

# The Accuracy Assessment of Determining the Axis of Railway Track Basing on the Satellite Surveying

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Received June 2012

## Abstract

In 2009, at the Gdansk University of Technology there have been carried out, for the first time, continuous satellite surveying of railway track by the use of the relative phase method based on geodesic active network ASG-EUPOS and NAVGEO service. Still continuing research works focused on the GNSS multi-receivers platform evaluation for projecting and stock-taking. In order to assess the accuracy of the railway track axis position, the values of deviations of transverse position XTE (Cross Track Error) were evaluated. In order to eliminate the influence of random measurement errors and to obtain the coordinates representing the actual shape of the track, the XTE variable was analyzed by signal analysis methods (Chebyshev low-pass filtering and fast Fourier transform). At the end the paper presents the module of the computer software SATTRACK which currently has been developing at the Gdansk University of Technology. The program serves visualization, assessment and design process of railway track, adapted to the technique of continuous satellite surveying. The module called TRACK\_STRAIGHT is designed to assess the straight sections. A description of its operation as well as examples of its functions has been presented.

## 1. Introduction

The development of satellite geodetic techniques, together with the increase of GPS (Global Positioning System) surveying precision, leads to taking an effort of

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application the GPS technology for the purpose of railway track inventory. Available at the beginning of the twenty-first century single-station measurement techniques as well as the development of RTK (Real Time Kinematics) GPS methods, allowed to obtain the accuracy of measurements of one centimeter for 1÷5 Hz frequency, with necessity of additional reduction of the height measurement to the orthometric height.

Significant qualitative change came together with the implementation of GPS/GPRS emissions (General Packet Radio Service) in the range of teletransmissive operation of the active geodesic network. In 2004, the RTCM (Radio Technical Commission for Maritime Services) introduced NTRIP protocol (Networked Transport of RTCM via Internet Protocol) [8], which determined the fate of the range of radio emissions VHF so far commonly used in the RTK systems and other solutions which use wireless radio lines in relation: base station – rover. Their small, several kilometers range have been preventing both implementation of GNSS (Global Navigation Satellite System) network solutions and the transfer of surface patches using various methods like VRS, MAX or FPC. The process of standardization of the format of emission has been supplemented by the RTCM standard - version 3.0 [9], which was also published in 2004.

Start of Polish geodesic active network in 2008 and a significant qualitative improvement in the GPS precise designations (use of integrated GPS-GLONASS receivers, increasement of designations fix rate up to 20 Hz, and implementation of the geoid model in receivers' system) led the interdisciplinary research team to undertake an investigation within attempts to assess the possibility of using GPS for geodetic service of railways [4, 5, 6].

The methodology of conducted research based on the passages through the experimental section of the railway line by rail vehicle WM-15 with a trailer (platform) PWM-15. On the surface of the trailer there were installed four antennas for the GPS satellite measurements, recording the coordinates with a fix rate of 5 Hz along with the accuracy between 1 to 3 cm. The initiator of started in 2009 research was Gdansk University of Technology – Department of Railway Engineering and Department of Geodesy with a participation of Naval Academy in Gdynia – Institute of Maritime Navigation and Hydrography. The technical possibilities of conducting the research program were created by the Department of Railways PKP PLK SA in Gdynia, while the company Leica Geosystems GA provided a proper measuring equipment.

In 2010, the measurement of the geometric shape of railway track was carried out on the section of the Gdansk – Gdansk Nowy Port railway line. Three receivers Leica System 1200 SmartRover, which consisted of Smart-type controllers RX ATX1230GG and 1250 were deployed on the longitudinal axis of the PWM-15 platform. Dynamic measurements were performed in real-time service NAWGEO (RTK measurement method), with the use of RTK corrections NAWGEO\_RTCM\_3\_1\_VRS stream. The measurement set used the internet access service via modems GPRS Siemens MC45-mode network NTRIP Simplus.

In order to write a raw observations the mode of autosave feature in a distance mode with 30 cm interval was used. After the dynamic measurement process, 40-minutes observation was made on the post-processing service POZGEO. Finally, the obtained data were sent to the Calculation Center in RINEX format.

