

# Acoustic field and the entropy mode induced by it in a waveguide filled with some non-equilibrium gases

Research Article

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**Abstract:** The non-linear propagation of an acoustic beam in a rectangular waveguide is considered. The medium of sound propagation, is a gas where thermodynamically non-equilibrium processes take place: such as exothermic chemical reactions or excitation of vibrational degrees of a molecule's freedom. The incident and reflected compounds of the acoustic field do not interact in the leading order in the case of periodic weakly nonlinear sound with zero mean value of velocity. The acoustic heating or cooling in a waveguide is discussed.

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## 1. Introduction

As a rule, sound waves propagate over bounded volumes. Many studies are devoted to resonators of different shapes, above all, those filled with newtonian fluids [1, 2]. Kaner et. al. introduced the analytical method of different scales to describe the acoustic field in one-dimensional resonators [3]. This method uses slow dependence of the shape of the progressive wave on nonlinearity and attenuation, and its fast dependence on the retarded time. In this way, it becomes possible to consider the wave field in a resonator as a sum of non-interacting planar waves, which travel in opposite directions if they are periodic and

their mean values are zero [4]. It has been established, that nonlinearity leads to discontinuities in the waveforms propagating over free space or bounded volumes [5]. That is why many studies are devoted to the shock waves in resonators [6, 7].

Non-equilibrium molecular physics developed quickly due to the laser revolution in physics and chemistry. Diatomic gases with vibrationally excited degrees of freedom, non-isothermal plasma, chemically active fluids, and suspensions of microparticles in a gas are examples of non-equilibrium media [8, 9]. The high-frequency sound velocity in the non-equilibrium fluids is smaller than the low-frequency one, that reflects their anomalous acoustic dispersion [10, 11]. The bulk viscosity takes negative values making a non-equilibrium fluid acoustically active. Studies made in the last few decades focused mainly on the non-linear hydrodynamics of non-equilibrium media

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[12–14]. Attention was also paid to the resonant interaction of waves in relaxing gases [15].

If the sound energy is to be transmitted over long distances, the only way to do it effectively, is to use waveguides. The theory of waveguides is well developed, including those over which nonlinear waves propagate, and those of long scale, such as atmospheric waveguides, where a wave propagates over stratified or layered background [16]. As far as the author knows, weakly nonlinear propagation of sound in a waveguide which is filled with a non-equilibrium gas, is a new subject of study. As well as in the case of resonators containing a newtonian fluid, the planar waves, incident and reflected, do not interact in the leading order in the volume of a waveguide if they are periodic and zero on average (Sec. 2). The acoustic field in a rectangular waveguide, is considered before and after formation of discontinuities. The generic parameter which describes equilibrium or non-equilibrium processes in a gas,  $B$ , is negative in the equilibrium and positive in the non-equilibrium regimes. Discontinuity is always formed in the non-equilibrium regime. Despite enlargement of sound magnitude, the nonlinear attenuation on the front of the saw-tooth wave occurs, and both these opposite phenomena lead to stabilization of the peak magnitude of the shock wave which tends to some positive value in dependence on  $B$  (Sec. 2). In the equilibrium regime, discontinuity may not form at all. If that happens, the peak pressure of the shock wave quickly drops to zero. The non-linear generation of non-acoustic modes, such as the entropy mode, is anomalous in the non-equilibrium regime and yields in acoustic cooling instead of heating (Sec. 3). In the subsections below, two different non-equilibrium gases are considered; the first with excited vibrational degrees of a molecule's freedom, and the second with exothermic chemical reactions. Despite the different mechanisms of non-equilibrium, they may be described by the generic parameter mentioned above,  $B$ , which makes it possible to describe the wave field in the both cases by means of the same equations.

