SILENT SONAR WITH MATCHED FILTRATION

ROMAN SALAMON, JACEK MARSZAL, JAN SCHMIDT,
MARIUSZ RUDNICKI

Gdansk University of Technology
11/12, G. Narutowicza St., 80-233 Gdansk, Poland
roman.salamon@eti.pg.gda.pl

Radars with continuous wave frequency (CW FM sonars) are used in radiolocation as 'silent radars'. They determine the distance to target by measuring the difference between the frequency of the sounding signal and echo signal. The article presents the principle of operation and parameters of silent CW FM sonars. Target distance determined by these sonars is based on the signal at the output of the matched filter. The Doppler effect is studied in detail to identify its effect on the sonar's parameters. The results of theoretical calculations are presented together with the results of the sonar's computer simulation.

INTRODUCTION

Stealth is the precondition of many of the military applications. Sonars will be difficult to detect by the enemy if the power emitted in the sounding signal is low, the signal is not a pulse signal and its spectrum is wide [1]. This suggests that sonars with continuous sounding signals and linear frequency modulation should be a good solution. They are in fact used in radiolocation and are known as the CW FM radar [2]. The reason why this concept cannot be easily applied in sonars is the Doppler effect with its significant and negative impact on the operation of CW FM sonars [1]. The article presents an analysis of the possibilities to build a silent sonar using filtration matched to the CW FM signal in place of the conventional solution which determines differential frequency between the transmitted and received signal.

1. SILENT SONAR WITH MATCHED FILTRATION

A matched filtration receiver is a practical and feasible version of the correlation receiver, [3]. The matched receiver acts as a filter with pulse response *k(t)* which is the inverse copy of the emitted signal *s(t)*. As we know signal *y(t)* at the filter's output is a combination of input signal *x(t)* and pulse response *k(t)* namely, [3]:

$$y(t)=\int_{-\infty}^{\infty}x(\tau)k(t-\tau)d\tau=k(t)*s(t) \tag{1}$$

The pulse response of a matched filter is equal to:

$$k(t)=x^{*}(-t) \tag{2}$$

where symbol (*) means the conjugate value of the complex signal.

When we put these relations into formula (1) and change the denotation of the variables, we obtain:

$$r_{xx}(\tau)=\int_{-\infty}^{\infty}x(t)x^{*}(t-\tau)dt \tag{3}$$

It is the auto-correlation function of signal *x(t)* hence the matched filter with pulse response (2) is equivalent to the correlation receiver. As we know, a receiver like that is the optimal detector of a known signal against the Gaussian noise [4].

In underwater acoustics we can accept that pulse response is equal to a time scale reversed emitted sounding signal: $k(t) = s^{*}(-t)$. The received signal without noise *x(t)* can be reduced to:

$$x(t)=K_{o}s(t-t_{o}) \tag{4}$$

where K_o is the fraction which shows the difference between the echo signal and emitted signal and t_o – is the delay versus the emitted signal *s(t)*.

When we put the above relation to formula (1) we obtain:

$$y(\tau)=K_{0}\int_{-\infty}^{\infty}s(t-t_{0})s^{*}(t-\tau)dt \tag{5}$$

Following the replacement $t'=t-t_0$ we have:

$$y(\tau)=K_{o}\int_{-\infty}^{\infty}s(t')s^{*}[t'-(\tau-t_{o})]dt' \tag{6}$$

Using relation (3) we finally obtain:

$$y(\tau)=K_{o}r_{ss}(\tau-t_{o}) \tag{7}$$

The signal at the input of the matched filter is proportional to the auto-correlation function of the emitted signal where the function is shifted on the time axis by the delay of the received signal versus the transmitted sounding signal.

