THE TASK GRAPH ASSIGNMENT FOR KASKADA PLATFORM[1]

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Keywords: Task assignment, Computation cluster, Multimedia processing.

Abstract: The paper describes a computational model of the KASKADA platform. It consists of two main elements: a computational cluster, and a task graph. The cluster is represented by a finite set of the nodes with the specific maximum loads. The graph contains nodes representing tasks to be executed, and the edges representing continuous data flow between the tasks. The tasks are executed concurrently and the data flows between them are directed and acyclic. For such a model, the problem of task-to-nodes assignment is analysed, and two optimisation goals are defined: low cluster fragmentation, and minimum processing latency. For both problems the heuristic algorithms are described. The simulation results of the described algorithms are provided, and their evaluation is performed. Finally, the future algorithm improvements are suggested.

1 INTRODUCTION

Context Analysis of the Camera Data Streams for Alert Defining Applications platform (Polish abbreviation: KASKADA, i.e. waterfall) is a platform supporting services used for multimedia stream processing. The simple services are directly related to the computational tasks implementing concrete processing algorithms, e.g. face recognition. A complex service is defined as a graph of simple services, which is transformed to a task graph (Krawczyk, 2010).

The typical problem of task scheduling is defined as assignment of a set of task into the set of computational nodes in a particular order. The tasks are grouped in the graph as the nodes, where a directed edge between two tasks t_1 and t_2 indicates the task t_1 to be finished before the task t_2 is started. It is proved, the above problem is NP-hard for the general case, and there exists a polynomial solution for task scheduling on two computational nodes (El-Rewini, 1994).

The proposed architecture assumes tasks are executed continuously, consuming and producing data streams. Each of them requires concrete computational power to support provided functionality. We believe that due to the character of multimedia stream processing, the tasks require concurrent execution, and they cannot be queued and started in the sequence one by one.

We introduce a specific definition of the task-to-nodes assignment regarding the above constraints. The task graph is to be assigned to the computational nodes, which can be free or already partially used, by already working tasks. In case the task graph cannot be completely assigned, the platform will refuse the whole graph. And in case there is more than one possible complete assignment, the platform should optimise its selection using one of the following criteria: minimizing the fragmentation, or processing latency.

The assignment described above is quite similar to the well known bin packing problem (BPP) (Garey, 1979), especially the version with the variable bin sizes (VBPP) (Haouari, 2009). Typical BPP minimizes a number of baskets (computational nodes in our case) used for packing a set of objects

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(tasks). The version with the basket variable sizes introduces additionally a finite set of basket types with different sizes. An exact algorithm for (V)BPP is NP-hard (Garey, 1979).

We would like to emphasize the difference between the VBPP, and our problem: for VBPP we can use as many baskets of possible type (size) as we need, however for the considered assignment, found in the real environment, the number of each type is finite. Moreover, the optimisation goals are different, VBPP minimizes the number of used baskets and we consider fragmentation (the number of partially used nodes) or latency in the processing.

In the next section we provide the formal description of the processing model and assignment problem. In the third section we consider the existence of a solution for the problem. The fourth and fifth sections contain the description of the proposed heuristic algorithms followed by the section with their evaluation and finally some conclusions are provided.

2 THE ASSIGNMENT MODEL

A task graph is defined as a pair G=(T, D), where T=$\{t_1, t_2, \ldots t_N\}$ is a set of tasks executed during the processing, and D=$\{d_1, d_2, \ldots d_M\}$ is a set of the directed edges representing data flow between the tasks. E.g. edge d_1=(t_1, t_2) indicates the task t_2 receives the data from a stream generated by the task t_1. Function Φ: T $\rightarrow$ N (where N is a set of natural numbers) denotes the task load: computational power required for the task to be executed, see figure 1. We assume the task graph is acyclic and directed (DAG), where the edge directions indicate data stream flows.

$$\Phi(t_1) = 1$$
$$\Phi(t_2) = 1$$
$$\Phi(t_3) = 2$$
$$\Phi(t_4) = 0$$

Figure 1: Example of tasks-to-nodes assignment.

