On geometrically non-linear FEA of laminated FRP composite panels

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ABSTRACT: The paper presents a state-of-art review on Finite Element Analysis (FEA) of geometrically non-linear problems for laminated composite plates and shells made as fibre reinforced polymer (FRP) laminates. Besides a subjective overview of the historical development of geometrically non-linear FEA of laminated FRP composite panels, some remarks on possible future issues in this research area are given.

1 INTRODUCTION

Fibre reinforced polymer (FRP) is a composite material that consists of high modulus fibres (typically they are glass, boron or graphite fibres) embedded in a polymer matrix (epoxy, resin or polyamide), see e.g. Jones (1999). As a result of a significant progress in manufacturing technologies in the second half of the 20^{th} century FRP composites became a promising class of engineering materials.

A typical FRP composite is a lightweight material being extremely strong in the direction of fibres and remaining considerably weaker in all off-fibre directions. To gain the full advantage of such composite one can consider a multi-layer structure (a laminate) constructed by bonding together a number of unidirectional fibre reinforced composite layers (laminas) with a varying orientation of fibre reinforcement in different layers. A resulted laminated FRP composite panel can be considered as an optimal structure with effective utilization of composite material directional properties (c.f. Vasiliev & Morozov 2001).

Due to their high strength-to-weight and stiffness-to- weight ratios laminated FRP composites are attractive solutions for applications in advanced lightweight structures, mostly in the aerospace industry, but also in the manufacturing of automobiles, sailing boats or pressure vessels. Such light-weight thin-walled structures can be extremely sensitive to buckling; therefore a proper examination of their behaviour should include a stability analysis, usually performed as a geometrically non-linear large deformation analysis.

When comparing mechanical characteristics of typical FRP composites with those of standard structural materials like steel or aluminium (see generalized strain-stress graphs in Fig. 1), it can be noticed, regardless a lower density of the former one, that while the FRP composites are superior in the strength, they have lower stiffness then steel. As a consequence, one can expect that elastic deformations of thin-walled structures made of laminated FRP composites will be larger than deformations of equivalent steel thin-walled structures at the same level of loading. On the other hand, due to the character of the strain-stress relation being almost linear, for a long time a linear elastic material model has been considered as fairly justified representation for the behaviour of FRP composites. As a consequence, the predominant approach utilized in examining of a performance of laminated FRP composites within the range of large deformations was geometrically non-linear but materially linear analysis (c.f. Kim & Voyiadjis 1999, Kreja 2011).

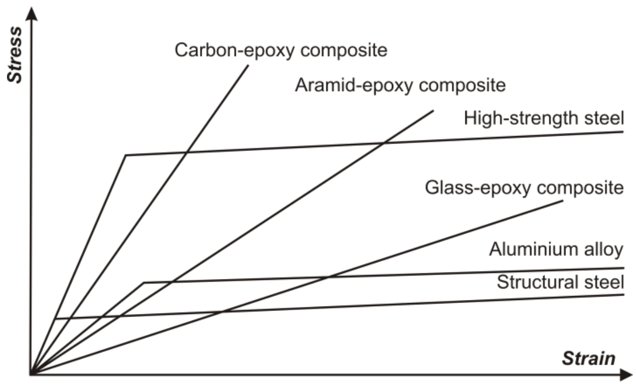

Figure 1. Generalized strain-stress relations in uniaxial tension.

The term "non-linear FEA" is routinely associated with the Finite Element Method (FEM) solution of large deformation problems (geometrically non-linear) or of an inelastic problem (materially non-linear). At present, after over 50 years of a vigorous development (cf. Clough 2004), the FEM can be considered without any doubt, as the most powerful tool in computational structural analysis (cf. Givoli 2001). It is worthy to notice here also a quite long history of the FEA of plates and shells (cf. MacNeal 1998, Chróścielewski, Gilewski & Kreja 2011), as well as FEM applications in the numerical analysis of composite panels (cf. Carerra 2002, Kreja 2011).

2 SCOPUS QUERY

During almost a half century of a development a number of published scientific papers dealing with a non-linear FEA of plates and shells considerably exceeded the level of perception of an average researcher. Quite fortunately, thanks to a strong progress of information technologies in the recent decade, now we can get a lot of information just at our fingertips by using search query services available in the internet. One of them, Scopus is advertised (cf. http://www.info.sciverse.com/scopus) as “the world’s largest abstract and citation database of peer-reviewed literature with smart tools that track, analyse and visualize research”, with one important additional remark “citations received since 1996”. To prepare a literature enquiry for publications related to the subject given in the title of the present work the author decided to perform an “Advanced Search” within Scopus looking for relevant words and phrases within titles, abstracts and key-words, by using a query in the form

TITLE-ABS-KEY (*string*) (1)

with *string* being an appropriate Boolean expression.

One should realize that the value of the response obtained in such search strongly depends on the quality of the question; therefore, a proper selection of phrases included in the *string* plays here a crucial role. Hence, while looking for publications related to non-linear analysis, the author defined the search area by using

string = S_{NL} = nonlinear OR "non-linear" OR "non linear" OR stability OR buckling OR "large deflection" OR "large deformation" OR "large displacement" OR “large rotation” OR “moderate rotation” OR “finite rotation” (2)

To limit the search to the area of plate and shell structures one should apply:

string = S_P = shell OR plate OR panel (3)

Similarly, an appropriate search query for the FEA or FEM related literature should include

string = S_{FE} = "finite element" OR FEM OR FEA OR "discrete element" OR “Direct Stiffness Method” (4)

One has to remember, that in 1960s and even in 1970s the name “Finite Element Method” was not so obvious for some authors dealing with FEA.

