TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics and Mechanics
Vol. 54, Issue I, 2011

MECHANISMS POSITIONED IN A NEIGHBOURHOOD OF THEIR SINGULAR POSITIONS - VELOCITY AMPLIFICATION IN THE ELECTROMAGNETIC DAMPING. PART 1 – BACKGROUND OF THE MECHANICAL STRUCTURE MODELING

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Abstract: *In the present paper, modelling methods are presented for a numerical model of a mechanical part of a hybrid (continuous/multibody) system. In the system, vibrations of a continuous mechanical part are present. To dampen them, their energy is transformed into the electrical current (by use of a DC generator) and dissipated. To amplify the damping, a double-bar mechanism is introduced between the vibrating part and the damping element. Two structurally different subsystems are considered. The elastic part is composed of finite elements. The mechanism is modelled as a multibody system. Constraint equations are used to joint the subsystems. The common final model effects in a numerical tool, useful when verification of electromagnetic damping is considered.* ***Key words:*** *electromagnetic damping; velocity amplification; singular position; multibody modelling; finite elements; constraint equations.*

1. INTRODUCTION

In the present paper, modelling methods are presented for a numerical model of a mechanical part of a hybrid (continuous/ multibody) system. In the second part, the resulting model is tested. As a final effect, common for the both parts, an effective damping method is searched for vibrations present in a continuous mechanical system. Announced mechanical vibrations are commonly observed in the everyday industrial practice, thus industrial significance of the problem is substantial. Except of some exceptional cases (when vibrations are generated intentionally), vibrations are non-required. Their negative outcomes are commonly cited. The uncontrolled motion of the effectors is the main outcome. The phenomenon complexity has to be underlined, as few sources can be responsible for such disturbances. The manual (or automatic) control of the system can be noised, as vibrations indicate a noise signal introduced in the control loop [1]. In-between the rest of the outcomes, significant acoustic noises have to be pointed. Motions of the surrounding elements can be evoked, when synchronized in frequencies. Some long-term effects are commonly cited. Harmful influence on the human health [2-4], and the fatigue of the mechanical elements are the examples. Summarising it, a search for an effective damping method is the essential topics in the most of the industrial design processes.

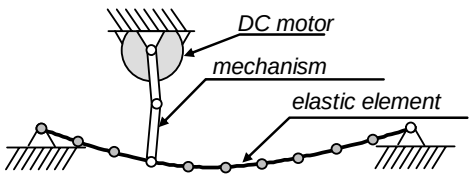

Fig. 1. Main elements of the considered system (a)

Within the presently used applications, the methods based on the viscous dampers look as the dominant solution. However, even if successfully applied in the medium size applications (as vehicles for example), they are difficult to operate in the small size

applications. There, the electromagnetic effects look as the challenging ones [5-7]. Such devices are favoured by their simpler construction, lower dimensions, easier semi active control and luck of the potential oil impurities.

As the internal structural damping is unavoidable in most of the industrial applications, high frequency vibrations (except of the self-excited or these present in the resonance cases) are damped well. None of the external actions is necessary. By contrast, when the low frequency vibrations are considered, the structural damping is ineffective. Velocities of the related motions are relatively low, and low damping forces are present. Such frequencies are damped badly, even when the external damping is present. To accelerate the damping, velocity amplification can be useful. The one considered focuses on a mechanism introduced between the vibrating element and the damper (Fig. 1).

In the present part, fundaments of a hybrid (continuous/multibody) numerical modelling are presented. Five sections are introduced. As a multibody system is present in the final model, fundaments of the classical (rigid body) multibody formalism are presented firs. Section two is devoted for it. To obtain the required dynamics equations, a method presented in [8, 9] is recalled. Three-like and closed loop systems are considered. The mass matrix formulas, as well as to the generalised forces formulas, are presented.

The continuous structure is considered as composed of short deformable elements, i.e. the finite elements, are used. The structure drift motion is absent. The nodes displacements (displacements at a set composed of few selected points) are considered as system's generalized coordinates. A method as proposed in [11-13] is used and required dynamics equations are obtained. These equations are linear in respect to the joint displacements, velocities and accelerations. Related formulas for the mass, dissipation and stiffness matrices are detailed in the third section.

To joint these structurally different sub-models, constraint equations are proposed for the contact between the mechanism and the vibrating element. Details of the constraint equations are presented in section four. To obtain dynamics of the closed-loop systems, classical coordinate partitioning is used as proposed in [10]. Finally, conclusions and perspectives are presented in section nine.

2. FUNDAMENTS OF THE USED MULTIBODY FORMALISM

In the present paper, classical multibody modelling is considered. The used multibody system is composed of *rigid bodies*. They are inertial and joined together. Massless *connections* are used to joint the bodies. Displacements are associated to connections only, and their magnitude can be significant. As a relative coordinate formulation is considered, these displacements are treated as system's generalized coordinates.

In the general theory, different types of the generic bodies can be used. The main set is composed of finite size and finite inertia elements. A term *material bodies* (*m-bodies*) can be used to denote such elements. By contrast, the *reference body* is motionless. Massless bodies, but finite in dimensions are useful to formulate constraint equations. Finally, the massless and dimensionless bodies are used as the reference elements or as the connecting elements. These last are called *point bodies* (*p-bodies*).

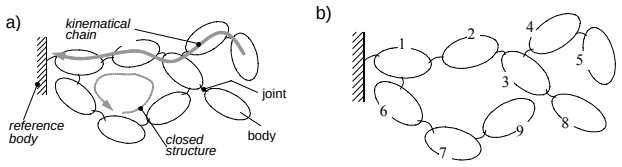

Fig. 2: Examples of the multibody systems: tree structure (a); body numbering (b);

In general, connections are some multi-degrees of freedom elements. When restricted to one-degree-of-freedom (moreover restricted to prismatic or revolute type), the reduced set is sufficient to describe the multibody system. Such elemental connections are called joint. Any of the multi-freedom connections can be modelled as composed of joints and p-bodies.

Then, when a set of bodies is considered (not independent but joint connected), a *kinematical chain* can be defined (Fig. 2a).

Among of the potential chains, the *reference kinematical chain* (i.e. the chain that connecting its generic body with the reference body) is the most significant. When its content is uniquely determinable, the chain is defined as an *open kinematical chain*. Otherwise, the chain is a *closed kinematical chain* (Fig. 2a). Moreover if none on the system chains is of the closed type, the system is called as a *tree structure* (Fig. 2b).