A computational cluster is represented as a set of nodes C=$\{c_1, c_2, \ldots c_H\}$, with the same initial computational power: γ. Each node can have different available computational power denoted by a function Γ: C $\rightarrow$ N. E.g. for a cluster with three nodes the computational power can be as follows: $\Gamma(c_1)$=1, $\Gamma(c_2)$=2, $\Gamma(c_3)$=2 and γ=3. We assume the inter-node connections are realized by a complete

graph of network links – communication between any two nodes always requires the same resources and takes the same time.
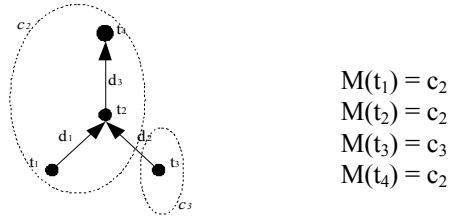

$$M(t_1) = c_2$$
$$M(t_2) = c_2$$
$$M(t_3) = c_3$$
$$M(t_4) = c_2$$

Figure 2: Example of tasks-to-nodes assignment.

A tasks-to-nodes assignment is defined as a function M: T $\rightarrow$ C, assigning a node c to the task t, fulfilling the following:

$$\sum_{\tau:M(\tau)=c}\Phi(\tau) \leq \Gamma(c) \text{ (1)},$$

so the sum of computational load of all nodes assigned to a node is lower or equal to the actual computational power. The example of a task graph (from figure 1) assignment to a cluster with three nodes is provided in figure 2.

For the assignment algorithms analysis we use the following assumptions:

Z1. A task graph is a tree.

Z2. A task graph is a tree and only the tasks being leaves perform computations:

$$\bigforall_{\tau \in T:\rho_*(\tau)>0}\Phi(\tau) = 0$$

where $\varrho_*(\tau)$ is a degree of τ node for incoming edges.

Z3. The computation load of all tasks is 0 or 1:

$$\bigforall_{\tau \in T}\Phi(\tau) \in \{0, 1\} \text{ (2)}.$$

3 THE ASSIGNMENT EXISTENCE

The first problem considered for the above assignment definition, is an examination of if it is possible to make any assignment of a given task graph to a cluster, formally, it is equal to the following question: Does there exist any function complying with the condition (1), for a given task graph G=(T, D) and a node set C? For instance, the graph in figure 1 cannot be assigned and executed using the following cluster: C=$\{c_1, c_2, c_3, c_4\}$, where $\Gamma(c_i)$=1 for i=1…4.

The above problem is NP-hard.

Proof: Even if we assume all cluster nodes have the same actual computational power equal to initial power:

$$\forall_{c \in C} \Gamma(c) = \gamma \text{ where } k \in N^+ \quad (3),$$

it is an equivalent to NP-hard optimisation bin packing problem (BPP). We treat the computational nodes as the bins, and the tasks as the packed objects. Let's assume we have the algorithm checking if h nodes can be used to pack T (objects), then iterating one-by-one natural numbers (maximally |T| times, or even $\log_2|T|$ if we use binary search instead) we would resolve BPP: the result is the minimal number for which the algorithm returned a positive answer.

With the assumption Z3, any task can be assigned for any node c, if $\Gamma(c)>0$. In such a simplified case, it is enough to compare the actual computational power of the cluster (sum of all nodes actual power) with the sum of task load, to check the existence of a solution:

$$\sum_{t \in T} \Phi(t) \le \sum_{c \in C} \Gamma(c) \quad (4).$$

4 OPTIMIZATION OF CLUSTER FRAGMENTATION

The fragmentation indicates the number of nodes partially engaged in task processing. Figure 3 shows a scenario where a new complex task cannot be assigned to any node.s
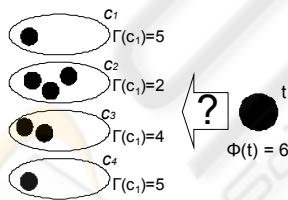

Figure 3: Example of the cluster fragmentation preventing from a new task assignment.

