and more practical method for determining more accurate bearings without having to rotate beams. Bearing is determined as a result of single calculations without equalising signals in deflected beams.

1. METHOD DESCRIPTION

We assume that the method for improving bearing accuracy will be applied in a sonar with a plane multi-element array with equally spaced *d* column centres. Let us assume that the number of columns *2M+1* is an odd number and the beams are deflected by the same angle approximately equal to the width θ_{3dB} of the narrowest central beam. The sonar emits "chirp" sounding pulses with linear frequency modulation and mid frequency f_0, bandwidth *B* and duration τ, propagating in water at velocity *c*. The beamformer generates a number of deflected beams with echo signal phase compensation at frequency f_0. Matched filters in the frequency domain are placed at beamformer outputs where we obtain the functions of echo signals correlated with the sounding pulse for each beam. The sonar operator watches the correlation functions and selects targets for more accurate bearing. This triggers off the computational procedure, which yields the targets' bearings. We will present the procedure for the simplest case with a single target in the central beam.

Quadrature sampling of echo signal produces *2M+1*, a sequence of complex samples *x(n,m)* which are the sum of the usable signal $S_0 s(n,m)$ and Gaussian noise where n means the sample number in time domain, and *m* – number of the channel (array column). Assuming that the distance between the centres of array columns d is equal to half the length of the acoustic wave for frequency f_0, we can assume with good approximation that the noises in the channels are not correlated. The sequences of samples are successively fed to the beamformer which generates two beams deflected from the central beam. The signals at beamformer outputs can be written as:

$$\begin{aligned} y_l(n) &= \sum_{m=-M}^{M} x(n,m)\cdot w_l(m) \\ y_r(n) &= \sum_{m=-M}^{M} x(n,m)\cdot w_r(m) \end{aligned} \tag{1}$$

where $y_l(n)$ are complex samples of signals in the left beam, $y_r(n)$ – in the right beam and $w_l(m)$ and $w_r(m)$ are beamformer coefficients for the left and right beam respectively. The coefficients are described with these formulas:

$$\begin{aligned} w_l(m) &= \exp(j\pi m \sin\beta) \\ w_r(m) &= w_l^*(m) \end{aligned} \qquad m = -M,....,M \tag{2}$$

where β is the angle of beam deflection relative to the direction of the central beam acoustic axis.

The outputs of both beams have filters matched to the sounding signal. The filtration applied in the sonar simulation is matched filtration in the frequency domain. It is equivalent to the correlation function when determined in the time domain. Signals $y_l(n)$ and $y_r(n)$ are transformed using the Fourier transform yielding two discrete spectra:

$$\begin{aligned} Y_l(k) &= \Im\{y_l(n)\} \\ Y_l(k) &= \Im\{y_l(n)\} \end{aligned} \tag{3}$$

The Fourier transform of the sounding signal is determined as well. Assuming that the signal is approximately equal to *s(n)=s(n,0)* we get:

$$S(k) = \Im\{s(n)\}\,. \tag{4}$$

Signals at the output of the matched filter are equal to:

$$
\begin{aligned}
z_l(n) &= \Im^{-1}\{Y_l(k)\cdot S^*(k)\} \\
z_r(n) &= \Im^{-1}\{Y_r(k)\cdot S^*(k)\}
\end{aligned} \tag{5}
$$

Formulas (2) show that the coefficients for the central beam *w(m)=1*. In the absence of noise, using formulas (1) and (3) we get:

$$Y(k) = (2M+1)S_0 S(k)\ . \tag{6}$$

Using formulas (5) we get:

$$z(n) = (2M+1)S_0\Im^{-1}\{S(k)\cdot S^*(k)\} = (2M+1)S_0\Im^{-1}\{|\,S(k)\,|\} = (2M+1)S_0 r_{ss}(n)\,, \tag{7}$$

where $r_{ss}(n)$ is the function of sounding signal autocorrelation.

The shape of the signal z(n) is shown in Fig. 1.

