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Quantum corrections to ϕ^4 model solutions and applications to Heisenberg chain dynamics

Research Article

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Abstract: The Heisenberg spin chain is considered in ϕ^4 model approximation. Quantum corrections to classical solutions of the one-dimensional ϕ^4 model within the correspondent physics are evaluated with account of rest $d-1$ dimensions of a d-dimensional theory. A quantization of the model is considered in terms of space-time functional integral. The generalized zeta-function formalism is used to renormalize and evaluate the functional integral and quantum corrections to energy in a quasiclassical approximation. The results are applied to appropriate conditions of the spin chain model and its dynamics, for which elementary solutions, energy and the quantum corrections are calculated.

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1. Introduction

There is a wide field of Heisenberg spin chain [1–3] realizations intensely studied as quantum integrable systems [4] and in the context of its static and dynamic properties in an external magnetic field [4, 5]. The original approach of W. Heisenberg is based on localized electrons as an initial approximation valid for metals with weak conductance. The second approximation accounts for quantum exchange (due to the Pauli principle) between electrons in different places. Starting from Heitler-London formula for the exchange and Coulomb integrals, one arrives at a (Heisenberg) Hamiltonian which, by construction, describes the spin system field of a solid. Investigations of symmetry in the first paper of Heisenberg [1–3] gives a fundamental approach to magnetics classification, as well as to the ferromagnetism phenomenon and, for example, its existence only in cubic crystals (eight neighbours necessity). All of this allows one to believe in further development of the whole model, and its particular cases and applications.

Some results of the known model applications are directly related to experiments in a thermodynamics context, e.g. in [6–8]. Easy-plane ferromagnetism and in-plane domain-wall form factor is theoretically studied in connection with neutron scattering effects in $CsNiF_3$ crystal [9]. Let us stress its quantum origin, which apart from well-known linear quasiparticles, provides a way to account for nonlinear collective phenomena as kinks, solitons and

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cnoidal waves. Generally speaking we have important applications of soliton theory aspects, as, e.g. in [10]. It is relevant to mention, that while continuum Heisenberg spin chain with both an anisotropy and a transverse magnetic field (4) is non-integrable [11] regardless of the values of equation parameters (as long as they are non-zero), in the easy plane limit it can be approximated by an integrable Sine-Gordon model [12]. Other option leads to a non-integrable ϕ^4 model, in which the simplest solutions are very similar. We would note a growing interest in a secondary quantization of such nonlinear quasiparticle fields, that, for example, allows one to obtain so-called quantum corrections to classical energy of the objects.

There is a method that is convenient for evaluation of the corrections. It is a Feynman integral by trajectories [13] (path integral). Quantum corrections are a topic of stable interest since the seminal paper of R.F. Dashen, B. Hasslacher and A. Neveu [14], see also L. D. Faddeev, L.A. Takhtajan and V. E. Korepin papers [15, 16]. The functional integral method becomes a practical tool for evaluation of quasiclassical corrections to the action from the time of the V.P. Maslov paper [17, 18].

One of principal results of the method is obtained in [19], where the general algorithm of corrections evaluation is elaborated for arbitrary background profiles, expressions for ground state energies were derived for a 3+1 dimensions theory with a potential dependent on a single variable. Generalization for the supersymmetric kink is given in [20]. Solutions for Sine-Gordon quasiperiodic potentials was given in [21].

Developing these results to arbitrary dimensions, investigating kink models and periodic solutions we demonstrated details of the Feynman integral construction and generalized zeta function evaluation as well as the renormalization realization [12, 22]. A general algebraic method of quantum corrections evaluation based on zeta-function [23] is used and the Green function for heat equation with an elliptic potential is constructed (see also [24]).

In this paper we continue our investigations of the problem in the spirit of [12] and fix our attention on Heisenberg chain model in a so-called ϕ^4 or Landau–Ginsburg [25] approximation (see also Gross–Pitaevski equation [26, 27]). The merit of the presented paper is the attempt to derive conditions of possible application to a realistic magnetic medium (Sec. 2). In the next section we describe general features of the Heisenberg spin chain model and its reduction in specific conditions of ϕ^4. In Sec. 3 we reproduce formulas resulting from [12] for the reader's convenience, namely, the space-time consideration close to the original Feynman papers [13]. As for terminology, we use the word renormalization [12] to exclude divergence terms while in some other papers the term regularisation is met. The final section is devoted to specific case of so-called zero "mass" $m^2 = 0$ condition, that is specified by a distinguished value of magnetic field, in which the field configuration drastically changes.

