

# Thermal-hydraulic issues of flow boiling and condensation in organic Rankine cycle heat exchangers

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**Abstract** In the paper presented are the issues related to the design and operation of micro heat exchangers, where phase changes can occur, applicable to the domestic micro combined heat and power (CHP) unit. Analysed is the stability of the two-phase flow in such unit. A simple hydraulic model presented in the paper enables for the stability analysis of the system and analysis of disturbance propagation caused by a jump change of the flow rate. Equations of the system dynamics as well as properties of the working fluid are strongly non-linear. A proposed model can be applicable in designing the system of flow control in micro heat exchangers operating in the considered CHP unit.

**Keywords:** Dynamics of heat exchangers; Flow stability; Heat exchangers

## Nomenclature

$A$  – cross-section area, m<sup>2</sup>  
 $B$  – blowing parameter  
 $Bo$  – boiling number,  
 $c_f$  – friction factor  
 $c_p$  – specific heat, J/kg K

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$d$	-	tube diameter, m
$f$	-	friction coefficient
$F_f$	-	friction force, N
$g$	-	gravity, $\text{m/s}^2$
$G$	-	mass velocity, $\text{kg/m}^2\text{s}$
$h$	-	enthalpy, J/kg
$L$	-	channel length, m
$\dot{m}$	-	mass flux, kg/s
$M$	-	molecular mass, kg/kmol
$m$	-	mass, kg
$m, n$	-	exponents
Nu	-	Nusselt number
$n$	-	number of tubes
Pr	-	Prandtl number
$p$	-	pressure, Pa
$R$	-	two-phase flow multiplier
Re	-	Reynolds number
$q$	-	heat flux, $\text{W/m}^2$
$Q$	-	heat
$s$	-	slip
$t$	-	time
$U$	-	perimeter, m
$w$	-	flow velocity, m/s
$v, u$	-	velocity components, m/s
$x$	-	quality, coordinate
$z$	-	coordinate

#### Greek symbols

$\alpha$	-	heat transfer coefficient, $\text{W/m}^2\text{K}$
$\mu$	-	dynamic viscosity, Pa·s
$\nu$	-	kinematic viscosity, $\text{m}^2/\text{s}$
$\lambda$	-	thermal conductivity, $\text{W/mK}$
$\xi$	-	friction factor
$\Phi$	-	dissipation function
$\rho$	-	density, $\text{kg/m}^3$
$\sigma$	-	surface tension, N/m
$\varphi$	-	void fraction
$\tau$	-	shear stress, $\text{N/m}^2$

#### Subscripts

$ac$	-	acceleration
$f$	-	friction
$h$	-	hydrostatic
$l$	-	liquid
$g$	-	gas, vapour
$LO$	-	liquid only
$lv$	-	liquid-vapour transition



<i>MS</i>	–	Müller-Steinhagen
<i>mt</i>	–	momentum-transfer
<i>PB</i>	–	pool boiling
<i>r</i>	–	reduced conditions
<i>s</i>	–	saturation
<i>TP</i>	–	two phase
<i>TPB</i>	–	two-phase boiling
<i>v</i>	–	vapour
<i>w</i>	–	wall
<i>in</i>	–	inlet
<i>out</i>	–	outlet
0	–	steady state
$\infty$	–	undisturbed flow

### Superscripts

+	–	non-dimensional
-	–	time-averaged

## 1 Introduction

A new promising direction of contemporary power engineering, supplementing the centralized power sector is the disperse power engineering, where electricity is produced in cogeneration with heat. At the Institute of Fluid-Flow Machinery PAS there arose the idea of a domestic micro combined heat and power (CHP) [1–4]. Such CHP operates according to organic Rankine cycle (ORC) and is capable of producing electricity and heat for the domestic needs. In future such micro CHP will replace conventional boilers for heating purposes. The advantage of such domestic unit is its compactness and small dimensions. Small dimensions of the unit are obtained when the micro heat exchangers will be utilized as well as modern microtechnologies. It is envisaged that dimensions of such micro CHP unit will not be much different from presents boilers and additionally electricity will be obtained apart from heat. The source of energy for micro CHP can be natural gas or renewable resources. The proposed micro CHP with low-boiling point fluid as working fluid operates as significantly lower temperatures than the combustion engine or gas turbine and therefore requires less of precious materials as well as the production technology is simpler. Using the postulated unit it is possible to generate electricity at prices close to the electricity prices from professional large power plants. The micro CHP utilized chemical energy in the fuel in almost 90%. That means that about 70–80% is heat for heating purposes and about 10–20% is the additional electrical power. Better utilization of chemical energy in micro CHP



units leads to reduction of harmful emissions accompanying the process of fuel combustion. The small CHP unit can be fully automated and would not require servicing. In such way the energy users who are in possession of such CHP unit become the producers of electricity.

The basic components of the micro CHP are: boiler (evaporator), steam turbine, condenser, electricity generator and circulation pump. The new CHP concept requires solution of several new problems such as selection of working fluids, investigation of two-phase flow instabilities, search for original designs of micro heating exchangers and many others [1]. The dimensions of heat exchangers primarily influence volume occupied by the micro CHP unit in the boiler. Dimensions of micro CHP are regarded as one of the fundamental criteria of micro CHP effectiveness. In the micro CHP there will be only two basic heat exchangers, that is the evaporator, where the thermal oil heated in boiler will heat the working fluid to the saturation temperature and subsequently evaporates and the condenser, where condensing working fluids will heat water for central heating purposes.

Issues of flow boiling, dynamic and thermal hydraulic stability of recuperator channel (evaporator) as well as vapour in evaporator as a whole [2–7] are indispensably connected with low boiling point fluids evaporation. Instability of two-phase-flow in a channel with evaporating fluid is closely related to the existence of pulsations of pressure and flow rate. Too significant pressures pulsations may be dangerous. They can lead to channel wall deformations and, in consequence, emergency operation of evaporator. Changes of mass flowrate accompanying pressure changes may, on the other hand, lead to boiling crisis, which in effect lead to reduction of heat transfer effectiveness in the recuperator. Instability of vapour flow produced in evaporator can lead to unstable operation of turbine. Therefore the issue of local stability (flow in the channel with evaporating fluids) as well as general evaporator stability is important to the designers of heat exchangers for the micro CHP.