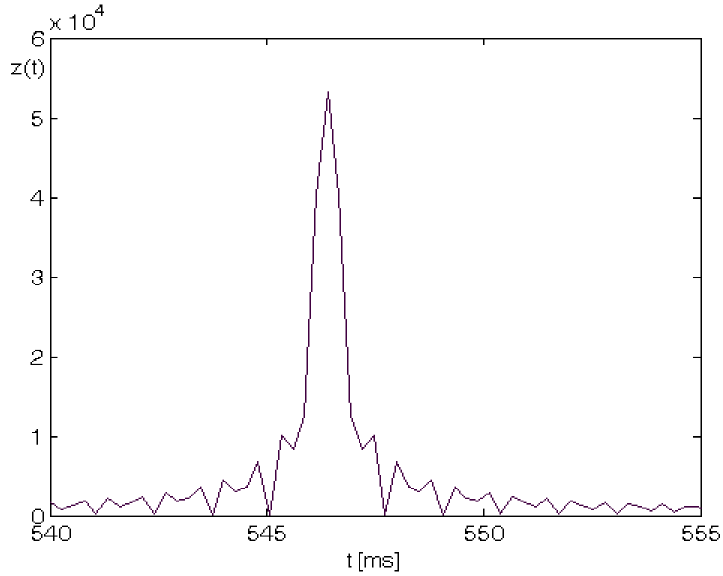

Fig.1. Signal at the output of a matched filter (f_0*=15 kHz, B=1.5 kHz, /=1.1s, 2M+1=13*)

While the signals in the right and left deflected beam are not fully correlated with the sounding signal, this does not significantly affect bearing accuracy as will be shown later.

In the absence of noise the maximal values of the signals $z_l(n)$ and $z_r(n)$ related to the maximum of the correlation function, depend on target bearing. The relation between maximal values of these signals and bearing θ are described with these beam patterns:

$$
\begin{aligned}
b_l(\theta) &= \frac{\max[z_l(n,\theta)]}{\max[z_l(n,\beta)]} \\
b_r(\theta) &= \frac{\max[z_r(n,\theta)]}{\max[z_r(n,\beta)]}
\end{aligned} \tag{8}
$$

Fig. 2 shows examples of beam patterns of deflected beams and the beam pattern of the central beam. Deflected beams are generated from a smaller number of array elements than the central beam, a requirement of the method in question. The same figure includes graphs of theoretical beam patterns with this formula:

$$b(\theta) = \frac{\sin[0.5\pi(2M+1)(\sin\theta - \sin\beta)]}{(2M+1)\sin[0.5\pi(\sin\theta - \sin\beta)]}. \qquad (9)$$

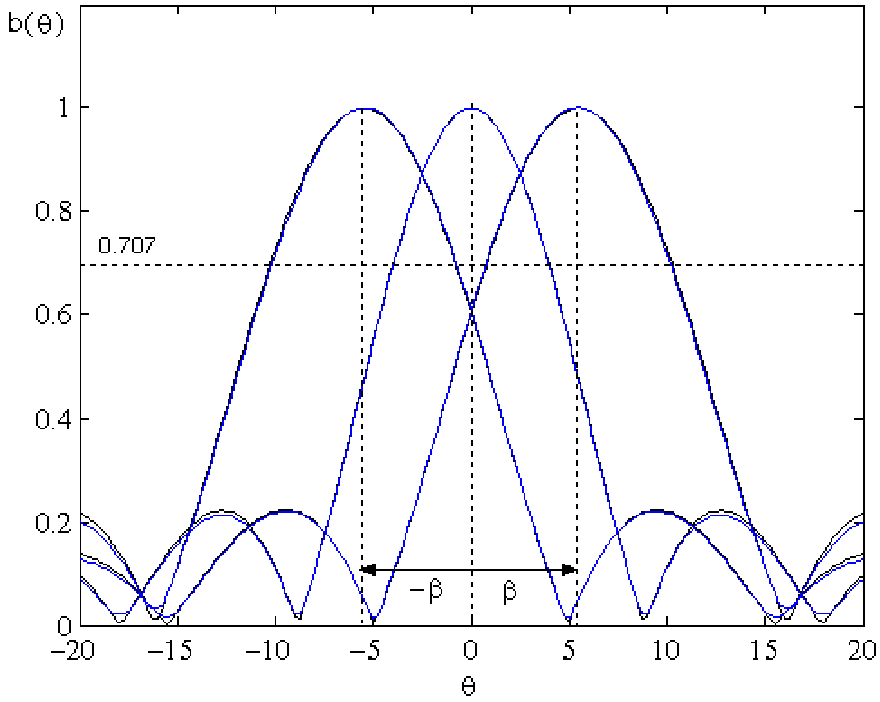

Fig.2. Beam patterns ($d=c/2f_0$, central beam $2M+1=13$, deflected beams $2M_d+1=11$, $\beta=5.5^0$)