2.1. Dynamics of the tree structures

When a tree structure is considered, a numbering can be introduced for the bodies (Fig. 2b). The numbering correlates with the succession order, observed in the reference chain. According to it, numbers are lower when bodies of the reference chain are considered. Then $a<b$ symbol is used when a belongs to the reference chain of the generic body b. A a^+ symbol denotes the complete set of direct successors and a a^- symbol denotes the direct predecessor of the body a. Finally, for the joint numbering, the joint that connect body a with the body a^- is numbered as a.

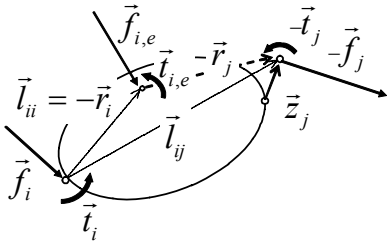

Fig. 3: The main distances and the main interactions of a selected body of the multibody system

To describe the geometrical properties, vectors are introduced for the generic body as presented in Fig. 3. Coordinates are constant for these vectors, when the coordinates are expressed in the body fixed coordinate system. By contrast, configuration dependent orientation matrices $\boldsymbol{T}^i$ are necessary to express them in the reference body system (*reference coordinate system*). The $\boldsymbol{T}^i$ matrices are calculated as the ordered products of the relative orientation matrices $\boldsymbol{R}^j$ present in the joints. Joints of the related reference chain have to be considered in the product. Simultaneously, vector $\vec{x}^i$ (it locates the mass centre of body i, with respect to the origin of the reference system) is a sum of dimensions of all the bodies. The distance has to be extended with the translations present in the joints, of the reference chain considered. It leads to [8, 9]:

$$\boldsymbol{T}^i = \prod_{j:j\le i} \boldsymbol{R}^j \quad ; \quad \vec{x}^i = \sum_{j:j\le i} \left(\vec{z}^{\,j} + \vec{d}^{\,ji}\right) \ , \tag{1}$$

When differentiated in respect to time and when a matrix form is used, ones obtain velocity and acceleration relations [8, 9]:

$$\vec{\omega}^i = \vec{\boldsymbol{A}}^{2,i} \cdot \dot{\boldsymbol{q}}^b \quad ; \qquad \vec{\dot{x}}^i = \vec{\boldsymbol{A}}^{1,i} \cdot \dot{\boldsymbol{q}}^b \ ; \tag{2}$$

$$\vec{\dot{\omega}}^i = \vec{\boldsymbol{A}}^{2,i} \cdot \ddot{\boldsymbol{q}}^b + \vec{\dot{\omega}}^{i,R} \quad ; \quad \vec{\ddot{x}}^i = \vec{\boldsymbol{A}}^{1,i} \cdot \ddot{\boldsymbol{q}}^b + \vec{\ddot{x}}^{i,R} \ , \tag{3}$$

where: $\boldsymbol{q}^b$ - column matrix of system generalized coordinates; $\vec{\boldsymbol{A}}^{1,i}$, $\vec{\boldsymbol{A}}^{2,i}$ - row matrices of vectors[1]; $\vec{\ddot{x}}^{i,R}$, $\vec{\dot{\omega}}^{i,R}$, - "remainders" independent of joint accelerations.

To obtain the dynamic equations, free body diagrams are composed (Fig. 2c). All the joints are cut and joint interactions are introduced to replace the cut joints. Then, the Newton/Euler equations of dynamics are used [8, 9]:

$$m^i \cdot \vec{\ddot{x}}^i = \vec{f}_i + \vec{f}_i^{\,e} - \sum_{j\in i^+} \vec{f}_j^{\,i} \ ; \tag{4}$$

$$\begin{aligned} &\vec{\omega}^i \times \left(\bar{\bar{I}}^i \cdot \vec{\omega}^i\right) + \bar{\bar{I}}^i \cdot \vec{\dot{\omega}}^i = \\ &= \vec{t}_{iC} + \vec{r}^{\,ii} \times \vec{f}_i + \vec{t}_{iC}^{\,e} - \sum_{j\in i^+} \vec{t}_{jC}^{\,i} - \sum_{j\in i^+} \vec{r}^{\,ij} \times \vec{f}_j^{\,i} \ , \end{aligned} \tag{5}$$

where: m^i - mass of the i^{th} body; $\bar{\bar{I}}^i$ - i^{th} tensor of moments of inertia about the mass centre; $\vec{f}^{\,i}, \vec{t}^{\,i}$ - force and torque at the joint; $\vec{f}_i^{\,e}$ - net external force; $\vec{t}_{iC}^{\,e}$ - net external torque about the mass centre.

Combining the dynamics equations (4) with the kinematics equations (3), ones obtain [8, 9]:

$$\vec{\boldsymbol{B}}^{1,i} \cdot \ddot{\boldsymbol{q}}^b + m^i \cdot \vec{\ddot{x}}^{i,R}(\boldsymbol{q}^b, \dot{\boldsymbol{q}}^b) = \vec{f}_i + \vec{f}_i^{\,e} - \sum_{j\in i^+} \vec{f}_j^{\,i} \ ; \tag{6}$$

$$\begin{aligned} &\vec{\boldsymbol{B}}^{2,i} \cdot \ddot{\boldsymbol{q}}^b + \vec{\omega}^i \times \left(\bar{\bar{I}}^i \cdot \vec{\omega}^i\right) + \bar{\bar{I}}^i \cdot \vec{\dot{\omega}}^{i,R}(\boldsymbol{q}^b, \dot{\boldsymbol{q}}^b) = \\ &= \vec{t}_{iC} + \vec{r}^{\,ii} \times \vec{f}_i + \vec{t}_{iC}^{\,e} - \sum_{j\in i^+} \vec{t}_{jC}^{\,i} - \sum_{j\in i^+} \vec{r}^{\,ij} \times \vec{f}_j^{\,i} \end{aligned} \ , \tag{7}$$

where: $\vec{\boldsymbol{B}}^{1,i} = m^i \cdot \vec{\boldsymbol{A}}^{1,i}$, $\vec{\boldsymbol{B}}^{2,i} = \bar{\bar{I}}^i \cdot \vec{\boldsymbol{A}}^{2,i}$ - matrices of coefficients (partial vectors).