## 2. Determination of the Actual Shape of the Track Axis in the Local Coordinate System

In order to determine the shape of analyzed straight track sections, the signal obtained by measuring is transformed into the local system of coordinates. In first step, the linear approximation of points measured on the route was performed (those coordinates were transformed from World Geodetic System WGS-84 into the state (local) system of spatial reference called 2000 [10]). On the base of the calculated line (its equation) the whole set of points is transformed as it is shown in the Fig. 1. Starting from the equation  $X = A + BY$  (where  $X$  and  $Y$  coordinates are expressed in meters), the  $Y$ -axis is shifted of the constant  $A$ , and then the points are rotated of an angle  $\phi$  which can be calculated from the inclination of estimated line. In the result the points and their approximation are aligned with the horizontal axis of system defining our route in the straight section. In the system  $X_1 (Y_1)$  the ordinates  $X_1$  can be understood as a deviation from that direction. Those values show us the horizontal inequalities occurring in the track (as well as measurement error).

The transformation formulas are shown in the following equations:

$$Y_1 = Y \cos\phi + (X - A) \sin\phi, \quad (1)$$

$$X_1 = -Y \sin\phi + (X - A) \cos\phi. \quad (2)$$

The values of  $\sin\phi$  and  $\cos\phi$  result from the relationships below:

$$\sin\phi = \pm \frac{B}{\sqrt{1+B^2}}, \quad \cos\phi = \pm \frac{1}{\sqrt{1+B^2}}$$

After transformation, made according to the algorithm described above, the signal is presented in the coordinate system, in which the horizontal axis states direction consistent with the axis of track on a straight section. Therefore, the values different from zero on the vertical axis state the deviation of the GPS signal from the direction of the measured route. If the measurement was conducted at the perfectly straight track (with a specified accuracy of horizontal inequality), the recorded samples would represent the value of the measurement error, which would determine the accuracy of adopted methodology. However in practice, the situation is that track's axis tends to be deformed as a result of operating and technological processes, finally resulting in the track maintenance. It is therefore expected that the measured signal transformed to such system (after transformation) will represent the deformation of the track in relation to the design assumptions.

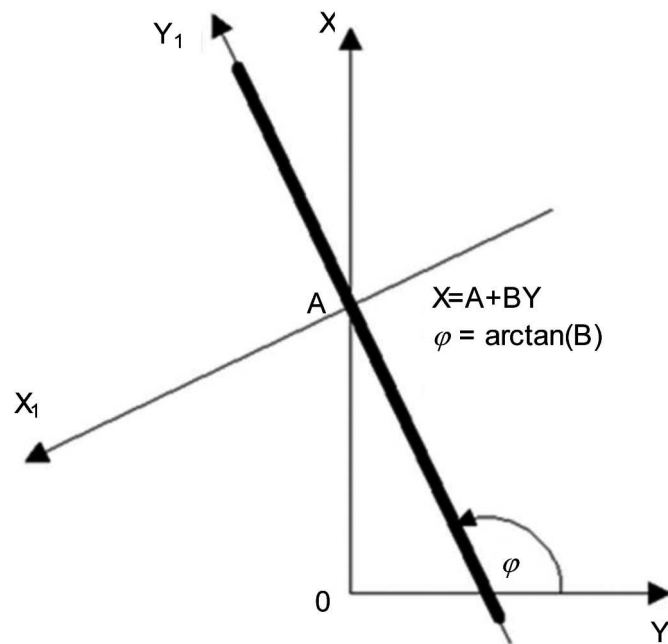


Fig. 1. The idea of coordinate system transformation

In navigational terms, the position of navigated object at a distance from the established course (as assumed direction) in the lateral direction is called as XTE (Cross Track Error), and it states one of the measures of the position errors of a moving object (Fig. 2). As we can see, on the railway a very similar phenomenon occurs. Therefore we can also describe the horizontal irregularity of the track with the use of the XTE function adapted for this purpose.

To illustrate above fact, let's consider the example of a straight section of track, determined by 321 points. The calculated equation of the line in the 2000 coordinates system is presented as follows:

$$X = 14711290.544 - 1.32765Y \quad (3)$$

while the angle  $\phi = 126.990^\circ$ . In the system  $Y_1, X_1$  beginning point of the section has coordinates  $(-10872997.753; 0.001983)$ , however, due to the substantial magnitude of  $Y_1$  coordinate values, it will be more convenient to operate in the local coordinate system in which the beginning point has the abscissa equal to zero. We will get it by adopting

$$Y_{loc} = Y_1 + 10872997.753 \quad (4)$$

$Y_{loc}$  values indicate the location of measurement points on the length of the section and in this case they are inside the range  $[0; 266.920 \text{ m}]$ . Preparation of the received signal XTE ( $Y_{loc}$ ) for further analysis still requires to be interpolated in order to obtain a constant interval step of  $Y_{loc}$  (adopted step was 0.2 m).