As you can see from the figure, there are some clear deviations between the theoretical and real beam patterns within the side lobes. This range of angles is not used for further calculations. This explains the insignificance of incomplete echo signal and sounding signal correlation in the method in question.

The next step in the method is to determine the function $f(\theta)$ defined as:

$$f(\theta) = \frac{b_r(\theta) - b_l(\theta)}{b_r(\theta) + b_l(\theta)}. \qquad (10)$$

Fig. 3 shows the shape of the function using parameters given in Fig. 2. The same figure includes the beam pattern of the central beam. Function $f(\theta)$ was determined from the beam patterns calculated from the signals at matched filter outputs as:

$$f(\theta) = \frac{\max[z_r(n,\theta) - \max[z_l(n,\theta)]}{\max[z_r(n,\theta) + \max[z_l(n,\beta)]}. \qquad (11)$$

For a specific bearing θ the fractional expression on the right hand side of the above equation has a certain numerical value A in the range of (-1,1). Once it is calculated, bearing θ can be determined as:

$$\theta = f^{-1}(A), \qquad (12)$$

where $f^{1}(\cdot)$ is the inverse function of the function $f(\cdot)$ ($f(\theta)=f[f^{-1}(A)]$).

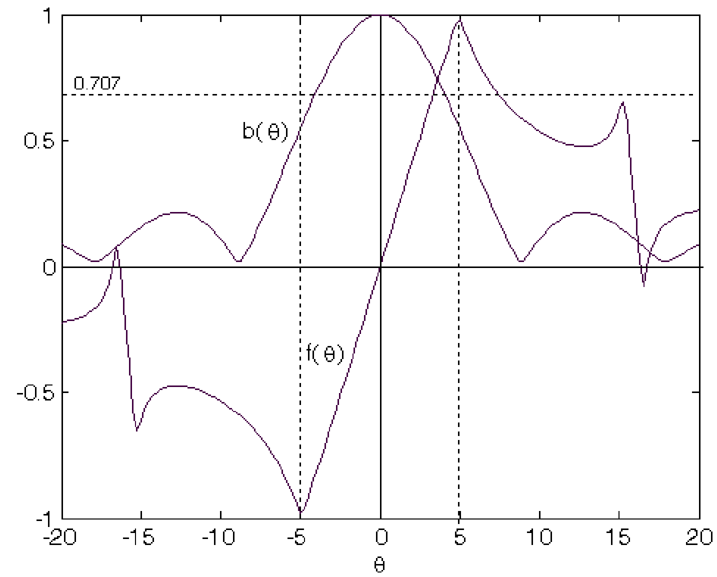

Fig.3. Function $f(\theta)$ and beam pattern of the central beam $b(\theta)$

To ensure that the calculation results are not ambiguous, the function $f(\theta)$ must be injective. This condition restricts the scope of the function $f(\theta)$ to the mid range which is almost linear. It must be wider than the range determined by adjacent intersecting beamformer beams. Because adjacent beams tend to intersect at –3dB or higher, we can assume that the range in question should be wider than the three decibel central beam. The width of the range depends on the width of deflected beams and their angle of deflection. Let us discuss how the parameters are determined.

Formula (10) shows that function $f(\theta)$ reaches 1 and –1 for angles θ_0 at which the beam patterns $b_l(\theta_0)$ and $b_r(\theta_0)$ are zero. We can determine the positive value of angle θ_0 from formula (9). It is equal to:

$$\theta_0 = \arcsin\left(\frac{2}{2M_d + 1} - \sin\beta\right). \tag{13}$$

where $2M_d+1$ is the number of array elements for deflected beams which does not have to be equal to the number of elements for the central beam .

