MODELING OF THE ACOUSTIC WAVES PROPAGATION IN NON-HOMOGENEOUS MEDIUM

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The aim of the paper is theoretical analysis of acoustic waves propagation in medium which are not homogeneous. The paper presents mathematical model and some results of numerical investigations. The mathematical model was built on the basis of the KZK equation. To solve the problem numerically the finite-difference method was applied. The on-axis pressure and power spectrum density were analysed both in homogeneous and non-homogeneous medium.

INTRODUCTION

Knowledge of transmission properties of different kinds of medium like sandy sediment or bubbly liquid in case of high amplitude acoustic waves is very important in practice. Figure 1 presents the sketch of the problem studied in this paper. We assume that a circular source of two-frequency acoustics waves with a radius equal a is placed in plane $y0z$. Waves are propagated in x direction. The layer of liquid with different as water nonlinear properties is located between $x=x_P$ and $x=x_K$. The receiver is placed at distances $x=X_{max}$ from the source. The pressure changes and power spectrum density on the beam axis were studied. Calculations were done for different values of physical parameters of the liquid in the layer.

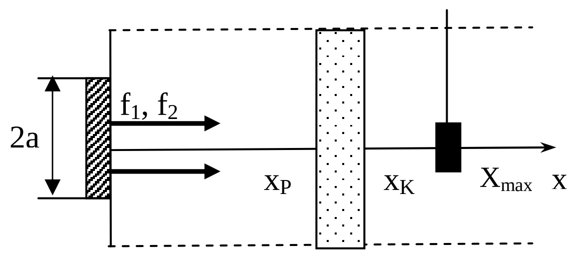

Fig.1. Sketch of the problem

1. MATHEMATICAL AND NUMERICAL MODEL

The problem of waves propagation can be modelled using the KZK equation:

$$\frac{\partial^2 p'}{\partial\tau\partial x} - \frac{\varepsilon}{2c_0^3\rho_0}\frac{\partial^2 p'^2}{\partial\tau^2} - \frac{b}{2c_0^3\rho_0}\frac{\partial^3 p'}{\partial\tau^3} = \frac{c_0}{2}\Delta_\perp p' \tag{1}$$

where c_0 and ρ_0 denote speed of sound and medium density in the equilibrium, b denotes dissipation coefficient of the medium, ε is nonlinear coefficient, variable $\tau = t - x/c_0$. Assuming axial symmetry of the problem the Laplace operator is given by $\Delta_\perp = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}$ where $r = \sqrt{y^2 + z^2}$. This equation describes the acoustic pressure changes $p' = p'(x,r,\tau)$ along the sound beam.

Assuming that the source generates two harmonic waves, the boundary conditions can be written in the following form:

$$p'(x=0,r,\tau) = \begin{cases} p_{01}\sin\omega_1\tau + p_{02}\sin\omega_2\tau & \text{for } r \le a \\ 0 & \text{for } r > a \end{cases} \tag{2}$$

where p_{0i} and $\omega_i = 2\pi f_i$, i=1,2 denote amplitudes and angular frequencies of primary waves respectively.

Additionally we assume that function p' is a periodic function of the time coordinate and

$$\frac{\partial p'}{\partial r}_{|r=0} = 0$$

$$\frac{\partial p'}{\partial r}_{|r=R_{\max}} = 0$$

We are seeking the solution of the problem (1), (2) inside of the cylinder with radius R_{max}, at distances from the source to X_{max}, i.e. in the space $D = \{(x,r) : x \in [0, X_{\max}], r \in [0, R_{\max}]\}$ for the fixed time interval (Fig.2). Because of diffraction this radius must be suitably great taking examined distances from the source into consideration.

