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Modelling and Identification of Tower Type Historic Buildings

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Abstract

In this paper the problem of parametric identification of a historic masonry tower model is discussed. The tower leans and its foundation stiffness is a concern to authorities. The authors identified some modal characteristics of the tower, natural frequencies and mode shapes. It is known, based on the first mode shape identified, that the structure behaves like a stiff solid on elastic foundation. Thus, a simple, five parameter plane model is considered. The unknown parameters are identified to be the solution to an optimisation problem, in which the sensitivity analysis and scatters of the modal identification are applied. A hierarchical process is formulated, where two natural frequencies are assumed to be the input data. In this approach, the number of unknown parameters increases incrementally, and the process changes from even-posed to under-posed successively. Such approach allows one to control the final under-posed identification problem and leads to an increasingly better solution.

Keywords: Identification, Modelling, Optimisation, Sensitivity analysis, Masonry tower

1. Introduction

This paper discusses the problem of parameters identification of the Vistula Mounting tower model (see Fig.1). The tower dates back to the 15th century, however it was damaged several times in military conflicts. Nowadays it is 22.65 m high, and its external diameter is 7.7 m. The structure has seven floors with concrete reinforced ceilings. Its walls were built using masonry and were restored at different times. The average wall thickness is 1.25 m. The tower was founded on weak and layered subsoil. The foundations were made of boulders and lie just below the ground level. This is probably a cause, why the tower leans. This behaviour of the structure is now a concern of authorities.

The author's task is to estimate foundation stiffness of the tower and create the model of the structure. For that purpose dynamic measurements were taken and some modal characteristics have been identified. Basing on the first mode shape a the type of a tower model was selected. A rigid solid body resting upon elastic foundation is considered to be a good approximation of the structure, since a considerable rotation - in comparison to the tower structural deformation - about the tower base is observable. Natural frequencies of the first and the second coplanar mode shapes, and two coordinates of the first mode shape are used as the data in the model parameters identification. In order to solve this problem, a least square error function was formulated as the objective function. Important elements of this task are scatters of the structure's measured modal characteristics. They are used to accurately define the optimisation problem.

2. Experimental modal identification

The Peak Picking method (see [1]) was used for modal identification of the tower. The method is suitable for any signals, also for low-energy vibrations, which occur in the tower. The method was selected for the investigation also because of the possibility of determining statistical errors of identified modal characteristics. This feature of the method was useful for this investigation.

The mode shapes errors arise from the fact that only estimates of the auto-spectra, which are basic functions in the Peak Picking method, can be calculated. Real values of the functions could be obtained for signals infinite in time and that is practically impossible. The estimates are affected by statistical errors, bias ε_b and random ε_r, which give a final error $\varepsilon = \varepsilon_b + \varepsilon_r$. They are presented in [1] and [2]. The formulae are:

$$\varepsilon_b\left[\hat{G}_{pp}(f)\right] \approx \frac{\Delta f^2}{24}\left[\frac{\left(\hat{G}_{pp}(f)\right)''}{\hat{G}_{pp}(f)}\right]; \qquad \varepsilon_r\left[\hat{G}_{pp}(f)\right] \approx \frac{1}{\sqrt{n_d}}, \tag{1}$$

where $\hat{G}_{pp}(f)$ is the estimate of auto-spectrum calculated for signal measured in a structural point p, Δf denotes the frequency resolution of the analyzed spectra, $\left(\hat{G}_{pp}(f)\right)''$ is the second derivative of the function $\hat{G}_{pp}(f)$ and n_d is a number of signals $p(t)$ analyzed.

If coordinates of a mode shape associated with the resonant frequency f_m are calculated according to the formula (2) (see [1]):

$$\hat{\phi}_p(f_m) = \sqrt{\frac{\hat{G}_{pp}(f_m)}{\hat{G}_{rr}(f_m)}}, \tag{2}$$

where $\hat{\phi}_p(f_m)$ denotes the estimated mode shape coordinate at a discretization point p and $\hat{G}_{rr}(f_m)$ is the auto-spectrum value for f_m, calculated for a signal $r(t)$, measured at the structural reference point r, then the statistical error of the mode shape coordinates is calculated from the following formula:

$$\varepsilon\left[\hat{\phi}_p\right] = \frac{1}{2}\left(\varepsilon\left[\hat{G}_{pp}\right] + \varepsilon\left[\hat{G}_{rr}\right]\right) \tag{3}$$

The error of the measured natural frequencies has two components: the digitalisation error equal to the half of the spectrum resolution, and the random error calculated using dispersion of the measured resonant frequencies.

Accelerations of points selected across the tower were measured during ambient vibrations according to the above-presented rules of the Peak Picking method. Wind and water waves from the nearby situated river (Fig. 1) caused major environmental excitation. The measuring points were arranged along two opposite walls at the tower height on nine levels. Accelerations in two horizontal directions, East-West (parallel to the wall surfaces) and North-South (perpendicular to the wall surfaces) were recorded at each point. Thus, 36 measuring points were set. Each measurement took 1024 seconds, 256 samples were collected per second, so each signal consisted of 262144 samples. In order to estimate the signal spectra, time histories were divided into 32 sections ($n_d = 32$).

