

## AN UPPER BOUND ON THE TOTAL OUTER-INDEPENDENT DOMINATION NUMBER OF A TREE

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**Abstract.** A total outer-independent dominating set of a graph  $G = (V(G), E(G))$  is a set  $D$  of vertices of  $G$  such that every vertex of  $G$  has a neighbor in  $D$ , and the set  $V(G) \setminus D$  is independent. The total outer-independent domination number of a graph  $G$ , denoted by  $\gamma_t^{oi}(G)$ , is the minimum cardinality of a total outer-independent dominating set of  $G$ . We prove that for every tree  $T$  of order  $n \geq 4$ , with  $l$  leaves and  $s$  support vertices we have  $\gamma_t^{oi}(T) \leq (2n + s - l)/3$ , and we characterize the trees attaining this upper bound.

**Keywords:** total outer-independent domination, total domination, tree.

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### 1. INTRODUCTION

Let  $G = (V(G), E(G))$  be a graph. By the neighborhood of a vertex  $v$  of  $G$  we mean the set  $N_G(v) = \{u \in V(G) : uv \in E(G)\}$ . The degree of a vertex  $v$ , denoted by  $d_G(v)$ , is the cardinality of its neighborhood. By a leaf we mean a vertex of degree one, while a support vertex is a vertex adjacent to a leaf. We say that a support vertex is strong (weak, respectively) if it is adjacent to at least two leaves (exactly one leaf, respectively). The path on  $n$  vertices we denote by  $P_n$ . Let  $T$  be a tree, and let  $v$  be a vertex of  $T$ . We say that  $v$  is adjacent to a path  $P_n$  if there is a neighbor of  $v$ , say  $x$ , such that the subtree resulting from  $T$  by removing the edge  $vx$  and which contains the vertex  $x$  as a leaf, is a path  $P_n$ . By a star we mean a connected graph in which exactly one vertex has degree greater than one. By a double star we mean a graph obtained from a star by joining a positive number of vertices to one of its leaves.

We say that a subset of  $V(G)$  is independent if there is no edge between every two its vertices. A subset  $D \subseteq V(G)$  is a dominating set of  $G$  if every vertex of  $V(G) \setminus D$  has a neighbor in  $D$ , while it is a total dominating set if every vertex of  $G$  has a neighbor in  $D$ . The domination (total domination, respectively) number of  $G$ , denoted by  $\gamma(G)$  ( $\gamma_t(G)$ , respectively), is the minimum cardinality of a dominating

(total dominating, respectively) set of  $G$ . Total domination in graphs was introduced by Cockayne, Dawes, and Hedetniemi [2], and further studied for example in [1]. For a comprehensive survey of domination in graphs, see [3, 4].

A subset  $D \subseteq V(G)$  is a total outer-independent dominating set, abbreviated TOIDS, of  $G$  if every vertex of  $G$  has a neighbor in  $D$ , and the set  $V(G) \setminus D$  is independent. The total outer-independent domination number of  $G$ , denoted by  $\gamma_t^{oi}(G)$ , is the minimum cardinality of a total outer-independent dominating set of  $G$ . A total outer-independent dominating set of  $G$  of minimum cardinality is called a  $\gamma_t^{oi}(G)$ -set. The study of total outer-independent domination in graphs was initiated in [5].

Chellali and Haynes [1] established the following upper bound on the total domination number of a tree. For every nontrivial tree  $T$  of order  $n$  with  $s$  support vertices we have  $\gamma_t(T) \leq (n + s)/2$ .

We prove the following upper bound on the total outer-independent domination number of a tree. For every tree  $T$  of order  $n \geq 4$ , with  $l$  leaves and  $s$  support vertices we have  $\gamma_t^{oi}(T) \leq (2n + s - l)/3$ . Moreover, we characterize the trees attaining this upper bound.

## 2. RESULTS

Since the one-vertex graph does not have a total outer-independent dominating set, in this paper, by a tree we mean only a connected graph with no cycle, and which has at least two vertices.

We begin with the following two straightforward observations.