Echo signal is detected when signal $y(\tau)$ reaches its maximal value. As we know, the autocorrelation function always reaches the maximal value for a zero argument [4]. Formula (7) shows that it occurs at moment $\tau = t_o$. At this moment of time which is equal to the delay in echo signal, the signal at the matched filter's output is:

$$y(t_{o})=K_{o}E_{t} \tag{8}$$

where E is the energy of the emitted signal equal to:

$$y(t_0) = \int_{-\infty}^{\infty} |\, s(t)\,|^2 \; dt \tag{9}$$

If the usable signal is received together with Gaussian noise with power spectrum density N, then the output signal to noise ratio is [4]:

$$SNR_m = \frac{K_o E_t}{N} \tag{10}$$

where E_t is the energy of the sounding signal.

For the purposes of detection the energy of the signals should be as high as possible and for the purposes of range resolution the autocorrelation function of the emitted signal should be narrow with possibly low levels of side lobes. Please note that signals with linear frequency modulation can meet both criteria.

If the objective is to build a silent sonar using matched filtration and FM signal, we must reduce the power of the emitted signal as much as we can. The underlying assumption of this postulate is that the enemy does not know the sounding signal we are using and hence cannot use a matched filter. During detection the enemy receiver's signal to noise ratio is equal to [4]:

$$SNR_{um} = \frac{P_r}{\sigma^2} \tag{11}$$

where P_r is the power of the received signal and σ^2 is the variance of noise in the receiver's transfer band.

The power of the received signal is:

$$P_r = \frac{K_r E_t}{T} = K_r P_t \tag{12}$$

where T is the duration of the sounding signal with energy E_t, P_t is the power of that signal and K_r is the coefficient of reduced power as a result of loss during propagation. Because $\sigma^2 = NB$ where B is the receiver's transfer band, hence:

$$SNR_{um} = K_r \frac{P_t}{BN} \tag{13}$$

As you can see, by reducing the power of the sounding signal we deteriorate the enemy receiver's signal to noise ratio hampering detection of the sounding signal. We can produce the same positive effect by increasing the width of the sounding signal spectrum. In the formula above the listening bandwidth B is equal to the width of the sounding signal spectrum. If the enemy uses a wider listening bandwidth because they do not know the location or width of the sounding signal spectrum, the level of noise is rising and deteriorates the signal to noise ratio. If, on the other hand, they choose a band which is too narrow (or if the band does not match the sounding signal spectrum), power P_t is decreasing which also reduces SNR_{um}.

A reduction in the power of the sounding signal which maintains the same duration causes a drop in energy E_s and deteriorates detection performance of the operator's sonar. This is shown in the quotient of both signal to noise ratios which is equal to:

$$\frac{SNR_m}{SNR_{um}} = \frac{K_0 E_s}{N} \frac{BTN}{K_r E_s} = BT \frac{K_o}{K_r} \tag{14}$$

The above relation shows that the enemy's detection capacity deteriorates when compared to a matched filtration sonar and that it continues to deteriorate as the duration of the sounding signal T grows longer and its spectrum B grows wider. Although power is not a key component of the above formula, this does not mean that it should not be as low as possible. For a high level of power P_t the signal to noise ratio SNR_{um} can be high enough for detection to occur even though it is significantly below the SNR_m.

Now let us consider the effect of coefficients K_0 and K_t on detection capacity. Coefficient K_0 (formula 4) refers to the amplitude of the echo signal. When used in an echolocation system, if we leave out absorption damping, the amplitude is inversely proportional to r_0^2, where r_0 is the distance between the target and sonar. The amplitude also depends on target strength TS [5] and so coefficient K_0 can be approximated as:

$$K_0 \cong \frac{r_1^2}{r_0^2} 10^{TS/20} \tag{15}$$

Coefficient K_r (formula 12) shows the signal power, hence:

$$K_r \cong \frac{r_1^2}{r_r^2} \tag{16}$$

We put the relations into formula (14) and obtain:

$$\frac{SNR_m}{SNR_{um}} \cong BT \left(\frac{r_r}{r_0} \right)^2 10^{TS/20} \tag{17}$$

