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Nonlinear increase in bubbles radii caused by sound in a bubbly liquid

Research Article

Anna Perelomova [1*], Weronika Pelc-Garska [1†]

1 Gdansk University of Technology, Faculty of Applied Physics and Mathematics, Narutowicza 11/12, 80-233 Gdansk, Poland

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Abstract: The nonlinear interaction of acoustic and entropy modes in a bubbly liquid is considered. The reasons for interaction are both nonlinearity and dispersion. In the field of intense sound, a decrease in the mixture density is predicted. That corresponds to the well-established growth of bubbles volumes due to rectified diffusion. The nonlinear interaction of modes as a reason for a bubble to grow due to sound, is discovered. The example considers variation in the mixture density and bubbles radii caused by acoustic soliton.

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1. Introduction

The dependence of phase speed on frequency is called dispersion. In general, dispersion in unbounded inhomogeneous acoustical media is a weak effect, in contrast to the strong dispersion of light in most optical media. In spite that dispersion accompanies attenuation and both phenomena are connected by the Kramers-Kronig relations [1, 2], acoustical dispersion can normally be neglected. Thermodynamic relaxation towards the equilibrium state, such as molecular relaxation in air and seawater, and boundary layers, are examples of weak relaxation [3, 4].

Among acoustical media with strong dispersion, waveguides and bubbly liquids are of great importance. The dispersion relations in waveguides are established by solving the eigenvalue problem for the normal modes. For certain amplitude modulations, the effects of dispersion and nonlinearity are in balance, and the pulse envelope propagates without change in shape [5]. Presence of even small concentrations of bubbles in a liquid dramatically increases the compressibility and thus reduces the sound speed. Ensemble of gas oscillators gives rise to dispersion. Nonlinearity due to bubbles can exceed, by orders of magnitude, the nonlinearity due to liquid alone [4]. This makes studies of nonlinear effects important not only relatively to sound itself, but in connection with nonlinear phenomena induced in the field of sound. Analysis of finite-amplitude sound in bubbly liquids is quite complicated and involves a number of theoretical models concerning thermodynamic processes in bubble itself and its surrounding liquid [4, 6–10].

As for nonlinear generation of non-wave modes, i.e., vorticity and entropy modes (these names come from the theory of flows of standard uniform fluids [11]) in the field

*E-mail: anpe@mif.pg.gda.pl
†E-mail: wpelc@mif.pg.gda.pl

of sound in a bubbly liquid, as far as the authors know, there is still unexplored domain. Studies in this area must start from equations describing fluid dynamics of the mixture as a whole. These equations schedule all motions which may exist in a bubbly liquid and should be consequently decomposed in order to yield equations governing every mode. Modes of infinitely small amplitude do not interact far from boundaries. As a result of the proper decomposition of equations, nonlinear terms become distributed between equations correctly. They include terms of all modes and may be considered as "specific forces" which are involved in any dynamic equation. As usual, it is possible to solve the system of coupling equations for interacting modes approximately under some simplifying conditions, for example, in the case when one mode is intense as compared with other. These general ideas concern a wide variety of fluid flows, not necessarily flows of bubbly liquids. The procedure was applied by one of the authors in problems of acoustic heating and streaming in newtonian and non-newtonian fluids [12–14]. In the theory of acoustic streaming, the acoustic nonlinear terms are called the "driving force" of streaming and actually possesses dimension of a mass force. In Sec. 4, the system of coupling nonlinear equations which describes interaction of modes in a bubbly liquid, is derived, as well as the approximate equation governing the "entropy" mode in the field of intense sound. The illustration in Sec. 5 considers exact solution of the Korteweg-de Vries equation which describes sound propagation over a bubbly liquid [6], and the entropy mode cased by it.