2. The ϕ^4 model of Heisenberg spin chain

2.1. General equation of motion

According to [5] (with $\vec{S}_n = (S_n^x, S_n^y, S_n^z)$ as unit vectors) the Heisenberg magnetic chain with anisotropy in the direction of the chain and external magnetic field perpendicular to the chain is described by a classical Hamiltonian

$$H = -J\sum_n \vec{S}_n \cdot \vec{S}_{n+1} + D\sum_n (S_n^z)^2 - g\mu_B B\sum_n S_n^x, \quad (1)$$

with corresponding equation of motion (here in SI units)

$$\hbar\partial_t \vec{S}_n = \vec{S}_n \times (-J(\vec{S}_{n+1} + \vec{S}_{n-1}) + 2DS_n^z\hat{z} - g\mu_B B\hat{x}), \quad (2)$$

where J, D are spin coupling constants and g is the effective electron g-factor, μ_B is the Bohr magneton, $\hat{x}$ and $\hat{z}$ are unit vectors and B is the magnetic field. After taking the continuum limit (with a as lattice constant) one obtains

$$\left\{\begin{array}{rcl} \hbar\partial_t S^x &=& -Ja^2(S^y\partial_z^2 S^z - S^z\partial_z^2 S^y) + 2DS^yS^z \\ \hbar\partial_t S^y &=& -Ja^2(S^z\partial_z^2 S^x - S^x\partial_z^2 S^z) - 2DS^xS^z \\ && -g\mu_B BS^z \\ \hbar\partial_t S^z &=& -Ja^2(S^x\partial_z^2 S^y - S^y\partial_z^2 S^x) + g\mu_B BS^y \end{array}\right. . \quad (3)$$

By substituting $\vec{S} = (\cos\theta\cos\phi, \sin\theta\cos\phi, \sin\phi)$ one can reduce equations of motion to

$$\begin{array}{rcl} \hbar\cos\phi\partial_t\theta &=& Ja^2(\partial_z^2\phi + \sin\phi\cos\phi(\partial_z\theta)^2) \\ &-& 2D\cos\phi\sin\phi - g\mu_B B\sin\phi\cos\theta, \\ \hbar\partial_t\phi &=& -Ja^2(\cos\phi\partial_z^2\theta - 2\sin\phi\partial_z\theta\partial_z\phi) \\ &+& g\mu_B B\sin\theta. \end{array} \quad (4)$$

2.2. Model ϕ^4 approximation

Stationary points of the system for $D < 0$ (easy axis anisotropy) are such pairs (θ, ϕ) for which $\theta = 0$, $\phi \in \{-\arccos(\frac{g\mu_B B}{-2D}), 0, \arccos(\frac{g\mu_B B}{-2D})\}$ with $\phi = 0$ being unstable. For $g\mu_B B$ close to $-2D$ both stable points are around $\phi = 0$. In such a situation it is valid to assume $\phi \approx 0$

(with ϕ^3 as the highest considered term) and with θ as the highest considered term.

$$\begin{aligned} \hbar\partial_t\theta &= Ja^2\partial_z^2\phi - 2D\left(\phi - \frac{2\phi^3}{3}\right) - g\mu_B B\left(\phi - \frac{\phi^3}{6}\right), \\ \hbar\partial_t\phi &= -Ja^2\partial_z^2\theta + g\mu_B B\theta. \end{aligned} \tag{5}$$

If we additionally assume $|Ja^2\partial_z^2\theta| << |g\mu_B B\theta|$, we obtain

$$\begin{aligned} \hbar\partial_t\theta &= Ja^2\partial_z^2\phi - 2D\left(\phi - \frac{2\phi^3}{3}\right) - g\mu_B B\left(\phi - \frac{\phi^3}{6}\right), \\ \hbar\partial_t\phi &= g\mu_B B\theta, \end{aligned} \tag{6}$$

which leads to

$$\begin{aligned} \frac{\hbar^2}{g\mu_B B}\partial_t^2\phi &= Ja^2\partial_z^2\phi - (2D + g\mu_B B)\phi \\ &+ \frac{8D + g\mu_B B}{6}\phi^3, \\ \theta &= \frac{\hbar}{g\mu_B B}\partial_t\phi. \end{aligned} \tag{7}$$

