

# On the modelling of aerodynamic force coefficients for over-shroud seals of turbine stages

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## Abstract

*This paper presents experimental investigations which made it possible to determine dynamic coefficients of labyrinth over-shroud seal of a model air turbine. The coefficients associate pressure forces with turbine rotor displacement, velocity and acceleration respective to turbine casing (linear model) and play important role in analyzing turbine-set dynamics. The obtained results indicated that involving serious errors can be expected in the case of application of the simplification consisting in neglecting inertia coefficients, proposed in the literature. It was simultaneously demonstrated that seals can be also met of weak damping qualities, for which to neglect damping coefficients is allowable.*

**Keywords:** rotor dynamics, self-excited vibrations, aerodynamic forces, blade seals, turbine shrouds

## Introduction

In engineering many cases are known of occurrence of aerodynamically generated vibrations of turbine rotors. Such phenomena can lead to dangerous vibrations which make correct operation of turbine set impossible. Self-excited aerodynamic forces occur as a result of phenomena associated with non-stationary flow of working medium through a turbine stage in the case of eccentric position of turbine's rotor relative to its casing. During operation of a rotary machine whose rotor axis is deviated from casing axis, local flows through particular passages between rotor blades are changed as well as working medium leakages along circumference of radial gap of over-shroud seal are also changed. As a result, a circularly unsymmetrical distribution of forces acting on rotor blades and shroud surface occurs. Hence in publications devoted to the problem two main components of aerodynamic forces resulting from different character of their occurrence, are distinguished:

- resultant force due to non-uniform distribution of circumferential forces in rotor blade ring, called blade-ring force,
- resultant force due to non-uniform distribution of pressure in radial gap over rotor blades, called pressure force.

In most cases of design solutions of turbine stages the pressure forces are of decisive influence on occurrence of self-excited rotor vibrations of aerodynamic origin. For this reason to know dynamic features of over-shroud seals in designing process of a new turbine set is very important. The features are tried to be determined in a theoretical way, however results of experimental investigations appear extremely valuable because of a highly complex character of flow through turbine stage.

### Pressure forces – linear model

Mutual relationship between pressure force and displacement of rotor against casing is usually described by means of the so called „seal dynamic coefficients” [1-8, 12-14] which can be incorporated into whole model

of turbine set during its designing process. Three kinds of the dynamic coefficients can be distinguished: stiffness, damping nad inertia ones. They are associated with displacement, velocity and acceleration of rotor relative to casing, respectively.

In such system, forces and moments generated in radial gap over turbine rotor blades are described by using the linear model which takes form of the following vectorial equation:

$$[F] = [K] \begin{bmatrix} x \\ y \\ \varepsilon_x \\ \varepsilon_y \end{bmatrix} + [C] \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\varepsilon}_x \\ \dot{\varepsilon}_y \end{bmatrix} + [M] \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\varepsilon}_x \\ \ddot{\varepsilon}_y \end{bmatrix} \quad (1)$$

where:

- F – vector of aerodynamic forces and moments generated in seal,
- x, y – rotor linear displacements in the assumed frame of coordinates,
- $\varepsilon_x, \varepsilon_y$  – angular displacements of rotor axis,
- K – matrix of stiffness coefficients,
- C – matrix of damping coefficients,
- M – matrix of inertia coefficients,
- $\dot{x}, \dot{y}, \dot{\varepsilon}_x, \dot{\varepsilon}_y$  – the first time-related derivatives of linear and angular displacements,
- $\ddot{x}, \ddot{y}, \ddot{\varepsilon}_x, \ddot{\varepsilon}_y$  – the second time-related derivatives of linear and angular displacements.