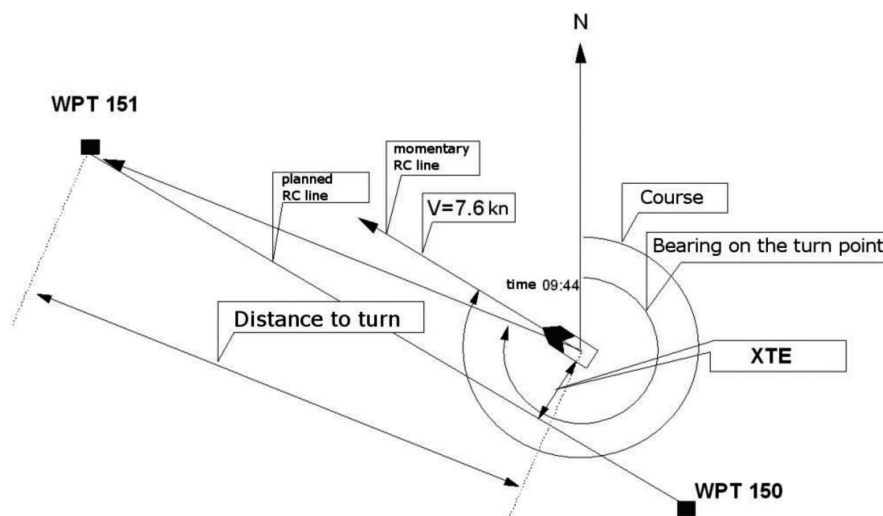


Fig. 2. Graphical interpretation of the XTE term

In the analyzed case, on the XTE value there is also superimposed the uncertainty associated with the technique of measurement. Therefore, the received signal has been analyzed in order to check the possibility to filter out certain components which can be regarded as secondary phenomena, having no direct relationship with a shape of the measured track.

### 3. The Analysis of XTE Signal

#### 3.1. The use of low-pass filtering process

Detailed analysis of the recorded measurement signal (after the transformation described earlier) leads to the conclusion that the signal gives not only information about the shape of track, which is characterized – by definition – by smooth curvatures. There is also a signal component which states a result of errors associated with the vibration of the moving vehicle as well as measurement errors.

In order to analyze the measurement signal in the frequency domain, Fourier transformation was used [12]. As the analyzed signal has discrete character, it has been used the so-called Discrete Fourier Transformation of the series of the measurement samples. The transformation is described by the formula:

$$X_k = \sum_{n=0}^{N-1} x_n \cdot e^{-\frac{2\pi i}{N}nk} \quad k = 0, \dots, N-1 \quad (5)$$

where:  $X$  – transformation result,  
 $x$  – signal's samples.

Modern algorithms for solving this problem, allow significant reduction in computing time. The most popular algorithm is the fast Fourier transformation of the base of 2. It reduces the number of operations from  $O(N^2)$  to  $O(N\log_2 N)$ . But to take advantage of this algorithm, it is necessary to prepare the data vector by establishing the right size of it, namely the number of samples should be  $2^k$ , where  $k$  is a natural number.

In order to analyze the signal in the frequency domain Scilab environment [11] was used. In the libraries of functions there are implemented functions of the Fourier transformation as well as the inverse transformation to the original domain. Calculating script written by the authors was based on the following algorithm:

- load the matrix of measurement signal (from a text file),
- extend the matrix to the size of  $2^k$  (empty positions of vector set as zero),
- create a vector of the frequency basing on the signal sampling frequency,
- perform and display the resulting FFT transform,
- specify the parameter of a low-pass filter and filter the transformed signal,
- perform the inverse transformation of the signal subjected to filtration and view the result on the background of the original signal.