Let us assume, as an example, that the minimal assumed target strength of the sonar is TS=–20 dB and that the sonar and detection listening system operate at the same sounding signal to noise ratio, we then obtain:

$$\frac{r_r}{r_0} \cong \sqrt{\frac{10}{BT}} \tag{18}$$

With the above relation we can determine the ratio between the distance of the listening system and target and the sonar at which the detection capacity of the sonar and listening system are the same. It can be treated as equivalent to the ratio of range in both systems. As you can see, the ratio improves as the BT product of the sounding signal grows. As an example, when B = 2.5 kHz, and T = 10 s then $r_r/r_0 \cong 1/50$. If the range of the sonar for TS = –20 dB is 15 km, then the listening system can detect sounding pulses at 300 m away from the sonar. This is a very satisfactory result.

The power of the sounding pulse can be determined for the sonar's assumed parameters from the range equation [5].

2. CW FM SONAR WITH MATCHED FILTRATION

Matched filtration sonars can use a variety of sounding signals featuring a high product of duration *T* and spectrum width *B*. Conventional sonars usually use the pulse signal with linear frequency modulation. Silent sonars can replace pulse signals with continuous signals comprising periodically repetitive pulses with linear frequency modulation. The changes in frequency are shown in Fig. 1 and the spectrum is given in Fig. 2.

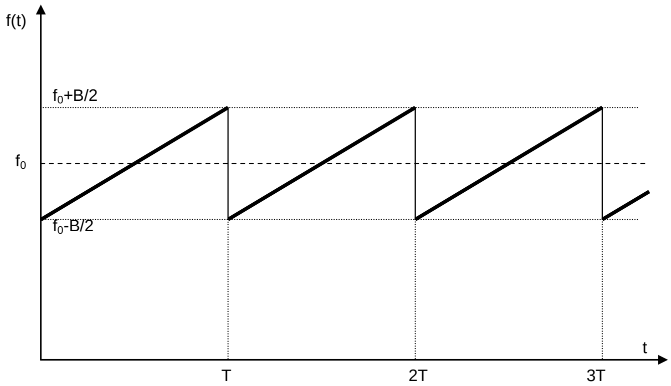

Fig. 1. Changing frequency of the sounding signal in a CW FM sonar

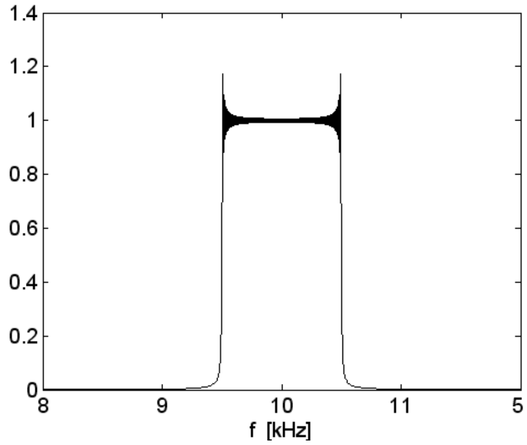

Fig. 2. Amplitude spectrum of a sounding signal (f_0 = 10 kHz, B = 1 kHz, T = 10 s)

Fig. 3 shows the shape of the signal at the output of a matched filter. It is assumed that the echo signal is a copy of the sounding signal delayed by t_o = 3 s Fig. 4 shows a fragment of the signal from Fig. 3 around its maximal value.