### 1.1. Gases with excited vibrational degrees of molecule's freedom

The first example of a fluid where equilibrium or non-equilibrium thermodynamic processes take place, is a gas whose steady state is maintained by pumping energy into the vibrational degrees of the molecule's freedom by power  $I$  withdrawal from the translational vibrational energy  $\varepsilon$  per unit mass has the form [8]:

$$\frac{d\varepsilon}{dt} = -\frac{\varepsilon - \varepsilon_{eq}(T)}{\tau} + I. \quad (1)$$

The equilibrium value of the vibrational energy at given temperature  $T$  is denoted by  $\varepsilon_{eq}(T)$ , and  $\tau(\rho, T)$  marks the vibrational relaxation time. The quantity  $\varepsilon_{eq}(T)$  in the case of a system of harmonic oscillators, equals:

$$\varepsilon_{eq}(T) = \frac{\hbar\Omega}{m(\exp(\hbar\Omega/k_B T) - 1)}, \quad (2)$$

where  $m$  is the mass of a molecule,  $\hbar\Omega$  is the magnitude of the vibrational quantum, and  $k_B$  is the Boltzmann constant. Eq. (2) is valid over the temperatures, where one can neglect anharmonic effects, i.e., below the characteristic temperatures, which are fairly high for most molecules [8, 9]. The quantity

$$B = -\frac{(\gamma - 1)^2 T_0}{2c^3} \left( \frac{C_v}{\tau} + \frac{\varepsilon - \varepsilon_{eq}}{\tau^2} \frac{d\tau}{dT} \right)_0 \quad (3)$$

is positive in the non-equilibrium regime of excitation of internal degrees of molecule's freedom, and negative in the equilibrium regime. It is the quantity evaluated at unperturbed  $p_0, T_0$ ;  $\gamma$  is the specific heat ratio in an ideal gas,  $c$  is the speed of sound of infinitely small magnitude in an ideal gas, and  $C_v = d\varepsilon_{eq}/dT$ . The non-equilibrium excitation is possible in principle, due to negative  $d\tau/dT$ . The relaxation time in the most important cases may be thought of as a function of temperature accordingly to Landau and Teller with some positive constants  $\tilde{A}$  and  $\tilde{B}$ ,  $\tau(T) = \tilde{A} \exp(\tilde{B}T^{-1/3})$  [8, 17]. There exists the threshold quantity of pumping magnitude  $I$ , starting from which the excitation is irreversible, since  $\varepsilon - \varepsilon_{eq} \approx I\tau$ .

### 1.2. Gases in which exothermic chemical reaction occur

For this kind of processes in a gas,

$$B = \frac{Q_0(\gamma - 1)(Q_\rho + (\gamma - 1)Q_T)}{2c^2 m} \quad (4)$$

is the quantity evaluated at unperturbed  $p_0, T_0, Y_0$ , where  $Y$  denotes mass fraction of a reagent  $A^*$  in  $A^* \rightarrow B^*$  exothermic reactions;  $Q$  is the heat produced in a medium per one molecule due to a chemical reaction,  $Q_0 = Q(T_0, \rho_0, Y_0)$  [18]. The dimensionless quantities  $Q_T, Q_\rho$  are conditioned by dependence of the heat produced due to a chemical reaction on temperature and density of the mixture:

$$Q_T = \frac{T_0}{Q_0} \left( \frac{\partial Q}{\partial T} \right)_{T_0, \rho_0, Y_0}, \quad Q_\rho = \frac{\rho_0}{Q_0} \left( \frac{\partial Q}{\partial \rho} \right)_{T_0, \rho_0, Y_0}. \quad (5)$$

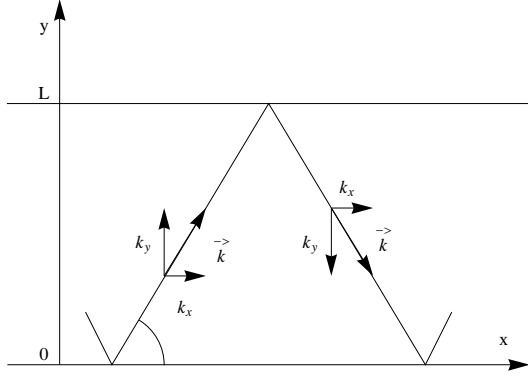


Figure 1. The geometry of a waveguide and sound beam path in it.