The research area related to composite FRP structures can be described by

string = S_{FRP} = FRP OR "fiber reinforce" OR "fibre reinforce" OR laminate OR composite (5)

With the variables defined above, one can formulate an appropriate query to perform searching for all peer-reviewed papers dealing with non-linear FEA of composite FRP panels (NLFEFRPP) published until 2012:

TITLE-ABS-KEY (S_{NL} AND S_P AND S_{FE} AND S_{FRP}) AND EXCLUDE(PUBYEAR, 2013) (6)

as well as for all peer-reviewed papers dealing with non-linear FEA of plates and shells (NLFEP) published until 2012:

TITLE-ABS-KEY (S_{NL} AND S_P AND S_{FE}) AND EXCLUDE(PUBYEAR, 2013) (7)

It is quite interesting to observe (cf. Fig. 2) that the subset of peer-reviewed papers dealing with NLFEFRPP became a substantial share of the peer-reviewed papers dealing with NLFEP – with the maximum reaching 39% in 1993 and the average level above 25%.

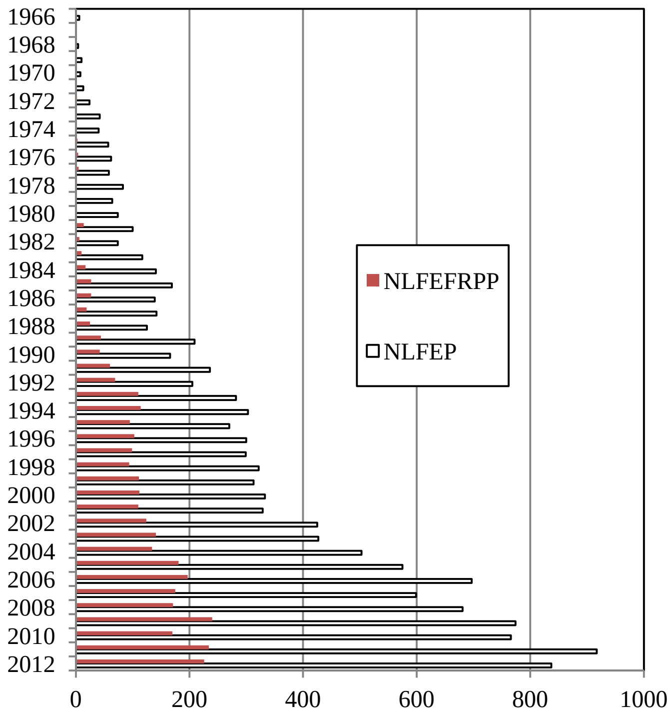

Figure 2. Annual numbers of peer-reviewed publications on NLFEFRPP among all peer-reviewed publications on NLFEP.

One should be aware of a limited precision of the achieved response – e.g. according to Scopus the most cited publication after 1995 within results of the search inquiry as described by (6) was paper Dvorkin & Bathe (1984), being one of millstones in the history of the development of shell finite elements, which, however, has nothing to do with composite FRP panels.

With just few additional operations available in Scopus one can compose the Top 10 ranking of the most recognized Authors writing about NLFEFRPP, as given in Table 1.

Table 1. Most Recognized Authors in NLFEFRPP.

Author	Number of citations*	Number of papers	Average citations per paper
J.N. Reddy	926	58	16.0
T. Kant	420	29	14.5
C. Mei	372	61	6.1
M. Ganapathi	221	22	10.0
J. H. Starnes	206	28	7.4
A. K. Noor	196	31	6.3
C. S. Hong	192	21	9.1
Z. Gürdal	188	27	7.0
A. N. Palazotto	187	56	3.3
D. R. Ambur	132	28	4.7

* Since 1996 and excluding self-citations of all authors.

Looking at the numbers showed in Table 1 one can notice, that Prof. J. N. Reddy from Texas A&M University with the total number of 926 citations is an unchallenged leader in this competition. In his oldest paper in that inventory (Reddy 1981) we can find a review on FEA of composite panels including large deflections and large amplitude vibrations. In the two most cited papers of J. N. Reddy in the topic of NLFEFRPP (Reddy 1989, Phan & Reddy 1985) the range of non-linearity was limited to von Kármán theory. However, in the next papers co-authored by J. N. Reddy the range of non-linear FEA of composite panels was extended to Moderate Rotations (cf. Kreja et al. 1997) and also to Finite Rotations (Arciniega & Reddy 2007). It is worth to remind that the subject of NLFEFRPP was included in the General Lecture given by Prof. Reddy during the SSTA2009 (Reddy et al. 2010).

It well known, that J. N. Reddy is a highly renowned author of scientific publications devoted to composite panels – by using the search query composed as

$$\text{TITLE-ABS-KEY } (S_P \text{ AND } S_{FRP}) \text{ AND EXCLUDE(PUBYEAR, 2013)} \tag{8}$$

one can find in Scopus 189 papers co-authored by J. N. Reddy, with an impressive citation score equal to 5593. Then, it is quite clear, that the research devoted to NLFEFRPP constitutes just a small fraction of the research activity of J. N. Reddy related to composite FRP panels.