2. Equations governing bubbly liquid

We consider one-dimensional motions of the mixture which consists of compressible liquid involving identical spherical bubbles of an ideal gas (along axis OX). All bubbles are of the same radii at equilibrium, there is no heat and mass transfer between liquid and gas. To simplify the analysis, we assume that motions of the bubbles do not influence each other (i.e., they are well separated), and that they pulsate in their lowest, radially symmetric mode. The characteristic scale of perturbation in the mixture is much larger than a bubble radius, so that the mixture as a whole may be treated as the homogeneous continuum. Pressure in the mixture coincides with pressure of the liquid [6, 15]. Quantities relating to gas, liquid or to the mixture, are marked by index g, l and m, correspondingly. The unperturbed quantities are marked by additional zero, and the disturbed ones are primed. Density of the mixture is given by

$$\rho_m = \frac{\rho_g \rho_l}{x\rho_l + (1-x)\rho_g}, \qquad (1)$$

where x is a constant mass concentration of gas in the mixture. It may be expressed by means of the initial volume concentration of gas in the mixture, α_0,

$$x = \alpha_0 \frac{\rho_{g0}}{\rho_{m0}}. \qquad (2)$$

Acoustics of incompressible liquids including bubbles was originally studied by Wijngaarden [6]. Involving of liquid compressibility enables us to account for effects of finite sound velocity in pure liquid, c_l, on the nonlinear phenomena associated with sound. In particular, that corrects the nonlinear parameter of sound [4, 16]. The requirement that the mixture as a whole is homogeneously continuous implies the possibility to use the conservation equations in the differential form. They declare conservation of momentum, energy and mass:

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + \frac{1}{\rho_m}\frac{\partial p'}{\partial x} = 0,$$

$$\frac{\partial p'}{\partial t} - c_l^2\frac{\partial \rho_l'}{\partial t} - \frac{c_l^2(\gamma_l - 1)}{\rho_{l0}}\rho_l'\frac{\partial \rho_l'}{\partial t} = 0, \qquad (3)$$

$$\frac{\partial \rho_m'}{\partial t} + \frac{\partial (v\rho_m')}{\partial x} = 0,$$

where v, p denote velocity and pressure in the mixture. Consideration of three-dimensional flow would complicate evaluations, because the vortex mode appears in flows exceeding one dimension. The vortex motion does not input in variations in the mixture density, so that it is beyond the scope of this study. The second equation in (3) is actually a result of linear combination of continuity and energy equations for pure liquid, with $\gamma_l = \frac{C_{p,l}}{C_{V,l}}\frac{\rho_{l0}}{p_{l0}}\left(\frac{\partial p_l}{\partial \rho_l}\right)_{T=const}$, C_p and C_V denote heat capacities at constant pressure and density. For water at normal conditions, it equals approximately 7. Some other equations complement the system (3). The first reflects constant mass of gas inside a spherical bubble,

$$R^3\rho_g = R_0^3\rho_{g0}, \qquad (4)$$

and the second one describes adiabatic behavior of gas in it,

$$p_g\rho_g^{-\gamma_g} = p_{g0}\rho_{g0}^{-\gamma_g}. \qquad (5)$$

Eq. (4) imposes also constant density over the bubble volume, and Eq. (5) imposes, among spatially homogeneous distribution of density and pressure in a bubble, no energy exchange between bubbles and surrounding liquid,

$\gamma_g = \frac{C_{p,g}}{C_{V,g}}$. Pulsation of each bubble is described by the Rayleigh-Plesset equation:

$$R\frac{\partial^2 R}{\partial t^2}+\frac{3}{2}\left(\frac{\partial R}{\partial t}\right)^2 -\frac{1}{c_l^2}\left(R^2\frac{\partial^3 R}{\partial t^3}+6R\frac{\partial R}{\partial t}\frac{\partial^2 R}{\partial t^2}+2\left(\frac{\partial R}{\partial t}\right)^2\right)=\frac{p'_g-p'_l}{\rho_l}. \quad (6)$$

The surface tension is not taken into account by (6), but it accounts for compressibility of a liquid [15, 17]. Eqs (4), (5), (6) permit to rearrange the second equation from the system (3) in terms of quantities describing the mixture p, ρ_m, v. Eq. (3) in the dimensionless quantities

$$v^d=\frac{v'}{c_m},\ p^d=\frac{p'}{c_m^2\rho_{m0}},\ \rho^d=\frac{\rho'_m}{\rho_{m0}},\ x^d=\frac{x}{\lambda},\ t^d=\frac{tc_m}{\lambda}, \quad (7)$$

where λ denotes the characteristic scale of perturbation, and c_m is the velocity of sound of infinitely small magnitude in a bubbly liquid [6],

$$\frac{1}{c_m^2}=\frac{(1-\alpha_0)^2}{c_l^2}+\frac{\alpha_0(1-\alpha_0)\rho_{l0}}{\gamma_g p_{g0}}, \quad (8)$$

take the form [16]

