Submitted to "Archives of Civil and Mechanical Engineering" 2009
Published Vol. XI (2011), no. 1, pp. 195-207

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# Stability analysis of multilayered composite shells with cut-outs

A. SABIK and I. KREJA
Gdansk University of Technology, G. Narutowicza 11/12, 80-233 Gdańsk, Poland

A numerical stability analysis of an axially compressed multilayered composite shell is presented. The authors examine how the stability performance of a panel can be influenced by a centrally located square cut-out. The computations are performed within FEM computer program NX-Nastran (ver. 6.0). The stability is investigated by means of a *linearized buckling analysis* as well as of a *non-linear large deformations incremental analysis*. To get more insight into the performance of the layered structure, the failure index according to Tsai-Wu criterion is monitored in the study.

Keywords: *multilayered shells, stability, cut-out*

## 1. Introduction

Composite materials play very important role in modern building engineering. The most attractive advantage of these materials is above all a combination of a light weight with a high strength. Most common in use are multilayered composites made of fibre-reinforced layers with various orientations of fibres in a stacking sequence that determines the global anisotropy of a body. This permits a range of possibilities in design process. There is no exaggeration in opinion that composites are most suitable materials for modern light structures. The best support for this statement is the fact, that the development of composites is strongly connected with the progress of aerospace technology.

On the other side, shells, which we focus on, are in general light thin-walled structures or members of other constructions. From this point of view, they are usually subjected to instability. Of course, the stability depends on many aspects, i.e. slenderness, boundary conditions, imperfections. If the influence of these factors is strong enough, the loss of stability can occur before other limit effects like failure or delamination arise. This fact should be taken into account during design process and analysis of shells. It is however understandable that multilayered composite shells are typically slender constructions, so that, in the relation to previous considerations, the necessity of stability analysis is fairly evident in this case.

In the present paper, we examine how the stability of a multilayered panel can be influenced by centrally located square cut-outs. Such cut-outs appear quite often in practical applications, because holes can serve as doors, windows or input of pipelines. The presence of a hole can remarkably change the structural response. Due to the reduction of a material volume significant changes of stiffness occur. A predictable decrease of the critical load in such a case is not a rule. Depending on the cut-out size, both a decrease as well as an increase of the buckling resistance is possible [1]. It is also worth noticing, that due to the more complicated stress states in multilayered composite structures than in isotropic media, some tendencies of the structure response caused by changes of the cut-out size, that can be observed in the case of homogeneous isotropic shells, are not directly applicable to multilayered structures [2].

## 2. Stability analysis

There are several ways to analyse the stability of a structure. In present study, two methods are used to estimate the critical load level, namely linearized buckling analysis and non-linear incremental large deformation analysis.

### 2.1. Linearized buckling analysis

The simplest way to get information about the critical load of a structure is to examine an appropriate linear eigenvalue problem. The system of equations, which has to be resolved, has a form:

$$[\mathbf{K}^{con} + \lambda \mathbf{K}^{\sigma}]\mathbf{v} = \mathbf{0}, \tag{1}$$

where $\mathbf{K}^{con}$ is the constitutive stiffness matrix, $\mathbf{K}^{\sigma}$ represents the geometrical stiffness matrix, $\lambda$ and $\mathbf{v}$ stand for an eigenvalue and the corresponding eigenvector, respectively. This approach can be very attractive due to its computational efficiency; however, as a linear formulation it is useless when the structure undergoes large deformations in the pre-buckling range.

### 2.2. Non-linear large deformations incremental analysis

Taking into consideration large deformations one has to carry out an incremental analysis, which states:

$$[\mathbf{K}_{\mathrm{T}}(\mathbf{q})]\mathbf{\Delta q} = \mathbf{R}(\mathbf{q}), \tag{2}$$

where $\mathbf{K}_{\mathrm{T}}(\mathbf{q})$ is the tangent stiffness matrix depending on actual displacements, $\mathbf{\Delta q}$ is the increment of displacement vector and $\mathbf{R}(\mathbf{q})$ indicates the vector of residual forces. This approach is much more expensive from the computational point of view than solving the linear eigenvalue problem; additionally, some experience is usually required from the user for a proper handling. On the other hand, non-linear large deformations incremental analysis combined with the arc-length strategy enables the examination of structures undergoing large displacements including also the post-critical behaviour.