A similar author's profile can be associated with Prof. Tarun Kant from Bombay's IIT – his 29 publications on NLFEFRPP represent just a small part of his research oeuvre. Among his most significant works dealing with NLFEFRPP are a frequently cited review paper (Mallikarjunaa & Kant 1993) and an acknowledged report on thermal buckling of composite panels (Kant & Babu 2000).

Prof. Chuh Mei from the Old Dominion University is known mainly for his works on the nonlinear flutter (cf. Zhou et al. 1994) and thermal buckling (cf. Shi et al. 1999) of composite panels.

Publications of M. Ganapathi from Mahindra Satyam Aerospace Engineering in Bangalore treat mostly on transient NLFEFRPP (cf. Ganapathi & Varadan 1995, Singha & Ganapathi 2004) within the range of von Kármán theory.

James H. Starnes Jr., from NASA Langley Research Center, was an internationally renowned researcher with expertise in the field of aerospace structures – among his most respected publications on NLFEFRPP one would consider a much appreciated review on shell stability (Arbocz & Starnes Jr. 2002) and many reports on buckling and imperfection sensitivity of composite panels (see e.g. Hilburger & Starnes Jr. 2002). In many of his papers the results of non-linear FEA of shells (obtained mainly with the program STAGS) were compared with the experimental results what made those publications especially valuable for the followers.

Prof. Ahmed K. Noor belongs to the group of the most eminent Authors writing about multi-layered plates and shells; his achievements related to NLFEFRPP are also significant - in his early paper in that field (Noor 1975) he indicated that transverse shear deformation should be taken under consideration in a proper buckling analysis of composite plates. Among the most appreciated papers of A. K. Noor one should include Noor (1986), where the application of global-local strategies in NLFEFRPP was discussed, as well as, Noor et al. (2000) presenting the influence analysis of variability of composite material properties on the non-linear response of composite structures.

Prof. Chang Sun Hong from Korea Advanced Institute of Science & Technology has an impressive output on composite structures applications in Aerospace Engineering; to his most influential works on NLFEFRPP one would include Jun & Hong (1988) and Kweon & Hong (1994), both treating on buckling analysis of composite cylindrical shells performed with degenerated FEs.

Prof. Zafer Gürdal is internationally recognized as an expert in designing and optimizing of composite materials. While working at Virginia Tech and Delft University of Technology he co-authored also a number of papers on NLFEFRPP; among them were Lee et al. (1995) on buckling of composite laminates with delamination studied with layer-wise FE model and Lopes et al. (2007) about non-linear damage analysis of variable-stiffness composite panels. The latter deserves a particular attention because FEA of the variable-stiffness composites (containing plies with spatially varying fibre orientations deliberately tailored to improve the structural efficiency of a composite) represents a current and computationally challenging problem.

Professor Anthony N. Palazotto from Air Force Institute of Technology is a recognized expert in Aerospace Engineering. In a series of papers on NLFEFRPP published in early 1990s, Palazotto promoted the Simplified Large Rotation (SLR) formulation for laminated shells modelled within the third-order shear deformation theory (cf. Tsai et. al 1991). Later Pai & Palazotto (1995) presented a co-rotational formulation accounting for large rotations and change of fibre directions during deformation of a laminate. However, one of the most cited papers of A. N. Palazotto concerned FEA of low-velocity impact on composite sandwich plates within the range of von Kármán non-linear theory but including failure detection and damage progression investigations (Palazotto et al. 2000).

In 1990s Dr. Damodar R. Ambur used to publish with his earlier mentioned colleague from NASA Langley Research Center, Dr. James H. Starnes Jr.; however, portfolio of research papers co-authored by D. R. Ambur is also quite impressive. His main activities within NLFEFRPP concerned the buckling of stiffened composite panels (cf. Jaunky et al. 1996) as well as progressive damage analysis (cf. Ambur et al. 2004).

It is quite symptomatic that the majority (eight out of ten) of the Authors listed in Table 1 represent the field of Aerospace Engineering - after all, the aerospace industry has been for a long period the main consumer of modern composite materials providing high strength at low self-weight. Nevertheless, the subject of NLFEFRPP attracted also many researches with roots in other engineering disciplines like Mechanical or Civil Engineering, represented in our list by J. N. Reddy and T. Kant, respectively.

The presented above ranking of the Top Ten Most Recognized Authors in NLFEFRPP has been prepared based on the citation statistics given by Scopus; however, even such procedure does not guarantee a totally objective assessment because the search results were considerably influenced by a somewhat arbitrary choice of the search query. It is quite obvious, that with a slightly different formulation of the question, one could obtain another response. On the other hand, probably every one researcher involved in NLFEFRPP has his own list of the most celebrated authors. After over 20 years of an exploration of this research area the Author of the current report also could find good reasons to formulate his own list of the most favourable writers in NLFEFRPP, however, instead, a quite different point of view has been adopted in a survey presented in the next chapter, which focused not on the Authors but on inspiring papers, assuming, that they could be quite possibly published by Authors with fairly moderate personal publication records in NLFEFRPP.