## 2. Potential and the wave perturbations in a waveguide

The non-linear equation which describes velocity potential in gases where the non-equilibrium relaxation takes place, in the leading order is

$$\frac{\partial^2 \varphi}{\partial t^2} - c^2 \Delta \varphi - 2cB \frac{\partial \varphi}{\partial t} = -2\nabla \varphi \left( \nabla \frac{\partial \varphi}{\partial t} \right) - (\gamma - 1) \Delta \varphi \frac{\partial \varphi}{\partial t}. \quad (6)$$

This wave equation in  $\varphi$  differs from the well-known equation describing lossless flows by the term including parameter  $B$ , which reflects attenuation (if  $B < 0$ ) or amplification of sound in a medium [19]. Some approximations should be introduced to obtain simplified wave equations that are more amenable to analysis, since even in the case of a lossless perfect gas, no analytical solutions are available for unsteady flow apart from those for planar waves. These approximations are introduced next. We will consider the velocity potential in a waveguide consisting of two parts,

$$\begin{aligned} \varphi_1(\tau_1) &= t - \frac{k_x}{\omega} x + \frac{k_y}{\omega} y, \quad T = \mu t, \\ \varphi_2(\tau_2) &= t - \frac{k_x}{\omega} x - \frac{k_y}{\omega} y, \quad T = \mu t, \end{aligned} \quad (7)$$

where  $\mu$  is a generic small parameter that characterizes the smallness of both nonlinearity and attenuation (or amplification),  $k_x, k_y$  are components of the wave vector, and  $\omega = c\sqrt{k_x^2 + k_y^2}$ . Fig. 1 illustrates the geometry of a flow. Our primary objective is to derive model equations valid at order  $\mu^2$  in a waveguide. Eq. (6) with account for (7) may be rewritten in the leading order as follows,

$$2\mu \left( \frac{\partial^2 \varphi_1}{\partial \tau_1 \partial T} + \frac{\partial^2 \varphi_2}{\partial \tau_2 \partial T} \right) - 2cB \left( \frac{\partial \varphi_1}{\partial \tau_1} + \frac{\partial \varphi_2}{\partial \tau_2} \right) =$$

$$\begin{aligned} & - \frac{\gamma + 1}{c^2} \left( \frac{\partial \varphi_1}{\partial \tau_1} \frac{\partial^2 \varphi_1}{\partial \tau_1^2} + \frac{\partial \varphi_2}{\partial \tau_2} \frac{\partial^2 \varphi_2}{\partial \tau_2^2} + \frac{\partial \varphi_1}{\partial \tau_1} \frac{\partial^2 \varphi_2}{\partial \tau_2^2} + \frac{\partial \varphi_2}{\partial \tau_2} \frac{\partial^2 \varphi_1}{\partial \tau_1^2} \right) + \\ & + \frac{4k_y^2}{\omega^2} \left( \frac{\partial \varphi_1}{\partial \tau_1} \frac{\partial^2 \varphi_2}{\partial \tau_2^2} + \frac{\partial \varphi_2}{\partial \tau_2} \frac{\partial^2 \varphi_1}{\partial \tau_1^2} \right). \end{aligned} \quad (8)$$

Returning to the variable  $t$  and averaging all terms over periods in  $\tau_2$  (the first equation in the set which follows) and in  $\tau_1$  (the second one), allows to subdivide equations governing  $\varphi_1(\tau_1, t)$  and  $\varphi_2(\tau_2, t)$ :

$$\begin{aligned} \frac{\partial^2 \varphi_1}{\partial \tau_1 \partial t} - cB \frac{\partial \varphi_1}{\partial \tau_1} &= - \frac{\gamma + 1}{2c^2} \frac{\partial \varphi_1}{\partial \tau_1} \frac{\partial^2 \varphi_1}{\partial \tau_1^2}, \\ \frac{\partial^2 \varphi_2}{\partial \tau_2 \partial t} - cB \frac{\partial \varphi_2}{\partial \tau_2} &= - \frac{\gamma + 1}{2c^2} \frac{\partial \varphi_2}{\partial \tau_2} \frac{\partial^2 \varphi_2}{\partial \tau_2^2}. \end{aligned} \quad (9)$$

