



POSSIBILITY OF VALUATION OF OPERATION OF MARINE DIESEL ENGINES

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Abstract

The paper provides a proposal of a quantitative interpretation of operation which (as the operation of Hamilton and Maupertius presented in the classical mechanics and the operation after change of the body momentum) is considered as a physical quantity with the measurement unit called a joule-second [joule×second]. An original method for analyzing and estimating the engine operation has been demonstrated in energetic aspect for operational needs. Herein it has been shown that the operation of this kind of engines considered in the proposed aspect enables obtaining essential information about energetic properties of the engines, that completes the information regarding energy conversion in the form of work and heat. Possibilities of analyzing the diesel engine operation have been demonstrated in deterministic and probabilistic aspects. Basing on deterministic aspect of operation of this kind of engines there have been presented possibilities of determining the operational usability of the engines through determining the possible operation and demanded operation to perform a task.

Keywords: operation, energy, technical state, diesel engine, Poisson process, semi-Markov process

1. Introduction

During operation of diesel engines we need to identify not only their technical states but also their energetic properties [5, 7, 8, 12, 13, 15]. The properties characterize the medium torque (M_o) and rotational speed of a crankshaft (n) of this type of engines. The torque M_o and rotational speed n (as measurable values) enable to define a useful power (N_e) [12, 13, 15]. The useful power (N_e) is a quantity that characterizes the stream of energy converted in a form of useful work (L_e) at a defined time (t). From this reason the work L_e can be interpreted as the output of the delivered power $N_e = \dot{L}_e$ at time t and therefore expressed with the formula:

$$L_e = N_e t . \quad (1)$$

From the formula (1) results that the power N_e is a quantity containing information how quickly the work L_e has been (or can be) performed by a combustion engine.

However in practice also a quantity is significant that provides information how long the work L_e must be delivered by engine to a receiver (screw propeller of a ship, generator,

compressor) in order the given task could be performed. This quantity can be called operation [5, 6, 7]. Because each type of work being performed by engine (ex. useful work, compression, expansion, etc.) is a form of energy conversion, thus just understood operation (D) is a quantity expressing the energy (E) released over the time (t), and that is the reason it can be defined (when $E = idem$ can be accepted) with the formula:

$$D = Et. \quad (2)$$

The operation (2) determines thus the energy released over the time during which this energy has been consumed. When the engine wear is considered, the operation equals to the energy drop (decrease) at time at which it proceeded [4, 8, 11]. The energy can reveal only when converted into form of work or heat [2, 6, 8, 13, 14].

In case of any diesel engine the useful energy (E_e) generated by the engine with a defined useful power (N_e), in strictly determined conditions, can be considered as a measure of its ability to perform the work L_e at a defined time t . Therefore the work as a form of energy conversion, generated by the engine, can be defined from the formula [13]

$$L_e = 2\pi n M_o t, \quad (3)$$

in the case when: $M_o = idem$ and $n = idem$.

When $M_o \neq idem$ and $n \neq idem$ the work performed in the time interval $[t_1, t_2]$ can be presented in the form of dependences:

$$L_e = 2\pi \int_{t_1}^{t_2} n(\tau) M_o(\tau) d\tau. \quad (4)$$

In the operating practice of diesel engines (main engines) being applied to marine propulsion systems it is extremely important how long the work L_e can be released for the needs of the propulsion system of the given ship. This refers especially to the ships of which propulsion systems are equipped with such engines. In the case when due to the wear, the main engine cannot be loaded with the demanded useful power (N_e) at time t , it is not able to perform in this time the demanded work L_e needed to ensure generation of the demanded pressure force (T) by the screw propeller of the ship. In consequence the ship is not able to perform the transportation task. Moreover, when the cruise runs in storm conditions, it can lead to a catastrophe [9].