$$\frac{\partial v}{\partial t}+\frac{\partial p}{\partial x}=-v\frac{\partial v}{\partial x}+\rho\frac{\partial p}{\partial x},$$

$$\frac{\partial p}{\partial t}+\frac{\partial v}{\partial x}-\frac{\alpha_0(1-\alpha_0)R_0^2\rho_{l0}^2c_m^4}{3(\gamma_g p_{g0})^2}\frac{\partial^3 p}{\partial t^3}= (1-\alpha_0)c_m^2\left(-\frac{\gamma_l+1}{c_l^2}\rho\frac{\partial v}{\partial x}-c_m^2\frac{\alpha_0(1-\alpha_0)\rho_{l0}^2(\gamma_g+1)}{(\gamma_g p_{g0})^2}p\frac{\partial v}{\partial x}\right) -v\frac{\partial p}{\partial x}+\rho\frac{\partial v}{\partial x}, \quad (9)$$

$$\frac{\partial \rho}{\partial t}+\frac{\partial v}{\partial x}=-v\frac{\partial \rho}{\partial x}-\rho\frac{\partial v}{\partial x}.$$

Starting from Eqs (9), upper indices by dimensionless quantities will be omitted. The largest, quadratic terms are held in the right-hand parts of all equations and everywhere below in this study. We will consider nonlinear and dispersive terms of the same order. That provides possibility of equilibrium between nonlinearity and dispersion. Eqs (9) describe dynamic of pure liquid when $\alpha_0 \to 0$.

3. Decomposition of sound and the entropy mode in the flow of infinitely small magnitude

The linear analogue of the system (9) takes the form

$$\frac{\partial \Psi}{\partial t}+L\Psi=0, \quad (10)$$

where $\Psi=\begin{pmatrix} v & p & \rho\end{pmatrix}^T$,

$$L=\begin{pmatrix} 0 & \frac{\partial}{\partial x} & 0\\ \frac{\partial}{\partial x}+D\frac{\partial^3}{\partial x^3} & 0 & 0\\ \frac{\partial}{\partial x} & 0 & 0\end{pmatrix},\ D=\frac{\alpha_0(1-\alpha_0)R_0^2\rho_{l0}^2c_m^2}{3(\gamma_g p_{g0})^2\lambda^2} \quad (11)$$

are linear matrix operators including spacial derivatives and parameter responsible for dispersion. We replace $\frac{\partial^3 p}{\partial t^3}$ by $-\frac{\partial^3 v}{\partial x^3}$ following from the first and second equations in the system (9) and valid in the leading order with respect to powers of D. Studies of motions of infinitely-small amplitudes begin usually with representing of all perturbations as a sum of planar waves:

$$f(x,t)=\int\widetilde{f}(k,t)\exp(-ikx)dk= \int\widetilde{f}(k)\exp(i\omega t-ikx)dk, \quad (12)$$

($\widetilde{f}(k,t)$ denotes the Fourier transform of $f(x,t)$, $\widetilde{f}(k,t)=\frac{1}{2\pi}\int f(x,t)e^{ikx}dx$). There are three roots of dispersion equation, the first two being acoustic, specifying to the sound progressive in positive and negative directions of axis OX (marked by indices 1 and 2, respectively), and the third dispersion relation describing stationary (or "entropy") mode:

$$\omega_1=k\sqrt{1-Dk^2},\ \omega_2=-k\sqrt{1-Dk^2},\ \omega_3=0. \quad (13)$$

They determine relations of perturbations specific for every mode:

$$\Psi_1=\begin{pmatrix}\sqrt{1-Dk^2}\\ 1-Dk^2\\ 1\end{pmatrix}\rho_1,\quad \Psi_2=\begin{pmatrix}-\sqrt{1-Dk^2}\\ 1-Dk^2\\ 1\end{pmatrix}\rho_2,\quad \Psi_3=\begin{pmatrix}0\\ 0\\ 1\end{pmatrix}\rho_3, \quad (14)$$

where ρ_n $(n = 1, 2, 3)$ are perturbations in density of a bubbly liquid correspondent to every mode. The overall perturbation in density is a sum of all them, $\rho=\sum_{n=1}^{3}\rho_n$. Eqs (13), (15) may be expanded in series with respect to powers of D. That significantly simplifies evaluations of projectors but implies smallness of parameter of dispersion. Projectors are matrix operators decomposing every mode from the total vector of perturbations,