Unfortunately, the first summands of (6) and (7) are not the lonely terms dependent on the joint accelerations. The forces and the torques

[1] All elements of these matrices are geometrical vectors

present in the successors' joints are dependent on them, too. To obtain the explicit form, backward evaluation is employed. The leaf bodies (i.e. the bodies without a successor) are used to start the process. At each of the evaluation step, the i^+ interactions are detected and replaced by the formulas obtained in the previous step. As the result, the i^{th} interactions remain the only one present in the i^{th} equations of dynamics. They are placed as the left side expressions in the dynamics equations, and the matrix expression is formulated as:

$$\vec{f}^{\,i} = \vec{\boldsymbol{C}}^{\boldsymbol{1,i}} \cdot \ddot{\boldsymbol{q}}^{\boldsymbol{b}} + \vec{\boldsymbol{D}}^{\boldsymbol{1,i}} + \vec{\boldsymbol{E}}^{\boldsymbol{1,i}} \; ; \qquad \vec{t}^{\,i} = \vec{\boldsymbol{C}}^{\boldsymbol{2,i}} \cdot \ddot{\boldsymbol{q}}^{\boldsymbol{b}} + \vec{\boldsymbol{D}}^{\boldsymbol{2,i}} + \vec{\boldsymbol{E}}^{\boldsymbol{2,i}} \; , \tag{8}$$

where:

$$\begin{aligned}
&\vec{\boldsymbol{C}}^{\boldsymbol{1,i}} = \textstyle\sum_{l:i\le l} \vec{\boldsymbol{B}}^{\boldsymbol{1,l}} \\
&\vec{\boldsymbol{C}}^{\boldsymbol{2,i}} = \textstyle\sum_{l:i\le l} \left[\vec{\boldsymbol{B}}^{\boldsymbol{2,l}} + \left(\sum_{k:i\le k\le l} \vec{l}^{\,kl} \right) \times \vec{\boldsymbol{B}}^{\boldsymbol{1,l}} \right] ; \\
&\vec{\boldsymbol{D}}^{\boldsymbol{1,i}} = \textstyle\sum_{l:i\le l} m^l \cdot \ddot{\vec{x}}^{\,l,R} \\
&\vec{\boldsymbol{D}}^{\boldsymbol{2,i}} = \textstyle\sum_{l:i\le l} \left[\vec{\omega}^{\,l} \times \left(\bar{\bar{I}}^{\,l} \cdot \vec{\omega}^{\,l} \right) + \bar{\bar{I}}^{\,l} \cdot \dot{\vec{\omega}}^{\,l,R} + m^l \cdot \left(\sum_{k:l\le k\le l} \vec{l}^{\,kl} \right) \times \ddot{\vec{x}}^{\,l,R} \right]; \\
&\vec{\boldsymbol{E}}^{\boldsymbol{1,i}} = -\textstyle\sum_{l:j\le l} \vec{f}^{\,l,e} \\
&\vec{\boldsymbol{E}}^{\boldsymbol{2,i}} = -\textstyle\sum_{l:i\le l} \vec{t}^{\,l,e} - \sum_{l:i\le l} \left(\sum_{k:i\le k\le l} \vec{l}^{\,kl} \right) \times \vec{f}^{\,l,e} \, .
\end{aligned} \tag{9}$$

To obtain the active components, interactions are projected onto joint mobility vectors ($\vec{e}^{\,i}$ for rotational and $\vec{a}^{\,i}$ for translational). As the $\vec{e}^{\,i}$ vector is zero for the translational joint, and the $\vec{a}^{\,i}$ vector is zero for the rotational joint, it turns the dynamics equations into a matrix from [8, 9]

$$\boldsymbol{M}^{\boldsymbol{b}}(\boldsymbol{q}^{\boldsymbol{b}}) \cdot \ddot{\boldsymbol{q}}^{\boldsymbol{b}} + \boldsymbol{F}^{\boldsymbol{b}}(\dot{\boldsymbol{q}}^{\boldsymbol{b}}, \boldsymbol{q}^{\boldsymbol{b}}) + \boldsymbol{Q}^{\boldsymbol{b}}(\dot{\boldsymbol{q}}^{\boldsymbol{b}}, \boldsymbol{q}^{\boldsymbol{b}}, \boldsymbol{f}_{\boldsymbol{e}}, \boldsymbol{t}_{\boldsymbol{e}}, t) = 0 \; , \tag{10}$$

where:

$$\boldsymbol{M}^{\boldsymbol{b}} = \vec{a}^{\,i} \circ \vec{\boldsymbol{C}}^{\boldsymbol{1,i}} + \vec{e}^{\,i} \circ \vec{\boldsymbol{C}}^{\boldsymbol{2,i}} \; ; \tag{11}$$

$$\boldsymbol{F}^{\boldsymbol{b}} = \vec{a}^{\,i} \circ \vec{\boldsymbol{D}}^{\boldsymbol{1,i}} + \vec{e}^{\,i} \circ \vec{\boldsymbol{D}}^{\boldsymbol{2,i}} \; ; \tag{12}$$

$$\boldsymbol{Q}^{\boldsymbol{b}} = \vec{a}^{\,i} \circ \vec{\boldsymbol{E}}^{\boldsymbol{1,i}} + \vec{e}^{\,i} \circ \vec{\boldsymbol{E}}^{\boldsymbol{2,i}} \, . \tag{13}$$