From the above considerations follows that it is reasonable to analyze not only the power N_e released in diesel engine's workspaces, and simultaneously the work L_e , but also the operation (D) of this type of engines, understood in this case as energy conversion in these workspaces that leads to obtaining the demanded useful work (L_e) at a defined time (t). This will enable to fix whether the possible engine operation (D_M) for the given conditions is at least equal to the demanded operation (D_W) being indispensable to perform a defined task Z .

2. Diesel engine operation as energy conversion in the form of heat and work

Operation of engines consists in converting and transferring the supplied energy. In case of diesel engines, first the chemical energy contained in fuel-air mixture, generated in



workspaces, is converted into thermal energy and then the thermal energy – into mechanical energy [2, 5, 12, 13, 14].

It is obvious that the energy conversion in the form of heat in workspaces of each diesel engine can proceed at a different time. In practice it is essential to make the performance of the work as greatest as possible or as quickly as possible at a defined time. If it is not possible to obtain such energy conversion which is favorable the engine is considered to work incorrectly and to be in the state of partial usability [7, 12, 14].

In case of diesel engines, conversion of chemical energy into thermal energy and then into mechanical energy, enables creation of a torque (M_o) of a crankshaft at a defined rotational speed (n) of each engine [12, 13]. Thus, the operation of engine, interpreted as energy conversion in form of useful work L_e expressed with the formula (4) can be defined by the equation as follows

$$D_{L_e} = \int_{t_1}^{t_2} L_e(\tau) d\tau = 2\pi \int_{t_1}^t n(\tau) M_o(\tau) \tau d\tau \quad (5)$$

Engine operation connected with energy conversion in the form of work like compression of fresh charge, expansion of combustion gases in a cylinder, etc. can be considered in a similar way.

Determination of engine operation consisting in conversion of chemical energy (E_{ch}) contained in fuel-air mixture generated in engine combustion chambers into thermal energy (E_c) is equally important. Such operation (Fig. 1) when conversion of this kind of energy proceeds in the form of heat (Q) can be defined by the formula:

$$D_Q = \int_{t_1}^{t_2} Q(\tau) d\tau \quad (6)$$

Because the operation of this kind of engines consists in converting the energy E in the form of work and heat, can be generally interpreted as follows

$$D = \int_{t_1}^{t_2} E(\tau) d\tau \quad (7)$$

where:

D – engine operation, E – converted (obtained) energy enabling realization of a task Z ,
 t – time of E energy conversion (consumption).

Usability of particular combustion engines can be inferred after making value calculations of their operations (7) which are, in the interpretation proposed herein, equaled to physical quantities with the measurement unit: „joule-second”. Apparently, the functional dependence of energy from time, so $E = f(t)$ must be known in order to determine the field of operation (D). Because $D = f(E, t)$ the operation of machines can be presented in the coordinate system „ $D-E-t$ ” [5, 6, 7].

Such understood operation defined by the formula (7) can be presented in the coordinate system „ $E-t$ ” so in the form of graph which I propose to call *operation graph*. An example of such an operation graph for the range of the energy transformation from E_1 into E_2 for any selected time $t_0 = 0$ and t is presented in Fig. 1.

From the formulas (2) and (7) follows that the functional dependence of energy (E) from time (t) must be known in order to determine the operation field (D). Because $D = f(E, t)$, the machine operation can be displayed in the co-ordinate system „ D, E, t ”.

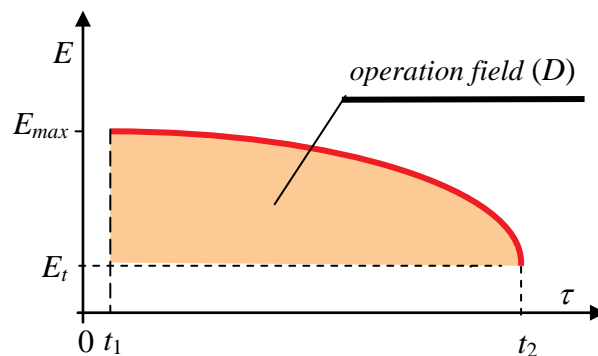


Fig. 1. An exemplary graph of engine operation: E [] energy, E_{max} – maximum energy, E_t – energy in the moment t , τ [] time