## 3. Composite shell modelling in NX-Nastran

The computations are performed within the commercial code NX-Nastran (ver. 6.0). The three-dimensional multilayered body is treated as a single layer with the equivalent stiffness of a multilayered cross-section. Such an approach is known as the Equivalent Single Layer (ESL) model [3]. As a consequence of this simplification, local effects like a delamination, matrix cracking or fibres breaking cannot be analysed.

The resulting two-dimensional model of a layered shell can be analysed adopting one of the theories established for homogeneous isotropic shells. In NX-Nastran, the so-called First Order Shear Deformation Theory (FOSD) is applied. It means that the straight line, normal to the reference surface, remains straight but not necessarily normal during the deformation. Such an approach can be also called Reissner-Mindlin type theory when comparing with the shell theories.

Another essential assumption with a reference to composites in NX-Nastran is the linear elastic material model. Nevertheless, according to previous considerations, the instability of slender structures can occur in elastic range; therefore, the NX-Nastran composite model seems to be sufficient for the present study.

In most common displacement formulations of finite elements, the assumption of a linear displacement profile through the shell thickness causes a necessity of a shear stiffness correction. This can be achieved by means of several ways. The simplest approach consists in using of pre-defined shear correction factors for each layer or for whole cross-section. It is however inconvenient to set the values of such pre-defined factors for laminated medium. More accurate for laminated bodies seem to be formulations in which shear factors for the whole cross-section or even the shear stiffness matrix are evaluated numerically. On the other hand, it is worth to mention, that such a method requires some essential presumptions. Very often, the basis for such an approach is an assumption of a cylindrical bending mode. This methodology is implemented also in NX-Nastran, and in the authors' opinion, this technique is very efficient due to its accuracy and simplicity [4].

For composite multilayered shells, only four-node flat elements are offered in NX-Nastran. The element possesses 6 degrees of freedom at each node. As a remedy for the locking phenomenon, the assumed strain field approach is applied. Stress recovery is possible only at the Gauss-point, which is located in the centre of element.

## 4. Tsai-Wu criterion

According to the previous sections, the load capacity of a structure can be very often strongly determined by other aspects than the strength of the material; hence, the stability study becomes the most important part of the analysis. Nevertheless, one should not forget of the stress state monitoring. In multilayered composites, due to the orthotropy of layers and various fibres orientations in the stacking sequence, a complicated stress state can appear, consequently appropriate strength or failure [5] criteria are required.

Similarly to homogeneous isotropic media, there exist several strength criteria for multilayered composites. The following theories are available in NX-Nastran: the maximum strain criterion, the Hill (or Tsai-Hill [5]) criterion, the Hoffmann criterion, and the Tsai-Wu criterion. The most general hypothesis has been proposed by Tsai and Wu [5]. This criterion enables the analysis of materials with different tensile and compressive strength. Moreover the theoretical results obtained by using of the Tsai-Wu hypothesis usually match very well the experimental data.

The failure surface of the Tsai-Wu criterion is described by the following equation:

$$\left(\frac{1}{X_t}-\frac{1}{X_c}\right)\sigma_1+\left(\frac{1}{Y_t}-\frac{1}{Y_c}\right)\sigma_2+\frac{1}{X_tX_c}\sigma_1^2+\frac{1}{Y_tY_c}\sigma_2^2+2F_{12}\sigma_1\sigma_2+\frac{1}{S^2}\sigma_{12}^2=1, \tag{3}$$

where $X_\mathrm{t}$, $X_\mathrm{c}$ are the longitudinal tensile and compressive strength respectively, $Y_\mathrm{t}$, $Y_\mathrm{c}$ stand for the transverse tensile and compressive strength correspondingly, $S$ represents the shear strength of a layer, whereas $\sigma_1$, $\sigma_2$, $\sigma_{12}$ symbolize stress components in the principal material coordinates. $F_{12}$ denotes the factor of an interaction between $\sigma_1$ and $\sigma_2$. Directly from (3) the formula for the failure index $FI$ can be obtained as:

$$FI=\left(\frac{1}{X_t}-\frac{1}{X_c}\right)\sigma_1+\left(\frac{1}{Y_t}-\frac{1}{Y_c}\right)\sigma_2+\frac{1}{X_tX_c}\sigma_1^2+\frac{1}{Y_tY_c}\sigma_2^2+2F_{12}\sigma_1\sigma_2+\frac{1}{S^2}\sigma_{12}^2. \tag{4}$$

The failure occurs, when the $FI$ index achieve the value equal to or greater than 1.

The value of $F_{12}$ should be determined experimentally in biaxial test. However, it is a little bit complicated [5]; therefore, very often $F_{12}$ is set to zero or evaluated from the formula given by Tsai and Hahn [6]:

$$F_{12}=\frac{-1}{2\sqrt{X_tX_cY_tY_c}}. \tag{5}$$

It will be shown later, that in spite of small value, $F_{12}$ can significantly influence the shape of failure surface.

## 5. Numerical example and discussion

The analysed example was proposed by Chaplin and Palazotto in [7]. The axially compressed cylindrical panel is studied without and with centrally located square cut-outs, as shown in Figure1.

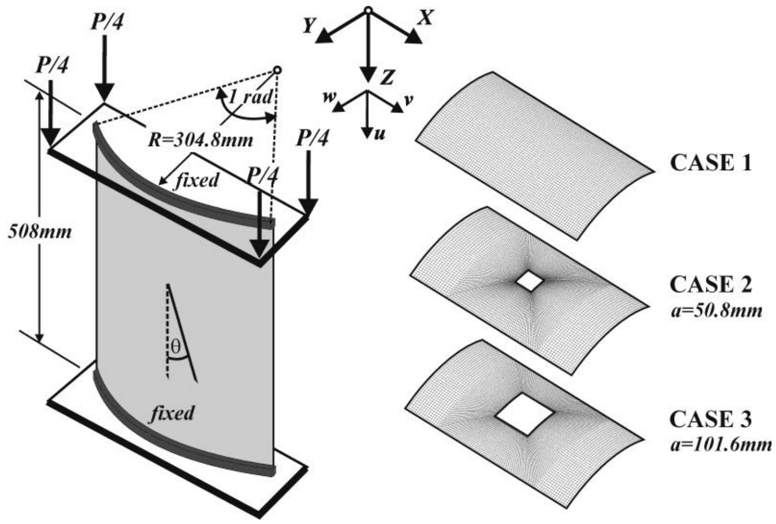


Figure 1. Geometry of the shell and cut-outs (*a* – length of cut-out's edge).

Due to the specific loading conditions, a rigid movement of the loaded edge is enforced. The panel is fixed along the curved edges and the straight edges are free. The shell consists of 16 layers with a quasi-isotropic lamination scheme $[0/45/-45/90]_{2s}$. All layers have equal thickness $h$=0.127 mm and they are made of the graphite-epoxy composite AS4/3501-6 with the following stiffness parameters: $E_1$=135.8·10$^3$ MPa, $E_2$=10.9·10$^3$ MPa, $G_{12}$=$G_{13}$= 6.4·10$^3$ MPa, $G_{23}$= 3.2·10$^3$ MPa, $\nu_{12}$=0.276. In order to investigate the failure indices, also the strength parameters are required. Since they were not given in [7], the appropriate data were adopted after [8]: $X_t$=1950 MPa, $X_c$=1480 MPa, $Y_t$=48 MPa, $Y_c$=200 MPa, $S$=79 MPa.

### 5.1. Linearized buckling analysis

Firstly the linear eigenvalue problem has been analysed. Figures 2-4 illustrate five critical modes with the corresponding critical load for the three considered cases. It can be observed, that the critical load decreases when the cut-out occurs, moreover, an additional decrease of the critical load takes place for the increasing cut-out size. While comparing the obtained eigenmode shapes, one can observe, that they are the same for all three cases and that they occur in the same sequence in case 1 and 2; however, the presence of large cut-out (case 3) interchanges the order of mode 2 and 3.