2.2. Dynamics of closed multibody structures

When a closed multibody structure is considered, its chains are cut. A *reference tree* structure is restored. The tree structure dynamics equations are obtained from (10). To preserve the chains consistency, the constraints equations are introduced. As proposed in [8, 9], their left side functions are grouped in a column matrix $\boldsymbol{h}$. Then, the constraint equations are differentiated twice to obtain the velocities and accelerations constraints. Then, the dynamics equations of the reference tree are accompanied with the Lagrange's multipliers. The resulting form of the dynamics equations is [8, 9]:

$$\boldsymbol{h}(\boldsymbol{q}^{\boldsymbol{b}}) = 0 \; ; \tag{14}$$

$$\dot{\boldsymbol{h}}(\boldsymbol{q}^{\boldsymbol{b}}) = \boldsymbol{J}(\boldsymbol{q}^{\boldsymbol{b}}) \cdot \dot{\boldsymbol{q}}^{\boldsymbol{b}} = 0 \; ; \tag{15}$$

$$\ddot{\boldsymbol{h}}(\boldsymbol{q}^{\boldsymbol{b}}) = \boldsymbol{J}(\boldsymbol{q}^{\boldsymbol{b}}) \cdot \ddot{\boldsymbol{q}}^{\boldsymbol{b}} + \boldsymbol{A}(\boldsymbol{q}^{\boldsymbol{b}}, \dot{\boldsymbol{q}}^{\boldsymbol{b}}) = 0 \; ; \tag{16}$$

$$\begin{aligned}
\boldsymbol{M}^{\boldsymbol{b}}(\boldsymbol{q}^{\boldsymbol{b}}, t) \cdot \ddot{\boldsymbol{q}}^{\boldsymbol{b}} + \boldsymbol{F}^{\boldsymbol{b}}(\dot{\boldsymbol{q}}^{\boldsymbol{b}}, \boldsymbol{q}^{\boldsymbol{b}}) + \\
+ \boldsymbol{Q}^{\boldsymbol{b}}(\dot{\boldsymbol{q}}^{\boldsymbol{b}}, \boldsymbol{q}^{\boldsymbol{b}}, \boldsymbol{f}_{\boldsymbol{e}}, \boldsymbol{t}_{\boldsymbol{e}}, t) + \boldsymbol{J}^T(\boldsymbol{q}^{\boldsymbol{b}}) \cdot \boldsymbol{\lambda} = 0 \, ,
\end{aligned} \tag{17}$$

where: $\boldsymbol{J}$ – Jacobian of the $\boldsymbol{h}$; $\boldsymbol{\lambda}$ – Lagrange's multipliers.

Not all the elements are independent in the $\boldsymbol{q}$ matrix (the independency was valid for the tree structure). According to the dependence, independent coordinates, $\boldsymbol{u}^{\boldsymbol{b}}$, as well as dependent coordinates, $\boldsymbol{v}$, are present in the $\boldsymbol{q}$ matrix, now. The dependent coordinates, as well as Lagrange's multiplier, have to be eliminated. Coordinate partitioning method proposed by Haug [10] can be employed. The constraint equations as well as the dynamics equations are partitioned. It leads to [10]:

$$\boldsymbol{J}_{\boldsymbol{v}}(\boldsymbol{u}^{\boldsymbol{b}}) \cdot \dot{\boldsymbol{v}} + \boldsymbol{J}_{\boldsymbol{u}}(\boldsymbol{u}^{\boldsymbol{b}}) \cdot \dot{\boldsymbol{u}}^{\boldsymbol{b}} = 0 \quad ; \; ; \tag{18}$$

$$\boldsymbol{J}_{\boldsymbol{v}}(\dot{\boldsymbol{u}}^{\boldsymbol{b}}) \cdot \ddot{\boldsymbol{v}} + \boldsymbol{J}_{\boldsymbol{u}}(\boldsymbol{u}^{\boldsymbol{b}}) \cdot \ddot{\boldsymbol{u}}^{\boldsymbol{b}} = -\boldsymbol{A}(\boldsymbol{u}^{\boldsymbol{b}}, \dot{\boldsymbol{u}}^{\boldsymbol{b}}) \; ; \tag{19}$$

$$\boldsymbol{M}_{\boldsymbol{vu}} \cdot \ddot{\boldsymbol{u}}^{\boldsymbol{b}} + \boldsymbol{M}_{\boldsymbol{vv}} \cdot \ddot{\boldsymbol{v}} + \boldsymbol{F}_{\boldsymbol{v}} + \boldsymbol{Q}_{\boldsymbol{v}} + \boldsymbol{J}_{\boldsymbol{v}}^T \cdot \boldsymbol{\lambda} = 0 \; ; \tag{20}$$

$$\boldsymbol{M}_{\boldsymbol{uu}} \cdot \ddot{\boldsymbol{u}}^{\boldsymbol{b}} + \boldsymbol{M}_{\boldsymbol{uv}} \cdot \ddot{\boldsymbol{v}} + \boldsymbol{F}_{\boldsymbol{u}} + \boldsymbol{Q}_{\boldsymbol{u}} + \boldsymbol{J}_{\boldsymbol{u}}^T \cdot \boldsymbol{\lambda} = 0 \, . \tag{21}$$

With the use of (19), the dependent accelerations are eliminated. Next, (20) is used to calculate the multipliers. They are eliminated from (21). The resulting equation is [10]

$$\begin{aligned}
\boldsymbol{M}^{\boldsymbol{bR}}(\boldsymbol{u}^{\boldsymbol{b}}, t) \cdot \ddot{\boldsymbol{u}}^{\boldsymbol{b}} + \boldsymbol{F}^{\boldsymbol{bR}}(\boldsymbol{u}^{\boldsymbol{b}}, \dot{\boldsymbol{u}}^{\boldsymbol{b}}) + \\
+ \boldsymbol{Q}^{\boldsymbol{bR}}(\boldsymbol{u}^{\boldsymbol{b}}, \dot{\boldsymbol{u}}^{\boldsymbol{b}}, \boldsymbol{f}_{\boldsymbol{e}}, \boldsymbol{t}_{\boldsymbol{e}}, t) = 0
\end{aligned} \; , \tag{22}$$

where:

$$\boldsymbol{M}^{\boldsymbol{bR}} = \boldsymbol{M}_{\boldsymbol{uu}} - \boldsymbol{M}_{\boldsymbol{uv}} \cdot \boldsymbol{J}_{\boldsymbol{v}}^{-1} \cdot \boldsymbol{J}_{\boldsymbol{u}} - \boldsymbol{J}_{\boldsymbol{u}}^T \cdot \left(\boldsymbol{J}_{\boldsymbol{v}}^{-1} \right)^T \left(\boldsymbol{M}_{\boldsymbol{vu}} - \boldsymbol{M}_{\boldsymbol{vv}} \cdot \boldsymbol{J}_{\boldsymbol{v}}^{-1} \cdot \boldsymbol{J}_{\boldsymbol{u}} \right); \tag{23}$$