The issue of two-phase flow stability based on investigations of hydraulic characteristics in the channel with evaporating fluid was analyzed in literature in the aspect of steam boilers. It has been concluded that the reason for flow instability in the channel is non-monotonic characteristics of pressure drop in the channel in relation to flowrate changes. Initially it was expected that it is sufficient to remove that aspect in order to ensure the stable flow, however it was shown later by means of experimental investigations that periodical type of flow instabilities will not be refrained [2–5]. However it

seems to be pertinent to study initial considerations of channel hydraulic characteristics which is the source of instability due to its simplicity. Such investigations are simple and in the case of finding non-monotonic channel hydraulic characteristics enables to prevent time consuming investigations of system dynamics. The flow boiling process is the complex phenomenon consisting of several complicated phenomena. Equations of dynamics as well as fluid properties are strongly nonlinear.

In the paper presented are considerations leading to better design two-phase heat exchangers (evaporator and condenser) together with a simple hydraulic model enabling for analysis of flow instabilities in such exchangers.

## 2 Calculation method of heat exchangers with phase change

It has been assumed that condensation and evaporation take place inside recuperator channels. For that reason in calculations must be considered the pressure drop caused by the two-phase flow. In theoretical considerations of two-phase flows either homogeneous model or separated flow models are used. In the first model the two-phase flow is treated as homogenous medium with averaged parameters (the velocity of gaseous phase and liquid are equal). The most effective description is attained by the homogeneous model for bubbly and dispersed flow regimes, which are characterized by a fairly uniform distribution of the dispersed phase in a continuous medium (liquid or vapor flow respectively). This model is also efficient for high pressures when the densities of liquid and vapor phases approach each other. The separated flow model allows for the difference in phase velocities and is efficient in description of stratified (in horizontal channel), annular, wave and other flows patterns.

The main parameters describing vapor-liquid flow are the local two-phase flow multiplier  $R$ , void fraction  $\varphi$  (relative volume or cross section of the channel occupied by vapor), and quality  $x$  ( $x = (h - h_l)/h_{lv}$ , where  $h$  is the flow enthalpy,  $h_l$  the saturated liquid enthalpy, and  $h_{lv}$  the latent heat of vaporization), and slip ratio  $s$ . All last three parameters are interconnected through the expression:

$$\frac{x}{1-x} = \frac{\varphi}{1-\varphi} s \frac{\rho_v}{\rho_l}, \quad (1)$$

where  $s = w_v/w_l$ , and  $\rho_v$ ,  $\rho_l$  are the densities of vapour and liquid phase, respectively.



## 2.1 Pressure drop

The overall pressure drop in the channel  $\Delta p_{TP}$  is the sum of three components, namely the pressure drop due to friction, pressure drop due to acceleration and hydrostatic pressure drop, respectively:

$$\Delta p_{TP} = \Delta p_f + \Delta p_{ac} + \Delta p_h. \quad (2)$$

In (2)  $\Delta p_f = R\Delta p_{LO}$ , is the pressure loss due to friction calculated as a product of the two-phase flow multiplier  $R$  and pressure drop of single phase fluid,  $\Delta p_{ac} = \Delta \left[ \frac{x^2}{\rho_v \varphi} + \frac{(1-x)^2}{\rho_l (1-\varphi)} \right]$  is the component of pressure drop due to the flow acceleration (of liquid and vapor phases) owing to the change of vapor quality, and  $\Delta p_h = [\rho_l (1 - \varphi) + \rho_v \varphi] g$  is the pressure drop brought about by overcoming the hydrostatic pressure, where  $g$  denotes the gravity. The pressure drop for flow when only liquid is flowing in amount equal to the total mass flow rate is defined as  $\Delta p_l = 4f_l \frac{G^2(1-x)^2}{2\rho_l} \left(\frac{L}{d}\right)$ , where  $G$  is the mass flow rate,  $L$  and  $d$  are the length and diameter of the tube. The friction factor is usually calculated from the Blasius equation  $f = 0.3164\text{Re}^{-0.25}$ , where Reynolds number  $\text{Re} = Gd/\mu$ , and  $\mu$  is the dynamic viscosity.

Two-phase flow resistance due to friction is greater than in case of single phase flow with the same flow rate. The two-phase flow multiplier is defined in the following way:

$$R = \frac{\Delta p_f}{\Delta p_0}. \quad (3)$$

In relation (3)  $\Delta p_f$  denotes pressure drop in two-phase flow, whereas  $\Delta p_0$  is the pressure drop due to friction in the flow with either liquid or vapour present.

Precise determination of void fraction for two-phase flow is crucial for evaluation of pressure drop and heat transfer coefficient. Solving (1) with respect to void fraction one can obtain:

$$\varphi = \frac{1}{1 + \left(\frac{1-x}{x}\right) \frac{\rho_v}{\rho_l} s}. \quad (4)$$

In literature there exist a number of relations describing void fraction. In the case of homogenous model of two-phase flow the slip is assumed as unity,  $s = 1$ . For separated phase model slip  $s$  can be determined from the Zivi [8] formula:

$$s = \sqrt[3]{\frac{\rho_l}{\rho_v}} = \sqrt[3]{\eta}, \quad (5)$$



where  $\eta$  is  $\rho_l/\rho_v$ . Chisholm relation for the slip yields:

$$s = [1 - x(1 - \eta)]^{0.5} . \quad (6)$$

Two-phase flow multiplier,  $R$ , is very often represented as a function of Martinelli's parameter  $X_{tt}$

$$X_{tt} = \frac{\left(\frac{dp}{dz}\right)_l}{\left(\frac{dp}{dz}\right)_g} = \left(\frac{1-x}{x}\right)^{0.9} \left(\frac{\rho_v}{\rho_l}\right)^{0.5} \left(\frac{\mu_l}{\mu_v}\right)^{0.1} , \quad (7)$$

where  $(dp/dz)_l$  and  $(dp/dz)_g$  are the frictional pressure gradients for the liquid and gas phase, respectively, flowing alone in the channel.