Diesel engine operation can be and sometimes must be considered as a stochastic process [1, 3, 6, 7]. Such operation can be then displayed in a form of stochastic process realization as the dependence $\{E(t); t \geq 0\}$, where energy E is a random value. The process is characterized by the expected value $E[E(t)]$ and the standard deviation $\sigma[E(t)]$ of energy E . Such approach follows from that the analysis and the resulted estimation of the operation of diesel engines can be presented in a probabilistic aspect by applying the theory of the stochastic processes. An exemplary graph of such engine operation is show in Fig. 2.

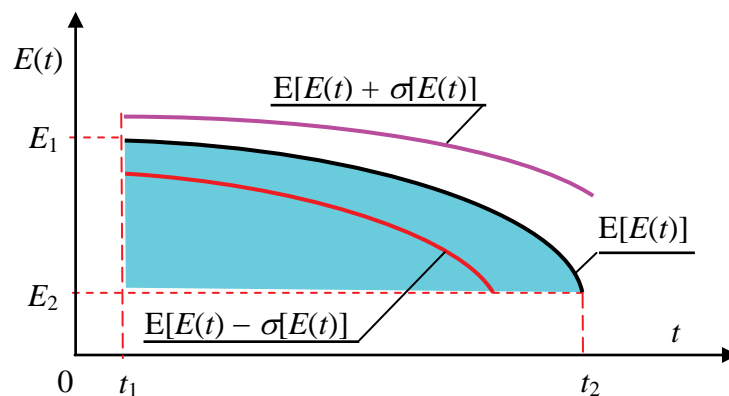


Fig. 2. An example of a stochastic process showing the dependence $E(t)$, where E is a random value: E [] energy, E_1 – energy assigned to time t_1 , E_2 – energy assigned to time t_2 , t [] time being a parameter of the process, $E[E(t)]$ – expected value E , $\sigma[E(t)]$ – standard deviation of E

A stochastic process is a random function of which the parameter is the time t . The time is not a random variable [1, 3]. This approach towards the issue of expressing the diesel engine operation as a value, results from the necessity of getting information what the operation can be in the interval defined by two arbitrary moments, ex. in the interval $[t_0, t_n]$. In this case, analyzing operation of each combustion engine, each time t from the considered time interval $[t_0, t_n]$ can be assigned by a state called momentary state of the process, which is a random variable X_t with the expected value $E(X_t)$ and variation $D^2(X_t)$ dependent from the t value. For the considerations the variable can be energy (E) or forms of its conversion, so

work (L_e) or heat (Q). Thus the stochastic process (a random function) is a set of random variables X_t for $t \in [t_0, t_n]$, so for $t_0 \leq t \leq t_n$. The function's expected value $E[X(t)]$ and variation $D^2[X(t)]$ are defined by the sets of expected values $E(X_t)$ and variations $D^2(X_t)$ for $t_0 \leq t \leq t_n$. It should be pointed here that the expected value $E(X_t)$ and the variation $D^2(X_t)$ of the random function $\{X(t): t \geq 0\}$ depend on time t because the values $E(X_t)$ and $D^2(X_t)$ can be different for different t values. They are not, however, random functions $X(t)$ because $E(X_t)$ and $D^2(X_t)$ are not random variables but the constants for the given t value and the given set of realizations of the random variable X_t [3].

An example of dependences of $E(X_t)$ and $D^2(X_t)$ from time t is shown in Fig. 2, at the assumption that the random variable X is the energy E supplied by a combustion engine to a receiver of the energy. In this Fig. the $\sigma[E(t)]$ quantity is a standard deviation of the random variable E . This quantity is a square root of the variation $D^2(E_t)$.