$$P_i\Psi = \Psi_i. \tag{15}$$

They take the approximate form

$$P_1 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2}-\frac{D}{4}\frac{\partial^2}{\partial x^2} & 0 \\ \frac{1}{2}+\frac{D}{4}\frac{\partial^2}{\partial x^2} & \frac{1}{2} & 0 \\ \frac{1}{2}-\frac{D}{4}\frac{\partial^2}{\partial x^2} & \frac{1}{2}-\frac{D}{2}\frac{\partial^2}{\partial x^2} & 0 \end{pmatrix},$$

$$P_2 = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2}+\frac{D}{4}\frac{\partial^2}{\partial x^2} & 0 \\ -\frac{1}{2}-\frac{D}{4}\frac{\partial^2}{\partial x^2} & \frac{1}{2} & 0 \\ -\frac{1}{2}+\frac{D}{4}\frac{\partial^2}{\partial x^2} & \frac{1}{2}-\frac{D}{2}\frac{\partial^2}{\partial x^2} & 0 \end{pmatrix},$$

$$P_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1+D\frac{\partial^2}{\partial x^2} & 1 \end{pmatrix}.$$

Projectors (16) were firstly derived in [16]. Only the first two terms in the Taylor series with respect to powers of D (i.e., proportional to D^0 and D^1) are retained in the elements of projectors. Projecting operators form a full set of orthogonal projectors,

$$\sum_{i=1}^{3} P_i=\mathbf{I},\ P_i\cdot P_j=\mathbf{0},\ i\neq j,\ P_i^2=P_i, \tag{16}$$

where $\mathbf{I}$ and $\mathbf{0}$ denote unit and zero matrix operators.

4. Coupled dynamic equations in the nonlinear flow

Eqs (9) accounting for nonlinear terms, take the form

$$\frac{\partial\Psi}{\partial t}+L\Psi=\Psi_{nl}. \tag{17}$$

The dynamic equations which govern any mode, may be readily decomposed applying the projectors on the linearized system (10). The projecting operators (16) point the way for successful decomposision of equations governing every mode also in the nonlinear flow. The projection results in dynamic equations with nonlinear terms responsible for the modes interaction, i.e, the "specific forces" for individual modes. In regard to the flow where sound is intense as compared to the entropy mode, the correct determination of sound itself is of importance. The effects of sound wave are nonlinear, and the "exchange force" caused by it in the equation for the entropy mode, is of the second order. So that sound modes should be evaluated within this accuracy, that add correctives of the second order in the linear modes, Eqs (15). These amendments make sound propagating in the positive or negative directions of axis OX isentropic in the leading order. The emended modes take the form

$$\begin{gathered}
v_1=\rho_1+\frac{1}{2}D\frac{\partial^2\rho_1}{\partial x^2}+\\
\left(\frac{(1-\alpha_0)c_m^2(\gamma_l+1)}{4c_l^2}+\frac{c_m^4\alpha_0(1-\alpha_0)^2\rho_{l0}^2(\gamma_g+1)}{4(\gamma_g p_{g0})^2}-1\right)\rho_1^2,\\
p_1=\rho_1+D\frac{\partial^2\rho_1}{\partial x^2}+\\
\left(\frac{(1-\alpha_0)c_m^2(\gamma_l+1)}{2c_l^2}+\frac{c_m^4\alpha_0(1-\alpha_0)^2\rho_{l0}^2(\gamma_g+1)}{2(\gamma_g p_{g0})^2}-1\right)\rho_1^2,\\
v_2=-\rho_2-\frac{1}{2}D\frac{\partial^2\rho_2}{\partial x^2}-\\
\left(\frac{(1-\alpha_0)c_m^2(\gamma_l+1)}{4c_l^2}+\frac{c_m^4\alpha_0(1-\alpha_0)^2\rho_{l0}^2(\gamma_g+1)}{4(\gamma_g p_{g0})^2}-1\right)\rho_2^2,\\
p_2=\rho_2+D\frac{\partial^2\rho_2}{\partial x^2}+\\
\left(\frac{(1-\alpha_0)c_m^2(\gamma_l+1)}{2c_l^2}+\frac{c_m^4\alpha_0(1-\alpha_0)^2\rho_{l0}^2(\gamma_g+1)}{2(\gamma_g p_{g0})^2}-1\right)\rho_2^2.
\end{gathered} \tag{18}$$