### 3 Model for pressure drop and heat transfer in flows with phase change

A fundamental hypothesis in the model under scrutiny here is the fact that heat transfer in flow boiling with bubble generation or condensation, regarded here as an equivalent flow of liquid with properties of a two-phase flow, can be modeled as a sum of two contributions leading to the total energy dissipation in the flow, namely the energy dissipation due to shearing flow without the bubbles,  $E_{TP}$ , and dissipation resulting from bubble generation/collapse,  $E_{PB}$ , Mikielwicz [9] in the following way:

$$E_{TPB} = E_{TP} + E_{PB} . \quad (8)$$

Energy dissipation under steady state conditions in the two-phase flow can be approximated as energy dissipation in the laminar boundary layer, which dominates in heat and momentum transfer in the considered process. Dissipation energy is expressed as power lost in the control volume. The term power refers to compensation of two-phase flow friction losses and is expressed through a product of shear stress and flow velocity. Analogically the energy dissipation due to bubble generation in the two-phase can be expressed. Substituting the definition of respective energies into Eq. (8) a geometrical relation between the friction factor in two-phase flow is obtained which forms a geometrical sum of two contributions, namely the friction factor due to shearing flow without bubbles,  $\xi_{TP}$ , and the friction factor due to generation of bubbles,  $\xi_{PB}$ , in the form:

$$\xi_{TPB}^2 = \xi_{TP}^2 + \xi_{PB}^2 . \quad (9)$$



It is difficult to imagine the flow resistance during the generation of bubbles, however in Russian literature there is a number of contributions, where such studies into flow resistance caused merely by the generation of bubbles on the wall were reported, see for example Ananiev [10]. That confirms that the modeling approach presented in the paper is reasonable. Expression (9) forms an underlying hypothesis of the present work. It enables subsequent derivation of the heat transfer model for a two-phase flow with phase change such as flow boiling and flow condensation.

## 4 Non-adiabatic effects in flow boiling and flow condensation

Analogy between the momentum exchange and heat transfer is by no means a complete one. There are effects related to the transfer of heat, i.e. so called non-adiabatic effects, which will be described below.

### 4.1 Non-adiabatic effects in annular flow

The shear stress between vapour phase and liquid phase is generally a function of non-adiabatic effects. That is a major reason why that up to date approaches, considering the issue of flow boiling and flow condensation as symmetric, are failing in that respect. The way forward is to incorporate a mechanism into the convective boiling term responsible for modification of shear stresses at the vapour–liquid interface. As our objective is to devise a model applicable both to flow boiling and flow condensation we will attempt to improve the model given by Eq. (9) by incorporation of the so called “blowing parameter”, which contributes to the liquid film thickening in case of flow condensation and thinning in case of flow boiling, Mikielewicz [11]. The devised formula for modification of shear stresses,  $\tau^+$ , in the boundary layer reads:

$$\tau^+ = 1 + \frac{B}{\tau_0^+} u^+ . \quad (10)$$

In Eq. (10)  $\tau^+ = \tau/\tau_w$ ,  $\tau_0^+ = \tau_w/\tau_{w0}$ , where  $\tau_w$  is the wall shear stress and  $\tau_{w0}$  is the wall shear stress in case where the non-adiabatic effects are not considered,  $u$  denotes the velocity in boundary layer region and  $B = 2\vartheta_0/(c_f u_\infty)$  is the so called “blowing parameter”. Additionally,  $\vartheta_0$  denotes the transverse velocity, which in case of condensation or boiling



is equal to  $q_w/(h_{lv}\rho_l)$ , and  $u_\infty$  is the undisturbed outer flow velocity. In case when  $Re \rightarrow \infty$  the relation (10) tends to that suggested earlier by Kutateladze and Leontiev [12], which reads:

$$\tau_0^+ = \left(1 - \frac{B}{4}\right)^2. \quad (11)$$

On the other hand, in case of small values of  $B$  the relation given by Eq. (10) reduces to that recommended by Wallis [13]:

$$\tau_0^+ = \left(1 - \frac{B}{2}\right). \quad (12)$$

The analyses due to Wallis [13] and Kutateladze and Leontiev [12] were carried out for the case of flow boiling.

There are also other studies which appeared only recently, where a way of describing the shear stress in flow condensation is presented, Bai *et al.* [14]. These authors recommended to consider the stresses as the interaction between the liquid and vapour phases including both frictional shear stress due to different velocities in the liquid,  $\tau_f$ , and vapour phases and a momentum-transfer shear stress due to vapour condensation,  $\tau_{mt}$ , in the following form:

$$\tau_0 = \tau_f + \tau_{mt}. \quad (13)$$

The term for the shear stress due to different velocity of phases in Eq. (13) reads:

$$\tau_f = \pm \frac{1}{2} f \rho_v (u_v - u_l)^2; \quad (14)$$

where  $u_v, u_l$  are the velocities of vapour and liquid phases, respectively. The “+” sign is applied when the average velocity of vapour phase is greater than that of liquid at the liquid/vapour interface, otherwise “-” is applied.

The momentum-transfer shear stress due to vapour condensation reads:

$$\tau_{mt} = \frac{q_w}{h_{lv}} (u_v - u_l). \quad (15)$$

Relation (13) can be expressed in the following form:

$$\tau_0^+ = \frac{\tau_0}{\tau_f} = \left(1 + \frac{\frac{q_w}{h_{lv}} (u_v - u_l)}{\pm \frac{1}{2} f \rho_v (u_v - u_l)^2}\right) = \left(1 + \frac{B'}{2}\right). \quad (16)$$



The general form of Eq. (16) reminds closely that of Eq. (12), confirming that the shear stresses at the liquid/vapour interface can be modified also in case of flow condensation.

Such approach is to be also incorporated in the present work. In the present paper the blowing parameter is defined in the following way:

$$B = \frac{2\vartheta_0}{c_{f0}u_\infty} = \frac{2q_w}{c_{f0}(u_v - u_l)h_{lv}\rho_l} = \frac{2q_w}{c_{f0}G(s-1)h_{lv}}d. \quad (17)$$

The considerations accomplished by Mikielewicz [11] were pertaining to both cases, i.e. flow boiling and flow condensation. Having acquired the way to modify the stresses in flow boiling and flow condensation it is relatively straightforward to implement Eq. (10) in the model of pressure drop, which will be presented in next subsection.

## 4.2 Non-adiabatic effects in other than annular flows

Flow resistance under non-adiabatic conditions differs from the adiabatic case. However, majority of research into that topic was related to adiabatic flow conditions as investigations were accomplished for air-water mixtures. Only a few studies have been devoted to vapour-liquid mixtures where the value of applied heat flux is important on the extent of flow resistance. In case of annular flow structure the influence of phase change was modeled using the so called “blowing parameter”. Such approach is, however, not possible in case of dealing with the bubbly flow structure or droplets flow. Presented below is author’s approach to incorporate the fact of non-adiabatic effects in other than annular structures.

