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USAGE OF SEMI-MARKOV PROCESS IN OPERATION EVALUATION OF DIESEL ENGINE

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Abstract

In paper, the proposition of quantitative evaluation of operation using semi-Markov processes theories has been presented. Basic assumptions, essential for creating mathematical model on example of diesel engine were shown. Special attention was given to practical aspects of using established mathematical model. Example of characteristic of analyzed process – temporary distribution of probability was assigned.

Keywords: *semi – markov process, operation, diesel engine*

1. Introduction

Precise establishing of task realized by analyzed technical object requires except for assumption of conditions in which it will be performed, also refining its duration. This issue is especially important in such domains as eg. sea transport, where specificity of tasks usually is connected to necessity for long-term functioning of essential mechanisms and devices (eg. ship).

Ipso facto, particularly important becomes not only what is worth of energy which can be disposed using specific energetic device (eg. diesel engine), but also time in which it can be provided.

Presented way of approach becomes possible to accomplish by examining engine's work (in following part of deliberation were restricted only to main ship propulsion engine) in that kind of evaluative view, to make it possible to be defined by energy and time simultaneously. Similar approach is also presented eg. in papers [5, 8, 9, 10].

Method of such appraisal of energetic device's work (in this case main ship propulsion engine) involves undoubted values of cognitive nature, but as in every case of real object of exploitation, a need of implementation of issues considered on theoretical field into practical use, appears as well.

During exploitation of ship, decisions about usage and servicing devices of marine power plant (especially power transmission system) are made permanently. Making any of that kind of decision, in particular moment, is to choose this one, from amongst possible to choose, which is considered to be the best. Choice of that decision, essential for defining rational operating strategy, including:

- proper planning of tasks

– providing required level of ship's safety
– planning preventive service adequate for needs
– right organization of service-remedial back-up facilities (eg. planning demand for replacements)

is possible to make after taking into consideration many different information, but in every case it should be based on possibly full information about exploitation object [7].

Especially precious in decision situation description is evaluative (quantitative) approach to this issue and searching for measures that would describe examined features of ship power plant's components at all and in this case, main propulsion engine, in most objective way.

Every new decision-making premise enhances probability of making apt decision [11, 12], so generally speaking, it influences safety level during realization of task.

One of many opportunities of gathering information about exploited engine is created by using semi-Markov's processes theories in mentioned before evaluative appraisal of working (in this case, main propulsion engine).

2. Description of engine's operation using semi-Markov's process theories.

Engine's operation (D) in time period [0, t] can be interpreted as physical quantity described as product of multiplication of energy variable in time E=f(t) and time, which can be generally depicted by following equation [5]:

$$D = \int_0^t E(\tau) d\tau = 2\pi \int_0^t M_0\, n\, t dt \tag{1}$$

where:
D – engine's operation
M_0 – torque
n – rotational speed

With elapsed time during engine's functioning, as an outcome of mostly disruptive influence of expenditure process, its general efficiency defined eg. as [3]:

$$\eta_e = \frac{1}{g_e \cdot w_d} \tag{2}$$

where:
g_e – specific fuel consumption
wd – fuel's lower calorific value

decreases, which obviously causes changes referring to operation value of possible D_M (which can be realized by engine in specific technical condition and specific functioning conditions).

In case of ship engine of main propulsion, in view of compliance so-called sea margin and spare exploit power for engine [13] used on nominal load and partial loads during design time, process of decreasing available power (functioning of possible D_M as well) will proceed in two stages:

- in first stage only increase of hour fuel consumption (with relatively constant value of developed torque) will occur, enhancing usage costs,
- in second stage as effect of structural restrictions and lack of possibility of increasing of fuel dose, limitation in useful power developed by engine will appear.

Graphically, this occurrence can be illustrated in a way shown on Fig.1.