To avoid the above problem, we propose the optimisation goal, where the number of the partially assigned computation nodes ($\Gamma(c)<\gamma$) is minimized. Below we provide the formal definition of the optimisation according the assumed model.

Let's define *M* as a set of all possible assignments M: T → C fulfilling condition (1). Optimal solution for optimisation of cluster fragmentation is a function M∈*M* for which the number of partially used nodes is minimal:

$$min_{M(.)} \left\| \left\{ c : \left(\gamma - \Gamma(c) - \sum_{t \in M(c)} \Phi(t) \right) > 0 \right\} \right\| \quad (5).$$

The existence of function M is NP-hard problem, so its optimisation has the same property.

Algorithm 1: Algorithm based on SSP.

```
1. T_R := T
2. create C*: sorted ascending set C,
   using Γ(.) function
3. for i = 0 to H
   4. c := C*_i
   5. T':= SSP(T_R, Γ(c))
   6. make an assignment:
      ∀_{τ∈T'} M(T*_i) := C*_i
   7. T_R = T_R\T'
8. success condition: T_R = Ø
```

We emphasize the difference between the above optimisation and the one defined for bin packing problem: minimizing of the used node number. The following example illustrates this: let's assume the following task graph: T={t_1, t_2, t_3}, where $\Phi(t_1)=2$, $\Phi(t_2)=3$, $\Phi(t_3)=5$ and the cluster nodes' set: C={c_1, c_2, c_3, c_4}, where $\Gamma(c_1)=5$, $\Gamma(c_2)=5$, $\Gamma(c_3)=6$, $\Gamma(c_4)=10$ and $\gamma=10$. The optimal solution for VBPP is as follow: {t_1, t_2, t_3} → c_4, however the fragmentation optimisation is: {t_1, t_2} → c_1 and {t_3} → c_2.

Algorithm 2: Algorithm FF/FFD.

```
1. create a list T* of elements from a set
   T
2. for FFD: sort descending T* by function
   Φ(.)
3. create a list C*: sorted ascending set
   C, by function Γ(.)
4. for i = 0 to N
   5. for j = 0 to H
      6. if Φ(T*_i) ≤ Γ(C*_j)
         7. make an assignment:
            M(T*_i) := C*_i
         8. continue with the next task
9. success condition: for each task
   condition in line 6 was true at least
   once
```

In algorithm 1, we define the heuristic using a subset sum function, based on (Haouari, 2009). SSP is a function solving subset sum problem. This problem is also NP-complete class, so in this case, the approximation solution can be used (Capara, 2004).

The next proposed group of heuristic algorithms is based on an intuitive approach, whose common property is a loop iterating the task set to fit them

one by one to the computation nodes. *First fit* (FF) algorithm assigns subsequent tasks to the first node having enough computational power for its execution, in the *first fit descending* (FFD) version of this algorithm it iterates the tasks in descending order, sorted by $\Phi(.)$.

Algorithm 3: Algorithm BF/BFD.

```
1.  create a list T* of elements from a set
    T
2.  for BFD: sort descending T* by function
    Φ(.)
3.  for i = 0 to N
    4.  find c∈C where Φ(T*ᵢ) ≤Γ(c) and
        Γ(c) - Φ(T*ᵢ) is minimal
    5.  make an assignment: M(T*ᵢ) := c
6.  success condition: for each task
    condition in line 4 was true at least
    once
```

The *best fit* (BF) algorithm, like FF, iterates the task set, however the nodes are not assigned one by one, but they are selected to make best fit: minimizing the actual power left in the assigned node. As well as for FFD there is also a version with the sorted task set. Algorithm 3 presents the detailed pseudo-code for BF/BFD.