The result represents the ϕ^4 model with the energy density

$$\begin{aligned} H &= \frac{\hbar^2}{2ag\mu_b B}\left(\frac{\partial\phi}{\partial t}\right)^2 + \frac{Ja}{2}\left(\frac{\partial\phi}{\partial z}\right)^2 \\ &+ \frac{2D + g\mu_B B}{2a}\phi^2 - \frac{8D + g\mu_B B}{24a}\phi^4. \end{aligned} \tag{8}$$

After rewriting the equations in dimensionless variables ($z = az'$, $t = Tt'$ with T as the time scaling parameter in the Feynman integral as in [12]) we obtain

$$\begin{cases} \frac{\hbar^2}{T^2 g\mu_B B}\partial_{t'}^2\phi = J\partial_{z'}^2\phi - (2D + g\mu_B B)\phi \\ \qquad + \frac{8D + g\mu_B B}{6}\phi^3 \\ H = \frac{\hbar^2}{2g\mu_b BT^2}\left(\frac{\partial\phi}{\partial t'}\right)^2 + \frac{J}{2}\left(\frac{\partial\phi}{\partial z'}\right)^2 \\ \quad + \frac{2D + g\mu_B B}{2}\phi^2 - \frac{8D + g\mu_B B}{24}\phi^4 \end{cases} . \tag{9}$$

For simplicity of further calculations we will write

$$\begin{cases} V^2 = \frac{6(2D + g\mu_B B)}{8D + g\mu_B B} \\ m^2 = -\frac{2D + g\mu_B B}{2J} \\ c^2 = \frac{Jg\mu_b BT^2}{\hbar^2} \end{cases} , \tag{10}$$

where m and V are parameters of the potential and c is a dimensionless propagation speed. Energy density takes the form

$$H = \frac{J}{2}\left(\frac{1}{c^2}\left(\frac{\partial\phi}{\partial t'}\right)^2 + \left(\frac{\partial\phi}{\partial z'}\right)^2 - 2m^2\phi^2 + \frac{m^2}{V^2}\phi^4\right). \tag{11}$$

2.3. Static kink of the ϕ^4 model

For the above described system there exists a well known static kink solution

$$\phi = V\tanh\left(mz'\right), \tag{12}$$

which in this case represents a crossection of a flat, uniform domain wall in direction of its normal vector. Classical energy of the kink is given by integration of the energy density (8)

$$E_c = \frac{4\sqrt{2}JmV^2}{3} = \frac{4\sqrt{-2J}(2D + g\mu_B B)^{\frac{3}{2}}}{8D + g\mu_B B}. \tag{13}$$

For a domain wall this represents the energy per single chain of atoms. It is of note that kink solutions vanish when $g\mu_B B$ reaches $-2D$. There are however static solutions, which are still present for $g\mu_B B \geq -2D$ and an example will be discussed in section (4).

3. Quasiclassical quantum corrections

3.1. Quantization scheme

For the purpose of this publication we will use a semiclassical quantization procedure explained in detail in [12]. For a given classical system described by action integral S with a static solution ϕ we derive energy corrections by expanding the action in path integral formulation of the propagator

$$\langle\phi|e^{-\frac{i}{\hbar}TH}|\phi\rangle = \int_{C^{0,T}_{\phi,\phi}} D\varphi(x,t)e^{\frac{i}{\hbar}S(\varphi)} \tag{14}$$

in a Taylor series around the classical solution and cutting it at the first non-trivial term with T as an arbitrary time period. Then the formal expression for quantum correction to energy is

$$\Delta E = -\frac{\hbar}{iT}\ln\left(\det[L]\right), \tag{15}$$

where L is the second derivative of the classical Lagrangian up to a multiplicative constant arising from the Gaussian integrals during the derivation process (see [17, 18] and [12]). We use zeta-function renormalization scheme ([12], compare with e.g. [19, 20, 23]) to deal with emerging infinities