Relations (19) result, among other, in the leading-order equation governing sound progressive in the positive direction of axis OX,

$$\frac{\partial\rho_1}{\partial t}+\frac{\partial\rho_1}{\partial x}+\varepsilon\rho_1\frac{\partial\rho_1}{\partial x}+\frac{D}{2}\frac{\partial^3\rho_1}{\partial x^3}=0, \tag{19}$$

where

$$\varepsilon=\left(\frac{(1-\alpha_0)c_m^2(\gamma_l+1)}{2c_l^2}+\frac{c_m^4\alpha_0(1-\alpha_0)^2\rho_{l0}^2(\gamma_g+1)}{2(\gamma_g p_{g0})^2}\right) \tag{20}$$

denotes the parameter of nonlinearity of sound in a bubbly liquid. Eq. (19) imposes intense rightwards propagating sound as compared with other modes. This equation is the well-known Korteweg-de Vries equation, which in the case of bubbly liquid has been discussed in details [6, 8, 9]. The parameter of nonlinearity, given by Eq. (20), coincides with this evaluated in [4]. In this last paper, the expression obtained for incompressible liquid is completed by the terms following from the nonlinearity in equations different from the pressure-density relation for the mixture. Unlike, Eq. (20) is immediate result of considering of the total system of conservation equations describing compressible liquid including bubbles. The system of equations which describes interaction of different modes, may be obtained by use of links specific for the modes, and by applying the projecting operators on the system (9). Projecting reduces terms of all other modes in the linear parts of equations and distributes the nonlinear "forces" between them correctly [12, 16]. That allows to rearrange Eqs (17) into the system in terms of reference quantities for every mode:

$$\frac{\partial \rho_n}{\partial t}+c_n\frac{\partial \rho_n}{\partial x}+\sum_{i,j=1}^{3}\beta^n_{i,j}\rho_i\frac{\partial \rho_j}{\partial x}+\sum_{i,j=1}^{3}\eta^n_{i,j}\rho_i\frac{\partial^3 \rho_j}{\partial x^3}+\frac{M_n}{2}\frac{\partial^3 \rho_n}{\partial x^3}=0. \quad (21)$$

The coefficients $\beta^n_{i,j}$ and $\eta^n_{i,j}$ are determined by Tables 1 and 2, respectively, where $c_1=-c_2=1$, $c_3=0$, $F=-(1-\alpha_0)c_m^2\frac{\gamma_l+1}{c_l^2}$, and $M_1=-M_2=D$, $M_3=0$. In studies of nonlinear effects connected with intense sound, both acoustic modes were re-defined in accordance to relations (19) and analogous ones for the second mode. The context of the nonlinear generation of non-acoustic modes in the field of sound, such as acoustic heating and streaming, imposes intense sound as compared with the other modes. As for the entropy mode caused by the first acoustic mode, that is valid over temporal and spatial domain where $|\rho_1|>>|\rho_3|$. The dynamics of sound itself is described in this domain by equation Eq. (19) which approximates the first equation from the set (21) when impact of other modes on the first one is ignored.

5. Growth in the bubbles radii induced by sound. Example of acoustic soliton

The example relates to generation of the entropy mode in the field of intense first acoustic mode (progressive in the positive direction of axis OX) comparatively with other modes. Application of the third verse of projector P_3 on the main system (9) reduces all terms belonging to the acoustic modes in the linear part of equation. In its nonlinear part, we will consider only the largest terms originated from the first acoustic mode. Finally, one obtains the dynamic equation governing an excess density of the entropy mode:

$$\frac{\partial \rho_3}{\partial t}=-D(\varepsilon-2)\rho_1\frac{\partial^3\rho_1}{\partial x^3}, \quad (22)$$

where ε and D are constants determined by equations (20) and (11), respectively. An acoustic excess density in the right-hand side of Eq. (22) must be solution of the leading-order equation, Eq. (19). The form of Eq. (22) reveals, that the reasons for excitation of the entropy mode by sound are both nonlinearity and dispersion. In the standard thermoviscous flows, the phenomenon of nonlinear generation of the entropy mode in the field of sound, is well-studied. It is known as acoustic heating [18]. Its origins are nonlinearity and the total attenuation, making nonlinear loss in acoustic energy. In the dispersive media like bubble liquid, dispersion is the prerequisite of modes interaction instead of attenuation, or it is better to say, in addition to attenuation. In this study, we do not consider neither viscosity of both pure phases, nor thermal conduction of gas (this is important for small bubbles) [19, 20], nor radiation attenuation connected with dynamics of a bubble, but only phenomena associated with dispersion.
The set of stationary solutions of Eq. (19) which tend to zero at infinite x (positive and negative), take the well-known form of solitons depending on constant $\widetilde{c}_1$:

$$\rho_1(\xi)=\frac{6(\widetilde{c}_1-1)}{\varepsilon}\left[1+\cosh\left(\sqrt{\frac{2(\widetilde{c}_1-1)}{D}}\,\xi\right)\right]^{-1}, \quad (23)$$

where $\widetilde{c}_1$ is velocity of the stationary waveform, exceeding the unit velocity of sound in the mixture for positive D, $\widetilde{c}_1>1$, and $\xi=x-\widetilde{c}_1t$. In fact, non-dimensional width of a pulse is unit, this determines $\widetilde{c}_1$:

$$\widetilde{c}_1=1+\frac{D}{2}. \quad (24)$$

Eqs (22), (23) may be readily rearranged as

$$\rho_1(\xi)=\frac{3D}{\varepsilon}\phi(\xi), \qquad \rho_3(\xi)=\frac{9D^3(\varepsilon-2)}{\varepsilon^2}\int_\infty^\xi\phi\frac{d^3\phi}{d\xi^3}d\xi,$$
$$\phi(\xi)\equiv[1+\cosh(\xi)]^{-1}. \quad (25)$$

The acoustic soliton and the entropy mode induced in its field, are shown in the Fig. 1(a, b). The perturbation in density ρ_3 is negative (in the bubbly water, ε may achieve 10^4 at $\alpha_0\approx10^{-4}$, and is smaller than 10 only for very high volume concentrations α_0 in the vicinity of 1 [4], so that the factor by integral is positive). In view of low compressibility of liquid as compared to gas, the bubble radii increase. The increase in volume concentration of bubbles $\Delta\alpha$, and that in a bubble radius, ΔR, equal approximately

$$\Delta\alpha\approx-\rho_3,\ \Delta R\approx-\frac{1}{3}\frac{\rho_3}{\alpha_0}R_0. \quad (26)$$

The equalities are exact in the case of incompressible liquid. Efficiency of generation of the entropy mode in the field of acoustic soliton is low (its amplitude is approximately D^2 times smaller compared to that of the soliton),

Table 1. The coefficients $\beta^n_{i,j}$.

	$\beta^1_{i,j}$				$\beta^2_{i,j}$				$\beta^3_{i,j}$		
i \ j	1	2	3	i \ j	1	2	3	i \ j	1	2	3
1	ε	$-\varepsilon$	$\frac{1}{2}$	1	0	$2-\varepsilon$	$\frac{1}{2}$	1	0	$2\varepsilon-2$	0
2	$\varepsilon-2$	0	$\frac{1}{2}$	2	ε	$-\varepsilon$	$-\frac{1}{2}$	2	$2-2\varepsilon$	0	0
3	$-\frac{1}{2}(F+2)$	$\frac{1}{2}F$	0	3	$-\frac{1}{2}F$	$\frac{1}{2}(F+2)$	0	3	$F+2$	$-(F+2)$	0

Table 2. The coefficients $\eta^n_{i,j}$

	$\eta^1_{i,j}$				$\eta^2_{i,j}$				$\eta^3_{i,j}$		
i \ j	1	2	3	i \ j	1	2	3	i \ j	1	2	3
1	$\frac{1}{4}D(\varepsilon-6)$	$\frac{1}{2}D(\varepsilon-2)$	$-\frac{1}{2}D$	1	$\frac{1}{4}D(3\varepsilon-4)$	$\frac{1}{2}D(\varepsilon-1)$	$-\frac{1}{2}D$	1	$D(2-\varepsilon)$	$\frac{1}{2}D(3-4\varepsilon)$	D
2	$\frac{1}{2}D(1-\varepsilon)$	$\frac{1}{4}D(4-3\varepsilon)$	$\frac{1}{2}D$	2	$\frac{1}{2}D(2-\varepsilon)$	$\frac{1}{4}D(6-\varepsilon)$	$\frac{1}{2}D$	2	$\frac{1}{2}D(4\varepsilon-3)$	$D(\varepsilon-2)$	$-D$
3	$\frac{1}{4}DF$	$-\frac{1}{4}D(F+2)$	0	3	$\frac{1}{4}D(F+2)$	$-\frac{1}{4}DF$	0	3	$-\frac{1}{2}D(2F+1)$	$\frac{1}{2}D(2F+1)$	0

but the example of acoustic soliton is remarkable because it is exact solution of Eq. (19). In general, nonlinear effects are hardly expected to be large if cased by a single waveform. They may be noticeable when cased by enough extended in time acoustic source like wavepackets. The example of acoustic soliton is also important in view of that any initial waveform transforms with time into a set of solitons in a dispersive medium [5, 19, 22].