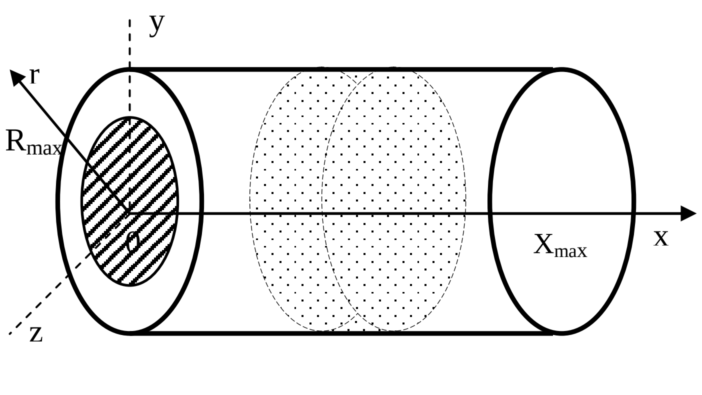

Fig.2. Geometry of the problem

To solve the problem numerically dimensionless variables are introduced

$$P = p'/p_0,\ X = \frac{c_0}{2\omega a^2}x,\ R = \frac{r}{a},\ \theta = \omega\tau$$

Then the equation (1) can be rewritten as

$$\frac{\partial^2 P}{\partial\theta\partial X} - N\frac{\partial^2 P^2}{\partial\theta^2} - M\frac{\partial^3 P}{\partial\theta^3} = \Delta_\perp P \tag{3}$$

where $N = \dfrac{\varepsilon p_0\omega^2 a^2}{c_0^4\rho_0}$ and $M = \dfrac{b\omega^3 a^2}{c_0^4\rho_0}$. The boundary condition (2) has now following form:

$$P(X = 0, R, \theta) = P_1 \sin l_1\theta + P_2 \sin l_2\theta \tag{4}$$

where P_i are amplitudes of the normalized primary waves and $l_i\theta = \omega_i\tau$ (i=1,2).

The finite-difference method is used to solve the problem (3), (4) numerically. As a result of computer calculations we obtain the pressure values inside the predetermined area and at fixed time interval.

2. RESULTS OF NUMERICAL INVESTIGATIONS

Numerical calculations were carried out assuming that circular source generates harmonic waves with frequencies equal f_1=30 kHz, f_2=35 kHz and identical amplitudes p_{0i}=5kPa. Figure 3 presents the generated signal.

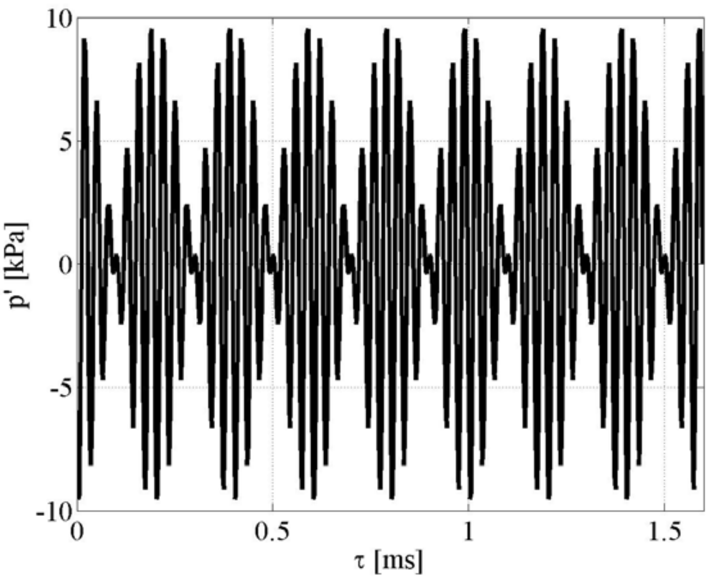

Fig.3. Pressure as a function of time

The aim of the theoretical investigations was analysis of pressure changes in the non-linear medium. In a first step of these investigations we have analysed the influence of varying acoustic parameters in water on the resulting acoustic field. Figure 4 shows on-axis pressure amplitudes of different frequency waves as a function of distance from the source. Left figure presents pressure amplitude of f_1 (solid line) and f_2 (dashed line) frequency wave. The pressure amplitudes of their second harmonic waves and sum frequency wave (dotted line) are shown in right figure respectively. Calculations were done for c_0=1450 m/s, ρ_0=1000 kg/m^3, ε=3.5 and b=0.004 what is equivalent with values of parameters N=7.8·10^{-4} and M=2.8·10^{-7}. It corresponds to waves propagation in water. The power spectrum density on the beam axis at distance x=0.3 m and x=1 m for this example is shown in Fig. 5.