In this case to define the operation D from the formula (7) the integral calculus can be applied because the integral defined by the formula is a definite Riemann integral with the integration range $[0, t]$ and the integrand $E(\tau)$. Because the function $E(\tau)$ is continuous for the examined exemplary range $[0, t]$, it can be stated in compliance with the second fundamental theorem of integral calculus (Newton-Leibniz Theorem) that [16]

$$\int_{t_1}^{t_2} E(\tau) d\tau = D(\tau) \Big|_{t_1}^{t_2} \quad (8)$$

whereas:

$$D(\tau) \Big|_0^t = D(t) - D(0)$$

Estimation of the expected value $E(E_t)$ for each value of time t requires application of statistical inference, so point or interval estimation [1, 3].

From the presented interpretation of combustion engine operation follows that the operation consists in converting and transferring the released energy in the form of work (L) and heat (Q), whereas both of the forms of energy conversion can be presented as the fields [13, 14]:

- in the Clapeyron diagram (diagram of work) when analyzing the work L (Fig. 3 and 4),
- in the Belpaire diagram (diagram of heat) when analyzing the heat Q (Fig. 5).

For instance, in case of a piston engine the absolute work (L_a) of exhaust decompression (i.e. work determined in relation to the ambient pressure $p_0 = 0$), of which the field is displayed in Fig. 3a in the p - V co-ordinate system, called the Clapeyron diagram, can be calculated from the formula

$$L_a = \int_{V_1}^{V_2} p(V) dV \quad (9)$$

where: p – pressure, V – volume.

That means that the integral (9) can be determined when the functional dependence $p = f(V)$ is known. The technical work of exhaust decompression (L_t), of which the field is displayed in Fig. 3b can be determined from the formula

$$L_t = - \int_{p_1}^{p_2} V(p) dp \quad (10)$$



Also in this case the integral (10) can be determined when the functional dependence $V = f(p)$ is known. The technical work of exhaust decompression can be however determined also by employing the definition of absolute work. Then the following formula is of application (Fig. 3)

$$L_t = p_1V_1 - p_2V_2 - \int_{p_1}^{p_2} V(p)dp \quad (11)$$

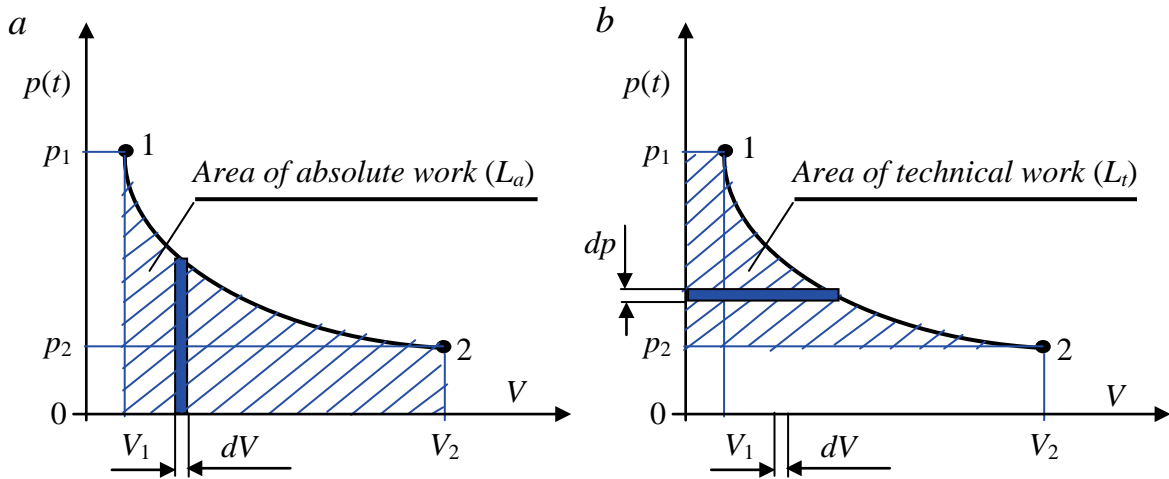


Fig. 3. Graphical examples of work: a) absolute, b) technical: p – pressure, V – volume

Taking into account that the work of exhaust decompression in the piston engine space is performed till the moment of opening the exhaust valve, when the exhaust gases are removed outside to the environment with the pressure $p_o = p_b$, the useful work can be then considered (Fig. 4).