$$\boldsymbol{F}^{\boldsymbol{bR}} = \boldsymbol{F}_{\boldsymbol{u}} - \boldsymbol{M}_{\boldsymbol{uv}} \cdot \boldsymbol{J}_{\boldsymbol{v}}^{-1} \cdot \boldsymbol{A} - \boldsymbol{J}_{\boldsymbol{u}}^T \cdot \left(\boldsymbol{J}_{\boldsymbol{v}}^{-1} \right)^T \left(\boldsymbol{F}_{\boldsymbol{v}} - \boldsymbol{M}_{\boldsymbol{vv}} \cdot \boldsymbol{J}_{\boldsymbol{v}}^{-1} \cdot \boldsymbol{A} \right); \tag{24}$$

$$\boldsymbol{Q}^{\boldsymbol{bR}} = \boldsymbol{Q}_{\boldsymbol{u}} - \boldsymbol{J}_{\boldsymbol{u}}^T \cdot \left(\boldsymbol{J}_{\boldsymbol{v}}^{-1} \right)^T \boldsymbol{Q}_{\boldsymbol{v}} \, . \tag{25}$$

3. FINITE ELEMENTS METHOD

When a finite element is considered, its generalized coordinates refer to the nodes displacements (in general, to the element endpoints). In the general case, six degrees of freedom are associated to each of the nodes (i.e. three translational and three rotational). In

particular cases, lower number is considered. In the present case, when a planar beam element is considered (Fig. 4a), two degrees of freedom are sufficient to describe the displacements (a vertical translation and an in-plane rotation).

To express its dynamics, a global coordinate system is fixed to the reference body. Some additional local systems coincide with the non-deformed elements. When the element parameters are expressed in the local coordinate system, ^ symbol (located above of the parameter) is used to mark it. Next, according to the generally accepted convention, the vertical displacements are performed along the y_2 axis, while the rotations are performed about the y_3 axis. Then, when the nodes of the element are numbered as i and j, the elements coordinates are [11-13]:

$$\hat{\boldsymbol{q}}_{\boldsymbol{e}} = col(\hat{q}_{i2}, \hat{q}_{i6}, \hat{q}_{j2}, \hat{q}_{j6}), \tag{26}$$

where: $\hat{q}_{i2}, \hat{q}_{j2}$ - translations of the beam nodes, $\hat{q}_{i6}, \hat{q}_{j6}$ - rotations of the beam cross sections (Fig. 4a).

In the general situation, the loads (when present in the system) can be distributed over all the particles of the element. In the paper case, they are attached in the nodes, only. Thus, the nodes loads are denoted as [11-13]:

$$\hat{\boldsymbol{P}}_{\boldsymbol{e}} = col(\hat{P}_{i2}, \hat{P}_{i6}, \hat{P}_{j2}, \hat{P}_{j6}), \tag{27}$$

where: $\hat{P}_{i2}, \hat{P}_{j2}$ - forces collinear to y_2; $\hat{P}_{i6}, \hat{P}_{j6}$ - torques collinear to y_3.

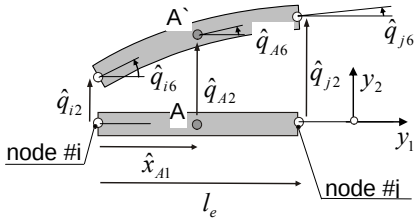

Fig. 4: Details of the finite element - displacements for a beam element;

In the presently considered case, it states that a linear function can be used to approximated relations between the point and the nodes displacements, when a potential displacement of a generic point $\boldsymbol{A}$ is considered. The used weight coefficients depend on the generic point relative position, i.e. depending on a parameter describing its relative position within the element. In the general case, nonlinear functions (polynomials for example) are used to express the coefficients. As some approximation errors are non-eliminable, elements size should be reduced in order to minimize the errors. Summarising it, when a displacement of the generic point $\boldsymbol{A}$ is considered, it can be express as [11-13]

$$\hat{\boldsymbol{q}}_A = \hat{\boldsymbol{N}}_{\boldsymbol{e}} \cdot \hat{\boldsymbol{q}}_{\boldsymbol{e}}, \tag{28}$$

where: $\hat{\boldsymbol{q}}_A$ – displacements of the generic point $\boldsymbol{A}$; $\hat{\boldsymbol{N}}_{\boldsymbol{e}} = col(N_1, N_2, N_3, ..., N_w)$ - matrix of shape functions; $\hat{\boldsymbol{q}}_{\boldsymbol{e}} = col(\hat{q}_1, \hat{q}_2, \hat{q}_3, ..., \hat{q}_w)$ - vector of the nodes displacements.

Next, the element relative strains, $\boldsymbol{\varepsilon}_A$, can be got from differential relation [11-13]

$$\hat{\boldsymbol{\varepsilon}}_A = \hat{\boldsymbol{B}}_l \cdot \hat{\boldsymbol{q}}_{\boldsymbol{e}} \quad , \tag{29}$$

$$: \quad \hat{\boldsymbol{B}}_l = \begin{bmatrix} \partial/\partial x_1 & 0 & 0 & \partial/\partial x_2 & 0 & \partial/\partial x_3 \\ 0 & \partial/\partial x_2 & 0 & \partial/\partial x_1 & \partial/\partial x_3 & 0 \\ 0 & 0 & \partial/\partial x_3 & 0 & \partial/\partial x_2 & \partial/\partial x_1 \end{bmatrix}^T \cdot \hat{\boldsymbol{N}}_{\boldsymbol{e}} \quad ,$$

where: $\hat{\boldsymbol{B}}_l$ - matrix that relates displacements and strains.

As a planar state of stresses is considered, the stress / strain relation is [11-13]

$$\hat{\boldsymbol{\sigma}} = \boldsymbol{D}_{\boldsymbol{e}} \cdot \hat{\boldsymbol{\varepsilon}} \quad : \quad \boldsymbol{D}_{\boldsymbol{e}} = \frac{E}{1-\upsilon^2} \begin{bmatrix} 1 & \upsilon & 0 \\ & 1 & 0 \\ sym. & & (1-\upsilon)/2 \end{bmatrix}. \tag{30}$$

where: E – elasticity modulus; υ - Poisson's number.