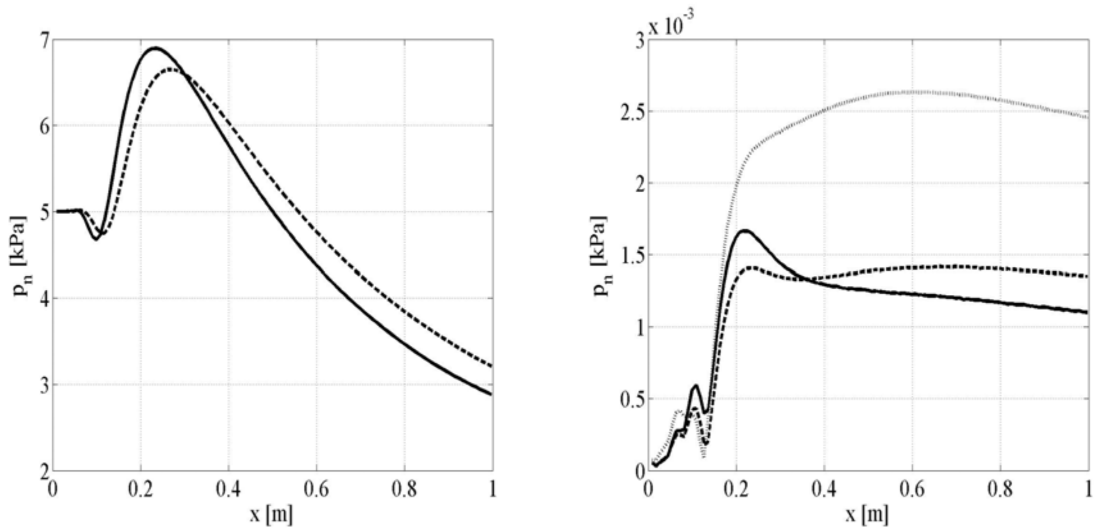

Fig.4. On-axis pressure amplitude of the different frequency waves as a function of a distance from the source

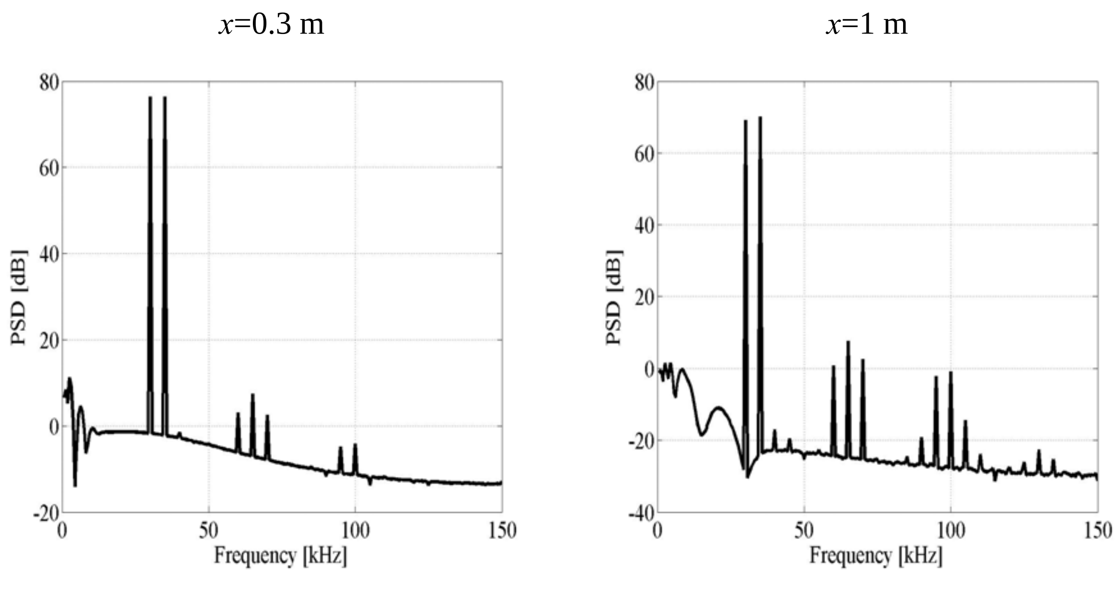

Fig.5. Power spectrum density on the beam axis for different distances from the source

Figure 6 presents the comparison of the pressure amplitudes at f_1 and $2f_1$ obtained for different values of parameter M ($p'_{f,M}$ denotes pressure amplitude of f frequency wave obtained for parameter M).