Expressing the flow resistance of a two-phase adiabatic flow through the two-phase flow multiplier in the form:

$$\xi_{TP} = R\xi_0. \quad (18)$$

On the basis of the thermal-hydraulic analogy we can assume that the friction factor for pool boiling is in proportion to the pool-boiling heat transfer coefficient,  $\alpha_{PB}$ , which therefore yields:

$$\xi_{PB} \sim \alpha_{PB}. \quad (19)$$

That approach is justified through the Stanton analogy originally postulated in the form:

$$\text{Nu} = \frac{\alpha d}{\lambda} = \frac{c_f}{2} \text{Re Pr} = \frac{\xi}{8} \text{Re Pr}, \quad (20)$$

where  $Nu$  and  $Pr$  denote the Nusselt and Prandtl number. The pool-boiling heat transfer coefficient, which is expressed as a function of wall heat flux, has the form, Cooper [15]:

$$\alpha_{PB} = 55p_r^{0.12} (-\log p_r)^{-0.55} M^{-0.5} (q_w)^{2/3} = C q_w^{2/3}, \quad (21)$$

where  $p_r$  is a reduced pressure,  $M$  molecular mass and  $C$  a constant. Substituting the result (21) into (9) we obtain a relation between friction coefficients:

$$\xi_{TPB}^2 = R^2 \xi_0^2 + \left( \frac{8\alpha_{PB}d}{\lambda RePr} \right)^2. \quad (22)$$

After re-arranging of (21) we can write the two-phase flow multiplier, incorporating the non-adiabatic effect for bubbly flows as:

$$R_{TPB} = \frac{\xi_{TPB}}{\xi_0} = \sqrt{R^2 + \frac{\xi_{PB}^2}{\xi_0^2}} = R \sqrt{1 + \frac{\left( \frac{8\alpha_{PB}d}{\lambda RePr} \right)^2}{\xi_0^2 R^2}}. \quad (23)$$

The two-phase flow multiplier presented by above equation reduces to the adiabatic formulation in case when the applied wall heat flux is approaching zero. Sample predictions using that model are presented in Fig. 1. As can be seen the modification is only pronouncing itself for small values of qualities, where for  $x = 0$  the modification is more than double and it almost disappears at  $x = 0.1$ .

## 5 General method for calculation of friction pressure drop in two-phase flows with phase change

Generalising the obtained above result it can be said that the two-phase flow multiplier inclusive of non-adiabatic effects can be calculated in the following way:

$$R_{TPB} = \frac{\xi_{TPB}}{\xi_0} = \begin{cases} R \left(1 \pm \frac{B}{2}\right) & \text{for annular flow boiling and condensation,} \\ R \sqrt{1 + \frac{\left( \frac{8\alpha_{PB}d}{\lambda RePr} \right)^2}{\xi_0^2 R_{MS}^2}} & \text{for other flow boiling flows,} \end{cases} \quad (24)$$

where  $R_{MS}$  is the Müller-Steinhagen multiplier.



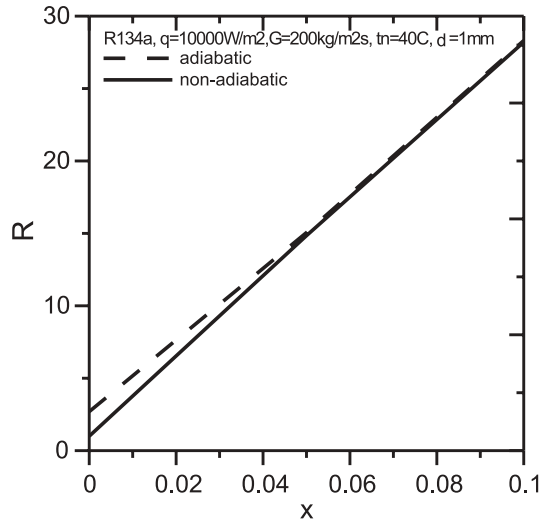


Figure 1. Sample prediction of non-adiabatic effect in bubbly flow,  $t_n$  – inlet saturation temperature,  $d$  – diameter.

### 5.1 Heat transfer coefficient in flow boiling and flow condensation

Making use of the analogy between the momentum and heat we can generalize the expression (9) to extend the postulated hypothesis over to heat transfer coefficients to yield heat transfer coefficient in flow boiling with bubble generation in terms of simpler modes of heat transfer, namely heat transfer coefficient in flow without bubble generation  $\alpha_{TP}$  and heat transfer coefficient for nucleate boiling  $\alpha_{PB}$ :

$$\alpha_{TPB}^2 = \alpha_{TP}^2 + \alpha_{PB}^2 . \quad (25)$$

Heat transfer without bubble generation,  $\alpha_{TPB}$ , which is applicable to flow boiling and flow condensation, can be modeled in terms of the two-phase flow multiplier. From the definition of the two-phase flow multiplier the pressure drop in two-phase flow can be related to the pressure drop of a flow where only liquid  $\Delta p_l$  is present:

$$\Delta p_{TP} = R \Delta p_l . \quad (26)$$

The pressure drop in the two-phase flow without bubble generation can also be considered as a pressure drop in the equivalent flow of a fluid flowing

with velocity  $w_{TP}$ :

$$\Delta p_{TP} = \frac{l}{d} \xi_{TP} \rho_l \frac{w_{TP}^2}{2}. \quad (27)$$

The pressure drop of the liquid flowing alone can be determined from a corresponding single phase flow relation:

$$\Delta p_l = \frac{l}{d} \xi_l \rho_l \frac{w_l^2}{2}. \quad (28)$$

In case of turbulent flow we will use the Blasius equation for determination of the friction factor, whereas in case of laminar flow the friction factor can be evaluated from laminar valid expression. In effect obtained is from (25) and (26) a relation enabling calculation of heat transfer coefficient in flow boiling without bubble generation in the form:

$$\frac{\alpha_{TPB}}{\alpha_l} = \sqrt{R^n}, \quad (29)$$

where  $\alpha_l$  is the heat transfer coefficient in single phase flow of liquid. That is also the form which will be used later in calculations of condensation inside tubes. In case of flow boiling in (29)  $n = 2$  for laminar flows, whereas for turbulent flows that value is taking up a value of 0.76. The two-phase flow multiplier  $R_{MS}$  due to Müller-Steinhagen and Heck [16] is recommended for use in case of refrigerants, Sun and Mishima [17].