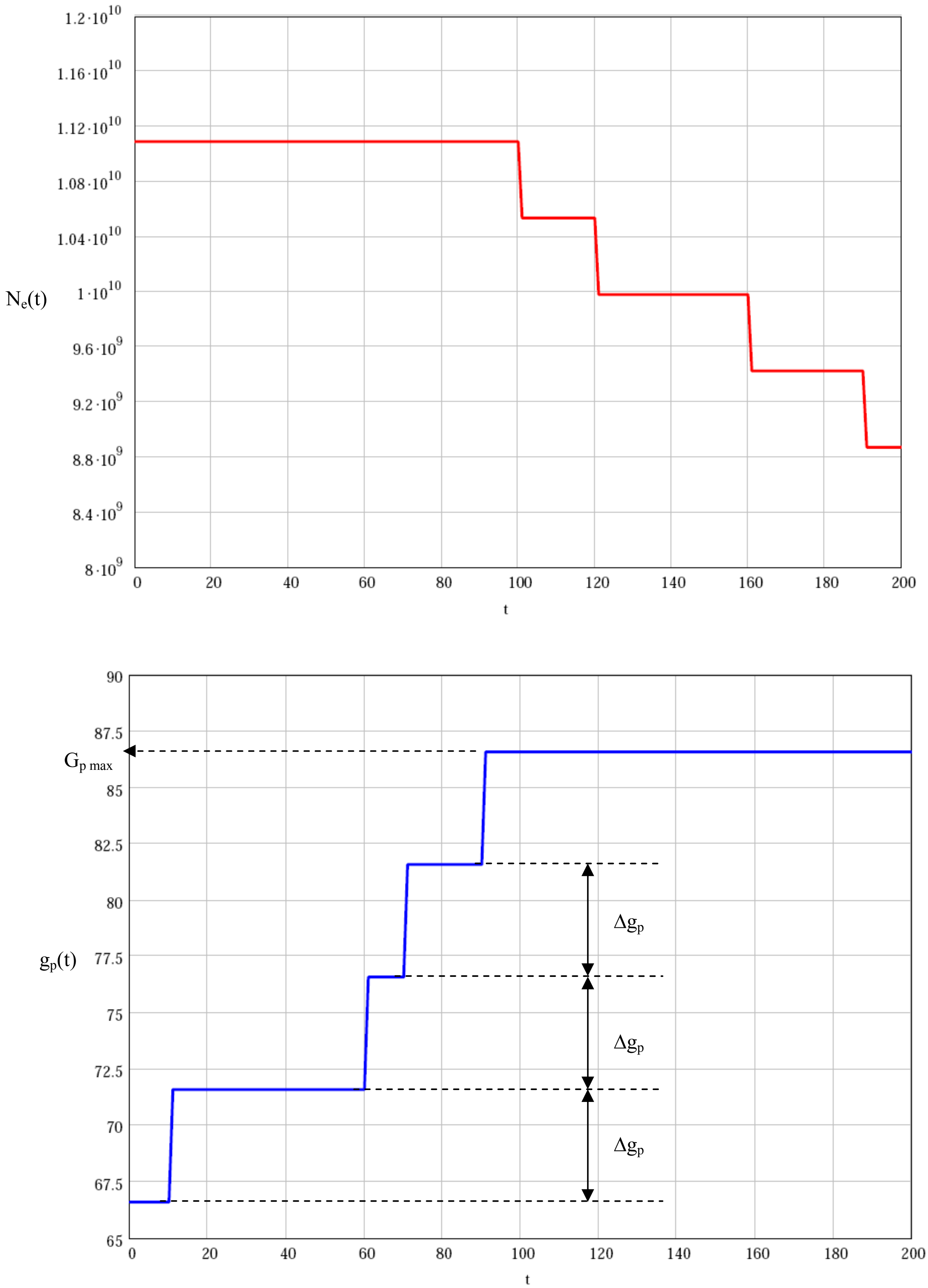

Fig.1 Stages of decreasing available power (functioning of possible D_M) process of diesel engine working on partial loads (description below)

Described occurrences are caused by control reaction of fuel injection apparatus, which be enhancing temporary fuel dose in particular interval of values – g_p ($g_p^{i\%}$ - dose of fuel for *i%* engine's load in technical efficiency state and full task ability, assuming that maximum engine's load is 110% of nominal load, *i <110*) until achieving it maximum value – G_{pmax}. Every following deceasing of engine's general efficiency value will cause noticeable decline of M_0.