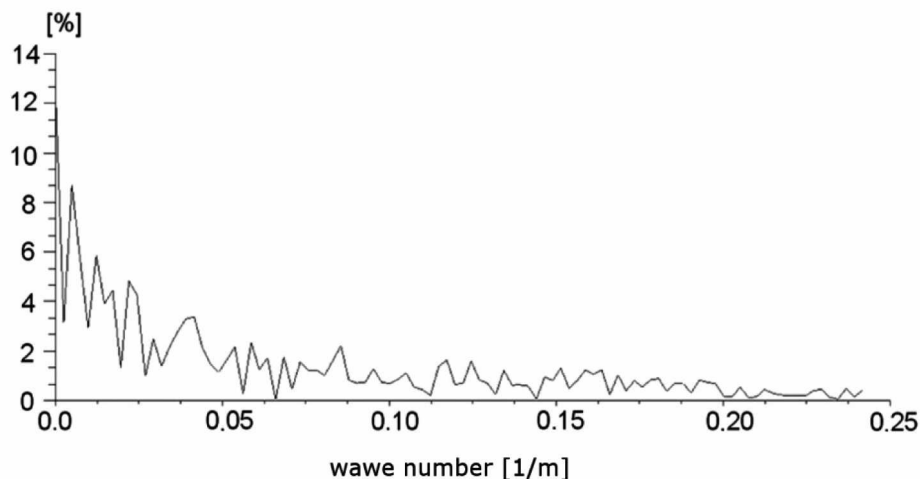


Fig. 3. Fourier transform for the analyzed section of the railway line

Figure 3 shows the result of transformation (FFT). We assumed low-pass filter with a cutoff frequency 0.15 [1/m]. This value is matched only by the qualitative assessment by the reverse transformation on the background of the input signal.

As it follows from the above algorithm and the presented results, the proper selection of the filter (its kind) and defining its parameters played a large role. At this stage, the authors do not yet determine the criteria that would allow automatic control of the filter and therefore the results presented are subject to uncertainty

of a loss of certain ranges of frequencies that affect the geometry of the analyzed track. Figure 4 shows a comparison between the original signal and filtered one.

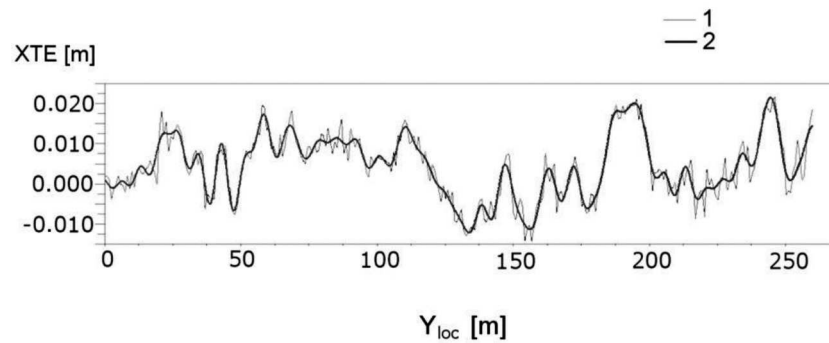


Fig. 4. Comparison of the signal before and after filtration:  
1 – signal before filtering, 2 – signal after filtering by low-pass filter  
with cutoff wave number of 0.15 [1/m]

If we assume that the filtered signal brings us closer to the real shape of the actual track, the difference between the both filtered and original signals will illustrate the measurement error. The signal for this deviation for the length of the analyzed section of the track is shown in Figure 5. The analysis shows that these values are small, their average value is 1.53 mm with a standard deviation equal to 1.22 mm.

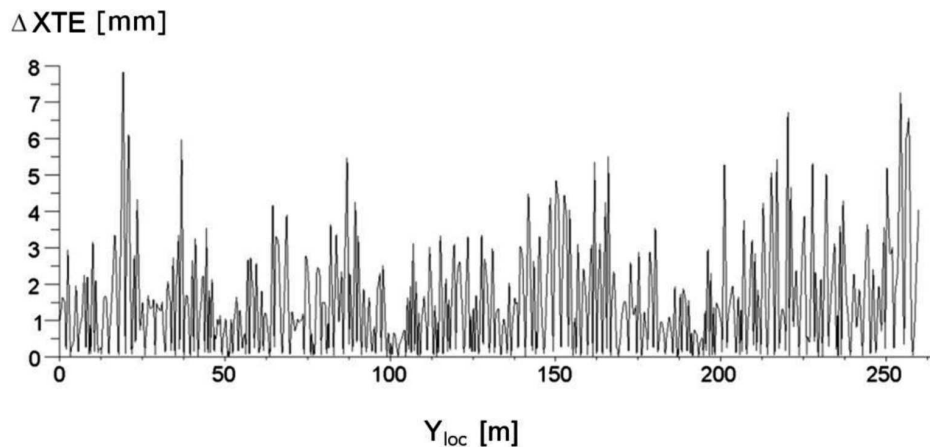


Fig. 5. The absolute values of deviations between the signal before and after filtration