3 PERSONAL VIEW

The significance of an every paper dealing with FEA deeply depends on the value of numerical results it contains. To the best knowledge of the author of this report, the first FE solution to a non-linear problem for FRP composite panel was presented by L.A. Schmit Jr. and G.R. Monforton at the 10th AIAA/ASME Conference in April 1969 and published some months later in Schmit Jr. & Monforton (1970). It is interesting to notice that mentioned paper is generally devoted to the non-linear FEA of sandwich panels with laminated faces; however, inside we can find also two examples of thin FRP composite laminated rectangular plates simply supported on all four sides and subjected to in plane compression in one direction. The analysed laminated plate was treated as an equivalent single layer (ESL) orthotropic plate; the FE model was consistent with the Kirchhoff-Love theory of thin shells/plates and the non-linear formulation based on the Marguerre–von Kármán shallow shell equations. A rather good agreement with the results of an experimental buckling test was indicated.

One can observe that the tactic proposed by Schmit Jr. & Monforton (1970) became a predominant methodology at early stage of a development of NLFEFRPP – most of contemporary computational models based on the ESL approach and the classical lamination theory (CLT). In the ESL model the entire laminate is represented by a single-layer panel with macro-mechanical properties estimated as a weighted average of the mechanical properties of each lamina. Accordingly, CLT represents the ESL concept used in conjunction with the classical Kirchhoff-Love theory of thin shells/plates. Among the most recognized papers on NLFEFRPP with CLT approach one should include the publication Saigal et al. (1986) where a non-linear FE formulation based on the Marguerre–von Kármán shallow shell equations was applied to solve eight examples of large deformation analysis of composite panels. One of those examples, a hinged cylindrical FRP composite laminated shell under central point load became a very popular benchmark test for geometric non-linear analysis of composite shells (cf. Section 4.1).

In the CLT models the effects of transverse shear strains were neglected, what constituted a severe limitation of those formulations (cf. Noor 1975, Jones 1999). A NLFEFRPP formulation accounting for transverse shear strains presented by Reddy & Chandrashekhara (1985) can be classified as the First Order Shear Deformation (FOSD) model. Nonetheless, concurrently Phan & Reddy (1985) applied a higher-order shear deformation (HOSD) theory to analyse buckling loads for laminated anisotropic composite plates. A comprehensive synthesis of those models can be found in Reddy (1989), where the universal (CLT, FOSD or TOSD) single-layer model was confronted with the layer-wise approach and continuum-based shell model.

The continuum-based model of laminated shells based on the application of degenerated shell elements which in 1980s became the predominant tactics in FE shell analysis (cf. Belytschko 1986). In fact, such treatment corresponded to the application of the FOSD theory. Degenerated shell elements were applied by Jun & Hong (1988) and Kweon & Hong (1994) to investigate the buckling phenomenon of FRP composite laminated cylindrical shells (cf. Section 4.2).

Tsai et al. (1991) presented a comprehensive study on non-linear response of cylindrical shells made of a glass-epoxy composite with various stacking sequences of a laminate. Their finite shell elements based on their own SLR TOSD theory of cylindrical laminated shells with parabolic distribution of the transverse shear stress through the shell thickness and linear strain-displacement relations for the transverse shear strains. However, as it was indicated by Kreja & Schmidt (2006), the solutions given by Tsai et al. (1991) suffered also from the lack of a proper treatment of the rotational parameters. The same SLR TOSD model was applied also by Chaplin & Palazotto (1996) to examine a buckling of cylindrical graphite-epoxy composite panels under axial compression. The influence of different sizes of centrally located rectangular cut-outs was studied for various shell thickness. Some of the examples from Chaplin & Palazotto (1996) were re-analysed with NX-Nastran by Sabik & Kreja (2011).

As far as the author of the present article mastered the subject the very first solution the of a finite rotation problem for FRP composite laminated panels we owe to Prof. Yavuz Başar from the Bochum University. Although his personal record of publications related to NLFEFRPP (just four items) cannot challenge the numbers presented in Table 1, Prof. Y. Başar was a renowned expert in non-linear shell analysis and his papers influenced many researchers involved in large rotation FEA of laminated FRP composite panels. One has to mention here mainly the paper Başar et al. (1993) presenting various FE models for NLFEFRPP in the range of finite rotations. Single-layer models (CLT, refined FOSD and TOSD) as well as layer-wise approach were considered in Başar et al. (1993) what explicitly corresponded with the collection included in Reddy (1989). Moreover, Başar et al. (1993) re-analysed Reddy's example of a simply supported asymmetric cross-plied laminated plate under uniform pressure providing a correct solution to the problem, which became a reference results for many followers (cf. Section 4.3).

To describe finite rotations of the shell director Başar et al. (1993) used two Euler angles; a similar concept was also applied by Wagner & Gruttmann (1994), Brank et al. (1995), and Kreja & Schmidt (2006). Başar et al. (2000) utilized a singularity free parameterisation of the rotations based on the Rodriguez rotation vector. Their formulation included 3 rotational dofs (i.e. degrees of freedom), with the third being a drilling rotation; similar approach was implemented by Chróścielewski et al. (2011). A slight different approach was used by Kim & Voyiadjis (1999); however, their co-rotational formulation also utilized 3 rotational dofs. Balah & Al-Ghamedy (2002) who also used the Rodriguez vector introduced additional constraints to eliminate a drilling rotation. Review of FE models for a large rotation analysis of laminated shells should not ignore the continuum based shell models where the behaviour of the shell-like construction is represented without any reference to rotational dofs. Such tactics was applied e.g. by Laschet & Jeusette (1990), Klinkel et al. (1999), Masud et al. (2000), Kulikov & Plotnikova (2003), Vu-Quoc & Tan (2003). The big advantage of that kind solution lies not only in a much less complicated treatment of finite rotations represented just by relative displacements between the nodes at the top surface and the reference surface, but also in a more natural and adjustable description of layered structure of the composite laminate including panels with ply drop-offs (cf. Klinkel et al. 1999, Vu-Quoc & Tan 2003)

4 SELECTED EXAMPLES

4.1 *Hinged cylindrical panel under point load*

The laminated cylindrical panel under the centrally located transverse force as presented in Figure 3 is hinged at the straight edges AD and BC, whereas curved edges AB and CD remain free.