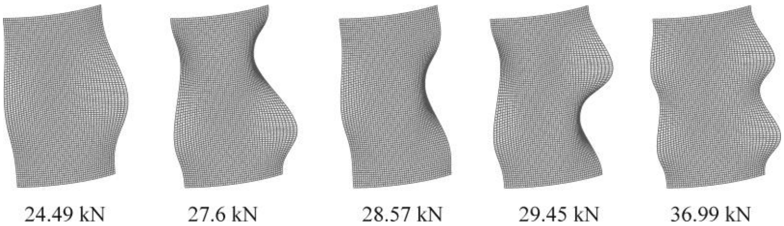


Figure 2. Linear eigenvalue problem solution, case 1.

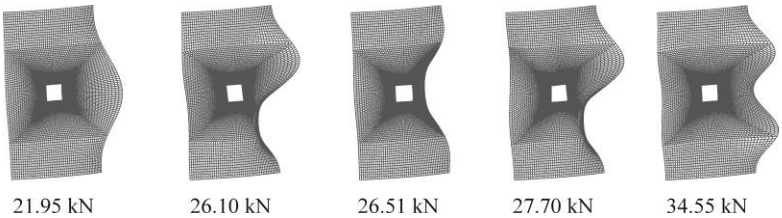


Figure 3. Linear eigenvalue problem solution, case 2.

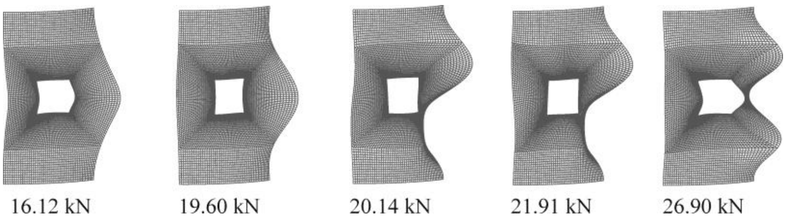


Figure 4. Linear eigenvalue problem solution, case 3.

### 5.2. Nonlinear solutions

The comparison of the results obtained by the use of linear eigenvalue problem and those of the nonlinear incremental analysis is presented in Figures 5-7. The critical load levels given by the linearized buckling analysis are depicted in the graphs by the horizontal lines. In all three cases, the maximum load limit point lays above the level of the first critical load.

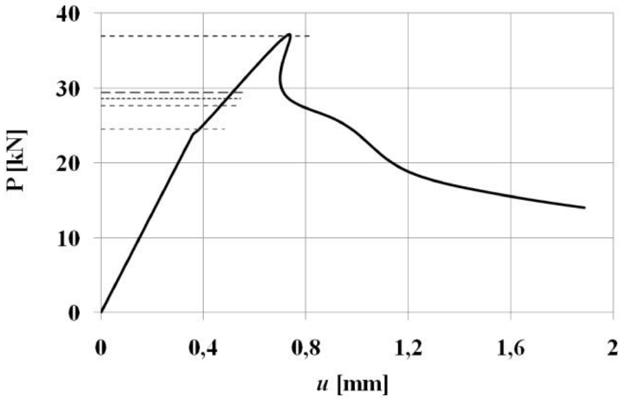


Figure 5. Nonlinear incremental analysis; case 1 – comparison with linearized buckling analysis results.

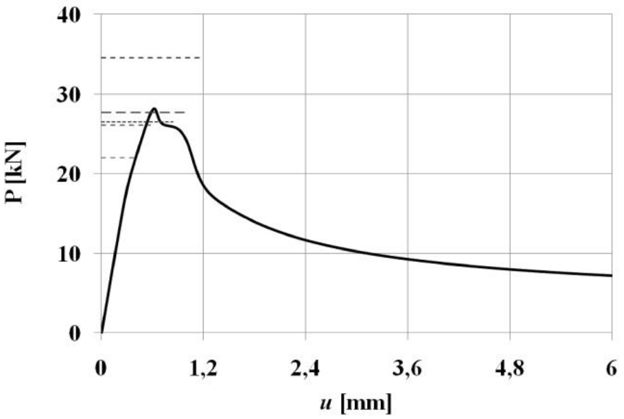


Figure 6. Nonlinear incremental analysis; case 2 – comparison with linearized buckling analysis results.