**Observation 2.1.** *Every support vertex of a graph  $G$  is in every  $\gamma_t^{oi}(G)$ -set.*

**Observation 2.2.** *For every connected graph  $G$  of diameter at least three there exists a  $\gamma_t^{oi}(G)$ -set that contains no leaf.*

We show that if  $T$  is a tree of order  $n \geq 4$ , with  $l$  leaves and  $s$  support vertices, then  $\gamma_t^{oi}(T)$  is bounded above by  $(2n + s - l)/3$ . For the purpose of characterizing the trees attaining this bound we introduce a family  $\mathcal{T}$  of trees  $T = T_k$  that can be obtained as follows. Let  $T_1$  be a path  $P_6$ , and let  $A(T_1)$  be a set containing all vertices of  $P_6$  which are not leaves. Let  $H$  be a path  $P_3$  with one of the leaves labeled  $u$ , and the support vertex labeled  $v$ . If  $k$  is a positive integer, then  $T_{k+1}$  can be obtained recursively from  $T_k$  by one of the following operations.

- Operation  $\mathcal{O}_1$ : Attach a copy of  $H$  by joining the vertex  $u$  to a vertex of  $T_k$  adjacent to a path  $P_3$ . Let  $A(T) = A(T') \cup \{u, v\}$ .
- Operation  $\mathcal{O}_2$ : Attach a copy of  $H$  by joining the vertex  $u$  to a vertex of  $T_k$  which is not a leaf and is adjacent to a support vertex. Let  $A(T) = A(T') \cup \{u, v\}$ .
- Operation  $\mathcal{O}_3$ : Attach a copy of  $H$  by joining the vertex  $u$  to a leaf of  $T_k$  adjacent to a weak support vertex. Let  $A(T) = A(T') \cup \{u, v\}$ .

Now we prove that for every tree  $T$  of the family  $\mathcal{T}$ , the set  $A(T)$  defined above is a TOIDS of minimum cardinality equal to  $(2n + s - l)/3$ .



**Lemma 2.3.** *If  $T \in \mathcal{T}$ , then the set  $A(T)$  defined above is a  $\gamma_t^{oi}(T)$ -set of size  $(2n + s - l)/3$ .*

*Proof.* We use the terminology of the construction of the trees  $T = T_k$ , the set  $A(T)$ , and the graph  $H$  defined above. To show that  $A(T)$  is a  $\gamma_t^{oi}(T)$ -set of cardinality  $(2n + s - l)/3$  we use induction on the number  $k$  of operations performed to construct the tree  $T$ . If  $T = T_1 = P_6$ , then  $(2n + s - l)/3 = (12 + 2 - 2)/3 = 4 = |A(T)| = \gamma_t^{oi}(T)$ . Let  $k \geq 2$  be an integer. Assume that the result is true for every tree  $T' = T_k$  of the family  $\mathcal{T}$  constructed by  $k - 1$  operations. Let  $n'$  mean the order of the tree  $T'$ ,  $l'$  the number of its leaves, and  $s'$  the number of support vertices. Let  $T = T_{k+1}$  be a tree of the family  $\mathcal{T}$  constructed by  $k$  operations.

First assume that  $T$  is obtained from  $T'$  by operation  $\mathcal{O}_1$ . We have  $n = n' + 3$ ,  $s = s' + 1$ , and  $l = l' + 1$ . The vertex of  $T'$  to which is attached  $P_3$  we denote by  $x$ . Let  $abc$  mean a path  $P_3$  adjacent to  $x$ , and such that  $a \neq u$ . It is easy to see that  $A(T) = A(T') \cup \{u, v\}$  is a TOIDS of the tree  $T$ . Thus  $\gamma_t^{oi}(T) \leq \gamma_t^{oi}(T') + 2$ . Now let  $D$  be a  $\gamma_t^{oi}(T)$ -set that contains no leaf. By Observation 2.1, we have  $v \in D$ . Each one of the vertices  $v$  and  $b$  has to have a neighbor in  $D$ , thus  $u, a \in D$ . Let us observe that  $D \setminus \{u, v\}$  is a TOIDS of the tree  $T'$  as the vertex  $x$  has a neighbor in  $D \setminus \{u, v\}$ . Therefore  $\gamma_t^{oi}(T') \leq \gamma_t^{oi}(T) - 2$ . Now we conclude that  $\gamma_t^{oi}(T) = \gamma_t^{oi}(T') + 2$ . We get  $\gamma_t^{oi}(T) = |A(T)| = |A(T')| + 2 = (2n' + s' - l')/3 + 2 = (2n - 6 + s - 1 - l + 1)/3 + 2 = (2n + s - l)/3$ .