$$\Delta E = -\frac{\hbar}{iT} \lim_{s\to 0_+} \frac{\partial}{\partial s} \frac{1}{\Gamma(s)} \int_0^\infty \tau^{s-1} \int (g_L(\tau, \vec{x}, \vec{x}) - g_{L_0}(\tau, \vec{x}, \vec{x})) d\vec{x}\, d\tau, \tag{16}$$

where $\vec{x}$ covers all variables of the classical system, g_L is the Green function of the heat equation

$$\left(\frac{\partial}{\partial \tau} + L\right) g_L(\tau, \vec{x}, \vec{x}_0) = \delta(\tau)\delta(\vec{x} - \vec{x}_0) \tag{17}$$

and L_0 is an operator analogous to L with a constant potential (representation of vacuum). Additionally, mass scale is used to cut logarithmic divergence in all relevant parameters (see [23] and [12]). Often the intermediate steps of (16) are defined explicitly as

$$\gamma(\tau) = \int (g_L(\tau, \vec{x}, \vec{x}) - g_{L_0}(\tau, \vec{x}, \vec{x})) d\vec{x} \tag{18}$$

and

$$\zeta(s) = \frac{1}{\Gamma(s)} \int_0^\infty \tau^{s-1} \gamma(\tau) d\tau. \tag{19}$$

It was shown in [24], with an important link to generalized zeta function theory, that if L can be written as a sum of operators acting on independent variables $L = \sum_i L_i$, heat equation Green function for L can be written as a product of Green functions for L_i. We are using this property to account for arbitrary number of spatial variables of the classical system.

3.2. Quantum corrections to ϕ^4 kinks

We now proceed to calculate quantum corrections for energy using the above described generalized zeta function renormalization scheme with following form of operators:

$$L_1 = A\left(\frac{\partial^2}{\partial z'^2} - 4m^2 + 6m^2 sech^2(mz')\right), \tag{20}$$

$$L_2 = -\frac{A}{c^2}\frac{\partial^2}{\partial t'^2}, \tag{21}$$

$$L_3 = \frac{A}{l^{d-1}}\Delta_{d-1}, \tag{22}$$

$$A = \frac{iTJ}{2\pi\hbar r^2}, \tag{23}$$

where d is the total number of spatial dimensions, Δ_{d-1} covers all spatial variables except for z', l is the range of all additional spatial dimensions in multiple of a (due to the same rescaling as for z) and r is the mass scale. Laplace transform of the Green function diagonal for L_1 was derived by use of algorithm described in [22]. For L_2 and L_3 spectra continuum approximation was taken. We obtained following corrections

$$\Delta E_{d=1} = \frac{\hbar c m}{2T\pi}\left(2 + \frac{\pi}{\sqrt{3}} - 2\ln(2) - 3\ln(-Am^2)\right), \tag{24}$$

$$\Delta E_{d=2} = -\frac{\hbar c m^2 l}{2T\pi}\left(3 + \frac{3}{2}\arcsin\left(\frac{1}{\sqrt{3}}\right)\right), \tag{25}$$

$$\Delta E_{d=3} = -\frac{\hbar c m^3 l^2}{8T\pi^2}\left(-6\ln(-Am^2) - 6 + \frac{2}{9}(-11 + 3\sqrt{3}\pi + 6\ln(2))\right). \tag{26}$$

Mass scale r is chosen so that any logarithmic contributions will vanish ($\ln(-Am^2) = 0$) for arbitrary T. Energy corrections exhibit a similar dependance on classic equation parameters as those of Sine-Gordon system [12]. Physical meaning of those parameters is however different. It is of note, that similar results were obtained by Konoplich in [23]. The difference comes from accounting for $\left(\frac{\partial\phi}{\partial t}\right)^2$ term in classical action. If we however compare the results by the total amount of accounted dimensions (in our work d counts only spatial variables), the results are qualitatively identical. Since Konoplich used an approximation of the Green function's diagonal, while we built its exact analytical form, there are some minor quantitative differences. After inserting proper forms of c and m we obtain

$$\Delta E_{d=1} = \frac{\sqrt{-g\mu_B B(2D+g\mu_B B)}}{2\pi}\left(6-6\ln(2)-\frac{\pi}{\sqrt{3}}\right), \tag{27}$$

$$\Delta E_{d=2} = \frac{\sqrt{g\mu_B B}(2D+g\mu_B B)l}{8\sqrt{J}\pi}\left(3+\frac{3}{2}\arcsin\left(\frac{1}{\sqrt{3}}\right)\right), \tag{28}$$

$$\Delta E_{d=3} = \frac{\sqrt{-g\mu_B B}(2D+g\mu_B B)^{\frac{3}{2}}l^2}{4J\pi^2}\left(18\ln(2)-18+\frac{\pi}{\sqrt{3}}\right). \tag{29}$$