With the obtained formulas, the total kinetic and potential energies can be formulated as

$$E_{\mathrm{e}} = \tfrac{1}{2}\int_{m_e} \dot{\hat{\boldsymbol{q}}}^T \cdot \dot{\hat{\boldsymbol{q}}} \cdot dm \quad ; \qquad V_{\mathrm{e}} = \tfrac{1}{2}\int_{V_e} \hat{\boldsymbol{\varepsilon}}^T \cdot \hat{\boldsymbol{\sigma}} \cdot dV \cdot \tag{31}$$

Next, when Eqs. (28), (29) and (30) are used, it can be rewritten to:

$$E_{\mathrm{e}} = \tfrac{1}{2}\dot{\boldsymbol{q}}_e^T \cdot \boldsymbol{A}_{\boldsymbol{e}} \cdot \dot{\boldsymbol{q}}_e \quad ; \qquad V_{\mathrm{e}} = \tfrac{1}{2}\hat{\boldsymbol{q}}_e^T \cdot \hat{\boldsymbol{C}}_{\boldsymbol{e}} \cdot \hat{\boldsymbol{q}}_e \,, \tag{32}$$

where:

$$\boldsymbol{A}_{\boldsymbol{e}} = \rho \cdot \int_{x_e}\int_{y_e}\int_{z_e} \hat{\boldsymbol{N}}_e^T \cdot \hat{\boldsymbol{N}}_e \cdot dx \cdot dy \cdot dz \;; \tag{33}$$

$$\hat{\boldsymbol{C}}_{\boldsymbol{e}} = \int_{x_e}\int_{y_e}\int_{z_e} \hat{\boldsymbol{B}}_l^T \cdot \boldsymbol{D}_{\boldsymbol{e}} \cdot \hat{\boldsymbol{B}}_l \cdot dx \cdot dy \cdot dz \cdot \tag{34}$$

When a beam element is considered, cubic functions can be set as the shape functions. As compatibility with the continuity condition has to be preserved, related shape matrix is [11-13]:

$$\tilde{\boldsymbol{N}}_{\boldsymbol{e}}(\zeta)=\begin{bmatrix} 2\zeta^3-3\zeta^2+1 & l_e(\zeta^3-2\zeta^2+\zeta) & -2\zeta^3+3\zeta^2 & l_e(\zeta^3-\zeta^2) \\ \dfrac{6(\zeta^2-\zeta)}{l_e} & 3\zeta^2-4\zeta+1 & \dfrac{6(-\zeta^2+\zeta)}{l_e} & 3\zeta^2-2\zeta \end{bmatrix}, \quad (35)$$

where: $\zeta=\tilde{x}_1/l_e$ - relative position of the considered point; l_e – length of the element (see Fig. 4a).

Then, displacements and velocities of the generic point can be obtained and expressed as:

$$col(\hat{q}_{A2},\hat{q}_{A6})=\tilde{\boldsymbol{N}}_{\boldsymbol{e}}(\zeta)\cdot\hat{\boldsymbol{q}}_{\boldsymbol{e}}\,; \quad (36)$$

$$col(\dot{\hat{q}}_{A2},\dot{\hat{q}}_{A6})=\tilde{\boldsymbol{N}}_{\boldsymbol{e}}(\zeta)\cdot\dot{\hat{\boldsymbol{q}}}_{\boldsymbol{e}}\,. \quad (37)$$

With the presented matrices, the mass and the elasticity matrices are calculated from Eqs. (32) and (34) [11-13]:

$$\hat{\boldsymbol{A}}_{\boldsymbol{e}}=\frac{\rho_e F_e l_e}{420}\begin{bmatrix} 156 & 22l_e & 54 & -13l_e \\ & 4l_e^2 & 13l_e & -3l_e^2 \\ & & 156 & -22l_e \\ sym. & & & 4l_e^2 \end{bmatrix}; \quad (38)$$

$$\hat{\boldsymbol{C}}_{\boldsymbol{e}}=\frac{E_e J_e}{l_e^3}\begin{bmatrix} 12 & 6l_e & -12 & 6l_e \\ & 4l_e^2 & -6l_e & 2l_e^2 \\ & & 12 & -6l_e \\ sym. & & & 4l_e^2 \end{bmatrix}. \quad (39)$$

To fulfil the list of the matrices, the damping matrix is approximated as a linear combination of the mass and the elasticity matrices [11-13]

$$\hat{\boldsymbol{B}}_{\boldsymbol{e}}=\alpha\,\hat{\boldsymbol{A}}_{\boldsymbol{e}}+\beta\,\hat{\boldsymbol{C}}_{\boldsymbol{e}}\,. \quad (40)$$

Then, when the local system and the global system are oriented identically (identity orientation matrix), the related matrices in the global system are [11-13]:

$$\boldsymbol{q}_{\boldsymbol{e}}=\hat{\boldsymbol{q}}_{\boldsymbol{e}};\quad \boldsymbol{P}_{\boldsymbol{e}}=\hat{\boldsymbol{P}}_{\boldsymbol{e}};\quad \boldsymbol{A}_{\boldsymbol{e}}=\hat{\boldsymbol{A}}_{\boldsymbol{e}};\quad \boldsymbol{C}_{\boldsymbol{e}}=\hat{\boldsymbol{C}}_{\boldsymbol{e}};\quad \boldsymbol{B}_{\boldsymbol{e}}=\hat{\boldsymbol{B}}_{\boldsymbol{e}}\,. \quad (41)$$

To join the set of the independent elements into the common construction, vectors of nodes' displacements and vectors of elements' loads are partitioned on sub-vectors of nodes displacements and nodes' loads. Then, they are collected into the common matrices:

$$\boldsymbol{q}^{*c}=col(\boldsymbol{q}_i)\;\;;\;\;\boldsymbol{P}^{*c}=col(\boldsymbol{P}_i)\;\;:\;\;i=1,2,...,w\,, \quad (42)$$

where: $\boldsymbol{q}_i$ – sub-vector of displacements at i^{th} node; $\boldsymbol{P}_i$ – sub-vector of loads at i^{th} node.