Due to phenomena presented before and regard as true following hypothesis *"degradation process of engine's technical condition (understood as random function, which argument is time and values are random variable, meaning both technical and energetic states existing simultaneously), is a process which state considered in any moment t_n (n = 0, 1, ..., m; $t_0 < t_1 < ... < t_m$) depends on state directly premising it and doesn't depend stochastically on states that occurred before and their lasting periods"* it becomes possible to formulate mathematical models useful for evaluative description of this engine's work, using semi-Markov's processes theories [4, 14].

Graph of states – transitions of analyzed $\{W(t): t \in T\}$ process, covering phenomena presented in Fig.1 can be presented in this way:

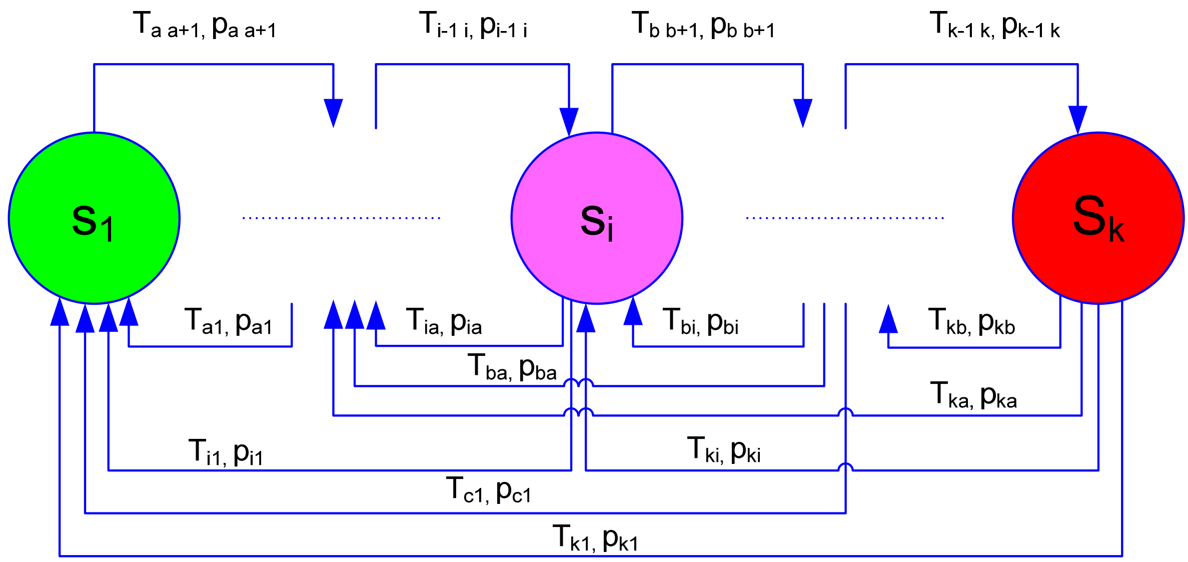

Fig.2 Graph of states – transitions of $\{W(t): t \in T\}$. process, s_a – state of engine in which increase of unit fuel consumption emerges without noticeable increase of N_e value, a=1, 2, ..., i-1, s_b – state of engine in which due to structural restrictions and lack of possibility to increase fuel dose, limitation in useful power developed by engine will appear, b= I, i+1, ..., k-1, s_k – state of engine in which due to degradation of structure, it's not used any more (no engine functioning – no operation)