After integration, they take the form

$$\begin{aligned} \frac{\partial \varphi_1}{\partial t} - cB \varphi_1 &= - \frac{\gamma + 1}{4c^2} \left( \frac{\partial \varphi_1}{\partial \tau_1} \right)^2, \\ \frac{\partial \varphi_2}{\partial t} - cB \varphi_2 &= - \frac{\gamma + 1}{4c^2} \left( \frac{\partial \varphi_2}{\partial \tau_2} \right)^2. \end{aligned} \quad (10)$$

Eqs(10) are valid if  $\varphi_1$  is a periodic function of  $\tau_1$ , and  $\varphi_2$  is a periodic function of  $\tau_2$ . Since  $u_x = u_{x,1} + u_{x,2} = \frac{\partial(\varphi_1 + \varphi_2)}{\partial x} = -\frac{k_x}{\omega} \left( \frac{\partial \varphi_1}{\partial \tau_1} + \frac{\partial \varphi_2}{\partial \tau_2} \right)$ , and  $u_y = u_{y,1} + u_{y,2} = \frac{\partial(\varphi_1 + \varphi_2)}{\partial y} = \frac{k_y}{\omega} \left( \frac{\partial \varphi_1}{\partial \tau_1} - \frac{\partial \varphi_2}{\partial \tau_2} \right)$ , the periodicity of potentials means that the averaged over periods components of velocity equal zero. Hence, the waves determined by oppositely directed transversal components of the wave vector, do not interact in the volume of waveguide. Equations describing parts of the velocity vector, follow from Eqs (9):

$$\begin{aligned} \frac{\partial u_{x,1}}{\partial t} - cB u_{x,1} &= \frac{\gamma + 1}{2c^2} \frac{\omega}{k_x} u_{x,1} \frac{\partial u_{x,1}}{\partial \tau_1}, \\ \frac{\partial u_{x,2}}{\partial t} - cB u_{x,2} &= \frac{\gamma + 1}{2c^2} \frac{\omega}{k_x} u_{x,2} \frac{\partial u_{x,2}}{\partial \tau_2}, \end{aligned} \quad (11)$$

$$\begin{aligned} \frac{\partial u_{y,1}}{\partial t} - cB u_{y,1} &= - \frac{\gamma + 1}{2c^2} \frac{\omega}{k_y} u_{y,1} \frac{\partial u_{y,1}}{\partial \tau_1}, \\ \frac{\partial u_{y,2}}{\partial t} - cB u_{y,2} &= \frac{\gamma + 1}{2c^2} \frac{\omega}{k_y} u_{y,2} \frac{\partial u_{y,2}}{\partial \tau_2}. \end{aligned} \quad (12)$$

The equations which describe the acoustic pressure,

$$p' = p_1 + p_2 \approx -\rho_0 \left( \frac{\partial \varphi_1}{\partial \tau_1} + \frac{\partial \varphi_2}{\partial \tau_2} \right), \quad (13)$$

are

$$\frac{\partial p_1}{\partial t} - cB p_1 = \frac{\gamma + 1}{2c^2 \rho_0} p_1 \frac{\partial p_1}{\partial \tau_1}, \quad \frac{\partial p_2}{\partial t} - cB p_2 = \frac{\gamma + 1}{2c^2 \rho_0} p_2 \frac{\partial p_2}{\partial \tau_2}. \quad (14)$$