In this system the difference between one-dimensional model and one accounting for two additional spatial dimension is especially visible near the $g\mu_B B = -2D$ border case. Since $\Delta E_{d=1}$ is proportional to $(2D+g\mu_B B)^{\frac{1}{2}}$ instead of $(2D+g\mu_B B)^{\frac{3}{2}}$, it would outweigh the classical energy significantly. If we look at the ratio of corrections to classical energy for $d=3$

$$\frac{\Delta E_{d=3}}{E_c} = \frac{\sqrt{g\mu_B B}(8D+g\mu_B B)}{16\sqrt{2}\pi^2 J^{\frac{3}{2}}}\left(18\ln(2)-18+\frac{\pi}{\sqrt{3}}\right) \tag{30}$$

we can see a particularly strong dependance on the J parameter, which represents interaction strength between neighboring electrons. The calculated ratio will be the highest for $g\mu_B B \to -2D$. In this limit we obtain

$$\frac{\Delta E_{d=3}}{E_c} \propto -\left(\frac{-D}{J}\right)^{\frac{3}{2}}. \tag{31}$$

It is worth noticing, that due to the way Planck constant enters propagation speed c, an energy correction is independent of its value. Regardless of the number of spatial dimensions taken into account, energy corrections still vanish along with the classical field, when $g\mu_B B = -2D$ (see the next section).

4. The special case of $2D + g\mu_B B = 0$

In this section we will discuss a static solution of ϕ^4 model, which does not vanish, when both stationary points of the potential coincide. Let us rewrite equation of motion and Hamiltonian in such case, when $m^2 = 2D + g\mu_B B = 0$.

$$\begin{cases} \frac{J}{c^2}\partial^2_{t'}\phi = J\partial^2_{z'}\phi + D\phi^3 \\ H = \frac{J}{2}\left(\frac{1}{c^2}\left(\frac{\partial\phi}{\partial t'}\right)^2 + \left(\frac{\partial\phi}{\partial z'}\right)^2 - \frac{D}{2J}\phi^4\right) \end{cases}. \tag{32}$$

This system has an interesting traveling wave solution

$$\phi(b(z' \pm vt')) = b\sqrt{\frac{2J(c^2-v^2)}{-Dc^2}}\,sn(b(z' \pm vt'); i) \tag{33}$$

with b as a wavenumber and sn denoting Jacobi SN elliptic function. The choice of b value is restricted only by long-wave approximation $b << 1$ (compared to lattice constant, "1" in our units). Real-valued amplitudes exist for $v < c$ due to $D < 0$. For now we will focus on the $v = 0$ case

$$\phi(z') = b\sqrt{\frac{2J}{-D}}\,sn(bz'; i). \tag{34}$$

It is particularly interesting, since it's a non-trivial static solution in a system with a single stationary point $\phi = 0$. Quantum corrections to the energy of this solution can be calculated through the same procedure as for a ϕ^4 kink with

$$L_1 = A\left(\frac{\partial^2}{\partial z'^2} + 6b^2 sn^2(bz')\right), \tag{35}$$

where A is the same as in (23) and other L_i as before. The operator (35) belongs to the class of Lame operators (with $n = 2$), which have periodic potentials with absolute continuous spectrum with finite number of gaps described by a hyperelliptic curve [28, 29]. The potential enters into the nonlinear equation for the Green function diagonal, that is studied in [22] and gives the direct link of gaps number and appropriate curve properties through the n parameter. Let us reproduce some formulas from [24]. Laplace transform of the Green function diagonal can be obtained in the same way as for ϕ^4 or Sine-Gordon kinks [22]

$$G_1(p,z) = \frac{p^2 - 3b^2p(1-z) + 9b^4(z-2)z}{2\sqrt{(3b^2+p)(3b^2-p)p(p^2-12b^4)}}. \quad (36)$$