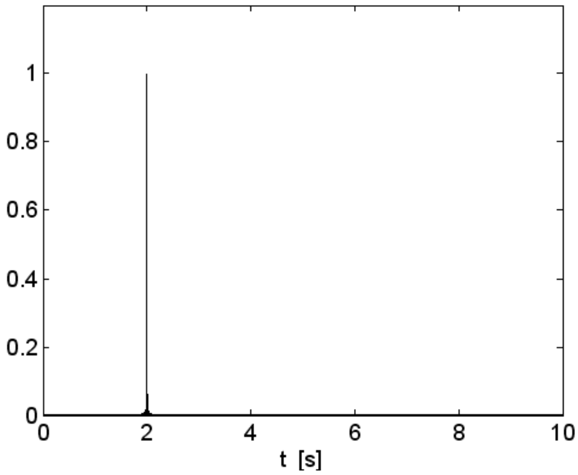

Fig. 3. Signal at output of matched filter maximal (f_0 = 10 kHz, B = 1 kHz, T = 10 s)

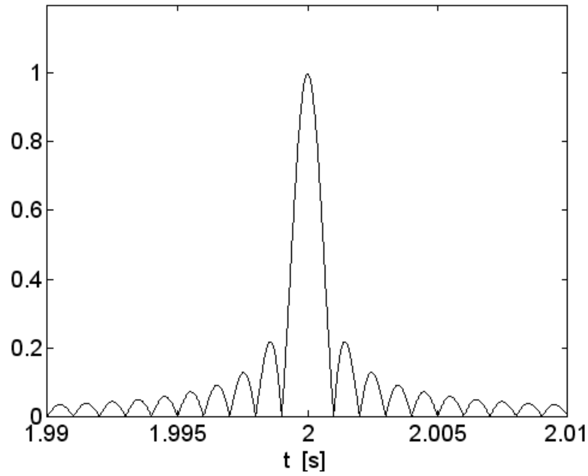

Fig. 4. Fragment of a signal around its value

As you can see in Fig.4, the shift of the first zero of the correlation function versus delay t_o= 2 s is 1 ms. The value is equal to the inverse bandwidth *B* = 1 kHz which is consistent with the theoretical results [4]. Value *1/B* is at the same time equal to the potential resolution of the sonar expressed here with units of time. Resolution expressed with units of distance is in this case ΔR = 0.5c/B ≅0 .75 m which is very good.

3. DOPPLER EFFECT, CW FM SONAR WITH MATCHED FILTRATION

As we know from the theory of matched filtration [4], and successful experiments, the Doppler effect has a negative impact on the operation of echolocation systems which use this method of detection. It reduces the maximal value of the correlation function and its shift over time. With a lower correlation function value, detection is hampered (a lower *SNR*) and target location is misinterpreted as a result of the shift in time of the correlation function maximum. The typical underwater acoustic systems are not really affected by either of these disadvantages and are acceptable in view of the benefits of matched filtration. The calculations we will be presenting further in the article suggest that if we extend signals with frequency modulation, the result is a significant change in the shape of the correlation and a strong effect on its position on the time scale.

Let the sounding signal have the following shape:

$$s(t) = S_0 \exp\{ j2\pi(f_0 - \frac{B}{2} + \frac{B}{2T} t)t] \qquad \text{for } t\in(0,T) \tag{19}$$

and outside the time range *(0, T)* it is equal to zero. We take this assumption to simplify the analysis and the results of numerical calculations further in the text will show that it has a major impact on the conclusions. In the formula above f_0 means the carrier frequency of the signal, B the width of its spectrum and T is the duration.

Let us assume that the sonar is stationary and the target is moving at velocity v. Let us assume that the target at time $t = 0$ is $x = x_0$ away from the sonar's receiver and is emitting a signal described with formula (19). The receiver receives a signal which we can write as follows:

$$x(t) = X_0 \exp\{ j2\pi[f_0 - \frac{B}{2} + \frac{B}{2T}(t - \frac{x_0 + vt}{c})](t - \frac{x_0 + vt}{c})\} \tag{20}$$

After some simple transformations, we have:

$$x(t) = X_0 \exp\{ j2\pi[f_0 - \frac{B}{2} + \frac{B}{2T}(et - t_0)](et - t_0)\} \tag{21}$$

where c is sound velocity, $t_0 = x_0/c$ and $e = 1-v/c$.