### 3.2. The use of the Chebyshev filter

Another, used by the authors, approach to the problem of filtering the signal was using the IIR filter class (Infinite Impulse Response). The authors have been analyzing the Chebyshev lowpass filters of type I and type II (inverse). As the type II Chebyshev filters are monotonic in the passband and equiripple in the stopband, the authors abandoned the type I (monotonic in the stopband). The type II filters do not roll off as fast as type I filters, but are free of passband ripple [7]. The type II Chebyshev filters of 2nd, 3rd, and 4th orders were checked for different values of the ratio of cutoff frequency  $\omega_c$  to the sampling frequency  $\omega_p$  (ranging from 0.01 to 0.3) as well as different slope in the transition band of 20dB. Filtration was performed according to the standard differential equation describing the type II Chebyshev filter in the form as below:

$$NX_i = b_0 \cdot X_i + b_1 \cdot X_{i-1} + b_2 \cdot X_{i-2} + \dots - a_1 \cdot NX_{i-1} - a_2 \cdot NX_{i-2} - \dots \quad (6)$$

The authors compared the results of filters of 2nd and 4th order. In case of 4th order a higher phase shift was observed (between the filtered signal and the original one). In the paper filtering results for 2nd, 3rd and 4th filter orders are presented in Fig. 6. The filter of odd order (3rd) turned out to be less fitting to the original signal in relation to the even orders. However the unsatisfactory response phase, is observed in case of all used filters, what is clearly visible in the figure 6. All signals obtained by the use of Chebyshev filters are simply shifted in relation to the original one. As the horizontal axis represents one of two spatial coordinates, the difference in phase would be an additional error to analyze. Moreover, despite the fact that filtering of the signal with the use of equation (6) does not require the transformation to the frequency domain, the assumption of the appropriate coefficients of the equation for Chebyshev filters is a critical task for the numerical stability of calculations. The coefficients utilized in the paper are presented in Tables 1, 2 and 3. The bolded values correspond with the signals in Figure 6.

Table 1

Coefficients of 2nd order filter

$\omega_c/\omega_p$	Coefficients of 2nd order filter (for various $\omega_c/\omega_p$ )				
	$b_0$	$b_1$	$b_2$	$a_1$	$a_2$
<b>0.01</b>	<b>8.663387e-4</b>	<b>1.732678e-3</b>	<b>8.663387e-4</b>	<b>-1.919129</b>	<b>9.225943e-1</b>
0.2	1.997396e-1	3.994792e-1	1.997396e-1	-4.291048e-1	2.280633e-1
0.3	3.8491163e-1	7.698326e-1	3.849163e-1	3.249116e-1	2.147536e-1



Table 2

$\omega_c/\omega_p$	Coefficients of 3rd order filter (for various $\omega_c/\omega_p$ )						
	$b_0$	$b_1$	$b_2$	$b_3$	$a_1$	$a_2$	$a_3$
0.05	1.213e-5	3.638e-5	3.638e-5	1.213e-5	-2.9710	2.9607	-0.989
0.06	2.095e-5	6.285e-5	6.285e-5	2.095e-5	-2.9608	2.9495	-0.987
<b>0.08</b>	<b>4.960e-5</b>	<b>1.489e-4</b>	<b>1.489e-4</b>	<b>4.960e-5</b>	<b>-2.9361</b>	<b>2.9198</b>	<b>-0.983</b>
0.1	9.696e-5	2.909e-4	2.909e-4	9.696e-5	-2.9055	2.8855	-0.979
0.2	7.930e-5	2.379e-4	2.379e-4	7.930e-5	-2.7026	2.6991	-0.995