Now assume that  $T$  is obtained from  $T'$  by operation  $\mathcal{O}_2$ . We have  $n = n' + 3$ ,  $s = s' + 1$ , and  $l = l' + 1$ . The vertex of  $T'$  to which is attached  $P_3$  we denote by  $x$ . Let  $y$  mean a support vertex adjacent to  $x$ . It is easy to see that  $A(T) = A(T') \cup \{u, v\}$  is a TOIDS of the tree  $T$ . Thus  $\gamma_t^{oi}(T) \leq \gamma_t^{oi}(T') + 2$ . Now let  $D$  be a  $\gamma_t^{oi}(T)$ -set that contains no leaf. By Observation 2.1 we have  $v, y \in D$ . The vertex  $v$  has to have a neighbor in  $D$ , thus  $u \in D$ . Let us observe that  $D \setminus \{u, v\}$  is a TOIDS of the tree  $T'$  as the vertex  $x$  has a neighbor in  $D \setminus \{u, v\}$ . Therefore  $\gamma_t^{oi}(T') \leq \gamma_t^{oi}(T) - 2$ . Now we conclude that  $\gamma_t^{oi}(T) = \gamma_t^{oi}(T') + 2$ . In the same way as in the previous possibility we get  $\gamma_t^{oi}(T) = (2n + s - l)/3$ .

Now assume that  $T$  is obtained from  $T'$  by operation  $\mathcal{O}_3$ . We have  $n = n' + 3$ ,  $s = s'$ , and  $l = l'$ . The leaf to which is attached  $P_3$  we denote by  $x$ . Let  $y$  mean a neighbor of  $x$  other than  $u$ . It is easy to see that  $A(T) = A(T') \cup \{u, v\}$  is a TOIDS of the tree  $T$ . Thus  $\gamma_t^{oi}(T) \leq \gamma_t^{oi}(T') + 2$ . Now let us observe that there exists a  $\gamma_t^{oi}(T)$ -set that does not contain the vertex  $x$ , and does not contain any leaf. Let  $D$  be such a set. By Observation 2.1 we have  $v \in D$ . The vertex  $v$  has to have a neighbor in  $D$ , thus  $u \in D$ . The set  $V(T) \setminus D$  is independent, thus  $y \in D$ . Let us observe that  $D \setminus \{u, v\}$  is a TOIDS of the tree  $T'$  as the vertex  $x$  has a neighbor in  $D \setminus \{u, v\}$ . Therefore  $\gamma_t^{oi}(T') \leq \gamma_t^{oi}(T) - 2$ . Now we conclude  $\gamma_t^{oi}(T) = \gamma_t^{oi}(T') + 2$ . We get  $\gamma_t^{oi}(T) = |A(T)| = |A(T')| + 2 = (2n' + s' - l')/3 + 2 = (2n - 6 + s - l)/3 + 2 = (2n + s - l)/3$ .  $\square$

Now we establish the main result, an upper bound on the total outer-independent domination number of a tree together with the characterization of the extremal trees.



**Theorem 2.4.** *If  $T$  is a tree of order  $n \geq 4$ , with  $l$  leaves and  $s$  support vertices, then  $\gamma_t^{oi}(T) \leq (2n + s - l)/3$  with equality if and only if  $T = K_{1,3}$  or  $T \in \mathcal{T}$ .*