Next, to obtain the element global matrices $\boldsymbol{A}_{\boldsymbol{e}}^{*},\boldsymbol{C}_{\boldsymbol{e}}^{*},\boldsymbol{B}_{\boldsymbol{e}}^{*}$ (matrices of the element dynamics expressed with generalized coordinates of the completed system), nodes corresponding cells are selected in the local matrices as well as in the global ones. Required cells can be found at the crossing places for rows and for columns with numbers matching to the numbers of the element nodes. Corresponding cells of the local $\boldsymbol{A}_{\boldsymbol{e}},\boldsymbol{C}_{\boldsymbol{e}},\boldsymbol{B}_{\boldsymbol{e}}$ matrices are placed in the identified cells of the global matrices $\boldsymbol{A}_{\boldsymbol{e}}^{*},\boldsymbol{C}_{\boldsymbol{e}}^{*},\boldsymbol{B}_{\boldsymbol{e}}^{*}$. The other elements are kept to be zero. Then, the system's global matrices are obtained as sums of the elements' global matrices (summation is performed over all the elements of the considered continuous system). It leads to the following formulas [11-13]:

$$\boldsymbol{A}^{*c}=\sum_{e=1}^{n_e}\boldsymbol{A}_{\boldsymbol{e}}^{*}\;\;,\;\;\boldsymbol{C}^{*c}=\sum_{e=1}^{n_e}\boldsymbol{C}_{\boldsymbol{e}}^{*}\;\;,\;\;\boldsymbol{B}^{*c}=\sum_{e=1}^{n_e}\boldsymbol{B}_{\boldsymbol{e}}^{*}\,, \quad (43)$$

where: n_e - number of elements in the considered continuous system.

Finally coordinates of the locked nodes have to be eliminated. With the zero values of their displacement, velocities and accelerations, related rows and columns are eliminated from the global matrices. The final form of the dynamics equation is [11-13]

$$\boldsymbol{A}^{c}\cdot\ddot{\boldsymbol{q}}^{c}+\boldsymbol{B}^{c}\cdot\dot{\boldsymbol{q}}^{c}+\boldsymbol{C}^{c}\cdot\boldsymbol{q}^{c}=\boldsymbol{P}^{c}\,. \quad (44)$$

4. CONSTRAINT EQUATIONS

With the multibody system joined to the elastic structure, a rotational joint constraint is introduced to model this connection. As within the elastic structure, the spherical joint position is constant, relatively simpler relation can be used when the point coincides with a system node. In such case, the shape functions are not necessary to formulate the constraint equation, and at the position level the constraint equations are:

$$h_1(\boldsymbol{q}^b, \boldsymbol{q}^c) = p_2(\boldsymbol{q}^b) - q^c_{j2}\ ;\ , \tag{45}$$

$$h_2(\boldsymbol{q}^b) = p_1(\boldsymbol{q}^b) - const\ , \tag{46}$$

where: $\vec{p}$ – vector that express position of the point at the multibody structure; p_i – i^{th} component of the vector $\vec{p}$.

When the $\boldsymbol{J}^b_v$ and $\boldsymbol{J}^b_h$ symbols are introduced (they express Jacobians of the vertical and the horizontal components of the vector $\vec{p}$ respectively, calculated in respect to multibody coordinates, only), the time derivatives of the constraint equations are:

$$\dot{h}_1 = \boldsymbol{J}^b_v \cdot \dot{\boldsymbol{q}}^b - \dot{q}^c_{j2} \quad ; \quad \dot{h}_2 = \boldsymbol{J}^b_h \cdot \dot{\boldsymbol{q}}^b \tag{47}$$

$$\begin{aligned} \ddot{h}_1 &= \boldsymbol{J}^b_v \cdot \ddot{\boldsymbol{q}}^b + A^b_2 - \ddot{q}^c_{j2}\ ; \\ \ddot{h}_2 &= \boldsymbol{J}^b_h \cdot \ddot{\boldsymbol{q}}^b + A^b_1 \end{aligned} \tag{48}$$

$$\boldsymbol{J}^b_v = \frac{\partial p_2}{\partial \boldsymbol{q}^b} \quad ; \quad \boldsymbol{J}^b_h = \frac{\partial p_1}{\partial \boldsymbol{q}^b}\ ; \tag{49}$$

$$A^b_2 = \frac{\partial (\boldsymbol{J}^b_v \cdot \dot{\boldsymbol{q}}^b)}{\partial \boldsymbol{q}^b} \cdot \dot{\boldsymbol{q}}^b\ ; \quad A^b_1 = \frac{\partial (\boldsymbol{J}^b_h \cdot \dot{\boldsymbol{q}}^b)}{\partial \boldsymbol{q}^b} \cdot \dot{\boldsymbol{q}}^b \tag{50}$$

In addition, the dynamics equations Eqs. (10) and (44) have to be accompanied with Lagrange's multipliers. According to it, the dynamics equations become:

$$\boldsymbol{M}^b \cdot \ddot{\boldsymbol{q}}^b + \boldsymbol{F}^b + \boldsymbol{Q}^b + \boldsymbol{J}^{bT} \cdot \boldsymbol{\lambda} = 0\ ; \tag{51}$$

$$\boldsymbol{A}^c \cdot \ddot{\boldsymbol{q}}^c + \boldsymbol{B}^c \cdot \dot{\boldsymbol{q}}^c + \boldsymbol{C}^c \cdot \boldsymbol{q}^c + \boldsymbol{J}^{cT} \cdot \boldsymbol{\lambda} = \boldsymbol{P}^c\ . \tag{52}$$

$$\boldsymbol{J}^b = \begin{bmatrix} \boldsymbol{J}^b_v \\ \hline \boldsymbol{J}^b_h \end{bmatrix} \quad ; \quad \boldsymbol{A}^b = \begin{bmatrix} A^b_2 \\ \hline A^b_1 \end{bmatrix} \tag{53}$$

$$\boldsymbol{J}^c = \begin{bmatrix} \underbrace{0 \ \cdots \ 0}_{2j-2} & -1 & \underbrace{0 \ \cdots \ 0}_{n^c-2j+1} \\ \underbrace{0 \ \cdots \ 0}_{2j-2} & 0 & \underbrace{0 \ \cdots \ 0}_{n^c-2j+1} \end{bmatrix}\ ; \tag{54}$$