Only one resonant frequency of the tower was identified using signals measured across the North-South plane, whereas three were determined using time series measured in the East-West direction. Nature of related mode shapes was also specified using the analysis of phase shifts between signals measured at different structural points. Additionally, coordinates of two first mode shapes in two perpendicular planes were determined. Hence, it is known that $f_1^{N-S} = 1.416$ Hz and $f_1^{E-W} = 1.446$ Hz refer to the first two lateral mode shapes in two perpendicular directions: North-South and East-West, respectively. The mode shape associated with f_1^{E-W} is presented in Fig.2. Then, frequencies identified in the East-West direction are $f_t = 4.485$ Hz , which relates to the torsional mode, and $f_2^{E-W} = 6.570$ Hz , connected with the second lateral mode shape in this plane.

The following values of the errors were obtained for the tower's natural frequencies $\varepsilon\left[f_1^{N-S}\right] = 0.00322$, $\varepsilon\left[f_1^{E-W}\right] = 0.00337$, $\varepsilon\left[f_t\right] = 0.00689$ and $\varepsilon\left[f_2^{E-W}\right] = 0.00871$. The error for all the modes is the same and amounts to $\varepsilon[\phi] = \varepsilon_r[\phi] = 0.177$, because $\varepsilon_b[\phi]$ is negligibly small as it is of the 0.001 order (see also [3]).

Figure 1. The Vistula Mounting Fortress

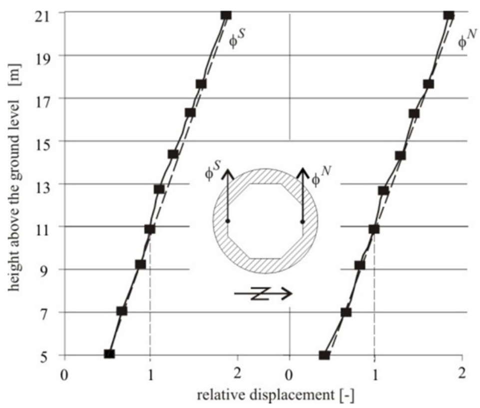

Figure 2. Mode shape of the first resonant frequency of the tower in the East-West plane

3. Mathematical model of the tower and its identification

In case of the Vistula Mounting Fortress tower the type of model is determined based on first mode shapes measured. The mode shape (Fig. 2) shows that the tower leans almost like a stiff solid therefore a model of a rigid solid body resting on an elastic foundations can be a reasonable mathematical approximation of the building's behaviour. A small number of parameters is the advantage of this model. It is convenient because only a few modal characteristics of the tower are to be used as state variables in the model parametric identification.

The plane model is the subject of interest. Therefore there are two dynamic degrees of freedom, namely: the displacement across the x axis and rotation φ, relative to the y axis. The foothold of the Cartesian coordinate system xyz is placed in the centre of gravity of the structure. The following equation of motion is valid:

$$\underbrace{\begin{bmatrix} m & 0 \\ 0 & J_y \end{bmatrix}}_{\boldsymbol{M}} \begin{Bmatrix} \ddot{x} \\ \ddot{\varphi} \end{Bmatrix} + \underbrace{\begin{bmatrix} k_x & -k_x z_c \\ -k_x z_c & k_\varphi + k_x z_c^2 \end{bmatrix}}_{\boldsymbol{K}} \begin{Bmatrix} x \\ \varphi \end{Bmatrix} = \mathbf{0}, \tag{4}$$

with the following five parameters: m mass of the tower, J_y the tower mass moment of inertia with respect to y axis, z_c coordinate of the tower's centre of gravity, and k_φ, k_x foundation stiffness modules. Those five parameters are to be determined based on measured tower modal characteristics

In the task of parametric identification of the mathematical model, an optimisation problem was formulated. The square error function is assumed to be the objective function:

$$F(\boldsymbol{b}) = \sum_{i=1}^{i=S} \alpha_i \left(s_i(\boldsymbol{b}) - \hat{s}_i \right)^2, \tag{5}$$

where $\boldsymbol{b}$ denotes a vector of the design variables (the sought-after parameters of the model), $s_i(\boldsymbol{b})$ stands for the state variables of the model, $\hat{s}_i$ represents measured state variables of the tower and α_i is a weight coefficient determined for each state variable. In order to find the minimum of the objective function (5) an iterative procedure is proposed and the optimization problem is reformulated as minimisation of the objective function in relation to the design variables vector variations:

$$\min_{\delta \boldsymbol{b}} F(\delta \boldsymbol{b}^k) = \min_{\delta \boldsymbol{b}} \sum_{i=1}^{i=S} \alpha_i \left(s_i^k(\boldsymbol{b}^k) + \delta s_i^k(\boldsymbol{b}^k, \delta \boldsymbol{b}^k) - \hat{s}_i \right)^2, \tag{6}$$