Table 3

$\omega_c/\omega_p$	Coefficients of 4th order filter (for various $\omega_c/\omega_p$ )									
	$b_0$	$b_1$	$b_2$	$b_3$	$b_4$	$a_1$	$a_2$	$a_3$	$a_4$	
<b>0.01</b>	<b>4.15e-7</b>	<b>1.659e-6</b>	<b>2.489e-6</b>	<b>b<sub>1</sub></b>	<b>b<sub>0</sub></b>	<b>-3.89</b>	<b>5.688</b>	<b>-3.695</b>	<b>0.901</b>	
0.075	9.73e-4	3.890e-3	5.835e-3	$b_1$	$b_0$	-3.10	3.774	-2.111	0.456	

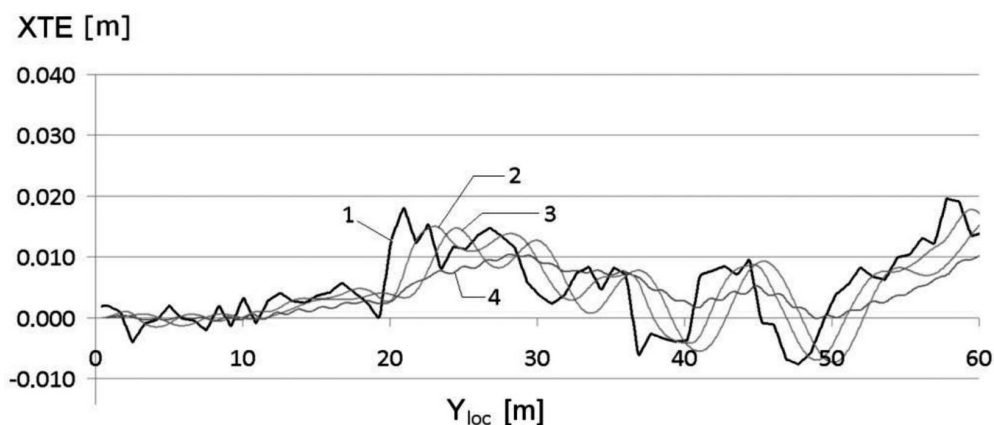


Fig. 6. Comparison of the filtration results of analyzed signal (a part) using of Chebyshev Type II filters: 1 – signal before filtering, 2 – Chebyshev filter of 2nd order with  $\omega_c/\omega_p = 0.01$ , 3 – Chebyshev filter of 4th order with  $\omega_c/\omega_p = 0.01$ , 4 – Chebyshev filter of 3rd order with  $\omega_c/\omega_p = 0.08$

#### 4. Computer Program for the Straight Sections of Railway Track Assessment

In the Department of Rail Transport in Gdansk University of Technology a computer program called SATTRACK for visualization, evaluation and design of the railway line is currently being developed. The algorithms are being adapted to the technique of continuous satellite measurements [2]. One of the elements of this program is a module for evaluating straight route segments – named TRACK\_STRAIGHT

[1]. This evaluation is based on making a proper interpretation and analysis of the measurement signal obtained by GPS method.

The first task is to extract the relevant parts of the measured signal, which correspond to straight track sections. This task is only seemingly trivial, since both sides of the straight line fluently change into unspecified transition curves or circular arcs, and what is more, the measured signal contains information about the unidentified deformation of the track as well as measurement error. All this means, that in order to precisely determine the direction of the main route (corresponding to the straight section), the factors disrupting the shape of the considered straight section should be isolated. SATTRACK program makes it possible, by presenting a route (set of measured points) on the grid of the 2000 coordinate system in the isometric way (the same scale in both directions) and by providing the function EXTRACT – which lets the user to extract and save the chosen range of data measuring signal by the indication the section's boundary ends [2]. So the extracted data recorded into relevant files will be further precisely analyzed.

In the presented module (TRACK\_STRAIGHT), user works with the data which were previously extracted from the general signal and saved in the proper folders. The main direction of the analyzed route is estimated in the program by the least squares method and the fitting degree of a correlation of the linear model and the input set of points describes a fit coefficient  $R^2$ . The program displays the set of points using non isometric projection, because it is the only way the user can visually capture the points of deviation from the assumed direction. These deviations can vary from a few millimeters to several meters [3]. The program allows the user to cut off those parts of the signal, which clearly do not belong to the straight part of the line.