*Proof.* First assume that  $\text{diam}(T) = 2$ . Thus  $T$  is a star  $K_{1,m}$  with  $m \geq 3$ . If  $m = 3$ , then  $T = K_{1,3}$ . We have  $\gamma_t^{oi}(T) = 2 = (8 + 1 - 3)/3 = (2n + s - l)/3$ . If  $m \geq 4$ , then  $(2n + s - l)/3 = (2m + 2 + 1 - m)/3 = (m + 3)/3 \geq (4 + 3)/3 > 2 = \gamma_t^{oi}(T)$ . Now let us assume that  $\text{diam}(T) = 3$ . Thus  $T$  is a double star. We have  $(2n + s - l)/3 = (2n + 2 - n + 2)/3 = (n + 4)/3 \geq (4 + 4)/3 > 2 = \gamma_t^{oi}(T)$ . Now assume that  $\text{diam}(T) = 4$ . Let  $v_1v_2v_3v_4v_5$  mean a longest path in  $T$ . If  $v_3$  is adjacent to a leaf, then all support vertices of  $T$  form a TOIDS of the tree  $T$ . Thus  $\gamma_t^{oi}(T) \leq s$ . Now we get  $\gamma_t^{oi}(T) \leq s = s/3 + 2s/3 = s/3 + 2(n-l)/3 = (2n + s - 2l)/3 < (2n + s - l)/3$ . Now assume that  $T$  is not adjacent to any leaf. It is easy to observe that all support vertices of  $T$  together with the vertex  $v_3$  form a TOIDS of the tree  $T$ . Thus  $\gamma_t^{oi}(T) \leq s + 1$ . We have  $n = l + s + 1$ . Now we get  $\gamma_t^{oi}(T) \leq s + 1 = s/3 + 2s/3 + 1 = s/3 + 2(n-l-1)/3 + 1 = (2n + s - 2l - 2)/3 + 1 = (2n + s - l)/3 + (1 - l)/3 < (2n + s - l)/3$ . Now let us assume that  $\text{diam}(T) = 5$ . Let  $v_1v_2v_3v_4v_5v_6$  mean a longest path in  $T$ . If both vertices  $v_3$  and  $v_4$  are adjacent to a leaf, then all support vertices of  $T$  form a TOIDS of the tree  $T$ . Thus  $\gamma_t^{oi}(T) \leq s$ . Now we get  $\gamma_t^{oi}(T) \leq s = s/3 + 2s/3 = s/3 + 2(n-l)/3 = (2n + s - 2l)/3 < (2n + s - l)/3$ . Now assume that exactly one of the vertices  $v_3$  and  $v_4$  is adjacent to a leaf. Without loss of generality we assume that  $v_3$  is adjacent to a leaf. It is easy to observe that all support vertices of  $T$  together with the vertex  $v_4$  form a TOIDS of the tree  $T$ . Thus  $\gamma_t^{oi}(T) \leq s + 1$ . We have  $n = l + s + 1$ . Now we get  $\gamma_t^{oi}(T) \leq s + 1 = s/3 + 2s/3 + 1 = s/3 + 2(n-l-1)/3 + 1 = (2n + s - 2l - 2)/3 + 1 = (2n + s - l)/3 + (1 - l)/3 < (2n + s - l)/3$ . Now assume that neither  $v_3$  nor  $v_4$  is adjacent to a leaf. It is easy to observe that all support vertices of  $T$  together with the vertices  $v_3$  and  $v_4$  form a TOIDS of the tree  $T$ . Thus  $\gamma_t^{oi}(T) \leq s + 2$ . We have  $n = l + s + 2$ . Now we get  $\gamma_t^{oi}(T) \leq s + 2 = s/3 + 2s/3 + 2 = s/3 + 2(n-l-2)/3 + 2 = (2n + s - 2l - 4)/3 + 2 = (2n + s - l)/3 + (2 - l)/3$ . If  $T$  has exactly two leaves, then  $T = P_6 = T_1 \in \mathcal{T}$ . By Lemma 2.3 we have  $\gamma_t^{oi}(T) = (2n + s - l)/3$ . Now assume that  $T$  has at least three leaves. We have  $\gamma_t^{oi}(T) \leq (2n + s - l)/3 + (2 - l)/3 < (2n + s - l)/3$ .

Now assume that  $\text{diam}(T) \geq 6$ . Thus the order of the tree  $T$  is an integer  $n \geq 7$ . The result we obtain by the induction on the number  $n$ . Assume that the theorem is true for every tree  $T'$  of order  $n' < n$ , with  $l'$  leaves and  $s'$  support vertices.