Half of the three decibel width of the central beam is approximately equal to:

$$\theta_{edB} / 2 = \arcsin\left(\frac{0.88}{2M + 1}\right). \tag{14}$$

As has been said above, the angle θ_0 should be greater from the angle $\theta_{3dB}/2$. Using formulas (13) and (14) the relation can be written as:

$$\frac{2}{2M_d + 1} - \sin\beta > \frac{0.88}{2M + 1}. \tag{15}$$

The above inequality requires an additional criterion, the result of the desired gradient of the function $f(\theta)$. To minimise bearing error the gradient should be relatively high. Because the gradient decreases as the width of deflected beams increases, it means that it decreases when the number of array elements $2M_d+1$ decreases. In addition the nearly linear shape of the function is obtained when angle θ_0 is smaller than the angle of deflection of beams β. This condition produces the following inequality:

$$\frac{1}{2M_d+1} < \sin\beta \,. \tag{16}$$

Inequalities (15) and (16) give us the following condition:

$$\frac{1}{2M_d+1} > \frac{0.88}{2M+1} \tag{17}$$

If either of the inequalities is satisfied for $M_d=M$, the width of the usable range of the function $f(\theta)$ is not much bigger than the width of the central beam. To obtain the required reserve we can reduce the number M_d. The example in Fig. 2 assumes that M=6, and M_d=5. We then have:

$$0.0909 < \sin\beta < 0.114\,,$$

that is

$$5.2^0 < \beta < 6.5^0\,.$$

To obtain a wide usable range of the function $f(\theta)$ it was assumed that the angle ® is close to the lower boundary of the above range and amounts to β=5.5^0.

Fig. 4 shows the function $f^{-1}(A)$ determined from theoretical beam patterns (formulas (9) and (10)) and the linear approximating function $f^{-1}(A)$ with this relation:

$$f^{-1}(A) \cong A \cdot \theta_0\,, \tag{18}$$

where θ_0 is given in the formula (13).

As you can see from the figure, the approximating function is not really different from the function $f^{-1}(A)$ and can be used for determining bearing. In this example for bearing θ=1^0 bearing error is 0.018^0 and for θ=2^0 bearing error is equal to 0.033^0. The error can be reduced by correcting the formula (18):

$$f^{-1}(A) \cong A \cdot (\theta_0 - \delta\theta_0)\,. \tag{19}$$

For an experimentally selected correction $\delta\theta_0$=0.06^0, the error for θ=1^0 was reduced to 0.005^0, and for θ=2^0 – to 0.008^0. Errors as small as these are technically insignificant because they are much smaller from those caused by noise in the system.

2. BEARING ERRORS

Before we analyse the effects of noise on bearing error, we will first define the values that characterise noise in the system. As has been said in the previous section, we will assume that uncorrelated white Gaussian noise is present at the outputs of sonar array elements. The input signal to noise ratio refers to the output of a single array element (receiver channel) and is equal to:

$$SNR_x = \frac{P_x}{\sigma_x^2}, \tag{20}$$

where P_x is the power of the useful signal and σ_x^2 is the noise variance in the band of receiver B.

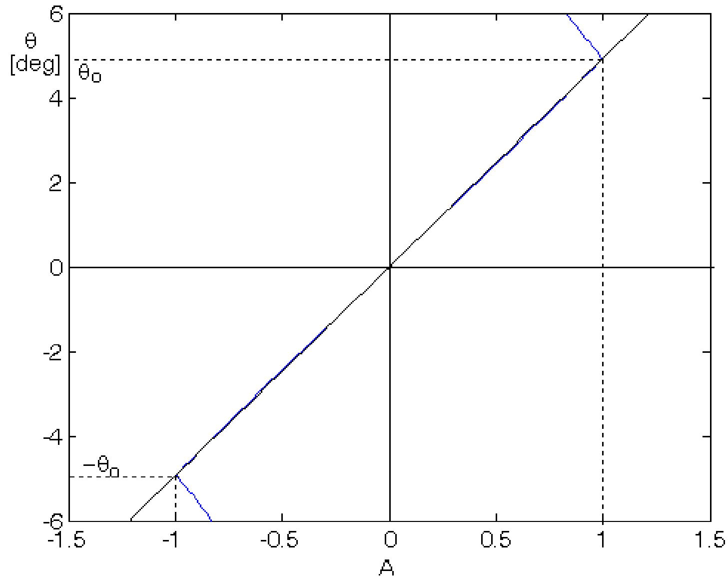

Fig.4. Relation between the bearing and number *A* (experimental curve –broken line, approximating relation – straight line)