As given in formula (3) the correlation function of the above signal with sounding signal takes this shape:

$$r_{xs}(\tau) = X_0 S_0 \int_{-\infty}^{\infty} \exp\{j2\pi[f_0 - \frac{B}{2} + \frac{B}{2T}(et - t_0 + \tau)](et - t_0 + \tau)\} \exp\{- j2\pi[f_0 - \frac{B}{2} + \frac{B}{2T} t]t\} dt \tag{22}$$

Let us introduce a new variable $\tau' = \tau - t_0$ and the following designations: $a = f_0 - B/T$, $b = B/2T$. We then have:

$$r_{xs}(\tau') = X_0 S_0 \int_{-\infty}^{\infty} \exp\{ j2\pi[a + b(et + \tau')](et + \tau')\} \exp\{- j2\pi(a + bt)t\} dt \tag{23}$$

After we have made the operations in the exponents and the elementary transformations, we obtain:

$$r_{xs}(\tau') = X_0 S_0 \exp[\, j2\pi[(a + b\tau')\tau'] \cdot \\ \cdot \int_{-\infty}^{\infty} \exp\{\, j2\pi(e-1)[a + (e+1)bt]t\} \exp(\, j4\pi be\,\tau' t) dt \tag{24}$$

We will not be making a significant mistake if we insert e+1 ≅ 2 and e-1 = -v/c. Having limited our interest to the module of the correlation function, we have:

$$| r_{xs}(\tau') | \cong X_0 S_0 \,| \int_{-\infty}^{\infty} \exp[-j2\pi(a + 2bt)\frac{v}{c} t] \exp(j4\pi be\,\tau' t) dt \tag{25}$$

As you can see in the formula above the module of the correlation function is the Fourier transform of the function describing linear frequency modulation. The equivalent of frequency in normal notation of the Fourier transform is variable φ which is equal to:

$$\varphi = 2b\tau' e \cong \frac{B}{T}\tau' \tag{26}$$

which is the outcome of reduction e = 1-v/c≅1. As you can see, the variable has frequency dimension.

When we insert the above designation, we obtain:

$$| r_{xs}(\tau') | \cong X_0 S_0 \,| \int_{\infty}^{-\infty} \exp[-j2\pi(a + 2bt)\frac{v}{c} t] \exp(-j2\pi\varphi t) dt \,| \tag{27}$$

As you can see in Fig. 2, the spectrum of the linear frequency modulation function is almost rectangular and contained between two frequencies of which the first is for time t = 0 and the second for time t = T (formula 19). In the case in question we have:

$$\varphi \cong \frac{d}{dt}[(a + 2bt)t\frac{v}{c}] = (a + 4bt)\frac{v}{c} = (f_0 - \frac{B}{2} + \frac{2B}{T}t)\frac{v}{c} \tag{28}$$

Hence the cut-off frequencies take the following values:

$$\varphi_1 \cong (f_0 - \frac{B}{2})\frac{v}{c} \qquad \varphi_2 \cong (f_0 + \frac{3B}{2})\frac{v}{c} \tag{29}$$

As shown in formula (26) the above frequencies in fact determine the delays of the correlation function. They go back to variable τ which describes the complete delay $\tau = \tau' + t_0$ and we obtain:

$$\tau_1 \cong t_0 + \frac{T}{B}(f_0 - \frac{B}{2})\frac{v}{c} \qquad \tau_2 \cong t_0 + \frac{T}{B}(f_0 + \frac{3B}{2})\frac{v}{c} \tag{30}$$

Hence the delay of the centre of the correlation function $\tau_0 = (\tau_1 + \tau_2)/2$ is:

$$\tau_0 \cong t_0 + \frac{T}{B}(f_0 + \frac{B}{2})\frac{v}{c} \tag{31}$$

The additional delay caused by the Doppler effect is approximately equal to:

$$\tau_d \cong \frac{T}{B}(f_0 + \frac{B}{2})\frac{v}{c} = T(\frac{f_0}{B} + \frac{1}{2})\frac{v}{c} \tag{32}$$

To ensure that target distance error is minimised, it is advisable to keep spectrum *B* of the sounding signal as wide as possible and time *T* as short as possible. In other words the straight lines shown in Fig. 1 should have the biggest possible inclination. As a reminder, detection performance improves with extended time *T*.