In the subject-matter literature a simplification which consists in neglecting the inertia coefficients in the model, can be found [5,6], [8÷11]. The vectorial equation of the linear model takes then the following form:

$$[F] = [K] \begin{bmatrix} x \\ y \\ \varepsilon_x \\ \varepsilon_y \end{bmatrix} + [C] \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\varepsilon}_x \\ \dot{\varepsilon}_y \end{bmatrix} \quad (2)$$

## Research stand

The above mentioned investigations were performed by using a research stand of model turbine installed in the Laboratory of the Ship Automation of Turbine Propulsion Department, Faculty of Ocean Engineering and Ship Technology, Gdańsk University of Technology. Schematic diagram of the stand is presented in Fig. 1. The turbine is fed with compressed air delivered from the compressor through the piping system. The compressor is driven by the electric motor through the reduction gear. The electric brake absorbs the turbine's output power. The electric energy generated in the electric brake is transformed into heat in the set of resistors.

The examined turbine is a single-stage impulse turbine with labyrinth over-shroud seal having two teeth in the shroud and casing (Fig. 2). Construction of the turbine makes it possible to change mutual position of casing and rotor during operation of the stand.

The measuring system of the stand makes it possible to measure non-stationary distribution of pressure along circumference of the over-shroud seal. The turbine's casing is adjusted to fastening the ENTRAN EPE miniature pressure sensors (Fig. 2). Altogether 88 pressure measurement points are provided for: 4 in each row, circumferentially distributed every 15°, except for the parting plane of the casing.

During the carried-out investigations a displacement of the turbine's casing (precisely, of the vane blade-ring with seal ring) against the turbine's rotor, was forced (during operation of the turbine). The casing

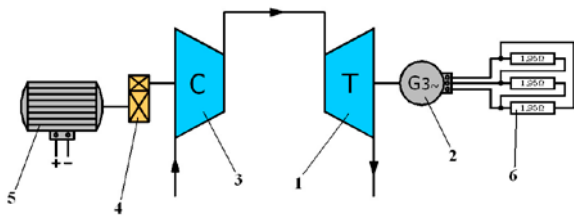


Fig. 1 Schematic diagram of the model air turbine stand: 1 – turbine, 2 – brake, 3 – compressor, 4 – reduction gear, 5 – electric motor, 6 – set of resistors

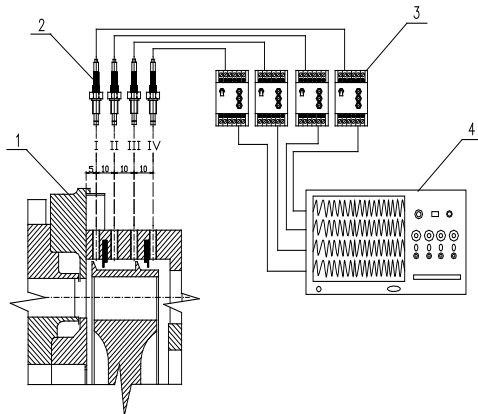


Fig. 2 Schematic diagram of the measurement system of pressure in over-shroud gap of the model turbine: 1 – axial cross-section of the turbine stage, 2 – pressure sensor, 3 – measurement amplifier, 4 – recording system of measurement data, I, II, III, IV – measurement planes perpendicular to rotor axis

displacement was measured by using an induction sensor of linear displacements. The sensor was fastened to the motionless part of the stand and its measuring tip was leaned against outer surface of the movable casing of the turbine.

## Experimental investigations

The research was aimed at verifying validity of the simplification of linear model, consisting in neglecting the inertia coefficients (Eq. 2), as proposed in the literature. To this end the dynamic coefficients of the over-shroud seal of the model air turbine were experimentally determined in compliance with the following vectorial equation: where:

$$\begin{bmatrix} F_x(t) \\ F_y(t) \end{bmatrix} = \begin{bmatrix} K_{xx} & K_{yy} \\ K_{yx} & K_{xy} \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} + \begin{bmatrix} C_{xx} & C_{xy} \\ C_{yx} & C_{yy} \end{bmatrix} \begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{bmatrix} + \begin{bmatrix} M_{xx} & M_{xy} \\ M_{yx} & M_{yy} \end{bmatrix} \begin{bmatrix} \frac{d^2x}{dt^2} \\ \frac{d^2y}{dt^2} \end{bmatrix} \quad (2)$$

- $F_x, F_y$  – components of pressure force vector,
- $t$  – time,
- $x, y$  – rotor displacements in the assumed frame of coordinates,
- $K_{xx}, K_{yy}, K_{xy}, K_{yx}$  – stiffness coefficients,
- $M_{xx}, M_{yy}, M_{xy}, M_{yx}$  – inertia coefficients,
- $C_{xx}, C_{yy}, C_{xy}, C_{yx}$  – damping coefficients.