6. Conclusions

The main result of this study, Eq. (22), reveals the new (once more) reason for a bubble to grow in the acoustic field, namely the nonlinear generation of the "entropy" mode in the field of sound. Enlargement of bubbles was well-studied experimentally and explained by shell and rectifying effects, see [19] and references therein. Diffusion of gas dissolved in liquid, occurs towards bubble during its expansion, and in the opposite direction during its compression. Since the surface of a bubble is smaller during compression, and the thickness of the liquid layer, which participates in aerogenesis, is larger (supporting smaller gradient of gas concentration) as compared to expansion, the averaged over the period flow of gas is directed towards bubble. That makes it to grow. The velocity of increase in mass of a bubble is found to be proportional to the squared sound pressure. In this study the discovered growth of a bubble radius in the field of sound is of different origin as compared with the rectified diffusion. The reason for that is the nonlinear interaction of entropy and sound modes which may take place without diffusion and does not connect with increase in the bubble mass. The analogous phenomenon in the standard thermoviscous fluids is isobaric acoustic heating which is followed by decrease in fluid density. The increase in the bubble mass m in unit time due to rectified diffusion which was derived by Neppiras [23], $\frac{dm}{dt}$, is proportional to radius of a bubble R_0, squared acoustic pressure, and to the factor depending on sound frequency, $((1-\Omega^2)^2+\Omega^2 d^2)^{-1}$, where Ω is the ratio of sound frequency and the natural frequency of a bubble $\omega_0=\sqrt{\frac{3\gamma_g p_{g0}}{\rho_{l0}R_0^2}}$, $\Omega=\omega/\omega_0$, d is the inverse Q-factor of a bubble as oscillator, which typically is about 10. So, the rate of mass variation achieves maximum approximately at the resonance frequency. As for the nonlinear generation of the entropy mode, its efficiency is also proportional to squared acoustic pressure and to radius of a bubble R_0. It is proportional to ω^3. In standard thermoviscous flows, the efficiency of heating is proportional to ω^2. This agrees with the general conclusion of nonlinear acoustics that nonlinear effects of sound are larger for big frequencies.

The entropy mode in the uniform fluid is specified by isobaric variations in density followed by variations in its temperature. As for the bubbly liquid, variations in temperature are different inside a bubble and in the surrounding liquid. However, the "entropy" mode exists in a bubbly liquid even if liquid itself is incompressible and there is no variations of temperature in it. In this case, density of the mixture varies solely by change in volumes of bubbles. In the linear flow, this mode is stationary. Nonlinearity and dispersion are the reasons for excitation of the entropy mode. The result of this is decrease in the mixture