The mentioned useful work (L_u) can be defined (Fig. 4) from the formula

$$L_u = \int_{V_1}^{V_2} p(V)dV - p_b(V_2 - V_1) \quad (12)$$

where: p – pressure, V – volume, p_b – barometric pressure (ambient pressure).

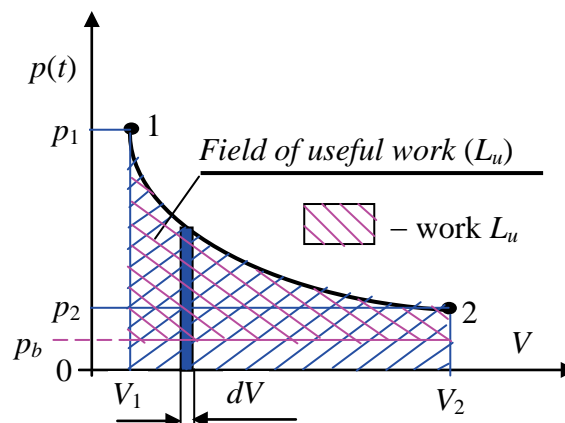


Fig. 4. An exemplary graph of useful work: p – pressure, V – volume, p_b – barometric pressure

A field of any other work (e.g. air compression in a cylinder, indicated work, useful work, etc.) can be presented in a similar way.

The carried away heat can be shown (Fig. 5) in the form of a graph by using the „ $T - S$ ” co-ordinate system (in the Belpaire diagram). The mentioned heat (Q) can be determined (Fig. 5) from the formula:

$$Q = \int_{S_1}^{S_2} T(S) dS \quad (13)$$

where: T – absolute temperature, S – entropy.

The following restrictions must be taken into account for the formula (13):

$$S_1 \leq S \leq S_2; \quad T_2 \leq T \leq f(S)$$

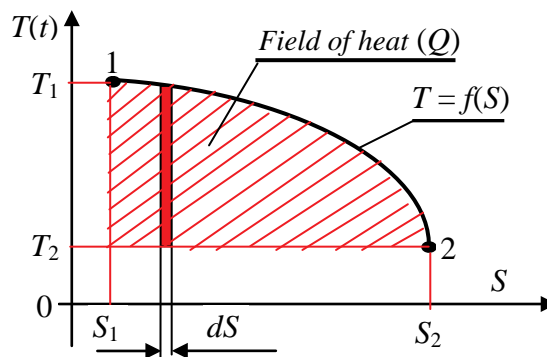


Fig. 5. An exemplary graph of heat: T – absolute temperature, S – entropy

In the paper's introduction it has been signaled that the operation in the interpretation presented herein will enable to determine if the possible engine operation (D_M) for the given conditions is at least equal to the demanded operation (D_W) being indispensable to perform the task (Z). That means the operation in the presented interpretation is of essential practical significance.

3. Practical significance of engine operation with value interpretation

Task for which a combustion engine has been designed and manufactured can be performed only when the following inequality is satisfied

$$D_M \geq D_W, \quad (14)$$

so when:

$$t_M \geq t_W, \quad \text{when at the same time } E_M \geq E_W.$$

where: t_M – possible operating time, t_W – demanded operating time, E_M – energy that can be converted by engine, E_W – demanded (desired) energy to perform the task Z (energy that must be converted to enable performance of the task).