The width of the output signal (correlation function) is approximately equal to:

$$\Delta\tau \cong 2T\frac{v}{c} \tag{33}$$

As a result of the above reductions, formula (33) shows that for velocity $v = 0$, the width of the correlation function is also equal to zero. In reality the width is then equal to $\Delta\tau = 1/B$.

With no Doppler shift ($v = 0$) delay $\tau = t_0$ and formula (27) is reduced as follows:

$$|r_{xs}(\tau')| \cong X_0 S_0 \left| \int_0^T \exp(-j2\pi\varphi t)dt \right| \tag{34}$$

Having calculated the elementary integral, we obtain:

$$|r_{xs}(\tau)| \cong TX_0 S_0 \left| \frac{\sin[\pi B(\tau - t_0)]}{\pi B(\tau - t_0)} \right| \tag{35}$$

This is the expected result consistent with Fig. 4.

Let us now calculate the shift of the correlation function for data used in Fig.2 and the others. By inserting f_0 = 10 kHz, B = 1 kHz, T = 10 s and v = 10 ms (c = 1500 m/s) we obtain: τ_1 = 2.633 s, τ_0 = 2.7 s, τ_2 = 2.766 s. Delays like this lead to major misinterpretations of the target distance. The centre of the correlation function is: Δr_0 = 525 m.

The results of the analysis are consistent with the results of the system's computer simulation. Fig. 5 shows the output signal for the example given above. Fig. 6 shows an enlarged fragment of the output signal for target velocity v = 10 m/s. The dotted lines show the delays determined from analytical formulas. As you can see, the consistency between numerical and analytical calculations is very good.

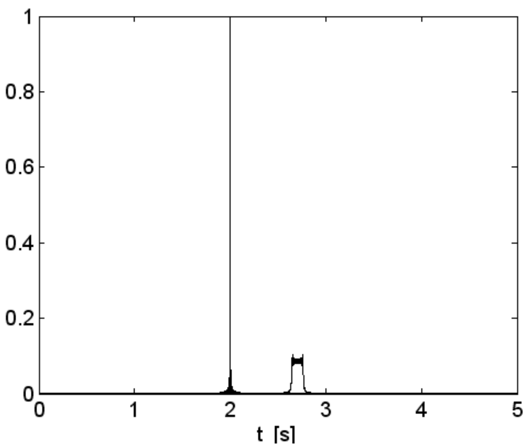

Fig. 5. Correlation function of the sounding signal and echo signal with Doppler shift (f_0 = 10 kHz, B = 1 kHz, T = 10 s, v = 10 ms)

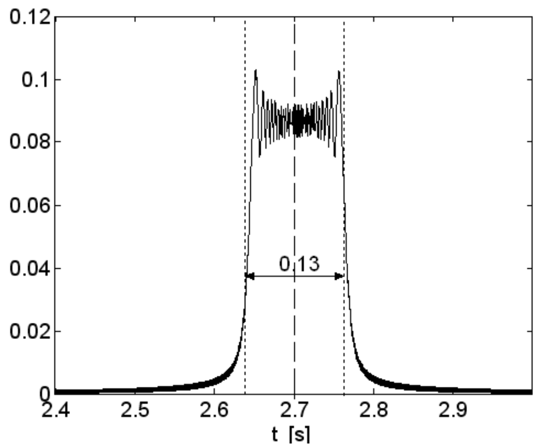

Fig. 6. A magnified fragment of Fig. 5

So far the analysis and simulations tested a simplified model of the sonar with the sounding signal and echo signal represented by single pulses. In order to test whether the results can be applied to periodical signals as well, the echo signal was replaced with a periodical echo signal in the simulation but the sounding signal kept its previous pulse shape. Fig. 7 shows the signal at the output of the matched filter for a stationary target. Its enlarged fragment is given in Fig. 8. A comparison of Fig. 3 and Fig. 4 shows that the results of the previous analysis can be fully applied to periodical echo signals.