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5 OPTIMIZATION OF PROCESSING LATENCY

Processing latency depends on cooperation among tasks which are executed. If a cooperated tasks are assigned to the same node the latency is low because instead of data marshalling, transfer and unmarshalling is used only copy operation, see figure 4.

For further consideration, we take the following simplifying assumptions:

Z4. All task graph nodes introduce the same processing latency: L_T.

Z5. The latency for the edge between two tasks executed on the same computational node is always the same and equals: L_N.

Z6. The latency for the edge between two tasks executed on different nodes is always the same and

equals: L_E.

Z7. The latency for the edge between two tasks inside one node is always lower than between tasks executed on different nodes: $L_E > L_N$.
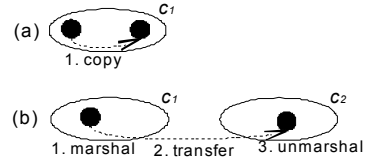
Figure 4: Example of the mappings with (a) low and (b) high latency due to task assignment to the different computation nodes.

Let's consider the task graph described in figure 1, it contains three edges and any one of them can be placed between computational nodes. Figure 5 presents example assignments: $M_1(.)$ and $M_2(.)$ - all edges are separated by the nodes, $M_4(.)$ - all edges are in the same node, and $M_3(.)$ where one edge is placed between the nodes.

According to the above assumptions, latency of the event generated in task t_3 after its propagation to task t_4 is as follows:

for M_1: $L(M_1, t_3, t_4) = L_E + L_T + L_E + L_T = 2L_E + 2L_T$

for M_3: $L(M_3, t_3, t_4) = L_E + L_T + L_N + L_T = L_E + L_N + 2L_T$

for M_4: $L(M_4, t_3, t_4) = L_N + L_T + L_N + L_T = 2L_N + 2L_T$

According to the assumption Z7:

$$L(M_1, t_3, t_4) > L(M_3, t_3, t_4) > L(M_4, t_3, t_4).$$

Below we define $L(M, t_1, t_2)$ for task graph G:

$$max_{(T^r, D^r) \in R(G, t_1, t_2)} \left\{ |T^r| \cdot L_T + S(D^r) \cdot L_N + (|D| - S(D^r)) \cdot L_E \right\} \quad (6)$$

where:

$$S(D) = |\{(t, t') \in D : M(t) = M(t')\}|,$$

$R(G, t_i, t_j)$ – is a set of subgraphs of graph G, being the routes between the tasks: t_i and t_j.

The above latency function is defined for two tasks between which its value is computed, we can also provide such a function L for the whole graph G and concrete assignment (mapping) M, it is a maximum latency for any pair of the tasks:

$$L(M, G) = max_{t_i, t_j \in T} L(M, t_i, t_j) \quad (7).$$

Assignment optimisation for the processing latency means selection of such mapping $M \in \mathbf{M}$, where the data propagation is minimal:

$$min_{M(.)} L(M, G) \quad (8).$$

Finding optimal assignment M(.) is NP-hard, because even checking for its existence has such property. Because of this, we propose a heuristic
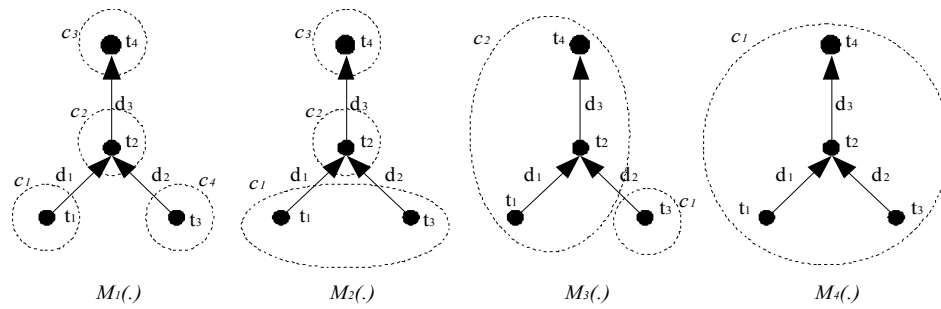
Figure 5: Task-to-node assignment example.

approach. Algorithm 4: HLT uses a set of tasks sorted by the topological order and the function `directNeighbours()` finding a set of neighbouring tasks – receiving data from the task provided as a function argument.