The solutions of Eqs (11), and (14) may include discontinuities, because the form of potential (7) agrees with a saw-tooth wave moving with speed  $c$ , if its magnitude slowly varies with time. That is valid for symmetric weakly attenuating (or rising) pulses for which the condition of zero mean values of all components of velocity is also true. In the new variables

$$\tau = \omega t, \quad \eta_1 = \omega \tau_1, \quad \eta_2 = \omega \tau_2, \\ X = \frac{\omega x}{c}, \quad Y = \frac{\omega y}{c}, \quad K_x = \frac{ck_x}{\omega}, \quad K_y = \frac{ck_y}{\omega}, \quad (15)$$

$$b = \frac{cB}{\omega}, \quad \vec{U} = \frac{\vec{u} \exp(-b\tau)}{Mc}, \\ P = \frac{\rho' \exp(-b\tau)}{Mc^2 \rho_0}, \quad \theta = \exp(b\tau) - 1,$$

where  $M$  is the acoustic Mach number, Eqs (11), and (14) take the form

$$\frac{\partial U_{x,1}}{\partial \theta} - \frac{G}{K_x} U_{x,1} \frac{\partial U_{x,1}}{\partial \eta_1} = 0, \quad \frac{\partial U_{x,2}}{\partial \theta} - \frac{G}{K_x} U_{x,2} \frac{\partial U_{x,2}}{\partial \eta_2} = 0, \quad (16) \\ \frac{\partial U_{y,1}}{\partial \theta} + \frac{G}{K_y} U_{y,1} \frac{\partial U_{y,1}}{\partial \eta_1} = 0, \quad \frac{\partial U_{y,2}}{\partial \theta} - \frac{G}{K_y} U_{y,2} \frac{\partial U_{y,2}}{\partial \eta_2} = 0, \\ \frac{\partial P_1}{\partial \theta} - K P_1 \frac{\partial P_1}{\partial \eta_1} = 0, \quad \frac{\partial P_2}{\partial \theta} - K P_2 \frac{\partial P_2}{\partial \eta_2} = 0, \quad (17)$$

where  $G = \frac{(\gamma + 1)M}{2b}$ . Note that  $K_y, K_x$  should be less than unit, and that their ratio is  $K_y/K_x = \tan \alpha$ . The solutions of Eqs (16), and (17) before formation of discontinuities, are [19]

$$U_x = U_{x,1} + U_{x,2} = 2 \sum_{n=1}^{\infty} \frac{J_n(n\sigma_x)(\sin(n\eta_1) + \sin(n\eta_2))}{n\sigma_x}, \\ U_y = U_{y,1} + U_{y,2} = 2 \sum_{n=1}^{\infty} \frac{J_n(n\sigma_y)(-\sin(n\eta_1) + \sin(n\eta_2))}{n\sigma_y}, \quad (18) \\ P = P_1 + P_2 = 2 \sum_{n=1}^{\infty} \frac{J_n(n\sigma)(\sin(n\eta_1) + \sin(n\eta_2))}{n\sigma}, \\ \sigma_x = \frac{G\theta}{K_x}, \quad \sigma_y = \frac{G\theta}{K_y}, \quad \sigma = G\theta. \quad (19)$$

The boundary condition at the planes  $y = 0$  and  $y = L$  are given by equalities

$$U_{y,1}(X, Y = 0, t) + U_{y,2}(X, Y = 0, t) = U_{y,1}(X, Y = \frac{2n\pi}{K_y}, t) +$$

$$+ U_{y,2}(X, Y = \frac{2n\pi}{K_y}, t) = 0. \quad (20)$$

The width of a waveguide equals the integer number of transversal wave lengths,  $L = n\lambda_y$ . That, in fact, determines the spectrum of vertical components of wavenumbers  $k_y$ . Fig. 2 shows the vertical velocity in a waveguide before formation of a discontinuity in the non-equilibrium regime. In calculations of (18), only ten first summands were taken into account.