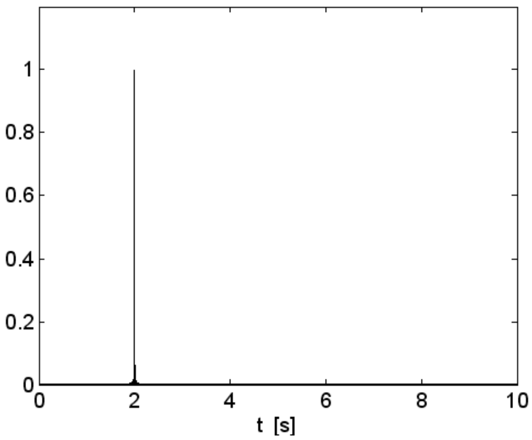

Fig. 7. Signal at output of matched filter of a periodical echo signal (f_0 = 10 kHz, B = 1 kHz, T = 10 s)

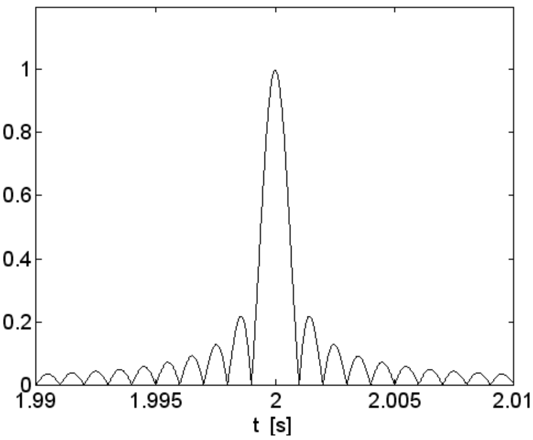

Fig. 8 Fragment of a signal around its maximal value

Analogous calculations were made for an echo signal with Doppler shift. The results showed that compared to a non-periodical signal the shift halves the height of the sounding signal at the output of the matched filter. This effect can be seen in Fig. 9 and 10 which are made for the same parameters as in Fig. 5 and Fig. 6. Another result of the comparison is that the duration of signals at the output of the matched filter is in both cases identical.

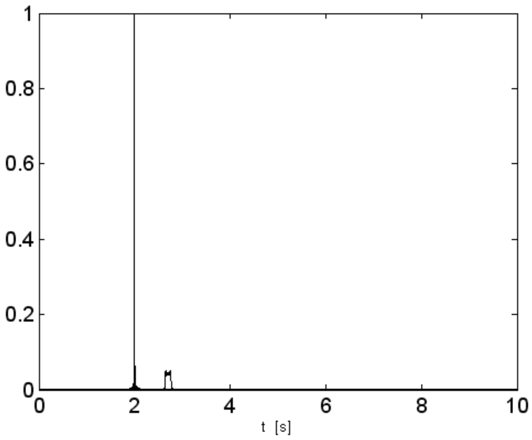

Fig. 9. Correlation function of a sounding signal and periodical echo signal with Doppler shift (f_0 = 10 kHz, B = 1 kHz, T = 10 s, v = 10 ms)