First assume that some support vertex of  $T$ , say  $x$ , is strong. Let  $y$  mean a leaf adjacent to  $x$ . Let  $T' = T - y$ . We have  $n' = n - 1$ ,  $s' = s$ , and  $l' = l - 1$ . Let  $D'$  be any  $\gamma_t^{oi}(T')$ -set. By Observation 2.1 we have  $x \in D'$ . Of course,  $D'$  is a TOIDS of the tree  $T$ . Thus  $\gamma_t^{oi}(T) \leq \gamma_t^{oi}(T')$ . Now we get  $\gamma_t^{oi}(T) \leq \gamma_t^{oi}(T') = (2n' + s' - l')/3 = (2n - 2 + s - l + 1)/3 = (2n + s - l)/3 - 1/3 < (2n + s - l)/3$ . Therefore every support vertex of  $T$  is weak.

We now root  $T$  at a vertex  $r$  of maximum eccentricity  $\text{diam}(T)$ . Let  $t$  be a leaf at maximum distance from  $r$ ,  $v$  be the parent of  $t$ ,  $u$  be the parent of  $v$ ,  $w$  be the parent of  $u$ , and  $d$  be the parent of  $w$  in the rooted tree. By  $T_x$  let us denote the subtree induced by a vertex  $x$  and its descendants in the rooted tree  $T$ .

First assume that  $d_T(u) \geq 3$ . Assume that among the descendants of  $u$  there is a support vertex, say  $x$ , different than  $v$ . Let  $T' = T - T_v$ . We have  $n' = n - 2$ ,  $s' = s - 1$ ,

and  $l' = l - 1$ . Let  $D'$  be a  $\gamma_t^{oi}(T')$ -set that contains no leaf. The vertex  $x$  has to have a neighbor in  $D'$ , thus  $u \in D'$ . It is easy to see that  $D' \cup \{v\}$  is a TOIDS of the tree  $T$ . Thus  $\gamma_t^{oi}(T) \leq \gamma_t^{oi}(T') + 1$ . Now we get  $\gamma_t^{oi}(T) \leq \gamma_t^{oi}(T') + 1 \leq (2n' + s' - l')/3 + 1 = (2n - 4 + s - 1 - l + 1)/3 + 1 = (2n + s - l)/3 - 1/3 < (2n + s - l)/3$ .

Now assume that some descendant of  $u$ , say  $x$ , is a leaf. Let  $T' = T - x$ . We have  $n' = n - 1$ ,  $s' = s - 1$ , and  $l' = l - 1$ . Let  $D'$  be a  $\gamma_t^{oi}(T')$ -set that contains no leaf. The vertex  $v$  has to have a neighbor in  $D'$ , thus  $u \in D'$ . It is easy to see that  $D'$  is a TOIDS of the tree  $T$ . Thus  $\gamma_t^{oi}(T) \leq \gamma_t^{oi}(T')$ . Now we get  $\gamma_t^{oi}(T) \leq \gamma_t^{oi}(T') \leq (2n' + s' - l')/3 = (2n - 2 + s - 1 - l + 1)/3 = (2n + s - l)/3 - 2/3 < (2n + s - l)/3$ .

Now assume that  $d_T(u) = 2$ . First assume that there is a descendant of  $w$ , say  $k$ , such that the distance of  $w$  to the most distant vertex of  $T_k$  is three. It suffices to consider only the possibility when  $T_k$  is a path  $P_3$ , say  $klm$ . Let  $T' = T - T_u$ . We have  $n' = n - 3$ ,  $s' = s - 1$ , and  $l' = l - 1$ . Let  $D'$  be any  $\gamma_t^{oi}(T')$ -set. It is easy to see that  $D' \cup \{u, v\}$  is a TOIDS of the tree  $T$ . Thus  $\gamma_t^{oi}(T) \leq \gamma_t^{oi}(T') + 2$ . Now we get  $\gamma_t^{oi}(T) \leq \gamma_t^{oi}(T') + 2 \leq (2n' + s' - l')/3 + 2 = (2n - 6 + s - 1 - l + 1)/3 + 2 = (2n + s - l)/3$ . If  $\gamma_t^{oi}(T) = (2n + s - l)/3$ , then obviously  $\gamma_t^{oi}(T') = (2n' + s' - l')/3$ . The tree  $T'$  has at least seven vertices. By the inductive hypothesis we have  $T' \in \mathcal{T}$ . The tree  $T$  can be obtained from  $T'$  by operation  $\mathcal{O}_1$ . Thus  $T \in \mathcal{T}$ .