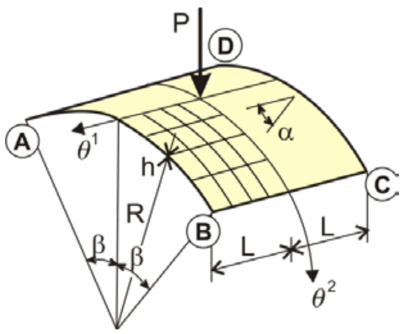

Figure 3. Cylindrical panel under point load

This problem originates from the family of isotropic cylindrical panels analysed by Sabir & Lock (1972) - the layered orthotropic variant was primarily proposed by Saigal et al. (1986) as a composite laminate of the thickness h=12.6 mm with the following material parameters: $E_a = 3.3$ kN/mm^2, E_b = 1.1 kN/mm^2, $G_{ab} = G_{ac} = G_{bc} = 0.66$ kN/mm^2 and $\nu_{ab} = 0.25$. Dimensions of the panel are taken as R = 2540 mm, L = 254 mm and $\beta = 0.1$.

This example was frequently used as a benchmark problem, see e.g. Laschet & Jeusette (1990), Brank et al. (1995), Lee & Kanok-Nukulchai (1998), Kim & Voyiadjis (1999), Sze et al. (2004), Kreja (2006). Another variant of the laminated panel with the thickness reduced by half (h = 6.3 mm) was analysed by Brank et al. (1995), Lee & Kanok-Nukulchai (1998), Sze et al. (2004), Kreja (2006), Kim et al (2007), Han et al. (2008) and Li et al. (2011).

Kreja (2006) considered also the cross-ply [0/90/0] panel of the thickness h = 3.15 mm and showed that even for such thin shell almost no difference could be observed between the results obtained with the Refined von Kármán (RVK5), Moderate Rotation (MRT5) or Large Rotation (LRT56) formulations. The corresponding results for the cross-ply [90/0/90] panel of the thickness h = 3.15 mm are presented in Fig. 4.

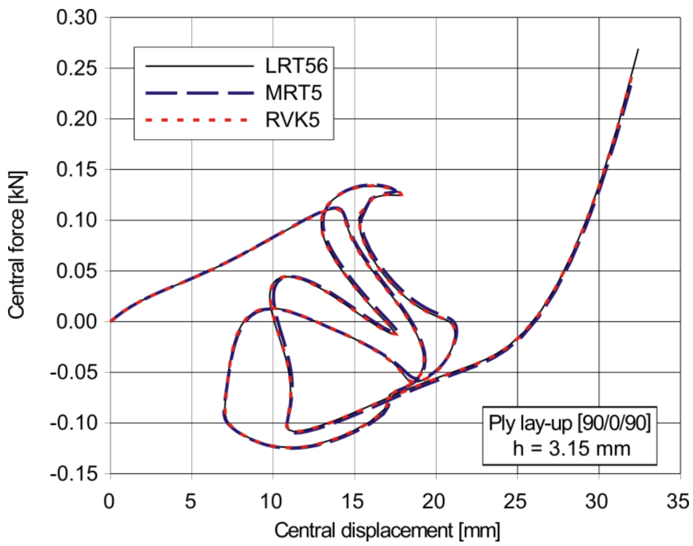

Figure 4. Central deflection for laminated panel 3.15 mm thick.

It is interesting to re-examine also the case of the angle ply lay-up [-45/+45]. Since this problem is asymmetrical it is necessary to model the whole panel. Own results for the panel 12.6 mm thick is presented in Fig. 5 together with the reference solutions of Saigal et al. (1986) and Laschet & Jeusette (1990). Here again, there are almost no differences between the results of the LRT56, MRT5 and the RVK5 analyses. All three our models give solutions that are very close to the response predicted by Laschet & Jeusette (1990); however, there is a visible disagreement with the reference solution given by Saigal et al. (1986). One should notice that Saigal et al. (1986) analysed just one-quarter of the shell, assuming biaxial symmetry, what was not right for the case of the angle-ply lamination. Kim & Voyiadjis (1999), who reported results very similar to those of Saigal et al. (1986) also performed calculations for one quarter of the shell. To allow for a more detailed examination of the angle ply laminate, additional calculations were performed with the LRT56 model for the whole panel and for one quarter of the shell assuming two different lay-ups [-45/+45] and [+45/-45].