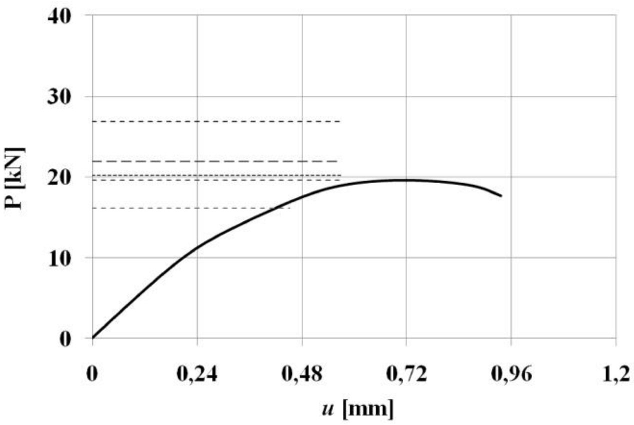


Figure 7. Nonlinear incremental analysis; case 3 – comparison with linearized buckling analysis results.

It is worth noticing, that in the case 1 the slope of the path arises in the vicinity of the first critical load level. This observation justifies the opinion, that there is a bifurcation point in the equilibrium path at the load level 24 kN, hence, the obtained path is not a fundamental one.

A more detailed study of this effect can be found in [3]. It has to be state, that the fundamental path cannot be obtained with the use NX-Nastran. The direct reason of this problem is the approximation of cylinder curvature with flat elements. It causes slight numerical imperfections, which the analysed panel, due to free straight edges, is sensitive to. On the other hand, the secondary path obtained in the present study is much more important for practical purposes.

One can observe in Figures 5-7, that with the occurring of the cut-out and with its growing size, the range of a linear shell response decreases. Only in the case 1 the structure behaves linearly up to the first critical force level. In other cases the nonlinear effects arise below the first critical force value, so the results computed in a linear eigenvalue problem are useless.

The cut-out influence on the limit load level is demonstrated in Figure 8. Likewise in the linear solution, the critical loads decrease with increase of the hole size. With dot-lines in Figure 8 the reference solution of Chaplin and Palazotto [7] is depicted. The observable quantitative discrepancies between the results of [7] and those of the present study can be explained by a too sparse FE discretization applied in [7].

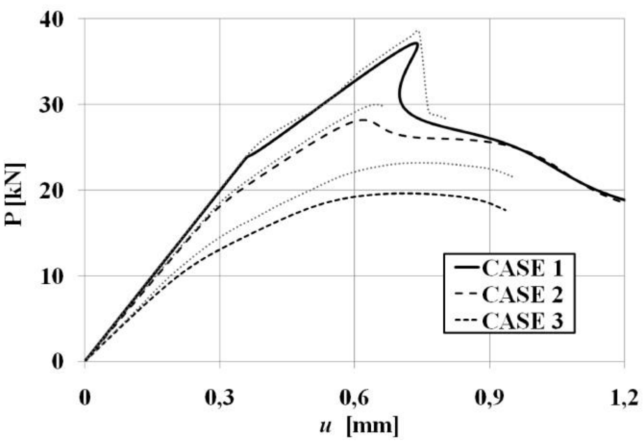

Figure 8. Influence of cut-out, nonlinear solutions. Comparison with [7].

### 5.3. Strength analysis

The range of the analysis can be further increased by taking into consideration the monitoring of the failure index *FI* for the three studied cases of the shell. The value of *FI* is calculated according to formula (4) resulted from the Tsai-Wu failure criterion.