After approval the new range of  $Y_{loc}$ , the program displays the full set of points in the local coordinate system whose horizontal axis coincides with the new (adjusted) main direction of the route estimated by the method of least squares. The user can conduct this operation of limiting the set scope many times – just as many as he/she needs. During the process, the user observes how the  $R^2$  coefficient varies. The displayed  $R^2$  is a helpful guidance, because when it starts to decrease then the user can stop cutting the signal so the user does not lose information. Of course, it is very important to save possibly greatest numerical range. The Example of finally adopted XTE ( $Y_{loc}$ ) signal of a railway route in straight shape is shown in Fig. 7.

Another option which has been implemented in the actual module TRACK\_STRAIGHT is the filtering of selected GPS signal in the frequency domain. Since the output signal domain in this case is not time, but the distance expressed in meters, the frequency will be understood here as a number of waves within the unit of length. Analysis of the signal requires the presentation of its value at equal intervals. Although the satellite measurement is performed with a constant speed of moving vehicle, in order to improve the precision of the process of filtering actual signal is interpolated with a correspondingly higher resolution by using polynomial splines of the third degree. This operation provides a signal with constant step

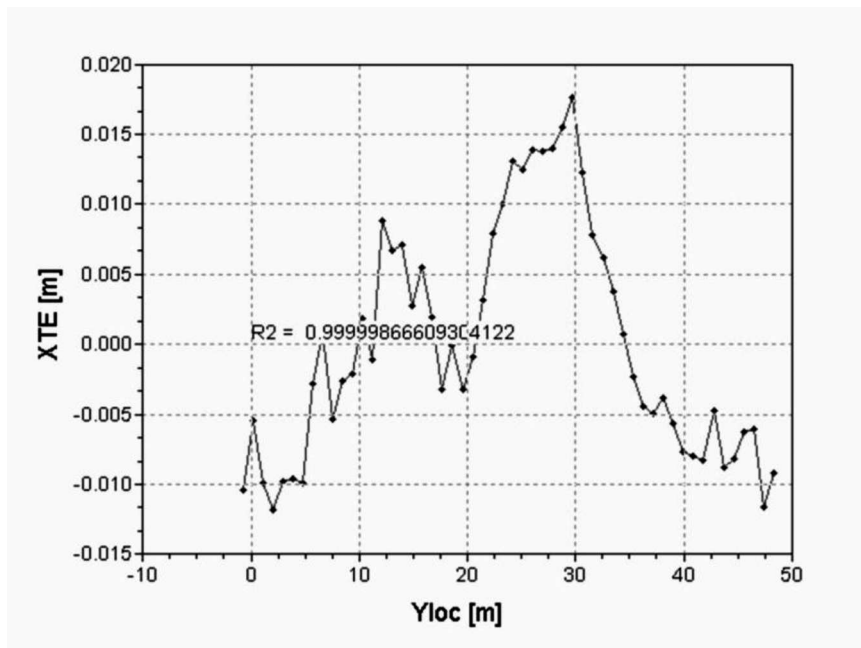


Fig. 7. Ultimately determined signal of  $XTE (Y_{loc})$  representing selected straight section of the route (non-isometric scale)

distance with keeping the original shape of the measured route. The user sees on a screen the Fourier transform chart, and the program lets him choose the frequency (in terms of number of waves per unit length) cut-off parameter which defines the low-pass filter. Once approved, the program makes the cut-off limit of the inverse transformation, and shows the result of filtering on the background of the original signal. User can repeat this operation as many times as he/she needs, each time assessing the outcome of filtering on the background of the results obtained earlier.

The final result of the signal filtering can then be treated as a picture of the actual real shape of the track. Of course, in this shape there can still be entered some frequencies which are not associated with the existing shape, but it is certainly much closer to the reality than in the initial stage, i.e. before the filtration. This fact results from elimination of high frequencies, which may not occur in the track because of its rigidity.

In this case study, it has been assumed that cut-off value equal to 0.1/m corresponds to the conditions of the actual work of the track. The existing shape of the track aligned in straight section, which can make the basis for determining the required set of values of transverse displacements (to the horizontal alignment project) is shown in Figure 8.