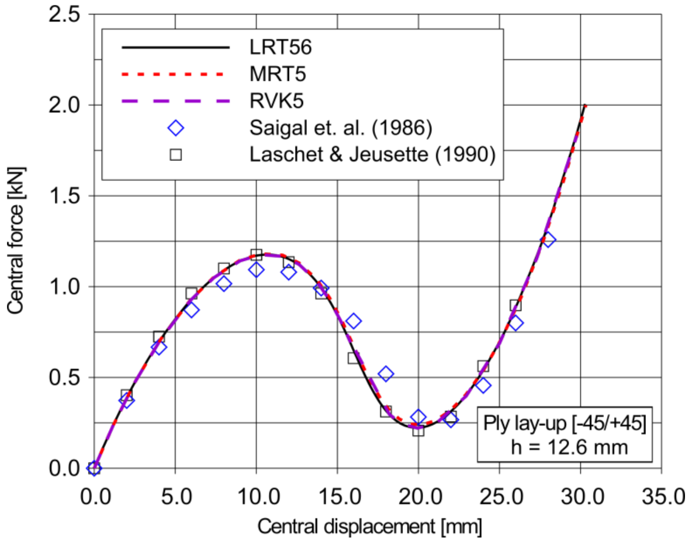

Figure 5. Central deflection for angle-ply panel, h = 12.6 mm.

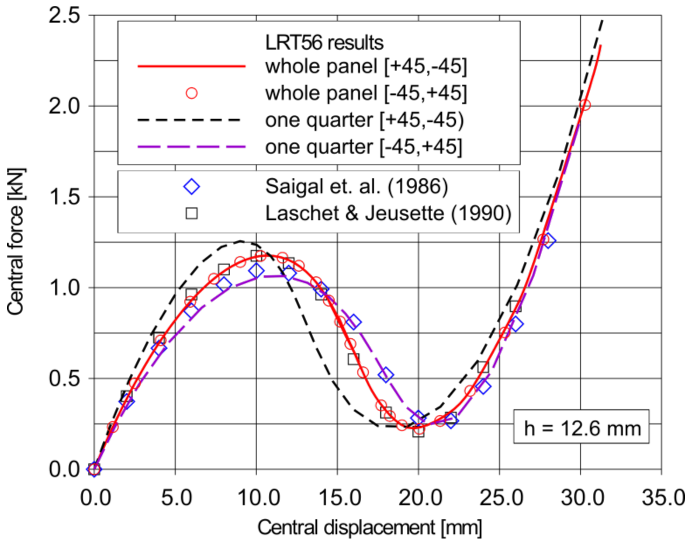

Figure 6. Additional study for angle-ply panel, h = 12.6 mm.

Looking at graphs presented in Fig. 6, one can observe that the same equilibrium paths were obtained for the both lay-ups ([-45/+45] and [+45/-45]), when the whole panel was represented in the FE model. On the other hand, the choice of the stacking sequence ([-45/+45] or [+45/-45]) was relevant when the reduced model of a one quarter with biaxial symmetry boundary conditions was used in calculations. Our results obtained for the whole panel agree quite well with the reference solution of Laschet & Jeusette (1990). However, when the reduced model was used for the stacking sequence [-45/+45] the obtained response resembles that of Saigal et al. (1986).

An analysis of the composite panel with the angle ply lay-up [-45/+45] and the thickness reduced to 6.3 mm is associated with some additional difficulties due to the presence of bifurcation points on the equilibrium path. In Fig. 7 one can find results of the

FEA performed for the whole panel – the symmetrical response was obtained by fixing horizontal displacements of the mid-point with additional constraints while the asymmetric response was enforced by applying an additional horizontal force 0.001 P as the load imperfection.

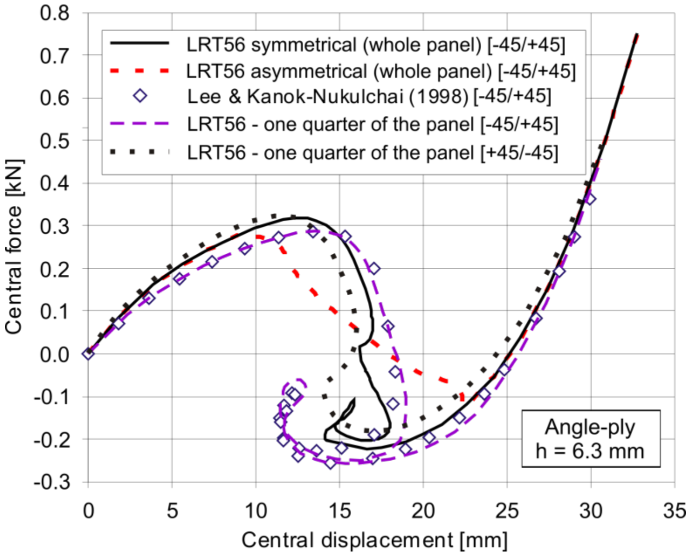

Figure 7. Central deflection for angle-ply panel, h = 6.3 mm.

Own results obtained for the FE model of the whole panel are noticeably different than the reference solution of Lee & Kanok-Nukulchai (1998). However, Lee & Kanok-Nukulchai (1998) performed their calculations for a reduced FE model consisting of one-quarter of the shell with boundary conditions reflecting biaxial symmetry. Quite surprisingly, such reduced model was applied also by Kim et al (2007), Han et al. (2008) and Li et al. (2011). Our additional computations performed with the LRT56 formulation for one-quarter of the shell resulted in the equilibrium path similar to that of Lee & Kanok-Nukulchai (1998) when the angle ply lay-up was assumed [-45/+45]; although, quite different response was obtained for the inverted lamina sequence [+45/-45] (cf. Fig. 7).