Figure 9 shows the projection of failure surface on the $\sigma_1$-$\sigma_2$ plane for two values of interaction parameter $F_{12}$, namely $F_{12}$= 0.0 and $F_{12}$= $-3.004 \cdot 10^{-6}$ (the second value evaluated according to formula (5)). As one can observe in Figure 9, the value of $F_{12}$ significantly influenced the shape of the failure surface.

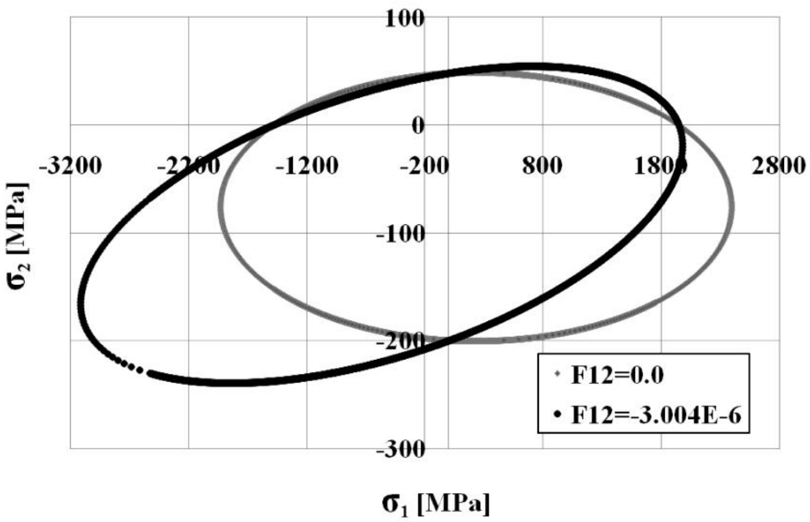


Figure 9. Influence of Tsai-Wu interaction parameter $F_{12}$ on Tsai-Wu failure surface.

The further results reported in the following were obtained for $F_{12}$= $-3.004 \cdot 10^{-6}$.
The distribution of the failure index *FI* at the limit load level for the three analysed cases are demonstrated in Figure 10. It is noteworthy that in the cases 1 and 2 the maximum values of *FI* occur in the first (bottom) layer, while in the case 3 the highest values are achieved in the ply 2.
In the case 1 the stress concentration takes place in the centre of the shell; however, the maximum value of *FI* is less than 0.5. As expected, the highest values of *FI* in the shells with cut-outs are achieved at the holes' corners. One can observe a non-symmetrical distribution of *FI* with maximum values at the upper right and the bottom left corner in the case 2 (*FI*=0.77) as well as in the case 3 (*FI*=1.3). The non-symmetrical distribution of *FI* (and stresses) is caused mainly by non-symmetrical deformation patterns – see Figures 11-13.