The dimensionless time of shock formation for the vertical profile of initially sinusoidal velocity, is  $T = \frac{1}{b} \ln \left( 1 + \frac{K_y}{G} \right)$ . For enough large times,  $\sigma_y > \pi/2$ ,  $U_{y,1}$  and  $U_{y,2}$  take the form of the saw-tooth shapes consisting of straight-line parts,

$$U_{y,1} = \frac{\tau_1}{1 + G\theta/K_y} \quad \text{if } -\pi \leq \tau_1 < \pi, \\ U_{y,2} = -\frac{\tau_2}{1 + G\theta/K_y} \quad \text{if } -\pi \leq \tau_2 < \pi. \quad (21)$$

Parts of acoustic pressure  $P_1$  and  $P_2$  take the form of the saw-tooth waves, if  $\sigma > \pi/2$ :

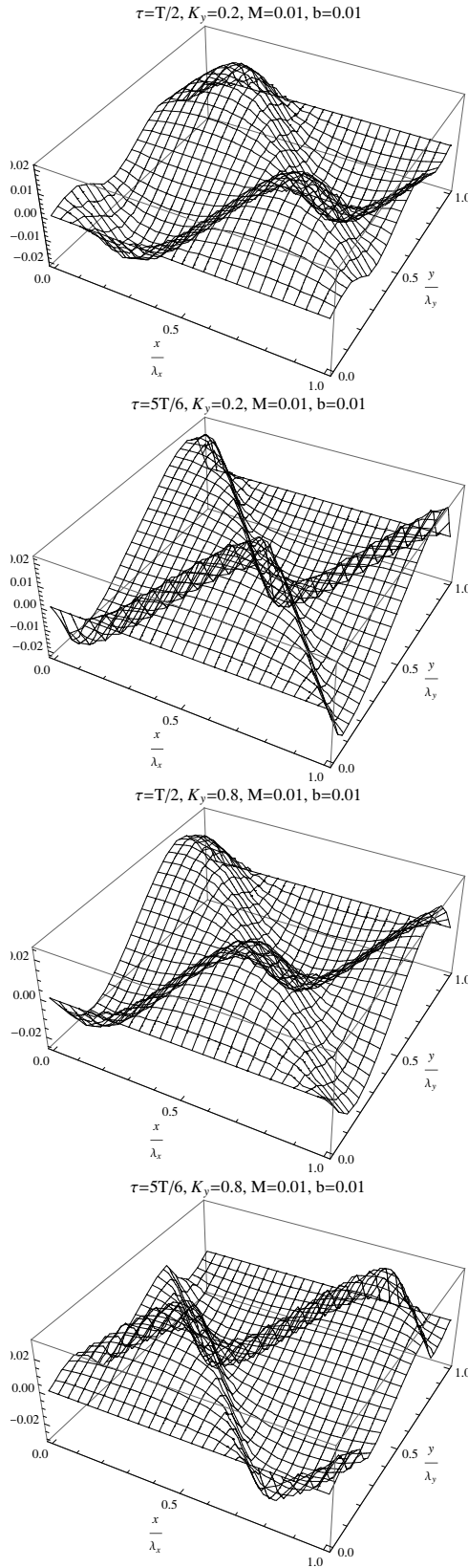
$$P_1 = -\frac{\tau_1}{1 + G\theta} \quad \text{if } -\pi \leq \tau_1 < \pi, \\ P_2 = -\frac{\tau_2}{1 + G\theta} \quad \text{if } -\pi \leq \tau_2 < \pi. \quad (22)$$

In a non-equilibrium gas, the peak magnitude of any saw-tooth waveform does not tend to zero when time increases. Enlargement of sound intensity is suppressed by nonlinear attenuation on the front of a saw-tooth wave. The peak magnitude of the dimensionless vertical velocity in the incident and reflected waves is  $G/K_y$ , and that of acoustic pressure is  $G$ . In the equilibrium regime, they quickly tend to zero. In an equilibrium gas, discontinuity in the profile of vertical velocity does not form at all if  $\frac{K_y}{G} \leq -1$ , that is, for large enough attenuation. The peak dimensionless magnitude of the vertical velocity at the same times as in the Fig. 3, for  $b = -0.01$  (that is, in the equilibrium regime) and  $K_y = 0.2$ , equals 0.08 at  $T = 2T_{saw}$  and 0.001 at  $T = 20T_{saw}$ , respectively.  $T_{saw}$  is the value calculated for  $b = 0.01$ . For  $b = -0.01$  and  $K_y = 0.8$ , discontinuity does not form at all.