4.2 *Axial compression of composite cylindrical panel*

In the second example, a an axial compression of a 16-layer composite cylindrical panel as presented in Fig. 8 is considered assuming that the straight edges AB and CD are simply supported with possibility to move along the generatrix, whereas the both curved edges are clamped. The lamination scheme can be described as $[45/-45_2/45/0_4]_S$, the total thickness is $h = 16\times0.125 = 2$ mm. Each lamina is made of carbon-epoxy composite with the following parameters: $E_a = 130{\cdot}10^6$ kPa, $E_b = 10{\cdot}10^6$ kPa, $\nu_{ab} = 0.3$ and $G_{ab} = G_{ac} = G_{bc} = 5{\cdot}10^6$ kPa. Geometry of the panel is characterized by the radius R = 250 mm, the length L = 540 mm, the opening angle $\beta = 1.6848$ rad.

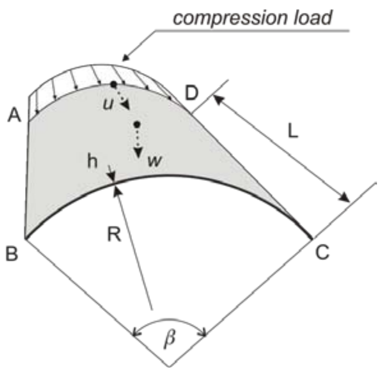

Figure 8. Composite cylindrical panel under axial compression.

The buckling behaviour of this composite cylindrical panel was analysed numerically e. g. by Jun & Hong (1988), Laschet & Jeusette (1990) and Kreja (2005). An interesting issue of this example is related to the loading conditions - two models were considered in present calculations: in the first approach (model 1) the panel was compressed by the axial load (pressure) uniformly distributed on the curved AD edge of the panel; in the second version (model 2) a rigid movement of the whole edge AD was enforced. Since Jun & Hong (1988) published only the magnitude of the buckling load without any graphical presentation of the equilibrium path, their results are confronted with other solutions in Table 2.

Table 2. Buckling load for compressed cylindrical panel

Solution	Buckling load [kN]
Laschet & Jeusette (1990)	137.8
Jun & Hong (1988)	143.2
LRT56 model 1	137.7
LRT56 model 2	140.9
Kreja (2005) – MSC Nastran LB	140.3
Kreja (2005) – MSC Nastran NL	140.4

Jun & Hong (1988) analysed this example using a displacement control approach – they enforced a uniform increase of displacements at the edge AD. The same procedure was declared by Laschet & Jeusette (1990), however, their results are much closer to the LRT56 solution obtained for model 1 than those of model 2. Nevertheless, a possible explanation of a observable discrepancy between numbers given by Jun & Hong (1988) and Laschet & Jeusette (1990) can be associated with a slightly different boundary conditions applied by Laschet & Jeusette (1990), who used elements possessing only translational dofs.

The graphs of the axial deflection vs. compression load for both models are given in Fig. 9 together with the reference solution of Laschet & Jeusette (1990).

As one can see in the graphs presented in Fig. 9, Laschet & Jeusette (1990) reported a visibly stiffer behaviour in the pre-buckling range than the response obtained with both LRT56 models. Distinctions related to a different interpretation of loading conditions in LRT56 models 1 and 2 are more evident in the post-buckling range, especially in graphs presenting the transverse deflection of the central point of the panel (cf. Fig. 10).

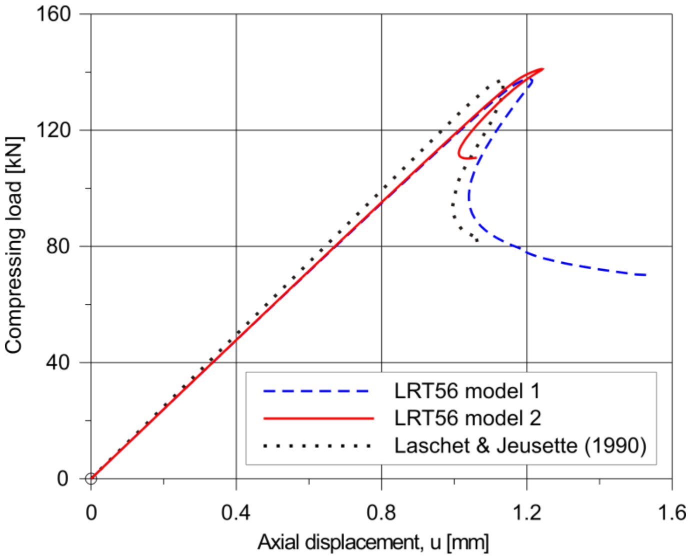

Figure 9. Axial deflection of the compressed cylindrical panel.

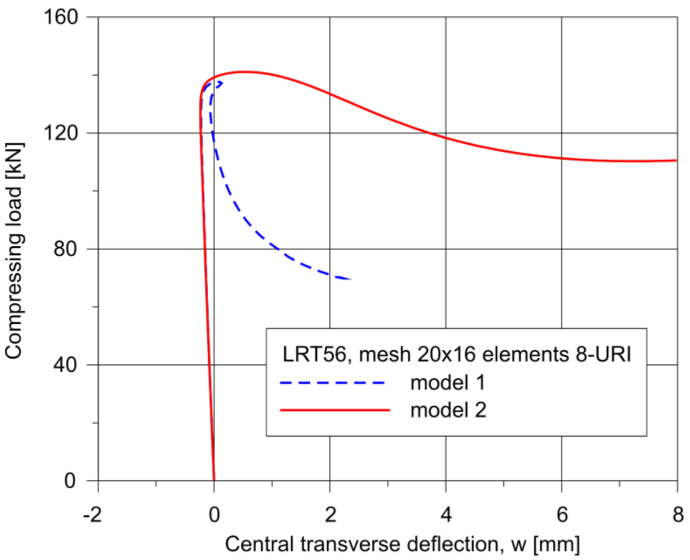

Figure 10. Transverse deflection of the compressed panel.