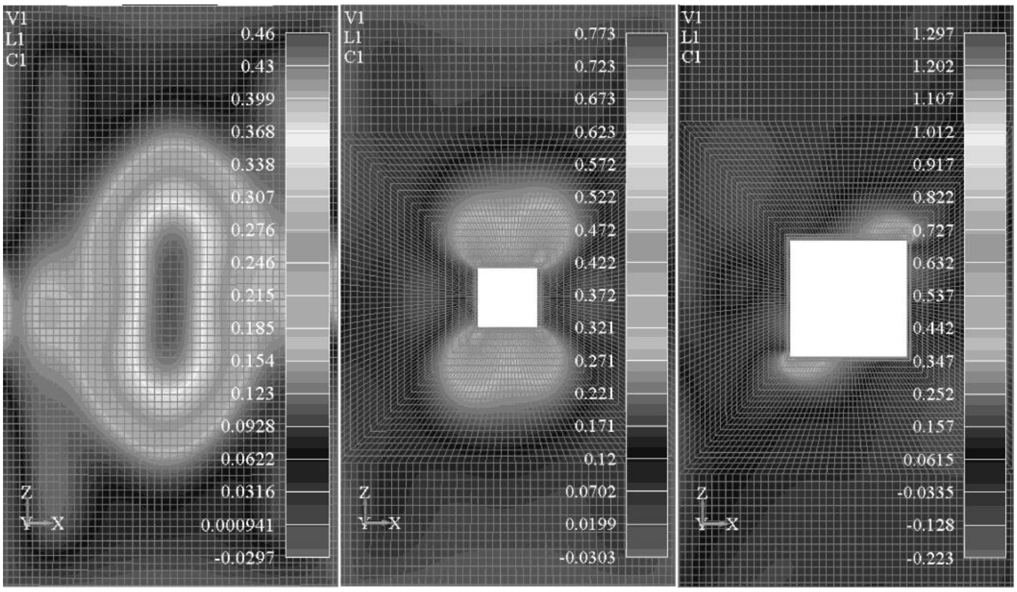


Figure 10. Failure index *FI* distribution at limit load levels, 3 cases.

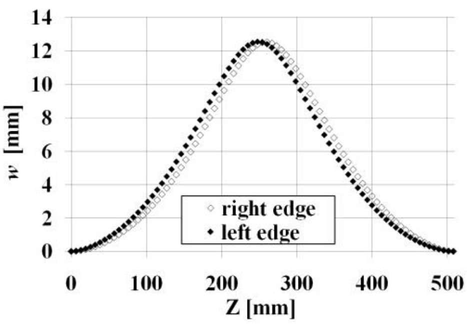


Figure 11. Deformation of straight edges at limit load level, case 1.

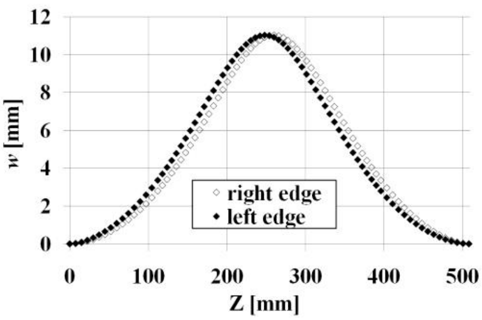


Figure 12. Deformation of straight edges at limit load level, case 2.

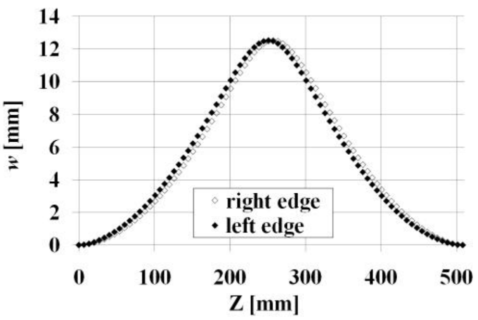


Figure 13. Deformation of straight edges at limit load level, case 3.

The increase of the cut-out's size causes a growth of the *FI* value. According to the obtained *FI* results, the failure at the limit point occurs only in the case 3.

It has to be mention, that NX-Nastran offers just a passive detection of a failure – the *FI* index value greater than 1 indicates the failure; however, the stiffness of the structure is not affected, what can be considered as a significant limitation of the program.

Since the failure has been detected for the case 3, an additional study has been performed to examine, how the response of the structure would change when the corners of the largest cut-out were rounded. The equilibrium paths (axial displacement vs. load) obtained for the rounding $x$=0%, $x$=5% and $x$=10% are presented in Figure 14a. The interpretation of $x$ parameter can be found in Figure 14a.

One can observe, that with the higher values of $x$ the limit load increases; however, the *FI* values for the limit load are still above 1, as listed in Figure 14a. On the other hand, when the failure index is monitored for the constant load level (equal to the limit load for the structure without rounded cut-out's corners), by applying the rounding parameter $x$=10% the value of *FI* can be cut down to 0.9 (Figure 14b).