## 5.2 General Method for calculation of heat transfer in two-phase flows with phase change

Following the derivation presented above the general expression for calculation of heat transfer coefficient in flow boiling and flow condensation can be devised in the form [18]:

$$\frac{\alpha_{TPB}}{\alpha_l} = \sqrt{(R_{TPB})^n + \frac{C}{1+P} \left( \frac{\alpha_{PB}}{\alpha_l} \right)^2}. \quad (30)$$

In the case of flow boiling modeling  $C = 1$  and  $C = 0$  for flow condensation. The correction term,  $P = 2.53 \times 10^{-3} \text{Re}^{1.17} \text{Bo}^{0.6} (R_{MS} - 1)^{-0.65}$ , has been established by a method of multiple regression fitting. The pool boiling heat transfer coefficient  $\alpha_{PB}$ , is to be calculated from the relation (21). The applied heat flux is incorporated through the boiling number Bo, defined as,  $\text{Bo} = q/(Gh_{lv})$ . For the same difference between the wall and saturation



temperature there is a different temperature gradient in the fluid in case of pool boiling and flow boiling. In the case of flow boiling the boundary layer is thinner and hence the gradient of temperature is more pronounced, which suppresses generation of bubbles in flow boiling. That is the reason why heat flux is included in modeling. That term is more important for conventional size tubes, but cannot be totally neglected in small diameter tubes in the bubbly flow regime, where it is important.

It should be noted, however, that the choice of the two-phase flow multiplier to be used in the postulated model is arbitrary. In the frame of works into modeling heat transfer to refrigerants the Müller-Steinhagen and Heck model has been selected for use as it is regarded best for refrigerants such as hydrocarbons in predicting appropriate consistency with experimental data, however, a different model could be selected such in case of dealing with other fluids as for example the Lockhart-Martinelli model, where the two-phase flow multiplier is a direct function of the Martinelli parameter, see Sun and Mishima [17]. The latter model is often found in correlations of flow boiling without bubble generation similar to Eq. (30). In the presented model the two-phase flow multiplier due to Muller-Steinhagen and Heck [16]  $R_{MS}$  is used which acts also in the correction  $P$  as a sort of convective number, known from other correlations. Here it is used in its original form.

It is a major drawback of most correlations developed for calculation of heat transfer in conventional size tubes that their accuracy significantly deteriorates when applied to small size tubes, regarded here as greater than  $600 \mu\text{m}$  [19]. In such situation the surface tension effects become to be more dominant and need to be reflected in the model. Most of experimental data indicate that most important for small channels is the convective flow boiling mode which, as stems from (30), is dependent on the flow resistance. In such case a great deal of care must be exercised in use of appropriate friction model. Again, available correlations of two-phase flow friction fail to be accurate for small diameter channels. For that reason the Muller-Steinhagen and Heck two-phase multiplier correlation was modified to incorporate the function  $f_{1z}$ . In case of modeling for small diameter passages the additional term responsible for surface tension effects, namely the constraint number  $\text{Con}$ , has also been applied to the discussed method [20].

In the form applicable to conventional and small diameter channels the model yields:

$$R_{MS} = \left[ 1 + 2 \left( \frac{1}{f_1} - 1 \right) x \text{Con}^m \right] (1 - x)^{1/3} + x^3 \frac{1}{f_{1z}}, \quad (31)$$

where  $\text{Con} = (\sigma/g/(\rho_l - \rho_v))^{0.5}/d$  and  $m = 0$  for conventional channels. Best consistency with experimental data, in case of small diameter and minichannels, is obtained for  $m = -1$ . In Eq. (31)  $f_1 = (\rho_l/\rho_v)(\mu_l/\mu_v)^{0.25}$  for turbulent flow and  $f_1 = (\rho_l/\rho_v)(\mu_l/\mu_v)$  for laminar flows. Introduction of the function  $f_{1z}$ , expressing the ratio of heat transfer coefficient for liquid only flow to the heat transfer coefficient for gas only flow, is to meet the limiting conditions, i.e. for  $x = 0$  the correlation should reduce to a value of heat transfer coefficient for liquid,  $\alpha_{TPB} = \alpha_l$  whereas for  $x = 1$ , approximately that for vapour, i.e.  $\alpha_{TPB} \cong \alpha_v$ . Hence:

$$f_{1z} = \frac{\alpha_v}{\alpha_l}, \quad (32)$$

where  $f_{1z} = (\lambda_v/\lambda_l)$  for laminar flows and for turbulent flows  $f_{1z} = (\mu_v/\mu_l)(\lambda_l/\lambda_v)^{1.5}(c_{pl}/c_{pv})$ . The correlation (30) seems to be quite general, as confirmed for example by the study by Chiou *et al.* [21].

## 6 Analysis of evaporator channel hydraulic characteristics

Investigations of hydraulic characteristics of the channel where the fluid evaporates and cooperates with the pump characteristics was considered by Ledinegg [6] for the case of boiler tubes. Monotonic character of hydraulic characteristics of the channel with evaporating fluid is a necessary condition for flow stability in the channel, Fig. 2. Characteristics **A** of a boiling channel is not monotonic in Fig. 2. In case of such channel there can occur instabilities of aperiodic type during transition from state **a** to **b** and **c**. On the other hand the channel characteristics **B** is stable as it has only one working point. Flow instability in micro channel of evaporator result from a sudden growth of steam bubbles. Of significant importance here are thermal and physical properties of the fluid such as density ratio of liquid to vapour as well as surface tension [6]. The static Ledinegg instability, as results from Fig. 2, is related to the flow characteristics in the channel and characteristics of the pump.

Let's consider a one dimensional homogeneous equation of momentum for the two-phase flow in the channel:

$$\frac{\partial \dot{m}}{\partial t} + \frac{\partial(pA)}{\partial z} + \frac{1}{A} \frac{\partial}{\partial z} \left( \frac{\dot{m}^2}{\rho} \right) + F_f + A \frac{\partial p_h}{\partial z} \sin \alpha = 0. \quad (33)$$



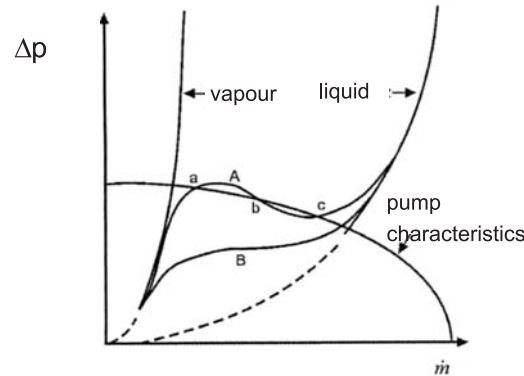


Figure 2. Characteristics of the pump and channel featuring the boiling fluid — Ledinegg instability [6].