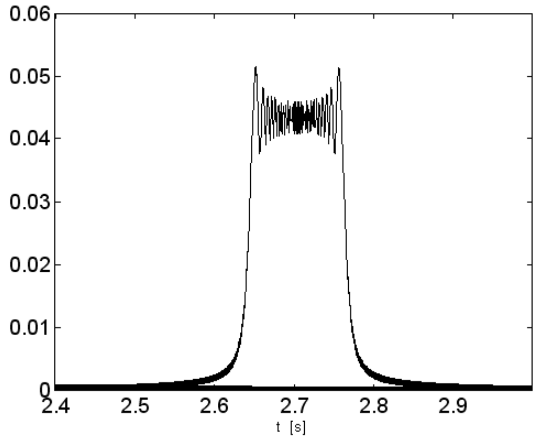

Fig. 10. A magnified fragment of Fig. 5

As you can see in the figures above the Doppler effect has one more negative impact. It increases the duration of the output signal and reduces its height. With a lower signal at the output of the matched filter, the signal to noise ratio decreases which deteriorates detection performance. This is the direct result of formula (10) in which energy E_t is proportional to the height of the function of the line shown in Fig. 5, Fig. 7 and Fig. 9 and Fig. 9 and 10 where the signal with the Doppler shift is significantly smaller.

Fig. 11 shows the relation between the height of the signal at matched filter output with the Doppler shift and the *v/c* for specific and constant values of the remaining sonar parameters. Fig. 12 shows the height of the signal in the function of *T* for a constant *v/c*.

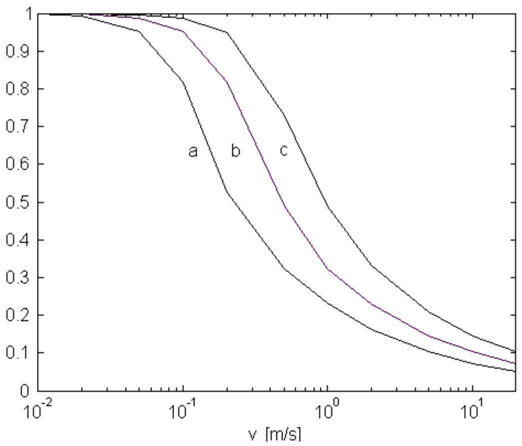

Fig. 11. Normalized maximum values of matched filter outpt: **a –** B = 2 kHz, **b** – B = 1 kHz, **c** = 0.5 kHz (f_0 = 10 kHz, T = 10 s)

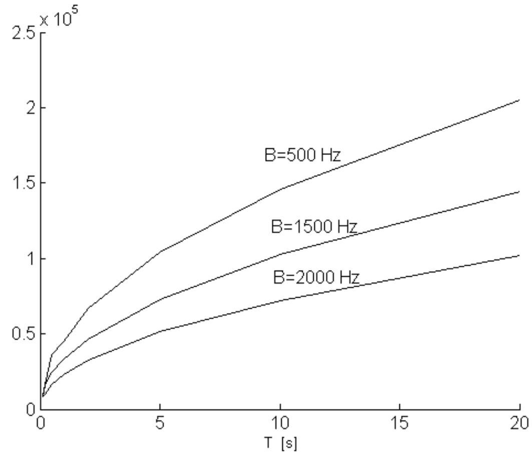

Fig. 12. Maximum values of matched filter output in the function of period T (f_0 = 10 kHz, v_0 = 10 m/s)

Charts show that the maximum value of the output signals decrease, when the bandwidth of the sounding signal frequency increases. The increase of period T results in increasing of the sounding signal energy, and consequently in increasing of amplitude of the signal at the output of the matched filter.

4. CONCLUSIONS

Matched filtration can be used as an alternative detection method in a CW FM sonar as opposed to the conventional method for determining differential frequency between the sounding signal and echo signal. Similarly to this method, sonars with matched filtration are also affected by the Doppler effect which misestimates the distance to the target and deteriorates detection performance. Our preliminary analysis has shown that there are ways to stop the negative effects of the Doppler effect partially or entirely by changing the form of sounding signals.

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