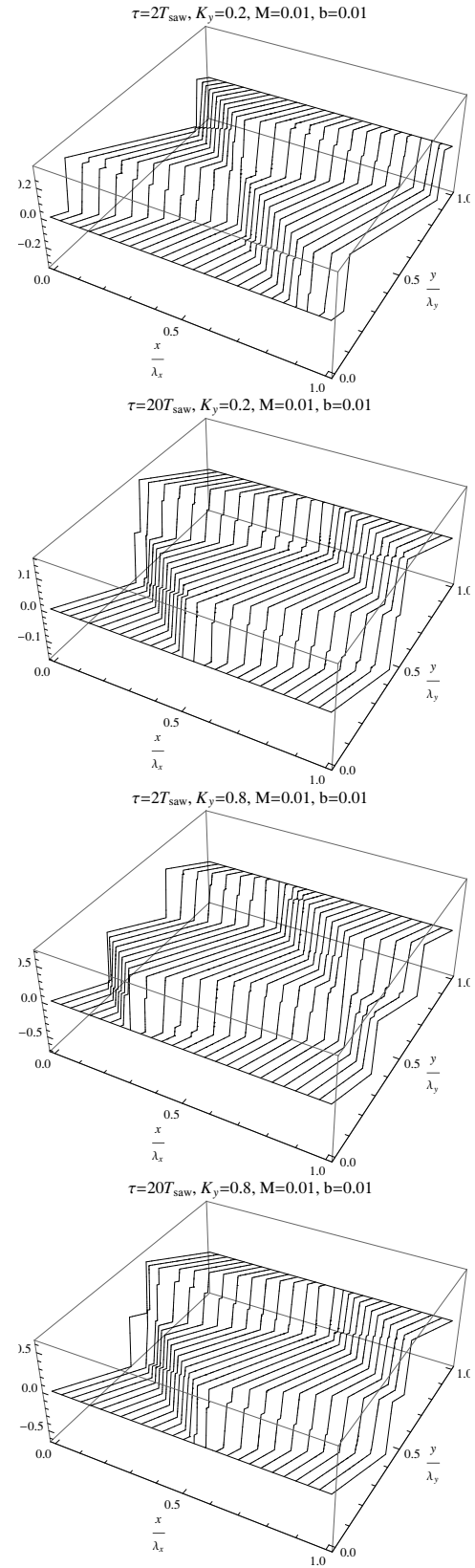
### 3. Acoustic heating

The excess density belonging to the non-wave entropy mode in the field of non-interacting incident and reflected wave, is governed by the following equation [20]:

$$\frac{\partial \rho_{ent}}{\partial \tau} = -\frac{1}{T_0} \frac{\partial T_{ent}}{\partial \tau} = bM^2(\gamma - 1) \exp(2b\tau) \left( P_1^2 + P_2^2 \right), \quad (23)$$



**Figure 2.** Vertical velocity  $u_y/(Mc)$  in a waveguide for different set of parameters before formation of discontinuity. For  $M = 0.01$ ,  $b = 0.01$ ,  $\gamma = 1.4$ , the dimensionless time of discontinuity formation  $T$  equals 15 if  $K_y = 0.2$  and 51 if  $K_y = 0.8$ .



**Figure 3.** Vertical velocity  $u_y/(Mc)$  in a waveguide for different set of parameters in a saw-tooth wave. For  $M = 0.01$ ,  $b = 0.01$ ,  $\gamma = 1.4$ , the dimensionless time of saw-tooth shape formation  $T_{saw}$  equals 23 if  $K_y = 0.2$  and 72 if  $K_y = 0.8$ .