Ipso facto, matrix of function $Q_{ij}(t)$ of analyzed process has (as it's shown on graph above) following shape:

$$Q_{ij}(t) = \begin{bmatrix} 0 & Q_{12}(t) & 0 & 0 & 0 & .... & 0 & .... & 0 \\ Q_{21}(t) & 0 & Q_{23}(t) & 0 & 0 & .... & 0 & .... & 0 \\ Q_{31}(t) & Q_{32}(t) & 0 & Q_{34}(t) & 0 & .... & 0 & .... & 0 \\ .... & .... & .... & .... & .... & .... & .... & .... & 0 \\ Q_{k-1\,1}(t) & Q_{k-1\,2}(t) & Q_{k-1\,3}(t) & Q_{k-1\,4}(t) & ... & Q_{k-1i}(t) & .... & 0 & Q_{k-1\,k}(t) \\ Q_{k\,1}(t) & Q_{k\,2}(t) & Q_{k\,3}(t) & Q_{k\,4}(t) & ... & Q_{ki}(t) & .... & Q_{k\,k-1}(t) & 0 \end{bmatrix} \quad (3)$$

Elements of that matrix depend on distribution of random variables that are time intervals of process being in convex conditions as follows [6]:

$$Q_{ij}(t) = P\{W(\tau_{n+1}) = s_j,\ \tau_{n+1}-\tau_n<t \mid W(\tau_n) = s_i\} = p_{ij} \cdot F_{ij}(t);\ s_i, s_j \in S;\ i, j = 1, 2, ..., k;\ i \neq j \quad (4)$$

where:

p_{ij} – probability of converting from state s_i to state s_j

$F_{ij}(t)$ – cumulative distribution function of random variable T_{ij}, which is duration of state s_i providing converting process into state s_j;

Initial distribution of process:

$$p_i = P\{W(0) = s_i\},\ s_i \in S;\ i=1, 2, 3, 4, \ldots, k \tag{5}$$

can be accepted depending on specific task situation, and eg. in case of analyzing engine in technical condition and full task ability:

$$\begin{aligned} p_1 &= P\{W\ (0) = s_1\} = 1, \\ p_i &= P\{W\ (0) = s_i\} = 0 \ \text{ dla } i = 2, 3, 4, \ldots, k \end{aligned} \tag{6}$$

Example of $\{W(t) : t \in T\}$ process realization has been shown on Fig. 3.