In the algorithm the tasks are assigned to the computational nodes using the recursion function `matchDepthFirst()`, traversing the graph using a depth-first approach.

Algorithm 4: Heuristic algorithm optimising processing latency: HLT.

```
1.  create list T*: the set T sorted
    descending in topological order
2.  create list C*: the set C sorted
    descending by Γ(.) function
3.  while T* ≠ ∅ do
  4.    c := C*₀
  5.    fv := head(T*)
  6.    if Φ(fv) > Γ(c) then
    7.      break
  8.    V := {fv}
  9.    matchDepthFirst(V, fv, Γ(c))
  10.   foreach cv in V
    11.     M(cv) := c
  12.   sort descending list C* by Γ(.)
        function
13. success condition: T* = ∅

14. procedure matchDepthFirst(V, v,
    max)
  15.   w := 0
  16.   foreach rv in V
    17.     w := w+Φ(rv)
  18.   foreach nv in
        directNeighbours(v)
    19.     if nv∈T* ∧ Φ(nv)+w≤max then
      20.       V := V∪{nv}
      21.       T* := T*\{nv}
      22.       match(V, nv, max)
```

6 EVALUATION OF THE PROPOSED ALGORITHMS

For the proposed algorithms we performed the simulation in conditions close to the real execution in the target environment – KASKADA platform. After its analysis we made the following assumptions:

1. the initial computational power of a node: γ=80 (integer values),
2. the task loads: $\Phi(t_i)$=1-80 (integer values, uniform distribution),
3. the number of not assigned nodes ($\Gamma()$=80): 20,
4. the available computational power of the partially assigned nodes: $\Gamma()$=1..79 (uniform distribution),
5. the number of partially assigned nodes: 8,
6. used SSP algorithm – exact (for the above condition we could use a dynamic programming solution)
7. latencies: L_E=16, L_N=1, L_T=32,
8. each task (except the termination ones) has 0-2 outgoing edges (communication with other tasks).

The above assumptions are related to the conditions for the assignment module in KASKADA platform used in project MAYDAY EURO 2012. However they are general enough to be used for other applications with possible slight modifications.

Table 1 and figure 6 present the simulation results for heuristics according to the above assumptions. They are grouped by the task set sizes: |T| and show the percent factor α_A, defined as follow:

$$\alpha_A = \frac{\textit{number of experiments when A gave the best solution for the fragmentation}}{\textit{number of experiments for A}}$$

Table 1: Simulation results of the heuristic algorithms for the factor α.

| |T|	FF	FFD	BF	BFD	SSP	BFDSS	LAT
2	97.49%	98.43%	98.28%	100.00%	98.43%	100.00%	5.11%
4	85.33%	90.74%	89.51%	99.03%	91.00%	99.41%	1.00%
6	65.93%	78.04%	75.03%	96.17%	79.60%	98.08%	0.29%
8	45.13%	63.64%	58.11%	91.85%	68.12%	96.80%	0.13%
10	27.57%	49.77%	42.53%	86.78%	58.89%	95.88%	0.05%
12	15.97%	38.85%	30.85%	82.52%	53.03%	96.03%	0.03%
14	9.10%	30.75%	22.83%	79.41%	49.24%	96.23%	0.01%
16	5.14%	25.06%	16.87%	76.79%	47.69%	96.64%	0.00%
18	3.01%	21.18%	13.02%	74.72%	46.94%	97.04%	0.00%
20	1.71%	18.53%	9.77%	73.03%	46.72%	97.33%	0.00%
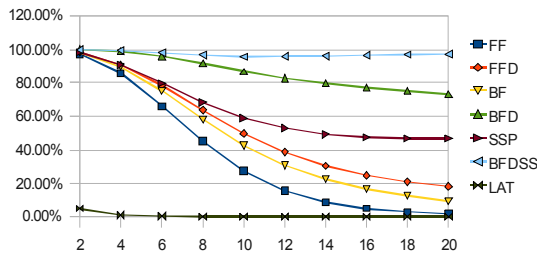
Figure 6: Simulation results of the heuristic algorithms for the factor α. Axis X: task number, axis Y: factor α.