In (33)  $\dot{m} = GA$  is the mass flux, where  $A$  is the cross-section area,  $F_f$  is a friction force and  $\alpha$  inclination angle of the channel and  $p_h$  is the hydrostatic pressure of liquid. Integrating along the channel length,  $L$ :

$$\frac{dG}{dt} = \frac{\Delta p_p}{L} - \frac{1}{L} \left( \frac{G^2}{\rho_{out}} - \frac{G^2}{\rho_{int}} \right) - \frac{\Delta p_f}{L} - \frac{\Delta p_h}{L} = \frac{\Delta p_p}{L} - \frac{\Delta p_k}{L}. \quad (34)$$

The first term on the right hand side of equation (34)  $\frac{\Delta p_p}{L}$  denotes the pressure drop induced by the pump where as the second one  $\frac{\Delta p_k}{L}$  is the pressure drop in the channel where boiling process takes place. The latter term consists of pressure drop due to acceleration, friction and hydrostatics, respectively. The acceleration term is positive in case of boiling and negative in case of condensation. The subscript  $k$  in Eq. (34) denotes all components of pressure drop due to phase change in the flow. Linearising Eq. (34) we get:

$$L \frac{dG}{dt} = \left[ \frac{\partial(\Delta p_p)}{\partial G} - \frac{\partial(\Delta p_k)}{\partial G} \right] dG. \quad (35)$$

The condition for the stable flow in the channel at the constant enthalpy at channel inlet and as well as constant amount of supplied heat is that:

$$\frac{\partial(\Delta p_p)}{\partial G} \leq \frac{\partial(\Delta p_k)}{\partial G}. \quad (36)$$

Flow instability in the channel with evaporating fluid appears when the increased flowrate is accompanied by a non-significant increase of flow re-



sistance and simultaneously small pressure drop necessary for fluid accelerations. In fact we observe the increase of the total pressure required to pump the fluid.

As mentioned earlier both condensation and evaporation take place inside the recuperator minichannel of ORC evaporator and condenser. Therefore in thermodynamic and flow calculations there ought to be considered pressure drop due to two-phase flow. The mean value of void fraction in the channel in the range  $0-x$  is:

$$\bar{\varphi}(x) = \frac{\eta}{\eta - s} x - \eta \frac{s}{(\eta - s)^2} \ln(x\eta + s - sx), \quad (37)$$

where  $\eta = \rho_l/\rho_v$ ,  $s$  represents the slip, and  $x$  is the flow quality. For the complete evaporation ( $x = 1$ ) we get:

$$\bar{\varphi}(1) = \frac{\eta}{\eta - s} - \eta \frac{s}{(\eta - s)^2} \ln \eta. \quad (38)$$

Introducing the expression for the slip  $s = (\rho_l/\rho_v)^{0.333}$  to (38) we get:

$$\bar{\varphi}(1) = \frac{\eta}{\eta - \eta^{1/3}} - \eta \frac{\eta^{1/3}}{(\eta - \eta^{1/3})^2} \ln \eta. \quad (39)$$

The biggest problem is determination of the pressure drop due to friction,  $\Delta p_f$ . In literature there are several empirical and theoretical correlations describing that term, however their applicability is restricted either to specific fluids or specific qualities [7]. Heat transfer in flow boiling can be determined for the example using the two-phase flow model due to Mikielewicz *et al.* [20].

Designing heat exchangers requires determinations of the length of a single channel, corresponding to the length of heat exchanger. Such exchanger consist of layers of channels connected in parallel, which increases the possibilities of attaining significant mass flow rates at small pressure drops, Fig. 3. The basic task of the micro CHP is to supply required and specified amount of heat for central heating purposes. For such reasons the calculation procedure starts from determination of the condenser dimensions, which must secure required amount of heat transferred to central heating installation.

From the energy balance for the whole of evaporator, accomplished at diathermal control volume border, results a relations between the exit quality from the evaporator channel with the flow velocity,  $w_0$ . Quality can



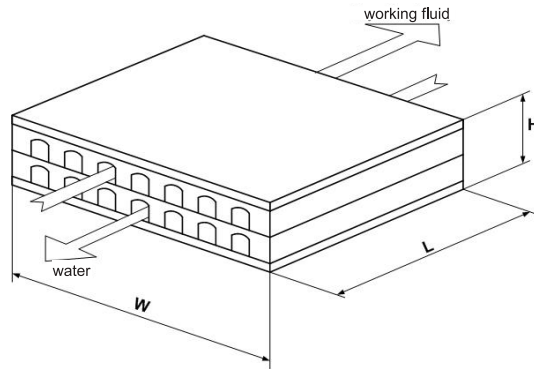


Figure 3. Two layers of the microchannel heat exchanger.

be determined at a constant amount of transferred heat and constant fluid enthalpies at evaporation inlet  $h_l$  and outlet  $h_v$ , as well as constant latent heat of evaporation:

$$x = \frac{4ql}{nd(w\rho)h_{lv}}, \quad (40)$$

where  $n$  notes the number of tubes in evaporator and the tube diameter.

The total pressure drop in the evaporator channel, neglecting the acceleration term and using (40) is function of friction and hydrostatic pressure drop in the form utilizing functions  $f_1$  and  $f_2$ :

$$\Delta p = f_1[x(w\rho)](\rho w)^2 + f_2[x(w\rho)]. \quad (41)$$

Applying the expression (35) to (41) the condition was obtained that the flow will be stable if:

$$2f_1(\rho w)^2 + \frac{\partial f_1}{\partial x} \frac{dx}{d(w\rho)} + \frac{\partial f_2}{\partial x} \frac{dx}{d(w\rho)} \geq 0. \quad (42)$$

Considering (40) in (42) we obtain the stability limit in the relation  $w\rho = w\rho(x)$ . Analysis can be significantly simplified, without incurring a significant error, by neglecting the hydrostatic pressure drop. Then the total pressure drop  $\Delta p$  is brought to the form:

$$\Delta p = f_1[x(w\rho)](w\rho)^2. \quad (43)$$

Applying now the condition (36) and the expression (42) the flow is stable when:

$$2f_1 + \frac{\partial f_1}{\partial x} \frac{dx}{d(w\rho)}(w\rho) \geq 0. \quad (44)$$

Determining the limiting mass velocity we get:

$$(w\rho) \geq -\frac{2f_1}{\frac{\partial f_1}{\partial x} \frac{dx}{d(w\rho)}}. \quad (45)$$

Utilising (40) we can determine from (42) or simplified relation (45) the limit of aperiodic flow stability in the evaporator channel in the iterative manner.