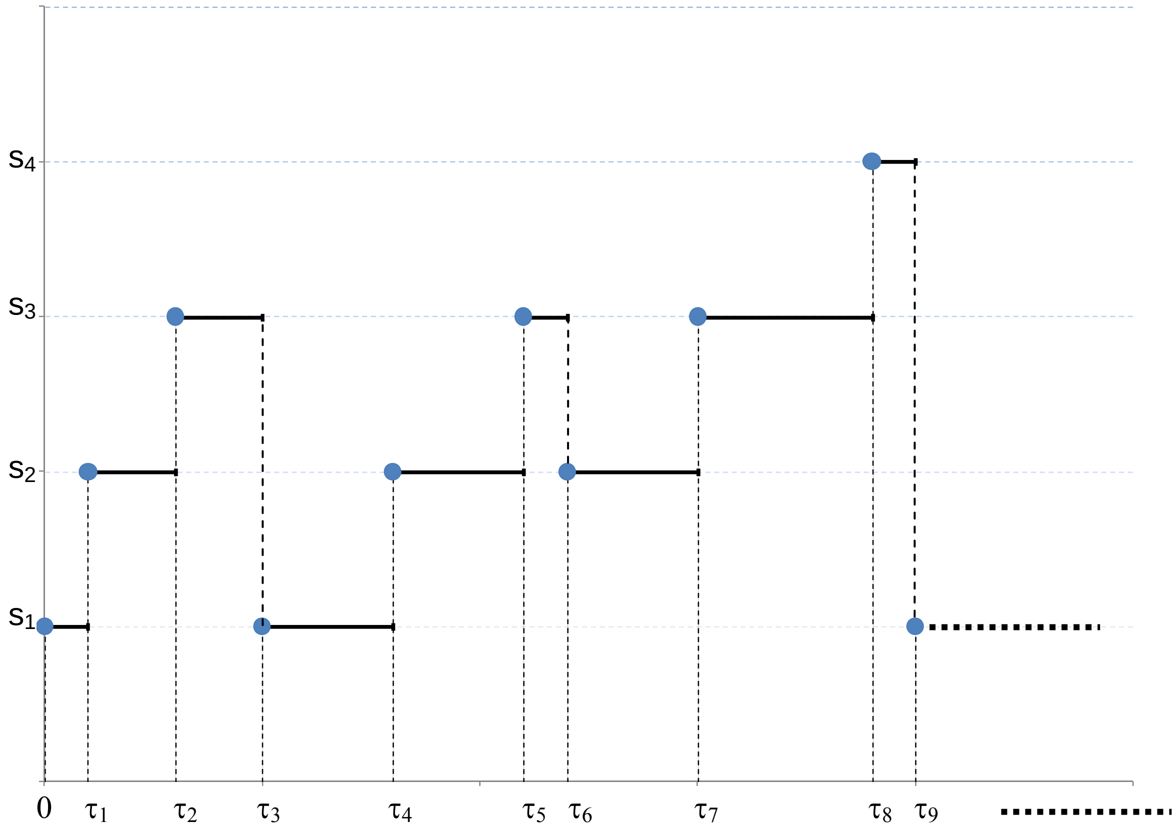

Fig.3 Example of $\{W(t) : t \in T\}$ process realization

Any definition of presented conditions determines entirely analyzed semi – Markov process, enabling assigning all of necessary characteristics of it (including time of first transition), application value of process defined this way is rather limited.

This limitation is effect of difficulty in model's verification, connected to assigning necessity of values of parameters of cumulative distribution function of random variable T_{ij}, describing duration of state s_i providing converting process into state s_j and estimations presented in formula due to probabilities p_{ij}. That activities require inter alia usage in exploit researches of advanced diagnosing systems, enabling identification of every possible state s_i, what in practice can be very difficult to implement or even impossible.

Due to that, to enhance utilitarian qualities of presented model, simplification enabling verification using standard control – measuring systems, which are equipment of majority ship power plants, seems necessary. That kind of verification will be possible as long as set of possible process states will be restricted to nominated subsets of states classes, important from engine's working and possible to identify using mentioned systems point of view.

3. Reduction of $\{W(t) : t \in T\}$ process' states set

Analyzing engine's operation in quantitative approach, practically speaking, four subsets of states classes of process presented in previous point will have meaning, namely:

- states class s'_1 – engine is in condition of technical efficiency and full task ability, no usable parameters restrictions occur and efficiency indexes achieve values established by producer,
- states class s'_2 – engine is in condition of technical inefficiency and deficient task ability, no usable parameters restrictions occur for loads less than nominal loads and efficiency indexes (eg. specific fuel consumption) have values different from those established by producer,
- states class s'_3 - engine is in condition of technical inefficiency and deficient task ability, restrictions in it's usable parameters for loads close to nominal and bigger occur and efficiency indexed (eg. specific fuel consumption) have values widely different from those established by producer
- states class s'_4 – engine is in condition of technical inefficiency and deficient task ability, restrictions in it's usable parameters for wide spectrum of loads prevent from using engine according to specifications.