During the investigations changes of pressure in the sealing, which result from mutual transverse displacement of rotor and casing, were recorded; next on the basis of the experimental data pressure forces

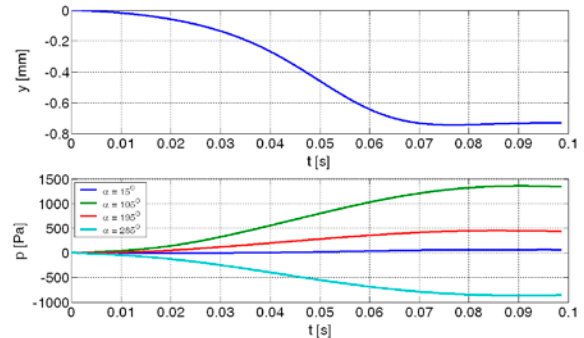


Fig. 3 Displacement of rotor against casing as well as change of pressure in the over-shroud seal, resulting from it and measured in 4 points distributed every 90° along circumference of the turbine stage;  $\alpha$  – angle which determines location of measurement point along circumference of the seal

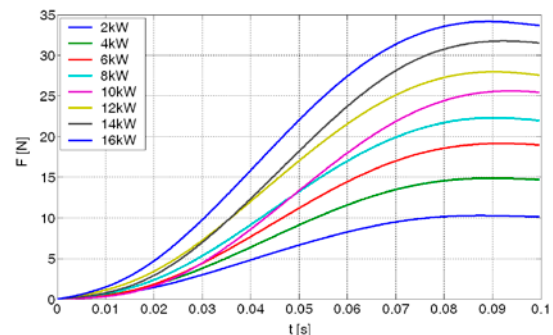


Fig. 4 Increase of the resultant pressure force at the step change of mutual position of rotor and casing by 0,74 mm and different turbine's loads

and dynamic coefficients of the sealing were calculated. The investigations were carried out at the constant rotor speed equal to 3600 rpm, and the turbine power values of 2 kW, 4 kW, 6 kW, 8 kW, 10 kW, 12 kW, 14 kW, 16 kW, as well as the values of eccentricity of rotor against casing, changed by 0,16 mm, 0,35 mm, 0,61 mm, and 0,74 mm. The nominal radial gap in the seal was equal to 1 mm (rotor co-axially aligned in casing).

In Fig. 3 is presented the example run of position of rotor against casing as well as that of change of pressure in the over-shroud seal, resulting from it and measured in 4 points located every 90° along circumference of the turbine stage, at the turbine's power of 12 kW and its rotor speed of 3600 rpm.

On the basis of the measurements the forces acting on the turbine's shroud were determined; the example of the obtained results is presented in Fig. 4.

As pressure force changes with time and rotor displacements against casing are known, it was possible to determine the stiffness, damping and inertia coefficients appearing in the linear model equations. The coefficients were determined by searching for solution of Eq. 3 with the use of the least squares method. The example results are shown in Fig. 5, 6, and 7. As observed on the basis of the obtained results, the relationship between the dynamic coefficients of the considered over-shroud seal and the turbine's output power, is linear.