where $\delta s_i^k \left(\boldsymbol{b}^k, \delta \boldsymbol{b}^k \right)$ is the first variation of the state variable with respect to the design variable vector. Variations $\delta \boldsymbol{b}^k$, calculated at each stage k are used for updating the $\boldsymbol{b}$ vector. Calculations continue until the relative variations $\delta \boldsymbol{b}^k$ are smaller than the assumed accuracies. The mathematically complicated relation $\delta s_i(\boldsymbol{b}, \delta \boldsymbol{b})$ is substituted by approximation $\delta s_i = \left(\boldsymbol{w}_{sb} \right)^{\mathrm{T}} \delta \boldsymbol{b}$ determined by means of sensitivity analysis.

In this approach the radial natural frequencies squared λ_i and the coordinates of the first mode shape $\boldsymbol{\phi}_{n1}$ are the state variables. Thus, the objective function is formulated as follows:

$$\min_{\delta \boldsymbol{b}} F(\delta \boldsymbol{b}^k) = \min_{\delta \boldsymbol{b}} \left(\sum_{i=1}^{i=2} \alpha_j \left(\frac{\lambda_i^k(\boldsymbol{b}^k) - \hat{\lambda}_i}{\lambda_i^k(\boldsymbol{b}^k)} + \left(\bar{\boldsymbol{w}}_{\lambda b}^k\right)^{\mathrm{T}} \delta \bar{\boldsymbol{b}}^k \right)^2 + \sum_{n=1}^{n=2} \alpha_n \left(\frac{\phi_{n1}^k(\boldsymbol{b}^k) - \hat{\phi}_{n1}}{\phi_{n1}^k(\boldsymbol{b}^k)} + \bar{\boldsymbol{W}}_{\boldsymbol{\phi} b}^k \delta \bar{\boldsymbol{b}}^k \right)^2 \right) \tag{7}$$

where the vector $\bar{\boldsymbol{w}}_{\lambda b}$ and the matrix $\bar{\boldsymbol{W}}_{\boldsymbol{\phi} b}$ consist of the relative first variations of the radial natural frequency squared λ and of the mode shape $\boldsymbol{\phi}$ relative to the variations of the design variables, respectively. The coefficients are derived from the equation of motion for a discrete system and are presented for example in [4]

The following values of state variables $\hat{\lambda}_1 = \left(2\pi f_1^{E-W}\right)^2 = 81.99\left(\mathrm{rad/s}\right)^2$ determined from experiments were used in the optimisation procedure: $\hat{\lambda}_2 = \left(2\pi f_2^{E-W}\right)^2 = 1703.34\left(\mathrm{rad/s}\right)^2$, $,\hat{\phi}_{t1}^{E-W} = 1.806\,[-]$, $\hat{\phi}_{b1}^{E-W} = 0.453\,[-]$. Errors in measured state variables are used to specify weighted coefficients of state variables so that their sum is equal to 1. Hence, the values are: $\alpha\left(\hat{\lambda}_1\right) = 0.869$, $\alpha\left(\hat{\lambda}_2\right) = 0.119$ and $\alpha\left(\hat{\phi}_{t1}^{E-W}\right) = \alpha\left(\hat{\phi}_{b1}^{E-W}\right) = 0.00588$.

The final criterion for identification is defined by a relative difference between the measured and the calculated state variables. For each variable this difference must be smaller than its error obtained from the modal identification. Therefore, the criterion consists of the following conditions: $\varepsilon\left(\phi_{n1}^{E-W}\right) = \dfrac{\hat{\phi}_{n1}^{E-W} - \phi_{n1}^{E-W}}{\hat{\phi}_{n1}^{E-W}} \le 0.177;\quad n = b, t$,

$\varepsilon\left(\lambda_1\right) = \dfrac{\hat{\lambda}_1 - \lambda_1}{\hat{\lambda}_1} \le 0.00337$ and $\varepsilon\left(\lambda_2\right) = \dfrac{\hat{\lambda}_2 - \lambda_2}{\hat{\lambda}_2} \le 0.00871$.

The result obtained in the optimization is assessed by calculating the Normalized Modal Difference (*NMD*) between the first mode shape calculated for the model and the measured mode shape of the tower.

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	Identified parameters / The parameter name	Starting value	Boundary condition	Identification result	Number of itera-tion steps	*NMD* [%]
Starting values	k_{φ} [Nm]	$1.00\cdot10^{10}$	100 - ∞	$1.239\cdot10^{10}$	10	4.666
	k_x [N/m]	$4.00\cdot10^{8}$	100 - ∞	$4.264\cdot10^{8}$		
	z_c [m]	10.00	0 – 15.00	9.921		
	J_y [kg·m^2]	$4.30\cdot10^{7}$	10^6- ∞	$4.115\cdot10^{7}$		
	m [kg]	$9.000\cdot10^{5}$	(9-10) $\cdot10^5$	$9.202\cdot10^{5}$		

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