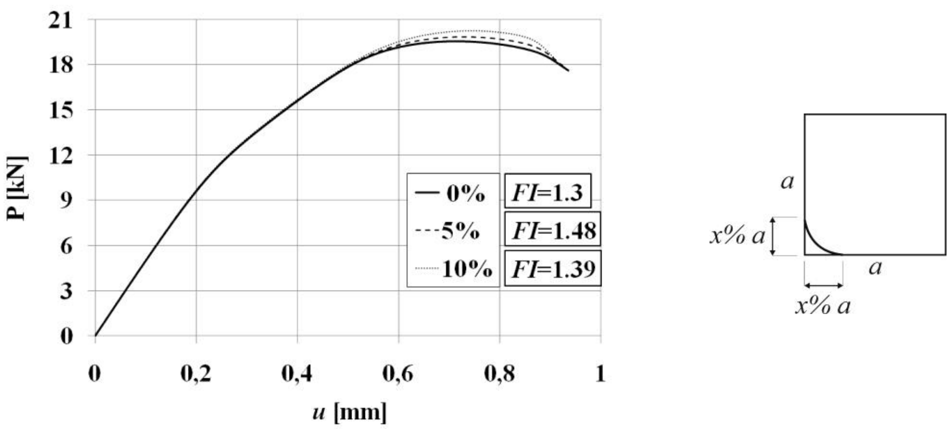


Figure 14a. Case 3, influence of corner' rounding.

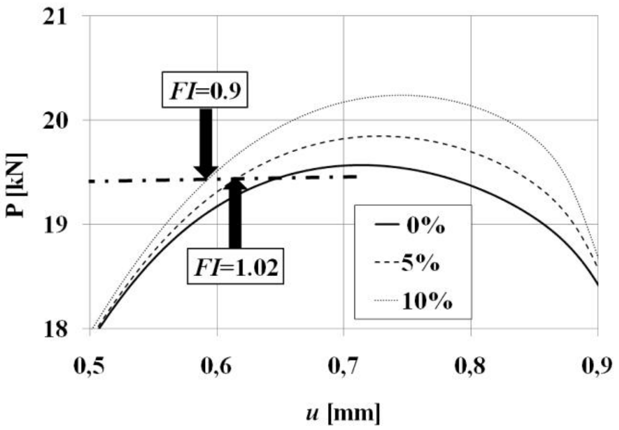

Figure 14b. Case 3, influence of corner' rounding.

## 6. Conclusions

A numerical stability study of an axially compressed multilayered composite shell has been presented. The authors have focused on the influence of a centrally located square cut-out on the structure instability. The analysis has been performed within NX-Nastran by using the four-node QUAD4 elements. The linearized buckling analysis as well as non-linear incremental analysis have been carried out to calculate critical load values. Additionally the material strength has been analysed by adopting the failure criterion of Tsai and Wu.

It has been shown, that cut-outs significantly change the structure behaviour. The increase of the cut-out's size reduces the critical load values and decreases the range of linear structure response. Furthermore, the presence of a cut-out causes considerably stress state changes when compared with a shell without any hole.

It is also noteworthy, that the analysis of composite failure in NX-Nastran has only a passive character, since the stiffness of the structure is not modified according to the results of the failure analysis.


## ACKNOWLEGMENTS

This work is financed by the Polish Ministry of Science and Higher Education as a research project no. N N506 254237.

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**Analiza stateczności kompozytowych powłok warstwowych z otworami**

W pracy analizowana jest stateczność ściskanej osiowo kompozytowej powłoki warstwowej. Badany jest wpływ usytuowanego centralnie w powłoce otworu kwadratowego na stateczność konstrukcji. Obliczenia wykonano w programie NX-Nastran (wersja 6.0). Poziom obciążeń krytycznych wyznaczony został w rozwiązaniu liniowym (liniowy problem własny) oraz na drodze analizy przyrostowej z uwzględnieniem dużych przemieszczeń (statyka nieliniowa). Dodatkowo analizowano wytężenie konstrukcji wykorzystując kryterium Tsai-Wu.

Z analizy rezultatów wynika, że wraz z pojawieniem się otworu i wzrostem jego wymiarów obniża się poziom obciążeń krytycznych oraz maleje zakres liniowej odpowiedzi konstrukcji. Ponadto obserwowany jest znaczny wzrost wskaźników wytężenia w obszarach koncentracji naprężeń.