That means that when analyzing the energetic properties of combustion engines (not only diesel ones) ability of this type of engines (as well as other energetic systems) to work can be considered in the following alternatives:

$$\left. \begin{aligned} t_M = t_W, & \text{ when simultaneously } E_M = E_W, \\ t_M = t_W, & \text{ when simultaneously } E_M > E_W, \\ t_M > t_W, & \text{ when simultaneously } E_M = E_W, \\ t_M > t_W, & \text{ when simultaneously } E_M > E_W. \end{aligned} \right\} (15)$$

In case when the inequality emerges:

$$D_M < D_W, \quad (16)$$

the engine is damaged and is not able to perform the task Z.

The inequalities (14), (15) and (16) are also true when dissipation of energy converted in the form of heat is considered. In such a case the heat is carried away from the engine and therefore (in accordance with the interpretation used by thermodynamics) gets a negative value.

Fig. 6. presents a case when the possible operation of engine $D_M = E_1 (t_2 - t_1)$ which while working can supply the energy $E_1 = idem$ being indispensable to perform a defined task at time t_1 . In this case however, in order to perform the task the demanded operation is required to be greater than the possible one, so $D_M < D_W$, where $D_W = E_1 (t_3 - t_1)$. That means that the possible operation (D_M) of engine will not ensure the task performance and that is why before starting the task realization the engine should be submitted to reconditioning through performing the adequate preventive service work.

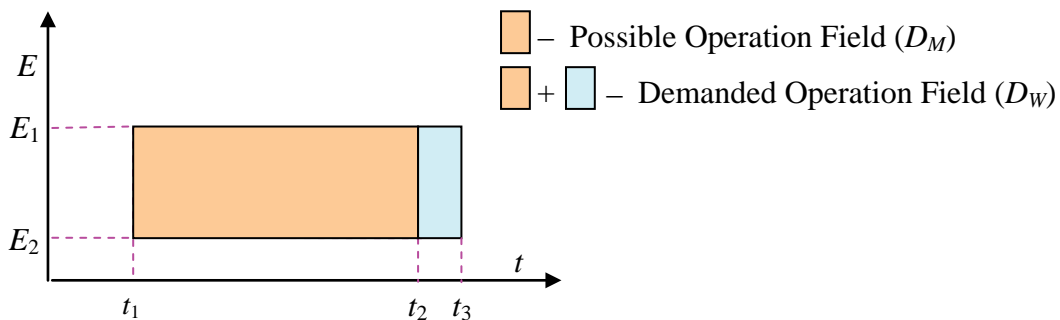


Fig. 6. An exemplary graph of possible operation (D_M) and demanded operation (D_W):
 E – energy, E_1 – energy assigned to time t_1 , E_2 – energy assigned to time t_2 , t – time

When no preventive service is carried out and any refurbishment on the machine is not done the performance of its operation can be presented in the form of a field provided in Fig. 7.

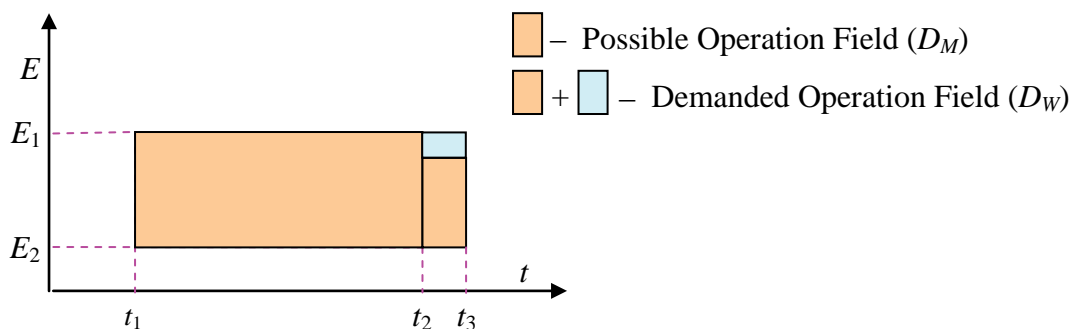


Fig. 7. An exemplary graph of possible operation (D_M) and demanded operation (D_W):
 E – energy, E_1 – energy assigned to time t_1 , E_2 – energy assigned to time t_2 , t – time