While looking at the graphs given in Figures 9 and 10, one can observe almost linear response of the structure in the pre-buckling range. This observation is consistent with a very good agreement between the results obtained with MSC-Nastran in a linear buckling (LB) analysis and non-linear (NL) incremental analysis as reported by Kreja (2005) (cf. Table 2). It can be also supported by negligible differences among values of the critical load calculated with the formulations RVK5, MRT5, and LRT56 - the influence of the applied FE mesh density was more significant here than the proper description of large rotations.

4.3 *Simply supported plate strip*

A simply supported asymmetric two layer laminated (0/90) panel under uniformly distributed transverse load, as shown in Figure 11 was analysed assuming $E_1 = 2.0 \times 10^7$ psi, $E_2 = 1.4 \times 10^6$ psi, $\nu_{12} = 0.30$, and $G_{12} = G_{23} = G_{13} = 0.7 \times 10^6$ psi, together with $a = 9.0$ in, $b = 1.5$ in, and $h = 0.04$ in.

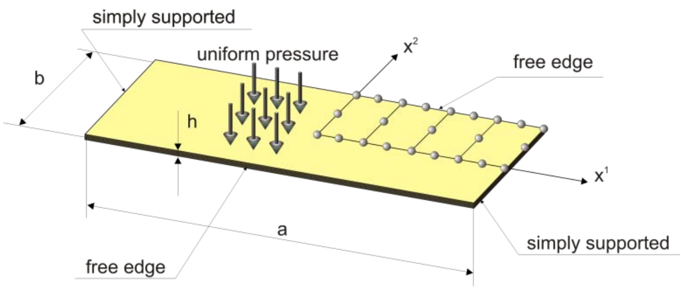

Figure 11. Simply supported plate under uniform pressure.

The graphs in Figure 12 show that the LRT56 results agree very well with the reference solution of Başar et al. (1993) who solved this problem using a fully non-linear formulation accounting for finite rotations. On the other hand, the LRT56 solution is evidently separated from the curves obtained for the models LRT5, MRT5 and RVK5. This observation stood behind a recommendation of this example as a proper benchmark example for large rotation analysis of composite panels given by Kreja (2006) and followed e.g. by Kim et al. (2007) and Li at el. (2011).

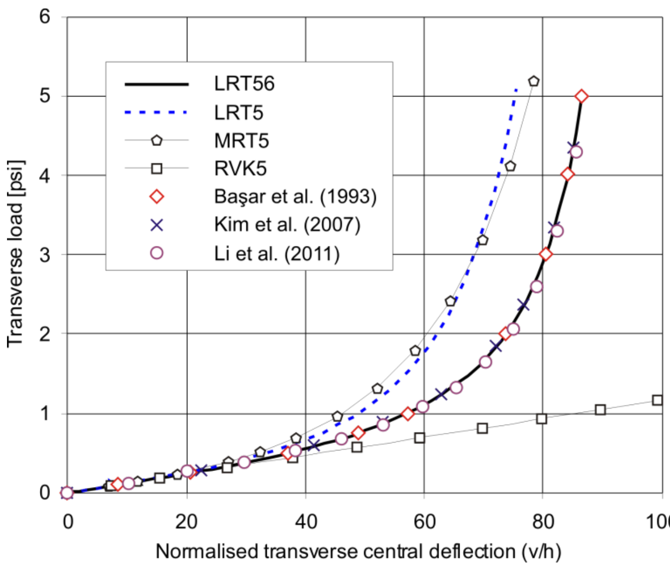

Figure 12. Central deflection of plate strip.

5 CONCLUSIONS

From the short survey presented above, one can realise that the development of the geometrically non-linear FEA of laminated FRP composite panels has already arrived at the very advanced level, therefore a vital question can appear of likely future research directions that may be identified as scientifically interesting and potentially beneficial for engineers.

Undoubtedly, the problem of a reliable and efficient prediction of the buckling loads for FRP composite structures is among the most significant topics. After Arbocz & Starnes Jr. (2002) one can expect here a growing contribution of stochastic stability analysis, where geometrical and material imperfections are taken into account by using random fields (cf. Broggi & Schuëller, 2011).

As structural reliability is not limited just to a buckling but also includes examination of a structural strength, realistic material failure conditions should be considered for FRP composite panels, especially for those undergoing very large deformations. The searching for the trustworthy and robust failure criteria and methods of prediction of progressive damage for composites have been still pursued (cf. Palazotto et al. 2000, Ambur et al. 2004, Lopes et al. 2007, Wagner 2010).

Another crucial issue is related to a proper treatment of the in-plane shear stiffness of a FRP composite panel. Due to a non-linear relation between in-plane shear stress and in-plane shear strain (cf. Jones, 1998) it seems quite obvious that this matter can be treated as material non-linearity (cf. Hu et al. 2006); however, Pai & Palazotto (1995) handled this problem exclusively within geometrical non-linearity. One can expect that this problem will be further examined.

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