a) b)

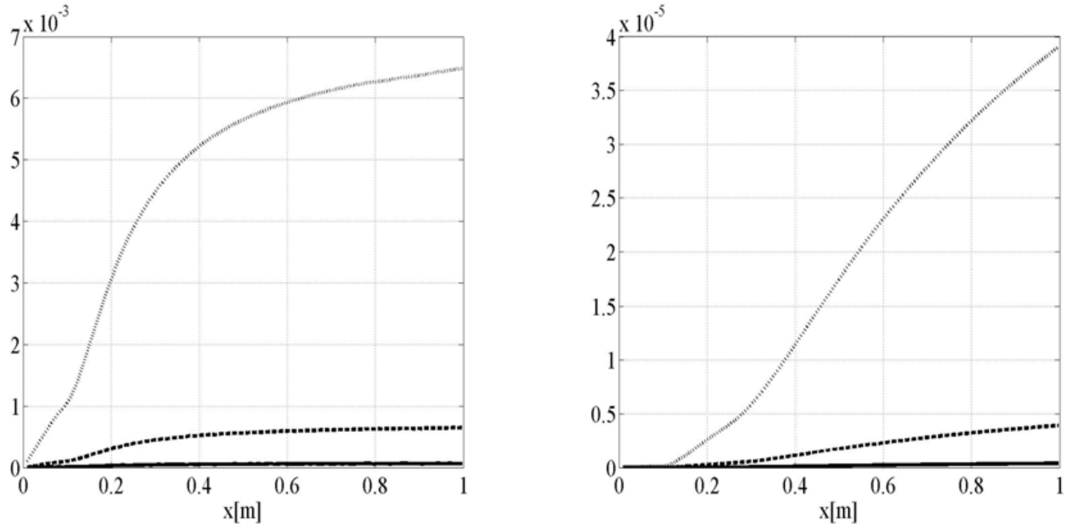

Fig.6. Comparison of on-axis pressure amplitudes of f_1 and $2f_1$ frequency wave calculated for different values of parameter M: a) $|p'_{f1,M}-p'_{f1,0}|$ (---); $|p'_{f1,10M}-p'_{f1,0}|$ (- - -); $|p'_{f1,100M}-p'_{f1,0}|$ (…), b) $|p'_{2f1,M}-p'_{2f1,0}|$ (---); $|p'_{2f1,10M}-p'_{2f1,0}|$ (- - -); $|p'_{2f1,100M}-p'_{2f1,0}|$ (…)

Theoretical analysis of the problem of waves propagation in medium which is not homogeneous is much more difficult. In the next step of theoretical investigation we studied that problem. Now we assume that layer of liquid with physical parameters different as water is located at distances from x_P=0.3 m to x_K=0.4 m.

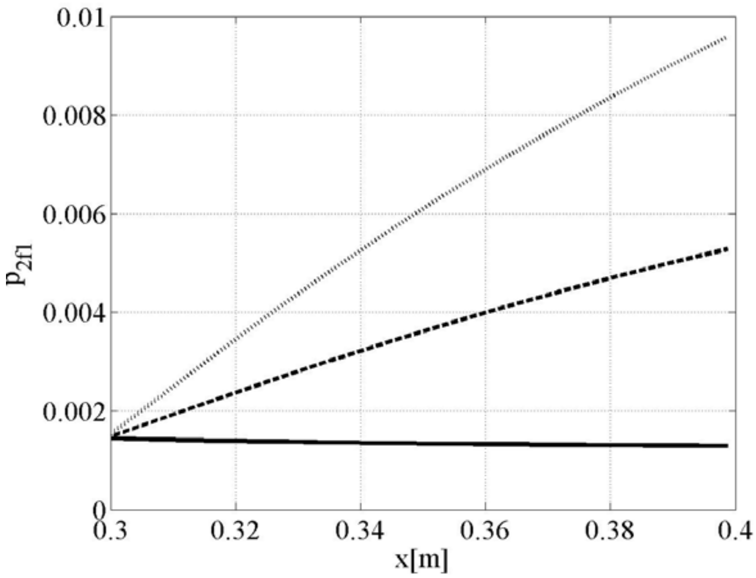

Fig.7. Pressure amplitude of $2f_1$ frequency wave as a function of distance from the source

Figure 7 presents on-axis pressure amplitudes of $2f_1$ frequency wave obtained for different values of parameter N_L. Solid line presents result obtained for N_L=7.8·10^{-4}, dashed line shows similar results obtained for N_L=0.011 and dotted one was obtained for N_L=0.022 (M_L=M).