## 7 Flow stability in evaporator channels at small velocities

At small flow velocities in vertical parallel channels the existing non-symmetry of thermal load can cause that in one of the channels there will develop the choked flow. That will, on the other hand, cause the increase of pressure difference between inlet and outlet chambers of evaporator channels and that in consequence will induce the liquid flow in the choked tube. That renders the pressure drop between chambers and subsequent choking of channels, etc. Such kind of flow pulsation (instability) can be refrained by inducing the flow with higher than minimum velocity, which can be determined from the condition:

$$\Delta p_h \leq \Delta p_{TPB}, \quad (46)$$

where  $\Delta p_h$  is the hydrostatic pressure drop of liquid between recuperation chambers and  $\Delta p_{TPB}$  is the total pressure drop in the channel with two-phase flow. Similar problem was analyzed for boiler tubes assuming that the pressure drop in the channel consist of only friction losses and fluid acceleration [9].

The Ledinegg instability enables determination of the stability limit for a large bundle of channels, which is the case for the domestic micro CHP evaporator. Then each individual channel has almost a constant pressure difference induced by the pump as of infinitive number of channels the conditions occurs  $\frac{\partial(\Delta p_p)}{\partial G} \rightarrow 0$  and the system is not stable for the characteristics with negative slope. In case of the single channel the pump delivers the rate of mass independently from the pressure drop  $\frac{\partial(\Delta p_p)}{\partial G} \rightarrow -\infty$ . The condition (36) is always obeyed and the system is stable [22].



## 8 Flow instability of periodic type

Models of dynamics feature different degree of complication. Model complication depends on assumed two-phase fluid model, number of conservation equations used in the model. The simple dynamic model of two-phase fluid is the fluid described by homogeneous model. More complex is the two-fluid model of the two-phase fluid. Description of evaporator or condenser dynamics can be carried out on the basis of mass balance and momentum balance equations. Such model describes only instabilities of hydrodynamic type. In order to account for thermal inertia of the fluid and the walls the models utilizing two equations of mass balance and energy balance for the fluid supplemented by energy equation for the wall or, the most complex model, described by three balance equation of mass, momentum, and energy.

For a full description of the dynamics model three elements are necessary:

- balance equations of mass, momentum and energy,
- constitutive equations (pressure drop, heat flux),
- thermal and physical properties of the fluids.

The models describing heat exchanger dynamics have two types of time constants. One of them are related to the flow, whereas the other one to thermal side. Hydraulic disturbances are fast and travel with the velocity of sound. The second type of constants, i.e. time constants related to thermal issues are connected with thermal disturbances, which are of significantly slower character.

### 8.1 Balance equations

The general mass balance equation reads:

$$\frac{\partial \rho}{\partial t} + \nabla(\rho \bar{v}) = 0, \quad (47)$$

where  $\rho$  and  $\bar{v}$  denote the fluid density and the time-mean flow velocity, respectively, which in one-dimensional case can be written as:

$$\frac{\partial A\rho}{\partial t} + \frac{\partial \dot{m}}{\partial z} = 0, \quad (48)$$

where  $A$  is the cross-section and  $\dot{m}$  the mass flux.  
The general momentum balance equation reads:

$$\frac{\partial \rho \bar{\vartheta}}{\partial t} + \bar{\vartheta} \nabla \rho \bar{\vartheta} = \nabla (\mu \nabla \bar{\vartheta}) - \nabla p . \quad (49)$$

The one-dimensional case reduces to:

$$\frac{\partial \rho \vartheta}{\partial t} + \vartheta \frac{\partial \rho \vartheta}{\partial z} = \frac{\partial \tau}{\partial z} - \frac{\partial p}{\partial z} . \quad (50)$$

where  $\tau$  is the shear stress. The general energy balance equation for the fluid reads:

$$\frac{\partial \rho h}{\partial t} + \nabla (\rho h \bar{\vartheta}) = \nabla q + \Phi . \quad (51)$$

where  $h$  is enthalpy,  $q$  is the heat flux, and  $\phi$  is the dissipation function. Neglecting axial conduction Eq. (51) in one dimensional case reads:

$$\frac{\partial (A \rho h)}{\partial t} + \frac{\partial \dot{m} h}{\partial z} = \frac{\partial q}{\partial z} + \Phi . \quad (52)$$

Energy balance for the channel wall reads:

$$c_w \rho_w A_w \frac{\partial T_w}{\partial t} = U q , \quad (53)$$

where  $c_w$  is the specific heat,  $T_w$  is the wall temperature, and  $U$  is the channel perimeter. One dimensional balance equations are often too complex to accomplish calculations. In such case the equation with lumped parameters, the so called zero-dimensional equations are used. In order to obtain equations with lumped parameters they are integrated along the coordinate. Presented below is a sample way of obtaining the zero-dimension mass balance equation for heating the fluid to saturation temperature. The length of that zone varies with the applied heating time. Account of that fact requires applications of the Leibniz rule. The procedure of derivation of mass balance equation is as follows. From (48) after integration along the channel length,  $L$ , we obtain:

$$\int_0^{L(t)} \frac{\partial A \rho}{\partial t} dz + \int_0^{L(t)} \frac{\partial \dot{m}}{\partial z} dz = 0 . \quad (54)$$



Then using the Leibniz rule we get:

$$A \frac{d}{dt} \int_0^{L(t)} \rho dz - A \rho(L) \frac{dL}{dt} + \dot{m} - \dot{m}_0 = 0, \quad (55)$$

where  $\dot{m}_0$  is the mass flux at steady conditions. From (55) the apparent mass,  $m$ , balance equations results:

$$\frac{dm}{dt} + \dot{m} - \dot{m}_0 = 0. \quad (56)$$

Similar procedure can be applied to other balance equations of momentum and energy.