Due to that we can define new semi-Markov $\{W'(t) : t \in T\}$, process, which graph of states - transitions will look as follows:

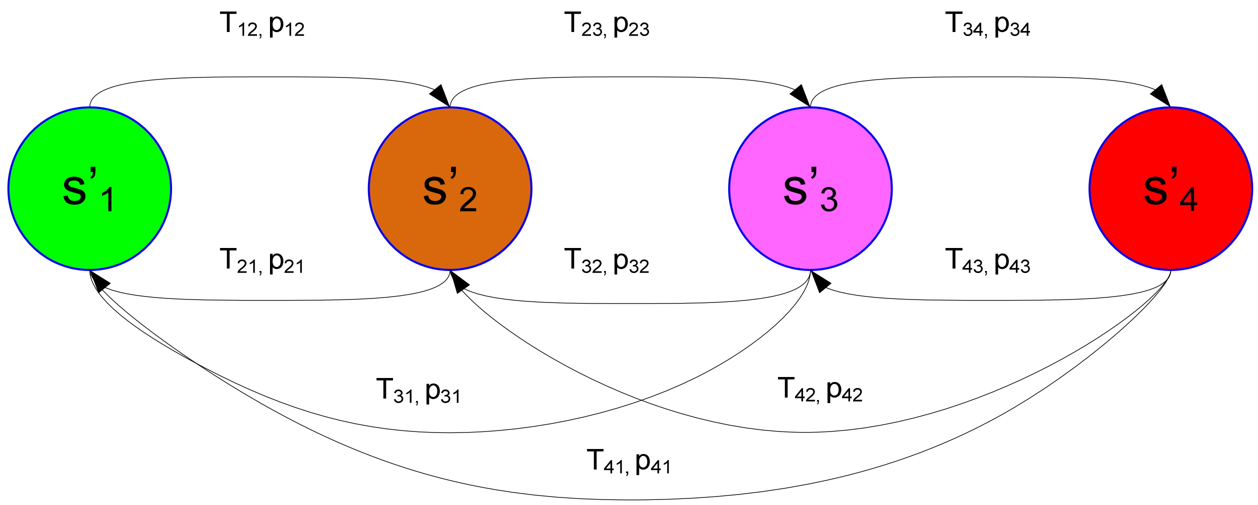

Fig. 4 States – transitions graph of $\{W'(t) : t \in T\}$ process

Matrix of function $Q'_{ij}(t)$ of analyzed process has (as it comes from graph) following form:

$$Q'_{ij}(t) = \begin{vmatrix} 0 & Q_{12}(t) & 0 & 0 \\ Q'_{21}(t) & 0 & Q'_{23}(t) & 0 \\ Q'_{31}(t) & Q'_{32}(t) & 0 & Q'_{34}(t) \\ Q'_{41}(t) & Q'_{42}(t) & Q'_{43}(t) & 0 \end{vmatrix}, \tag{7}$$

which can be presented basing on formula (4) as well as:

$$Q'_{ij}(t) = \begin{vmatrix} 0 & p_{12}\cdot F_{12}(t) & 0 & 0 \\ p_{21}\cdot F_{21}(t) & 0 & p_{23}\cdot F_{23}(t) & 0 \\ p_{31}\cdot F_{31}(t) & p_{32}\cdot F_{32}(t) & 0 & p_{34}\cdot F_{34}(t) \\ p_{41}\cdot F_{12}(t) & p_{42}\cdot F_{42}(t) & p_{43}\cdot F_{43}(t) & 0 \end{vmatrix} . \tag{8}$$

Using the formula (…) requires analysis of individual random variables T_{ij} assembled during exploit realization researches, estimation of parameters of cumulative distribution function and determining sequence and number of conversions n_{ij} of process from s'_i to s'_j state (i, j =1, 2, 3, 4 $i \neq j$).

Realization of that conditions enables estimating individual probabilities p_{ij} values, using following statistics [4]:

$$p_{ij}^{*} = \frac{n_{ij}}{\sum_{j} n_{ij}} \tag{9}$$

where:
n_{ij} – number of conversions from s'_i to s'_j state (i, j =1, 2, 3, 4 $i \neq j$).

Final defining of initial distribution of $\{W(t):t \in T\}$ process, eg. in case of analyzing engine in technical efficiency and full task ability as:

$$p_1 = P\{W(0) = s_1\} = 1,$$
$$p_i = P\{W(0) = s_i\} = 0 \text{ dla } i = 2, 3, 4 \quad , \tag{10}$$

completely defines analyzed process, ipso facto enables assigning of its characteristics, including one of the most important from practical point of view – instantaneous distribution of $P_j(t)$ process, meaning probability of finding process in s_j state, in moment t.