Now assume that there is a descendant of  $w$ , say  $k$ , such that the distance of  $w$  to the most distant vertex of  $T_k$  is two. Thus  $k$  is a support vertex. Let  $T' = T - T_u$ . In the same way as in the previous possibility we get  $\gamma_t^{oi}(T) \leq (2n + s - l)/3$ . If  $\gamma_t^{oi}(T) = (2n + s - l)/3$ , then  $\gamma_t^{oi}(T') = (2n' + s' - l')/3$ . The tree  $T'$  has at least six vertices. By the inductive hypothesis we have  $T' \in \mathcal{T}$ . The tree  $T$  can be obtained from  $T'$  by operation  $\mathcal{O}_2$ . Thus  $T \in \mathcal{T}$ .

Now assume that some descendant of  $w$ , say  $k$ , is a leaf. Let  $T' = T - t - k$ . We have  $n' = n - 2$ ,  $s' = s - 1$ , and  $l' = l - 1$ . Let  $D'$  be a  $\gamma_t^{oi}(T')$ -set that contains no leaf. By Observation 2.1 we have  $u \in D'$ . The vertex  $u$  has to have a neighbor in  $D'$ , thus  $w \in D'$ . It is easy to observe that  $D' \cup \{v\}$  is a TOIDS of the tree  $T$ . Thus  $\gamma_t^{oi}(T) \leq \gamma_t^{oi}(T') + 1$ . Now we get  $\gamma_t^{oi}(T) \leq \gamma_t^{oi}(T') + 1 \leq (2n' + s' - l')/3 + 1 = (2n - 4 + s - 1 - l + 1)/3 + 1 = (2n + s - l)/3 - 1/3 < (2n + s - l)/3$ .

Now assume that  $d_T(w) = 2$ . First assume that  $d$  is adjacent to a leaf. Let  $T' = T - T_u$ . We have  $n' = n - 3$ ,  $s' = s - 1$ , and  $l' = l$ . Let  $D'$  be any  $\gamma_t^{oi}(T')$ -set. It is easy to see that  $D' \cup \{u, v\}$  is a TOIDS of the tree  $T$ . Thus  $\gamma_t^{oi}(T) \leq \gamma_t^{oi}(T') + 2$ . Now we get  $\gamma_t^{oi}(T) \leq \gamma_t^{oi}(T') + 2 \leq (2n' + s' - l')/3 + 2 = (2n - 6 + s - 1 - l)/3 + 2 = (2n + s - l)/3 - 1/3 < (2n + s - l)/3$ .

Now assume that  $d$  is not adjacent to any leaf. Let  $T' = T - T_u$ . We have  $n' = n - 3$ ,  $s' = s$ , and  $l' = l$ . Let  $D'$  be any  $\gamma_t^{oi}(T')$ -set. It is easy to see that  $D' \cup \{u, v\}$  is a TOIDS of the tree  $T$ . Thus  $\gamma_t^{oi}(T) \leq \gamma_t^{oi}(T') + 2$ . Now we get  $\gamma_t^{oi}(T) \leq \gamma_t^{oi}(T') + 2 \leq (2n' + s' - l')/3 + 2 = (2n - 6 + s - l)/3 + 2 = (2n + s - l)/3$ . If  $\gamma_t^{oi}(T) = (2n + s - l)/3$ , then  $\gamma_t^{oi}(T') = (2n' + s' - l')/3$ . The tree  $T'$  has at least four vertices and is different from  $K_{1,3}$  as  $T'$  has no strong support vertex. By the inductive hypothesis we have  $T' \in \mathcal{T}$ . The tree  $T$  can be obtained from  $T'$  by operation  $\mathcal{O}_3$ . Thus  $T \in \mathcal{T}$ .  $\square$

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