In final part of the analysis there has been presented the differences between the original signal and the signal which states the result of filtering in the frequency

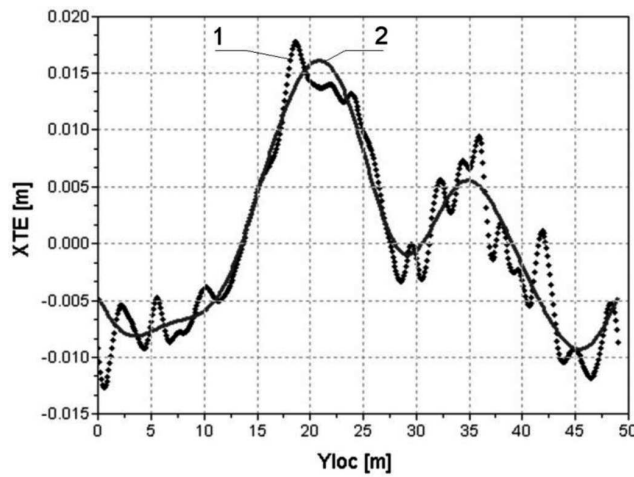


Fig. 8. The obtained hypothetical shape of the track (non-isometric scale) on a straight track section (with a cut-off value of 0.1/m):  
 1 – obtained shape, 2 – points which represent track section

domain on a separate chart. That signal (absolute values) can illustrate a measurement's error. In the graph there are also shown the arithmetic mean value (MEAN) and standard deviation (SD) of the obtained differences. The graph of the absolute values of the  $\Delta XTE$  for the presented example of the evaluation of straight track section is shown in Figure 9.

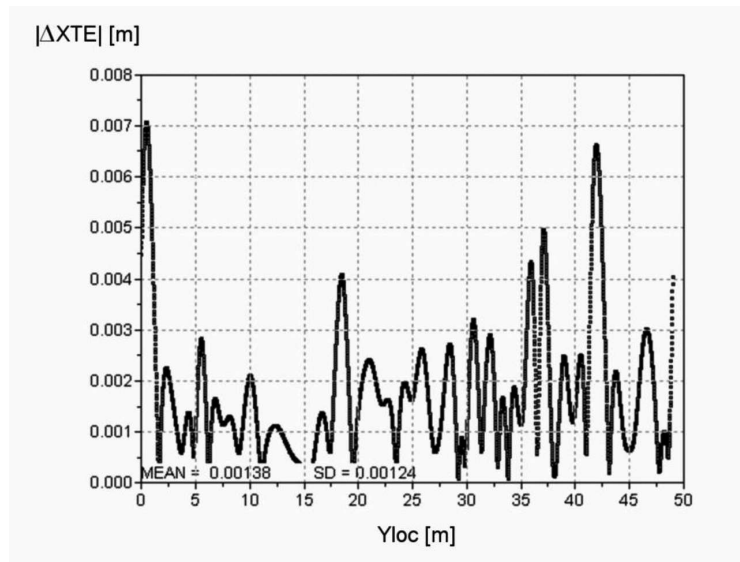


Fig. 9. The absolute values of XTE differences for the evaluated straight (non-isometric scale)

## 5. Conclusions

The authors believe that a radical improvement in the process of geometrical designing of a railway track will be available after the application of continuous satellite measurements with GPS receivers installed on a moving rail vehicle. Such methodology of measurements will allow reconstructing the position of the track axis in an absolute system of coordinates, and the number of recorded samples in output data will depend only on the established sampling frequency of the signal.

To determine the location of straight sections of the track the concept XTE (Cross Track Error), existing in navigation terminology may be used, which states a measure of the error position of a moving object. In the analyzed case the XTE is also applied to the uncertainty associated with the technique of measurement.

In order to evaluate the accuracy of determining the axis of the railway track by satellite surveying, obtained XTE signal must be analyzed in order to separate the signal's components which result from measurement error, not having a direct relationship with the shape of the measured track. As indicated initially, the use of Fourier Transform (FFT) gives optimistic results in this field.

The implementation of the present procedure requires the development of appropriate computer aided system. The paper presents a description and application example of already functioning modul called TRACK\_STRAIGHT, intended to assess the position of railway track straight sections. The program is adapted to the technique of continuous satellite measurements.

In the presented computational examples, the authors found that the differences between measured values and the values after applying the filtering in post-processing (representing the approximation of the actual track shape) were very small (i.e. a row of single millimeters). This fact can indicate a high-precision of the position determinations using continuous satellite measurements and their applicability for the design and inventory of the railway track.

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