Similarly table 2 and figure 7 present the simulation results for the algorithms, which are grouped by the task set sizes: |T|, but showing the percent latency factor βA, defined as follow:

$$\beta_A = \frac{\text{number of experiments when A gave the best solution for the latency}}{\text{number of experiments for A}}$$

Table 2: Simulation results of the heuristic algorithms for the factor β.

|T|	FF	FFD	BF	BFD	SSP	BFDSS	LAT
2	52.62%	52.62%	56.05%	59.40%	52.62%	54.20%	100.00%
4	56.42%	56.45%	59.68%	62.55%	56.46%	58.32%	97.66%
6	53.31%	53.34%	56.85%	59.54%	53.35%	55.77%	95.64%
8	51.38%	50.77%	54.93%	56.57%	50.74%	53.56%	93.79%
10	49.69%	48.41%	53.16%	53.36%	48.38%	51.07%	91.91%
12	48.18%	46.27%	51.53%	50.86%	46.14%	48.88%	90.24%
14	47.25%	45.05%	50.54%	48.78%	44.78%	47.18%	88.93%
16	45.74%	43.48%	48.81%	46.82%	43.07%	45.38%	88.07%
18	44.40%	42.24%	47.57%	45.16%	41.88%	43.99%	87.47%
20	43.33%	41.13%	46.63%	43.72%	40.95%	42.79%	87.00%

The number of executed experiments for each algorithm is 100,000, for each experiment there were randomly generated task graphs G and cluster states C: computational node set. In the table with results, there is an additional column SSP/BFD containing the results for the combined heuristics SSP and BFD, where the better solution for factor α is chosen.

Comparing the evaluation results for both types of optimisation, we can see that the BFD (except BFD/SSP together) is the best for the fragmentation optimisation, and works quite well for latency. The HLT algorithm, as you could expect, is the best for latency optimisation, but performs extremely poorly for the fragmentation.
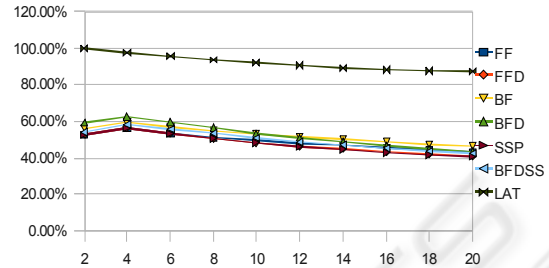

Figure 7: Simulation results of the heuristic algorithms for the factor β. Axis X: task number, axis Y: factor β.

7 CONCLUSIONS

We present a heuristic solution for the task-to-node assignment problem defined in the context of KASKADA platform. We consider six algorithms and evaluate them by a simulator of the platform. Based on the results we selected BFD heuristic as the best solution.

In the future works we can combine the above criteria and create a hybrid algorithms. It seems that HLT algorithm can be modified (e.g. by changing sorting order of the nodes) to obtain compromise between fragmentation and latency.

Alternative approach is to use several algorithms according to the current svalues of fragmentation and latency characteristics. If one of the values is not acceptable we use algorithm improving that value.

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