Signal to noise ratio at beamformer output increases and with the wave incident from the direction of beam axis it amounts to:

$$SNR_y = (2M+1) \cdot SNR_x = (2M+1)\frac{P_x}{\sigma_x^2}, \tag{21}$$

where $2M+1$ is the number of array elements. The improved signal to noise ratio is the result of summing of signal amplitudes (signal power increases $(2M+1)^2$ times) and noise variances (noise variance increases $(2M+1)$ times).

The signal to noise ratio continues to improve at the output of the matched filter and amounts to:

$$SNR_z = \frac{[S_0 r_{ss}(0)]^2}{G r_{ss}(0)} = \frac{S_0 r_{ss}(0)}{G} = \frac{(2M+1)^2 P_x \tau}{G(2M+1)} = (2M+1)B\tau \frac{P_x}{\sigma_x^2} = B\tau SNR_y, \tag{22}$$

where G is the spectral noise power density and $BG=\sigma_x^2$.

In order to estimate bearing error, we could try and make the statistical parameters of error analytically dependant on statistical noise parameters. But because the system performs non-linear operations, the analysis becomes complicated and the results are not clear either. This is why we will present the results of simulation tests only. Developed in MATLAB environment, our programme simulates the functioning of a complete system with all the important parameters selected freely. In particular we can select signal amplitude, its band and duration, array parameters, noise variations and target bearing. Below are some selected results of the simulation and general conclusions about the effects of noise on bearing error.

Fig. 5 shows the spread of bearing for three different bearing values. The results include 1000 attempts for each bearing. The input signal to noise ratio is SNR_x =2.5, band width B=1500 Hz, pulse duration τ =68 ms, number of array elements $2M+1$=11. The output signal to noise ratio is determined from the formula (22) and is equal to SNR_z =2812. Mean values of the bearing are given in the figure and standard deviations of the bearing are: for $\theta=0^0$ – 0.082^0, for $\theta=1.5^0$ – 0.085^0, for $\theta=3^0$ – 0.104^0. If we assume that beam width is the measure of beamformer bearing error and a double standard deviation of the bearing is the measure of error in this method, the improvement in accuracy has been about 40 times the original accuracy.

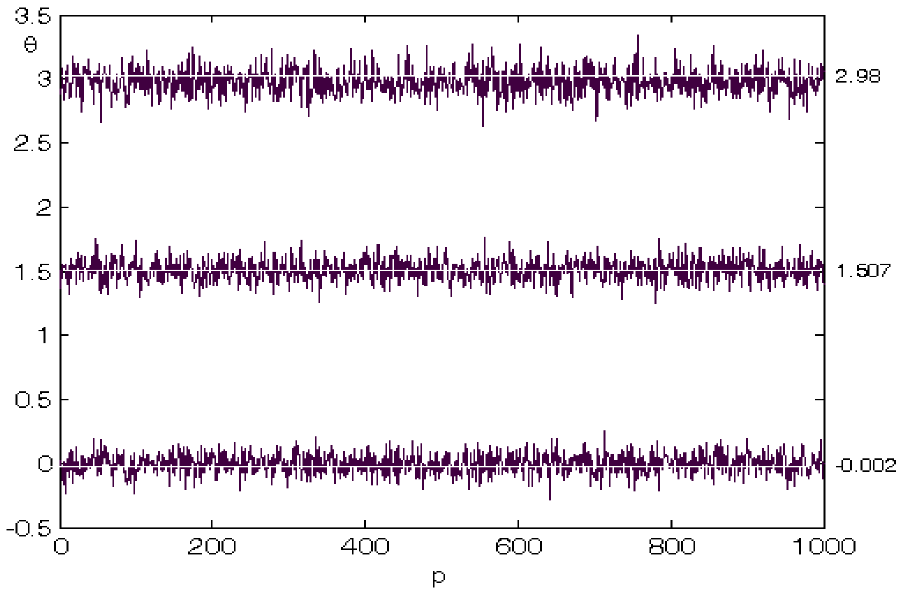

Fig.5. Bearing spread (p – number of test)

The simulation shows that all changes of the band width B, pulse duration τ,, signal amplitude and noise variance σ_x with a constant output signal to noise ratio SNR_z=2812 (34.5 dB) have practically no effect on the above standard deviations of bearing. This suggests that bearing error in this method depends on the output signal to noise ratio given in the formula (22).