On-axis pressure amplitude of $2f_1$, $2f_2$ and sum frequency waves as a function of distance from the source presents Fig. 8 (N_L=0.022). Power spectrum density calculated at distance x=1 m for this example is shown in next figure. On-axis pressure at distance x=0.3 m and x=1 m shows Fig 10.

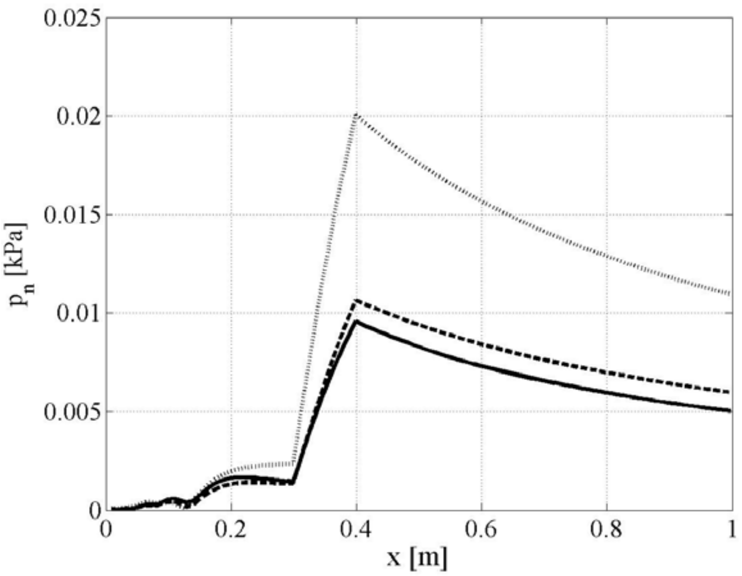

Fig.8. On-axis pressure amplitude of different frequency waves as a function of distance from the source