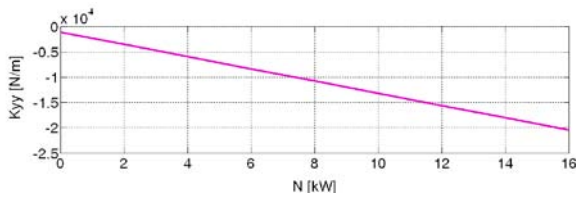


Fig. 5 Stiffness coefficients of the seal in function of output power of the turbine

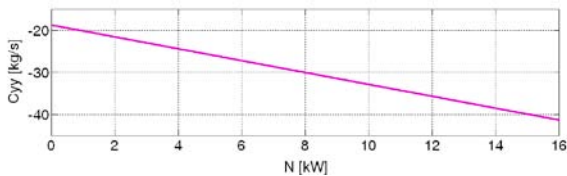


Fig. 6 Damping coefficients of the seal in function of output power of the turbine

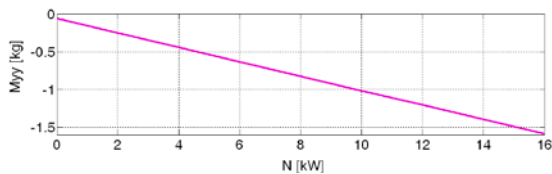


Fig. 7 Inertia coefficients of the seal in function of output power of the turbine

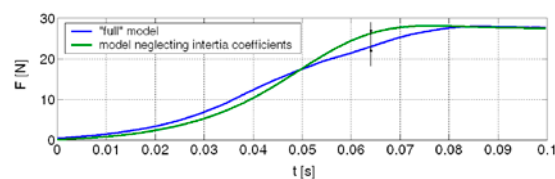


Fig. 8 Pressure force calculated by means of the linear model and that calculated by means of the model neglecting inertia coefficients.

By making use of the determined dynamic coefficients of the seal it was possible to determine pressure force for a given value of displacement of turbine's rotor against its casing. The example run of the pressure force calculated by means of the linear model with the use of the determined coefficients is presented in Fig. 8. In the same figure is also drawn the similar run but determined with the use of the model neglecting inertia coefficients.

The diagram in Fig. 8 shows that the neglecting of inertia coefficients in the linear model equations can lead to distinct differences in runs of the calculated force. In the presented example the differences reached as far as 15 %.

On the other hand, rather small influence of the inertia coefficients on the determined forces has been revealed. In

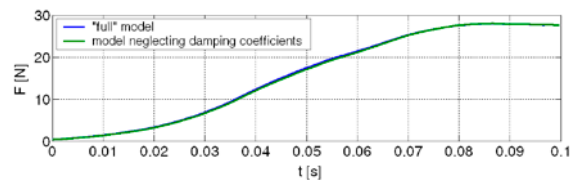


Fig. 9 Pressure force calculated by means of the linear model and that calculated by means of the model which neglects damping coefficients.

Fig. 9 are presented the example runs of the pressure force calculated by means of the complete linear model, i.e. that in which all three kinds of the coefficients have been taken into account, and that calculated by using the simplified model neglecting the term associated with damping coefficients -they differ only a little to each other. Such situation repeated within all the range of values of turbine's load and change of rotor displacement against casing, for which measurements have been performed. This suggests that the considered type of seal (i.e. the labyrinth over-shroud seal with two teeth on the shroud and two on the casing) is of very low damping qualities.

Therefore in the case of this kind of seal it was proposed to simplify the model which describes pressure forces, by omitting its part associated with velocity of rotor displacement against casing. In such case the equation which describes generation of pressure forces would take the following form:

## Conclusions

$$[F] = [K] \begin{bmatrix} x \\ y \\ \varepsilon_x \\ \varepsilon_y \end{bmatrix} + [M] \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\varepsilon}_x \\ \ddot{\varepsilon}_y \end{bmatrix} \quad (4)$$

- in the case of labyrinth over-shroud seal, values of dynamic coefficients of the seal change proportionally along with turbine's power changing;
- in the case of seals of low damping qualities, omittance of inertia coefficients can lead to noticeable

errors in determining the aerodynamic forces;

- the labyrinth over-shroud seal with two teeth on the shroud and two on the casing has as much low damping qualities that in the case of the seal in question the linear model can be simplified by omitting the term associated with damping coefficients.

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