For the above SNR_z the distribution of bearing probability density is similar to Gaussian distribution as is shown in the histogram in Fig. 6. It was determined based on 10000 attempts with the parameters as given in Fig. 5.

The simulation shows that as the output signal to noise ratio increases, bearing accuracy continues to improve. If we denote the bearing error variance from the above example as $\sigma_{\theta 0}^2$, the error variance as a result of the improved signal to noise ratio amounts to:

$$\sigma_{\theta 1}^2 = \sigma_{\theta 1}^2 \frac{SNR_{z0}}{SNR_{z1}}, \tag{23}$$

where SNR_{z0}=2812, a SNR_{z1} denotes an increased signal to noise ratio. As an example, for a tenfold improvement of the signal to noise ratio, the bearing variance also decreases ten times and standard deviation of the bearing decreases $\sqrt{10} = 3.16$ times.

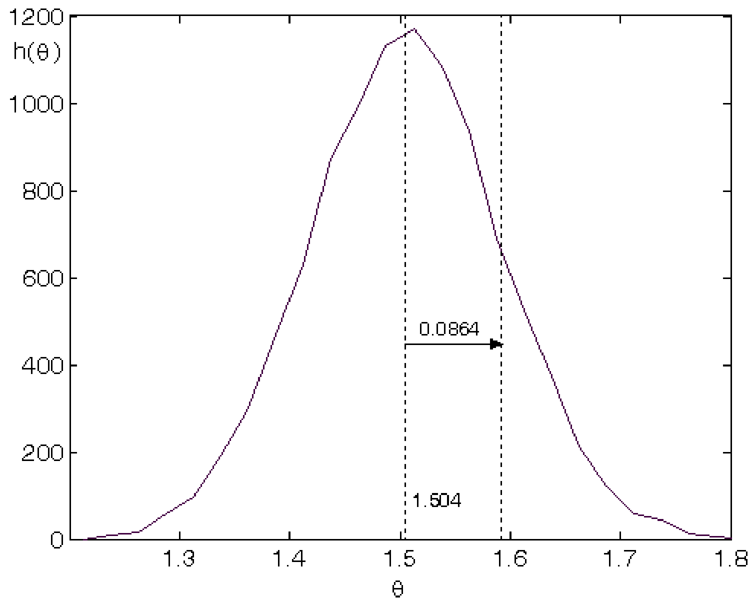

Fig.6. Distribution of bearing probability density (θ=*1.5*^{0})

A deteriorated output signal to noise ratio has two negative effects. It changes the mean value of bearing and increases its standard deviation. This is illustrated in Fig. 7 which shows the changes in the mean value of bearing (solid lines) and the curve of standard deviation of bearing (broken lines) in the function of output signal to noise ratio. When bearing is equal or close to the direction of the central beam's acoustic axis, its mean value does not in fact change while its standard deviation grows. The bigger the bearing, the faster are the changes in its mean value. While standard deviation does not increase much, determining bearing using this method is pointless when the output signal to noise ratio is lower by about 25 dB.