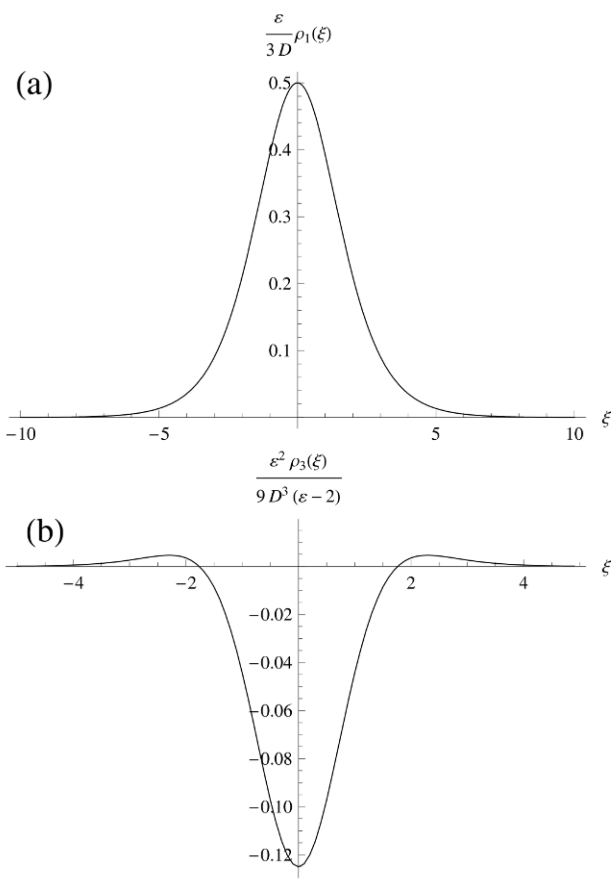

Figure 1. The stationary solution of Eq. (19), $\frac{\varepsilon}{3D}\rho_1(\xi)\equiv\phi(\xi)$, (b) variations in density in the entropy mode, $\frac{\varepsilon^2}{9D^3(\varepsilon-2)}\rho_3(\xi)\equiv\int_\infty^\xi\phi\frac{d^3\phi}{d\xi^3}d\xi$ (Eqs (25)).

density and correspondent increase of the bubbles radii. The example in Sec. 5 considers acoustic soliton and nonlinear excitation of the entropy mode in its field. As for efficiency of the entropy mode generation by the periodic in time sound, it is low. Simple evaluations of averaged over the sound period (2π in dimensionless quantities) velocity of density variation, follow from Eq. (22):

$$\left\langle\frac{\partial\rho_3}{\partial t}\right\rangle\approx D(\varepsilon-2)\left\langle\rho_1\frac{\partial^3\rho_1}{\partial t^3}\right\rangle= \tag{27}$$

$$-D\frac{(\varepsilon-2)}{4\pi}\left(\frac{\partial\rho_1}{\partial t}\right)^2\Bigg|_t^{t+2\pi}=0, \tag{28}$$

where angle brackets denote averaging over the sound period, $\langle f\rangle\equiv\frac{1}{2\pi}\int_t^{t+2\pi}f\,dt$. In the standard attenuating uniform fluids, the periodic sound produces non-zero quantity. The right-hand side of the correspondent equation includes $\rho_1\frac{\partial^2\rho_1}{\partial t^2}$ standing by total attenuation [21]. The averaged over the sound period decrease in a medium density is followed by increase in its temperature. This phenomenon in the field of periodic sound is known as acoustic heating [18, 22].

The results of this study account for the liquid compressibility. The limit of incompressible liquid ($c_l\to\infty$), among other, does not permit to account for nonlinear features of wave motion in a liquid. Neither heat and mass transfer between bubbles and surrounding liquid, nor non-uniformity of pressure and temperature inside bubble, nor vaporization in the case of bubbles including vapor, were considered. The numerical studies have taken into account these phenomena [20, 24], and the features due to them are well-established. Without giving fundamentally new results, consideration of these phenomena would significantly complicate mathematic content of the study. Account for higher-order nonlinearity or nonlinear dispersion may be a reason for some peculiarities in the sound propagation, they are, among other, non-elasticity of interaction of solitons or weak destroying of an acoustic soliton due to radiation [4]. We do not consider these phenomena. Since the thermal conductivity should be taken into account for small bubbles (which radius is smaller than resonant one) [19, 20], but is out of the scope of the present study, in concrete evaluations enough large quantities should be used. If the characteristic length of the thermal wave in a gas is larger than radius of a bubble, $\sqrt{\frac{2\chi}{\rho_{g0}C_{p,g}\omega}}>R_0$, the viscous losses in studies of a bubble oscillations are important (χ is the thermal conductivity of gas). For large bubbles, $R_0>\sqrt{\frac{2\chi}{\rho_{g0}C_{p,g}\omega}}$, the radiation caused by compressibility of liquid and viscous losses are of importance. It was experimentally established, that liquid compressibility is dominant as compared to viscosity at frequencies 10 KHz and $R_0>2$ mm. In the simple evaluations for bubbly water, we may use the following data: $R_0=2$ mm, $\rho_{l0}=10^3$ kg/m^3, $p_{g0}=10^5$ Pa, $\gamma_g=1.4$, $\gamma_l=7$, $c_l=1500$ m/s. Value of the initial volume concentration of gas in the mixture $\alpha_0=10^{-4}$, results in maximum increase in bubbles radii cased by acoustic soliton about 16%, and $\alpha_0=10^{-5}$ yields 3.6%.

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