4. Temporary distribution of $\{W'(t) : t \in T\}$ process' probability

Assignation of probability distribution $P_j(t) = P\{W'(t) = s'_j\}, j \in S'$ first of all requires appointing of conditional probabilities $P_{ij}(t)$ with following interpretation [4, 6]:

$$p_{ij}(t) = P\{W'(t) = s'_j \,/\, W(0) = s'_i\}. \tag{11}$$

Basing on [6] unknown Laplace transforms of $P_{ij}(t)$ function can be achieved by solving system of equations :

$$\widetilde{P}(s) = \frac{1}{s}\cdot(I - \widetilde{g}\ (s)) + \widetilde{q}(s)\cdot\widetilde{P}(s) \tag{12}$$

where :

$$\widetilde{P}(s) = \begin{bmatrix} \widetilde{p}_{11}(s) & \widetilde{p}_{12}(s) & \widetilde{p}_{13}(s) & \widetilde{p}_{14}(s) \\ \widetilde{p}_{21}(s) & \widetilde{p}_{22}(s) & \widetilde{p}_{23}(s) & \widetilde{p}_{24}(s) \\ \widetilde{p}_{31}(s) & \widetilde{p}_{32}(s) & \widetilde{p}_{33}(s) & \widetilde{p}_{34}(s) \\ \widetilde{p}_{41}(s) & \widetilde{p}_{42}(s) & \widetilde{p}_{43}(s) & \widetilde{p}_{44}(s) \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{13}$$

$$\tilde{q}(s) = L\left\{\begin{bmatrix} 0 & p_{12}\cdot\frac{dF_{12}(t)}{dt} & 0 & 0 \\ p_{21}\cdot\frac{dF_{21}(t)}{dt} & 0 & p_{23}\cdot\frac{dF_{23}(t)}{dt} & 0 \\ p_{31}\cdot\frac{dF_{31}(t)}{dt} & p_{32}\cdot\frac{dF_{32}(t)}{dt} & 0 & p_{34}\cdot\frac{dF_{34}(t)}{dt} \\ p_{41}\cdot\frac{dF_{41}(t)}{dt} & p_{42}\cdot\frac{dF_{42}(t)}{dt} & p_{43}\cdot\frac{dF_{43}(t)}{dt} & 0 \end{bmatrix}\right\}, \tag{14}$$

$$\tilde{g}(s) = L\left\{\begin{bmatrix} \frac{d(p_{12}\cdot F_{12}(t))}{dt} & 0 & 0 & 0 \\ 0 & \frac{d(p_{21}\cdot F_{21}(t)+p_{23}\cdot F_{23}(t))}{dt} & 0 & 0 \\ 0 & 0 & \frac{d(p_{31}\cdot F_{31}(t)+p_{32}\cdot F_{32}(t)+p_{34}\cdot F_{34}(t))}{dt} & 0 \\ 0 & 0 & 0 & \frac{d(p_{41}\cdot F_{41}(t)+p_{42}\cdot F_{42}(t)+p_{43}\cdot F_{43}(t))}{dt} \end{bmatrix}\right\} \tag{15}$$

with notation :

$$\tilde{q}_{ij}(s) = L\left\{p_{ij}\cdot\frac{dF_{ij}(t)}{dt}\right\}, \text{ i, j} \in \text{S'}, \quad \text{i} \neq \text{j} \tag{16}$$

$$\tilde{g}_i(s) = L\left\{\frac{d\left(\sum_j p_{ij}\cdot F_{ij}(t)\right)}{dt}\right\}, \text{ i, j} \in \text{S'}, \ \text{i} \neq \text{j} \tag{17}$$

solution of system of equations (12) can be shown as [6]:

$$\tilde{P}(s) = \frac{1}{s}\cdot[I - \tilde{q}(s)]^{-1}\cdot[I - \tilde{g}(s)] \tag{18}$$