in the case of vibrationally excited gas, and

$$\begin{aligned} \frac{\partial \rho_{ent}}{\partial \tau} &= -bM^2(\gamma - 1) \exp(2b\tau) \left( 2 \left( \frac{\partial P_1}{\partial \tau_1} \int_{-k_x x/\omega + k_y y/\omega}^{\tau_1} P_1 d\tau_1 + \frac{\partial P_2}{\partial \tau_2} \int_{-k_x x/\omega - k_y y/\omega}^{\tau_2} P_2 d\tau_2 \right) + \frac{\gamma - 2}{\gamma} (\overline{P_1^2} + \overline{P_2^2}) \right) = \\ &= bM^2 \exp(2b\tau) \frac{(\gamma - 1)(\gamma + 2)}{\gamma} (\overline{P_1^2} + \overline{P_2^2}) \end{aligned} \quad (24)$$

in the case of chemically reacting gas [21]. The top line denotes average over the sound period. Equations (23), and (24) are valid for periodic acoustic pressures with zero mean values. Before forming of discontinuity, the rate of heat release (it is proportional to  $-\partial \rho_{ent}/\partial \tau$ ) varies with time as  $b \exp(-2b\tau)$ , since  $\overline{P_1^2} = \frac{-\partial P_1}{\partial \tau_1} \int_{-k_x x/\omega + k_y y/\omega}^{\tau_1} P_1 d\tau_1 = \overline{P_2^2} = \frac{-\partial P_2}{\partial \tau_2} \int_{-k_x x/\omega - k_y y/\omega}^{\tau_2} P_2 d\tau_2 = 0.5$ . It depends on the sign of  $b$ : temperature of the medium of sound propagation increases if  $b < 0$  and decreases otherwise. Velocity, associated with the entropy mode, is zero and therefore does not disturb the boundary conditions (20). In the case of the saw-tooth wave, the acoustic heating is

$$\frac{\partial \rho_{ent}}{\partial \tau} = 2M^2 \pi^2 (\gamma - 1) \frac{b \exp(2b\tau)}{3(1 + G(\exp(b\tau) - 1)^2)} \quad (25)$$

in the chemically reacting gases, and

$$\frac{\partial \rho_{ent}}{\partial \tau} = 2M^2 \pi^2 \frac{(\gamma - 1)(\gamma + 2)}{3\gamma} \frac{b \exp(2b\tau)}{(1 + G(\exp(b\tau) - 1)^2)} \quad (26)$$

in the gases with excited internal degrees of a molecule freedom. In the non-equilibrium regime, propagation of sound is followed by cooling of a medium.

## 4. Concluding remarks

In this study, the peculiarities of a weakly nonlinear periodic acoustic beam which propagates in a waveguide filled with relaxing medium, which may be thermodynamically non-equilibrium, are studied. If the mean value of acoustic pressure is zero, the incident and reflected waves do not interact in a volume of resonator in the leading order. The periodic symmetric waves with discontinuities also do not interact. That allows the evaluation of perturbations in the sound field (pressure, velocity) for different

set of wavenumbers in a waveguide. Anomalous increase in the sound amplitude as a beam propagates, along with the nonlinear attenuation, result in the stabilizing of the peak magnitude in the shock wave in a non-equilibrium gas. Vice versa, in the equilibrium regime, the peak acoustic pressure rapidly decreases and discontinuity may not form at all for large enough attenuation connected with relaxation in a gas.

The nonlinear effects of sound in the non-equilibrium media also behave atypically. The theory of anomalous cooling of the medium (in contradistinction to acoustic heating) and streaming (with streamlines inverted as compared with direction of sound propagation) has been recently developed in reference to aperiodic and periodic in time sound beams, including beams with discontinuities in unbounded volumes of gases [21–23]. In waveguides, an increase (or decrease) in the temperature of a gas does not disturb the boundary conditions since velocity associated with the entropy motion, is insignificant. Since the squared speed sound is proportional to the background temperature, the vertical wave number varies in course of time. For constant angle of incidence  $\alpha$ , the vertical wave number  $k_y$  changes in inverse ratio to  $c$  in order to hold the sound frequency. Hence, it enlarges in the non-equilibrium regime and gets smaller otherwise proportionally to initial magnitude of acoustic pressure in the incident wave and according to  $B$ .

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