It is important to note, that for calculation convenience, we use rescaling $\tau \to \frac{\tau'}{A}$ for the Green function equation (17). The polynomial in the denominator has five simple roots. We will set the vacuum counterpart to coincide with the highest one

$$G_0(p,z) = \frac{1}{2\sqrt{2b^2\sqrt{3}-p}}. \quad (37)$$

We can now integrate the Green function diagonal over the period $\frac{4K(i)}{b}$, where K is the complete elliptic integral of the first kind

$$\hat{\gamma}(p) = \int_0^{\frac{4K(i)}{b}} (G_1(p, cn^2(bx)) - G_0(p, cn^2(bx)))dx, \quad (38)$$

$$\hat{\gamma}(p) = \frac{6b^4K(i) + 2p^2K(i) + 36b^4(K(i)-E(i)) - 3b^2p(E(i)-3K(i))}{b\sqrt{(3b^2+p)(3b^2-p)p(p^2-12b^4)}} - \frac{2K(i)}{b\sqrt{2b^2\sqrt{3}-p}} \quad (39)$$

To properly define the inverse Laplace transform, we need the $\hat{\gamma}$ function to be smooth in a $o_l < \Re(p) < o_p$ area with $o_l < o_p$ as constants [30]. Since the $\hat{\gamma}$ function has five distinct singularities (all on the real axis), we have six potential inverse Laplace transforms (disregarding the choice of the sign for the square roots), but only one of them fulfills the necessary condition $\forall_{\tau<0}\ \gamma(\tau) = 0$ arising from the definition (17) - it occurs, when $o_p \to \infty$. Thus the γ function as defined in (18) will take form

$$\gamma(\tau) = \frac{1}{2\pi i}\int_{o-i\infty}^{o+i\infty} e^{pA\tau}\hat{\gamma}(p)dp, \quad (40)$$

where $o > 2b^2\sqrt{3}$. At this point, we add all relevant variables as for ϕ^4 case $\gamma(\tau) \to \gamma(\tau)\gamma_2(\tau)\gamma_3(\tau)$. The energy corrections per wave period are finally obtained by performing the Mellin transform and taking the derivative at $s = 0$

$$\Delta E = -\frac{\hbar}{iT}\lim_{s\to 0_+}\frac{\partial}{\partial s}\frac{1}{2\pi i\Gamma(s)}\int_0^\infty \tau^{s-1}\gamma_2(\tau)\gamma_3(\tau)\int_{o-i\infty}^{o+i\infty} e^{pA\tau}\hat{\gamma}(p)dpd\tau, \quad (41)$$

Plugging expressions from (39) and relevant forms of γ_2 and γ_3 (see [12]) and inserting $p' = \frac{p}{b^2}$ for simplification yields

$$\begin{aligned}\Delta E = & -\frac{\hbar^{\frac{d}{2}}J^{\frac{1-d}{2}}\sqrt{g\mu_B B}r^d l^{d-1}}{(-2i)^{\frac{d}{2}}T^{\frac{d}{2}}}\lim_{s\to 0_+}\frac{\partial}{\partial s}\frac{1}{2\pi i\Gamma(s)}\int_0^\infty \tau^{s-\frac{d}{2}-1} \\ & \int_{o-i\infty}^{o+i\infty} e^{\frac{iTJb^2p'\tau}{2\pi\hbar r^2}}\left(\frac{2p'^2K(i) + 6(7K(i)-6E(i)) - 3p'(E(i)-3K(i))}{\sqrt{(3+p')(3-p')p'(p'^2-12)}} - \frac{2K(i)}{\sqrt{2\sqrt{3}-p'}}\right)dp'd\tau\end{aligned} \quad (42)$$

There still remains the problem of finding an explicit analytic or numeric solution of the shown integrals. Applications of the distinguished condition of the elliptic field configuration may be interesting from the point of measurements realization. The case may be realized simply by the magnetic field B value choice. Such configuration may be more easily noticed (recognized in experiments).

5. Conclusion

Energy corrections for easy axis domain walls would be particularly interesting in the case of thin ferromagnetic films, where they should dominate the classical energy. Formally, the results of the last section partially coincide with the case recently investigated (see e.g. [24], where the so-called Nahm model of Yang-Mills theory is studied) but the physical sense is different.
The space-time consideration we develop in our publications has obvious intentions to include mutisoliton configuration into the quantum quasiclassical picture.

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