System of equations (18) is a system of linear equations in transforms field. By solving it and then replacing particular transforms and reversed Laplace transform – L^{-1} [2] analytical dependencies can be obtained, which show $P_{ij}(t)$ values as matrix:

$$P(t) = \begin{bmatrix} P_{11}(t) & P_{12}(t) & P_{13}(t) & P_{14}(t) \\ P_{21}(t) & P_{22}(t) & P_{23}(t) & P_{24}(t) \\ P_{31}(t) & P_{32}(t) & P_{33}(t) & P_{34}(t) \\ P_{41}(t) & P_{42}(t) & P_{43}(t) & P_{44}(t) \end{bmatrix} \tag{19}$$

That dependencies enable to make assignation of $P_{ij}(t)$ distribution, because basing on complete probability formula [1] and for estimated initial distribution for {W'(t): t≥ 0) this distribution can be shown in following way:

$$P_j(t) = P_{1j}(t) \tag{20}$$

$P_j(t)$ probabilities are indeed elements of first line of $P(t)=[P_{ij}(t)]$ matrix.

In this case, very important is analysis of probabilities, which for $t\to\infty$ can be interpreted as $P_{11}(t) = P_1(t)$, $P_{12}(t) = P_2(t)$, $P_{13}(t) = P_3(t)$ and $P_{14}(t) = P_4(t)$, and mean probabilities of finding process in engine in following states: s'_1, s'_2, s'_3 and s'_4, under condition that engine's initial state was s'_1.

Practical advantage of obtained $P_j(t)$ values in presented aspect of analysis of engine's main power transmission work lies in assigning value of probability of occurring situation in which:

- there is no restrictions in engine's usable parameters for loads no bigger than nominal loads and efficiency indexes (eg. unit fuel consumption) have values different from those established by producer, increasing using costs,
- enhancement of $g_p^{i\%}$ fuel dose to G_{pmax} occurs, causing limitations in ship's ability to move.

5. Summary

Semi-Markov processes theory gives us many useful methods and tools helpful in technical objects' examination. Semi-Markov processes as models of real exploit processes eg. marine engines, seem to be useful in quantitative description of work as well, which results from fact that in case of analyzing processes with constant time parameter and finite set of states, intervals of that processes' presence in particular states are random variables with facultative distribution. Practical use of described models requires as well:

- gathering appropriate statistics during exploit researches,
- reasonable complication of model, differentiate as little necessary number of class states as possible and rather simple in mathematical sense its matrix of function – Q(t).

Second condition is important in case of calculating instantaneous distribution of $p_k(t)$ process' states. That distribution can be, as is well known, calculated knowing initial distribution of process and $p_{ij}(t)$ function. Calculating the $p_{ij}(t)$ probability lies in solving Voltera's system of equations of second type, in which known quantities are $Q_{ij}(t)$ functions, elements of Q(t) function's matrix of process. In case, where number of process' states is small and function's matrix of this process – uncomplicated, this system can be solved by using Laplace transformation. However, when number of process' states is big or its function's matrix (core of the process) is very complex, only numeric solution of this system of equations can be calculated. That solution doesn't give an answer for very important for exploit practice question: how do probabilities of semi-Markov process' states change, when t approaches to the infinity? From the theory of semi-Markov processes comes, that those probabilities, in case of ergotic semi-Markov processes, with the passage of time, aim to specific, constant numbers. Those numbers are called terminal probabilities of states and sequence of those numbers creates terminal distribution of process, which can be used during decision-making procedure, with use of probabilistic decision-making models. In those models, as is well known, in spite of defining a repertory of possible decisions to make, there is, amongst others, necessity of assigning values of conditional probabilities – $p(s_i)/d_j$, which describe probability of achieving si state by analyzed object, but only when d_j decision has been made. Estimations of these values can be terminal probabilities of states, which create terminal distribution of process.

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