## 8.2 A simple hydraulic model

In the paper presented is a simplified description of the evaporator based on two-balance equations of mass and momentum:

$$\frac{dm}{dt} + \dot{m} - \dot{m}_0 = 0, \quad (57)$$

$$\frac{dm}{dt} = \frac{A}{L} (\Delta p_0 - \Delta p), \quad (58)$$

where  $\Delta p_0$  and  $\Delta p$  are the pressure drops in steady conditions or instantaneous ones, respectively. A homogeneous model of two-phase flow has been assumed with the possibility of considering the slip between liquid and vapour phase. Assuming additionally:

- a two-zone model (subcooled liquid and two-phase zone),
- length of the channel is constant and equal to  $L$ ,
- channel is uniformly heated with a constant heat flux  $q$ .

In the analysis neglected has been the fact that the superheated vapour can be found at the end of the channel. That zone would not influence the channel dynamics to a significant extent, as it contains relatively small mass and for that reason can not be considered. The zone of fluid heating length  $L_1$ , can be determined from the energy balance:

$$\dot{m}(h_l - h_0) = qUL_1, \quad (59)$$



where  $h_l$  and  $h_0$  are enthalpies of saturated liquid and starting enthalpy, respectively. For the boiling zone, the energy balance equation yields the length of boiling zone  $L_2$ :

$$\dot{m}h_w x = qUL_2. \quad (60)$$

To prevent formation of superheated steam the following condition must be obeyed:

$$L \leq L_1 + L_2. \quad (61)$$

It stems from the mass balance (57) that:

$$m = V_l \rho_l + V_{TP} \bar{\rho}_{TP} = A(\rho_l L_1 + \rho_{TP} L_2), \quad (62)$$

where  $V_l$  is the volume of saturated liquid,  $V_{TP}$  is the volume of two-phase mixture, and  $\rho_{TP}$  denotes two-phase density.

Hence, taking into account Eqs. (59) and (60) and also:

$$\rho_{TP} = \varphi \rho_v + (1 - \varphi) \rho_l, \quad \frac{dL_1}{dt} = -\frac{dL_2}{dt}, \quad (63)$$

where  $\varphi$  is void fraction, we get expression for the rate of change of mass:

$$\frac{dm}{dt} = A \left( \rho_l \frac{dL_1}{dt} + \frac{d\rho_{TP}}{dt} L_2 + \rho_{TP} \frac{dL_2}{dt} \right) \quad (64)$$

and then:

$$\frac{dm}{dt} = A \frac{\dot{m}_0 h_w}{qU} \frac{dx}{dt} \left( \rho_{TP} - \rho_l + \frac{d\rho_{TP}}{d\varphi} \frac{d\varphi}{dx} x \right) = \dot{m}_0 - \dot{m}. \quad (65)$$

From (65) the range of change of quality  $x$  yields:

$$\frac{dx}{dt} = \frac{qU}{A \dot{m} h_w} \frac{\dot{m}_0 - \dot{m}}{(\rho_v - \rho_l) \frac{d(\varphi x)}{dx}}. \quad (66)$$

The total pressure drop in momentum balance Eq. (58)

$$\Delta p = \Delta p_l + \Delta p_{TP} \quad \text{where} \quad \Delta p_{TP} = R \Delta p_l. \quad (67)$$

Taking into account that:

$$\Delta p = \left( 1 + \frac{L_2}{L_1} R \right) \frac{\lambda L_1 \rho w^2}{2d} = \left( 1 + \frac{L_2}{L_1} R \right) \frac{\lambda L_1}{2\rho A^2} \dot{m}^2, \quad (68)$$



where  $w$  is mean flow velocity we get from Eq. (58):

$$\frac{dy}{d\tau} = 1 - \alpha f(x)y^2, \quad (69)$$

where

$$f(x) = 1 + \frac{h_{lv}x}{h_l - h_0}R(x), \quad \alpha = \frac{\lambda \dot{m}_0^2}{2A^2 \Delta p_0 \rho_l}, \quad \tau = \frac{A \Delta p_0}{L \dot{m}_0}t. \quad (70)$$

Introducing the non-dimensional numbers to (57) we get:

$$\frac{dx}{d\tau} = \frac{\beta(1-y)g(x)}{y}, \quad (71)$$

where

$$y = \frac{\dot{m}}{\dot{m}_0}, \quad \beta = \frac{qUL\dot{m}_0}{A^2 h_{lv} \Delta p_0 (\rho_v - \rho_l)}, \quad g(x) = \frac{1}{\frac{d(\varphi x)}{dx}}. \quad (72)$$

In case of steady-state operation we obtain from (68):

$$y_0 = 1 \quad f(x_0) = \frac{1}{\alpha}, \quad (73)$$

where  $x_0$  is the initial quality. Linearising Eqs. (68) and (71) around the steady-state condition we obtain the set of equations:

$$\frac{d}{d\tau} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 & -\beta g(x_0) \\ -\alpha f'(x_0)y_0^2 & 2\alpha f(x_0)y_0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \beta g(x_0) \\ 0 \end{bmatrix} u. \quad (74)$$

The system of Eqs. (73) enables to investigate the system stability and the influence of disturbance  $u$  on the course of distribution of variables  $x$  and  $y$ . Transforming (73) by means of the Laplace transformation we can obtain the characteristic equation in the form:

$$p^2 - 2\alpha f' y_0 p - \alpha \beta f' y_0^2 g = 0. \quad (75)$$

The condition of limiting stability is attained when the real part of the root of Eq. (75) is equal zero, and  $p = j\varpi$ , i.e. the imaginary part of solution. For positive values of  $\alpha$ ,  $\beta$ ,  $g$  the system is stable if:

$$f' < 0. \quad (76)$$

That condition is obeyed for large values of quality  $x$ , close to unity.



## 9 Conclusions

In the paper presented have been the issues related to design and operation of micro heat exchangers, where phase changes are present. Application of such heat exchangers is found in the domestic micro CHP units. Analysed were boiling and condensation processes in heat exchanger minichannels as well as presented was a methodology of investigations of micro CHP stability. A simple hydraulic model presented in the text enables to analyse the system stability and propagation of disturbances induced by a jump change of flow rate. Equations of dynamics as well as fluid properties are strongly non-linear. The discussed model can be used in designing the control systems of micro heat exchangers operating in the domestic micro CHP.

**Acknowledgements** Authors would like to appreciate the funding from the National Project POIG.01.01.02-00-016/08 “Model agroenergy complexes as an example of distributed cogeneration based on a local renewable energy sources”.

*Received 4 January 2012*

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