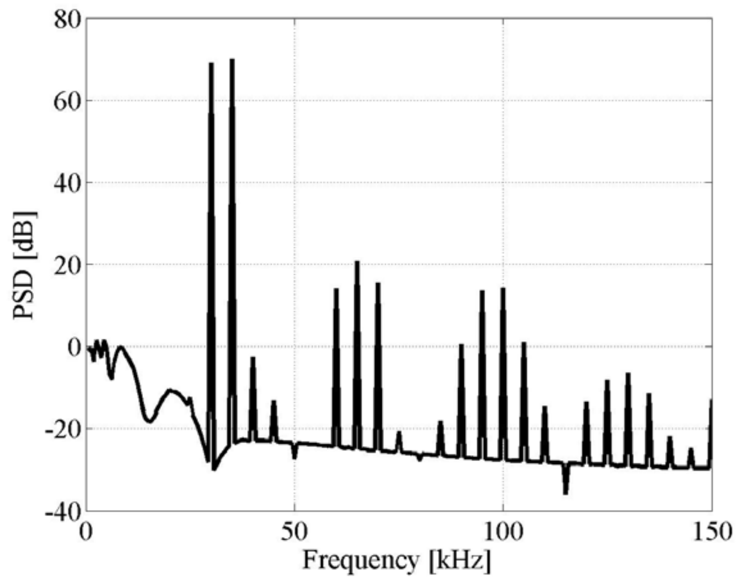

Fig.9. Power spectrum density on the beam axis at distance x=1 m

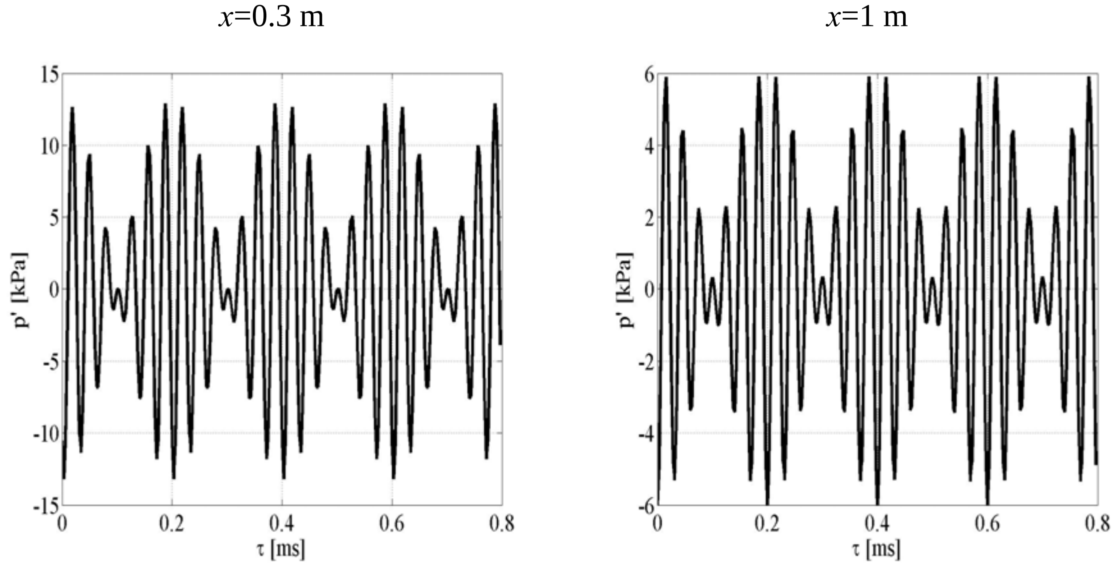

Fig.10. On-axis pressure as a function of time at different distances from the source

Last step of theoretical investigation was connected with influence of layer thickness *L* on the pressure distribution. Example of theoretical investigations is shown in Fig. 11. This figure presents power spectrum density obtained at distance *x*=1 m. Solid lines present the results obtained for f_1 and $2f_1$ frequency wave respectively. Results obtained for f_2 and $2f_2$ frequency waves show dashed lines. Dotted line illustrates the power spectrum density obtained for sum frequency wave.

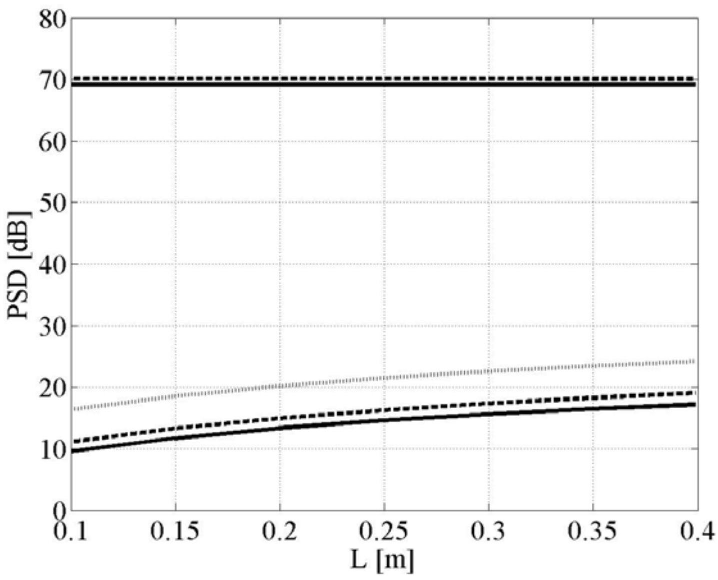

Fig.11. Power spectrum density at distance *x*=1 m for different layer thickness

3. CONCLUSIONS

The paper presented the results of theoretical investigation of nonlinear wave propagation in non-homogeneous medium. Mathematical model was built on the basis on the

KZK equation which allows including nonlinearity, dissipation of medium and sound beam diffraction. However presented in this paper mathematical model do not cover all physical properties. For example it does not include dispersion.

Correct choice of numerical parameters is very important for the output of numerical calculations [1-2]. It influences accuracy and correctness of results. But it is also important to remember that correct choice of values of physical parameters is also very important during numerical calculations. It is worth to say that if values of physical parameters are well known for propagation in water (for example [3]) than another situation is for example in bubble layer. It is possible to find information about speed in pure liquid but there are no sufficient data on values of nonlinear and in particular dissipation coefficient depending on bubbles size distribution and their concentration.

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