When diesel engine energy transmitted to a receiver is constant ($E = \text{idem}$) in the time interval $[t_1, t_2]$, the operation follows in accordance with the formula (7)

$$D = \int_{t_1}^{t_2} E(t)dt = E \int_{t_1}^{t_2} dt = Et \Big|_{t_1}^{t_2} = E(t_2 - t_1) \quad (17)$$

Taking into account the ways of energy E conversion (7) in energetic machines, work (L) and heat (Q), their operation can be determined by using the formulas (5) and (6), as follows:

$$D_L = \int_{t_1}^{t_2} Ldt = L(t_2 - t_1); \quad D_Q = \int_{t_1}^{t_2} Qdt = Q(t_2 - t_1) \quad (18)$$

Dependences (17) and (18) own interesting cognitive attributes, but may be also of utility significance. Applying them it is very easy to determine the demanded operation (D_W) as well as the possible operation (D_M) for each energetic machine and to obtain preliminary information on its usefulness (operational) to perform a defined task. Moreover, the formulas enable making simple graphs of demanded operation (D_W) and possible operation (D_M), which are presented in Fig. 6.

The characterized operation of diesel engines and the examples of its demonstration in the form of operation fields refer to the case when important is to determine energy or its possible conversions in forms of work and/or heat being indispensable to ensure performance of the given task. However, each operation of an energetic system at determined time is followed by energy dissipation in accordance with the second law of thermodynamics. In case of diesel engines, a part of the produced energy is used to overcome their mechanical resistances. From this reason for the piston diesel engines we distinguish indicated work (L_i) and connected with it indicated power (N_i) and useful work (L_e) and connected with it effective power (N_e). The indicated work of engine is a power being produced by the engine in its working spaces (in cylinders), without regard to its own mechanical resistances. The effective power, however, is a power which can be delivered to a power receiver in any conditions of engine operation. The difference between the two powers is the power (N_m) lost for overcoming the mechanical resistances of the engine. Taking this power into account, the energy or forms of its conversion, which are work and heat, can be considered from the conventional zero level. Then, the operation of a diesel engine can be presented with the same formulas but in the form of a field determined by the function $E(t)$ or $L(t)$ or $Q(t)$ and the time axis (t), what is displayed in Fig. 8. for the case when change in energy at time t is considered.

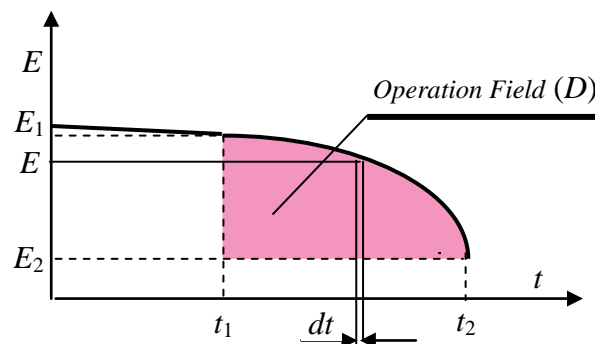


Fig. 8. An exemplary graph of operation for the case when $E = f(t)$: E – energy, E_1 – energy assigned to time t_1 , E_2 – energy assigned to time t_2 , t – time

In case, when the energy average value is considered, so when it can be accepted that $E = \text{idem}$ in the time interval $[t_1, t_2]$, the operation field can be presented as in Fig. 9.