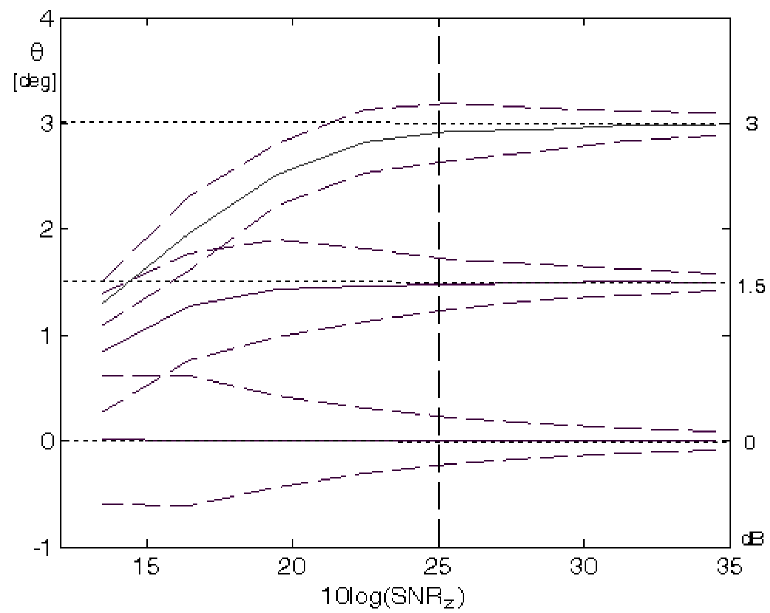

Fig.7. Mean values and standard deviations of bearings in the function of the output signal to noise ratio (θ=0^0 , θ =1.5^0, θ =3^0)

The reason why the mean value and standard deviation change is because the distribution of bearing error probability density changes as the output signal to noise ratio is decreasing. Fig. 8 shows an example of a histogram of bearing error probability density for a small signal to noise ratio. As you can see from the figure, the distribution of probability density is different from Gaussian distribution and its mean value has been significantly shifted.

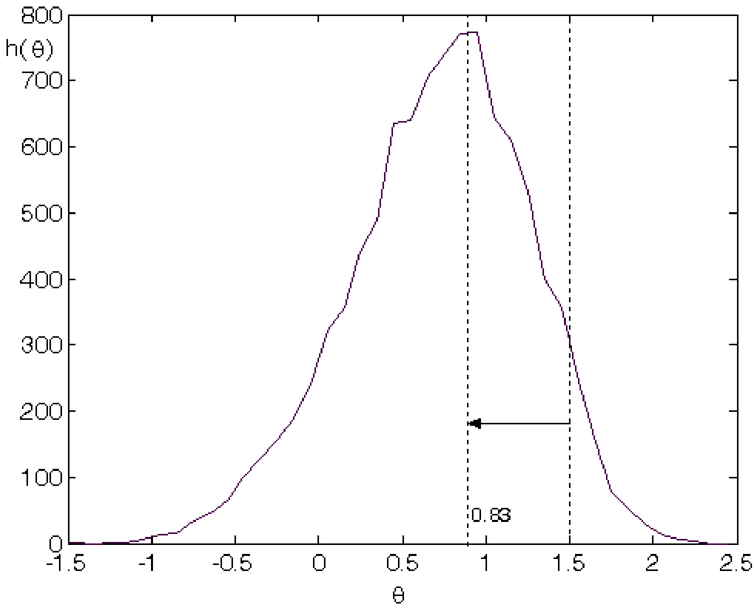

Fig.8. Distribution of bearing probability density (⟨ =1.5^0 , SNR_z=13.4 dB)

3. CONCLUSIONS

The simulation has shown that the proposed method for determining target bearing can be used in sonars when the output signal to noise ratio is relatively high. To ensure good detection conditions the output signal to noise ratio usually amounts to some ten or more decibels. Consequently, the method in question requires a signal to noise ratio higher by about 10 dB. This does not render the method useless because as a rule the requirements of output signal to noise ratio for target estimation (bearing in this case) keep growing. For a high signal to noise ratio, the method ensures a significant improvement in bearing accuracy. The advantage of the method lies in the simple algorithm for bearing determination without major computational effort.

Real multibeam sonars track several targets in deflected beams. The algorithm presented here would only have to be slightly modified for that purpose with some multiplication of the computational effort.

REFERENCES

[1] D. R. Rhodes, Introduction to Monopulse (Radar Library), Artech House Publishers, 1980.
[2] M. A. Richards, Fundamentals of Radar Signal Processing, McGraw-Hill, 2005.
[3] R. Salamon, Systemy hydrolokacyjne, Gdańskie Towarzystwo Naukowe, Gdańsk, 2006.
[4] W. Leśniak, J. Marszal, R. Salamon, M. Rudnicki, A. Schmidt, Digital mono-pulse method in cylindrical antenna, Archives of Acoustics, Vol. 31, 2006, No 4, s 357 - 363.