The introduced Lagrange's multipliers, as well as the dependent coordinates, have to be eliminated. A modified version of the classical coordinate partitioning method (presented in section 3.2) is proposed for it. For simplicity, the multibody coordinates are considered as the dependent ones (in the present case, all the multibody coordinates have to be eliminated). Thanks of it, the following formulas can be obtained after this elimination

$$\boldsymbol{A}^{cR} \cdot \ddot{\boldsymbol{q}}^c + \boldsymbol{B}^{cR} \cdot \dot{\boldsymbol{q}}^c + \boldsymbol{C}^{cR} \cdot \boldsymbol{q}^c = \boldsymbol{P}^{cR}\ :. \tag{55}$$

$$\boldsymbol{A}^{cR} = \boldsymbol{A}^c + \boldsymbol{J}^{cT} \cdot \boldsymbol{J}^{b-T} \cdot \boldsymbol{M}^b \cdot \boldsymbol{J}^{b-1} \cdot \boldsymbol{J}^c \tag{56}$$

$$\boldsymbol{B}^{cR} = \boldsymbol{B}^c \quad ; \quad \boldsymbol{C}^{cR} = \boldsymbol{C}^c \quad ; \tag{57}$$

$$\boldsymbol{P}^{cR} = \boldsymbol{P}^c - \boldsymbol{J}^{cT} \cdot \boldsymbol{J}^{b-T} \cdot \boldsymbol{A}^b \cdot \boldsymbol{J}^{b-1} \cdot \boldsymbol{J}^c + \boldsymbol{J}^{cT} \cdot \boldsymbol{J}^{b-T} \cdot (\boldsymbol{F}^b + \boldsymbol{Q}^b)\ . \tag{58}$$

5. CONCLUSIONS AND PERSPECTIVES

The goal of the paper is associated to a search of an effective damping method. Vibrations present in a continuous mechanical system have to be damped. A damping method is base on the electromagnetic effects. To increase the damping effectives, velocity amplification is introduced. Proposed amplification could be possible when an amplification mechanism is set between the vibrating element and the DC generator.

The proposed modelling method combines the multibody modelling and the finite elements modelling. The background of the proposed methodology is presented in the paper. It is expected that the modelling method can be found as an effective. The proposed model is based on the constraint equations. It is an interesting an effective alternative to the classical models of elastic contacts.

6. REFERENCES

[1] Soylu R., Cetin M., *Linearization of Hybrid Robots and Input Torque Balancing of Mechanisms*, Mech. Mach. Theory 33(5), pp. 611-624, Elsevier, ISSN: 0094-114X, 1998.

[2] Arakelian V., Dahan M., *Dynamic Balancing of Mechanism*, Mechanics Research Communications 27(1), pp. 1-6, Pergamon-Elsevier Science Ltd, 2000.

[3] Arghir M., Macovescu S. Truta A., *Transmitted Vibrations to the head from the vibratory Environment*, Acta Technica Napocensis, Series: Applied Mathemathics and Mechanics 52, vol III, pp. 17-20, ISSN 1221-5872, 2009.

[4] Urs S., Arghir M., The Consequences of Hand – Arm Vibrations. The Raynaud Phenomenon, Acta Technica Napocensis, Series: Applied Mathemathics and Mechanics 52, vol III, pp. 75-80, ISSN 1221-5872, 2009.

[5] Broel-Plater B., Domek S. Parus A., *Permanent Magnet Chatter Absorber with Fuzzy Logic Control*, Solid State

Phenomena, Vol. 147-149, pp.290-295, ISSN: 1012-0394, 2009.
[6] Hadas Z., Singule V., Ondrusek C., *Optimal Design of Vibration Power Generator for Low Frequency*, Solid State Phenomena, Vol. 147-149. pp.426-231, ISSN: 1012-0394, 2009.
[7] Lipinski K., *Studies on the electromagnetic damping – amplification based on a mechanism sets in a near to a singularity position*, 6th International Conference Mechatronic System and Materials : MSM 2010, Opole, 5-8 July 2010 eds. K. Kluger, E. Macha, R. Pawliczek, Wydaw. Politech. Opolskiej, 2010. pp. 1-12, ISBN: 978-83-60691-78-6, 2010.
[8] Fisette P., Samin J.C., *Symbolic Modeling of Multibody System*, Kluwer Acad. Pub., ISBN 978-1-4020-1629-8, 469 ps., 2003.
[9] Fisette P., Lipinski K., Samin J.C., *Symbolic Modelling for the Simulation, Control and Optimisation of Multibody Systems*, Advances in Multibody Systems and Mechatronics, Kecskeméthy, A. Et al. (eds.) Institüt für Mechanik und Getriebelehre Technische Universität Gratz, pp. 139–174, ISBN: 395-0110-801, Gratz, (Austria), Septembre, 1999.
[10] Haug E.J., Yen J., *Generalized Coordinate Partitioning Methods for Numerical Integration of Differential-Algebraic Equations of Dynamics*, NATO ASI Series 69, pp. 97–114, Springer-Verlag, ISBN: 3-540-53280-3, 1990.
[11] Gawroński W., Kruszewski J., Ostachowicz W., Tarnowski J., Wittbrodt E., *Metoda elementów skończonych w dynamice konstrukcji*, Arkady,(in Polish), Warszawa, ISBN: 83-213-3159-9, 1984.
[12] Kruszewski J., Wittbrodt E., Walczyk Z., *Drgania układów mechanicznych w ujęciu komputerowym*, Vol 2, WNT, (in Polish), Warszawa, ISBN: 83-204-1650-7, 1993.
[13] Zienkiewicz, O.C., Taylor, R.L., *The Finite Element Method*, Vol. 1: The Basis, fifth ed., Butterworth-Heinemann, Oxford, 2000.

LES MECANISMES SITUES SUR LE VOISINAGE DE LA POSITION SINGULIERE – L'AMPLIFICATION DE LA VITESSE DANS L'AMORTISSEMENT ELECTROMAGNETIQUE DES VIBRATIONS

Dans cet article, des méthodes de modélisation sont présentées pour un modèle numérique d'une part mécanique d'un système hybride (continu / multicorps). Dans le système, on présente les vibrations d'une part continue. Pour introduire l'amortissement, leur énergie est transformée en courant électrique (par l'utilisation d'un générateur de courant continu) et dissipée. Pour amplifier l'amortissement, le mécanisme de double-barre est introduit entre la partie vibrante et l'élément d'amortissement. Deux sous-systèmes structurellement différents sont considérés. La partie élastique est composée des éléments finis. Le mécanisme est modélisé comme un système multicorps. Les équations de contrainte sont utilisées pour joindre ces sous-systèmes. Le modèle final va être utilisé comme un outil numérique, efficace pour la vérification de l'amortissement électromagnétique.

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