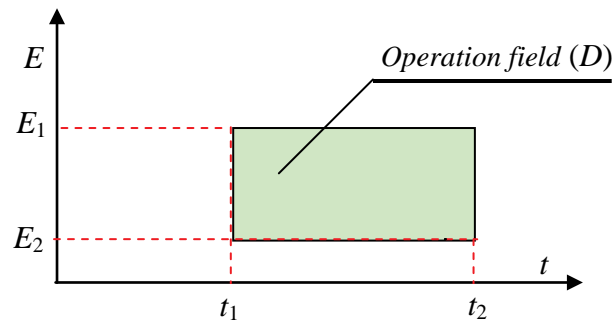


Fig. 9. An exemplary graph of operation for the case when $E = \text{idem}$: E – energy, E_1 – energy assigned to time t_1 , E_2 – energy assigned to time t_2 , t – time

Dependences (17) and (18) follow from that the integral expressed with the formula (7) or (5) or (6) is the Riemman integral with the interval $[t_1, t_2]$ in this case and the sub-interval function $E(\tau) = E$. This function is integrable in terms of Riemman in the mentioned time interval according to the formula:

$$\int_{t_1}^{t_2} E dt = E(t_2 - t_1) = (E_1 - E_2)(t_2 - t_1) \quad (19)$$

However, in practice it is not always possible to accept that energy supplied by an energetic machine to a receiver is constant [8, 12, 17, 18, 19]. Then we need to define the functional dependence of energy (E) from the time (t) of machine operation, so the dependence $E = f(t)$. In case when this is possible because the function $E = f(t)$ is continuous in the considered exemplary interval $[t_1, t_2]$, following the second fundamental theorem of calculus (theorem by Newton and Leibniz) we can write in accordance with the formula (8) that (Fig. 8).

$$\int_{t_1}^{t_2} E(t) dt = D(t_2) - D(t_1) \quad (20)$$

Application of the theorem by Newton and Leibniz is here necessary because it enables effective determination of the definite integral of the continuous functions if determination of any primitive for the functions is possible.

Generally, the functional dependence $E = f(t)$ is composite. In case, when for such a function the internal function derivative is a constant function, the definite integral of the function $f(t)$ can be determined by application of integration consisting in substitution [22].

Not always however the elementary function describing the dependence of energy from time is possible to be defined with elementary functions. Then determination of the definite integral from the Newton-Leibniz formula is troublesome, and sometimes even impossible. The trouble is that determination of a primitive is connected then with necessity of making difficult transformations. In such cases, just like when sub-integral function is determined in a table form, we can calculate an approximate value of the operation of an energetic machine, being the definite integral value, applying the method of trapezoids or the Simpson method. The attention must be paid that at a fixed n the Simpson method enables obtaining more accurate results in integration than the method of trapezoids.

9. Conclusion

Operation of combustion engines is understood as generating the energy E by them at a defined time t . It has been equated to a physical quantity which can be expressed with a numerical value and the measurement unit called *joule-second* [joule×second]. Such understood operation gets worse with the growing wear of this type of engines. This means that the operation value at a defined time decreases in the result of decreasing energy generated by the engines. It has been signaled herein that in case of application of the theory of stochastic processes to the analysis of changes of such understood operation the integral calculus can be applied. Two stochastic models of decreasing useful energy generated by the engines have been proposed for defining the range of worsening operation. The first model has been presented in the form of a homogenous Poisson process and the second – in the form of a discrete-state, continuous-time semi-Markov process.

Operation in such interpretation depends on the technical state of the engines and is characterized simultaneously by the energy converted by the engines and the energy generation time.

The advantage of the engine operation in the presented interpretation is that it can be tested through doing precise measurements and then expressed in the form of:

- a number with the measurement unit called *joule-second* [joule×second] (formulas 5, 6 and 7);
- a graph, as a field of operation (Fig. 2, 3 and 4).

Operation in the presented interpretation, although formulated for diesel engines, refers also to spark-ignition engines. Similar interpretation of operation can be provided for turbine combustion engines and other energetic machines.

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