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Lax-Wendroff and McCormack Schemes for Numerical Simulation of Unsteady Gradually and Rapidly Varied Open Channel Flow

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Abstract

Two explicit schemes of the finite difference method are presented and analyzed in the paper. The applicability of the Lax-Wendroff and McCormack schemes for modeling unsteady rapidly and gradually varied open channel flow is investigated. For simulation of the transcritical flow the original and improved McCormack scheme is used. The schemes are used for numerical solution of one dimensional Saint-Venant equations describing free surface water flow. Two numerical simulations of flow with different hydraulic characteristics were performed – the first one for the extreme flow of the dam-break type and the second one for the simplified flood wave propagation problem. The computational results are compared to each other and to arbitrary solutions.

Key words: open channel flow, mathematical modeling, numerical simulation, FDM schemes

1. Introduction

In recent years considerable effort has been devoted to modelling one-dimensional open channel flow. The free surface one-dimensional, unsteady water flow is governed by the mathematical model called the Saint-Venant equations (Cunge et al 1980). Quite a number of numerical methods of solving this equations system have been proposed and successfully applied, so far. Numerous schemes of finite difference method (FDM) and finite element method (FEM) are widely used (Szymkiewicz 2010) for simulation of the gradually varied flow. However, the FDM and FEM standard algorithms are often inefficient for modelling rapidly varied transcritical flow. For this kind of flow, shock-capturing methods should be implemented to solve the conservative form of the Saint-Venant equations. Many of these methods are based on the finite volume method (FVM). Numerous analysis of FVM schemes for hydrodynamics problems are available in the technical literature (LeVeque 2002). The FVM schemes are usually reported as efficient and robust.

The aim of this research is to analyze some numerical schemes of FDM in a case of gradually and rapidly varied flow in open channels. This analysis will allow to assess the usefulness of selected schemes to simulate the transitions between gradually and rapidly varied flow with local effects like steep wave fronts, which can be observed during sudden and flush floods.

The paper presents results of simulation of rapidly varied flow in open channel using the Lax-Wendroff scheme, as well as standard and improved McCormack scheme. Improvement of the latter is based on the theory of total variation diminishing (TVD) schemes that are capable of capturing sharp discontinuities without generating spurious oscillations of the numerical results. This technique was originally presented by Garcia-Navarro et al (1992). In order to assess the applicability and numerical features of particular schemes they were implemented to simulate an extreme, rapidly varied flow occurring in horizontal and frictionless open channel due to sudden, catastrophic dam collapse. The same schemes were also applied to simulate gradually varied flow related to the standard flood wave propagation problem in a prismatic open channel.

2. Governing Equations

Generally, the free surface water flow in channels and rivers is a phenomenon varying in time. Therefore, water motion in open channels is usually described using unsteady flow equations – often the Saint-Venant model is applied (Cunge et al 1980) for flow simulation. As various local phenomena, like hydraulic jumps, bores and steep surface fronts can occur during intense floods, a special form of flow model equations must be used. To properly reproduce the local phenomena, the conservative equations of water flow should be used in the hydraulic calculations. The Saint-Venant system, written in the conservative form for the rectangular channel of unit width, can be presented as:

$$\frac{\partial \boldsymbol{U}}{\partial t} + \frac{\partial \boldsymbol{F}}{\partial x} + \boldsymbol{S} = 0, \tag{1}$$

where the vectors $\boldsymbol{U}$, $\boldsymbol{F}$ and $\boldsymbol{S}$ are given as:

$$\boldsymbol{U} = \begin{pmatrix} h \\ uh \end{pmatrix}, \ \boldsymbol{F} = \begin{pmatrix} uh \\ u^2 h + 0.5 g\, h^2 \end{pmatrix}, \ \boldsymbol{S} = \begin{pmatrix} 0 \\ -gh\left(S_o - S_f\right) \end{pmatrix} \tag{2}$$

and:

- x, t – distance and time,
- h – water depth,
- u – flow velocity,
- g – acceleration due to gravity,
- S_o – slope of the bottom,
- S_f – slope of energy grade line (friction slope).

The friction slope can be defined by the Manning's formula:

$$S_f = \frac{n^2 u\ |u|}{h^{4/3}} \tag{3}$$

where n denotes the Manning's friction coefficient. Such approach to hydraulic losses in open channel is generally valid for steady and unsteady gradually varied flow. When the rapidly varied flow occurs in the channel the Manning theory does not represent energy loss precisely due to multidimensional characteristic of the flow for example. Howeveer, it is often used in hydraulic modeling as the simplest friction representation technique.

3. Solution Method and Numerical Tests

The Saint-Venant model (1, 2) is a system of partial differential equations and its solution for the given boundary conditions is composed of functions $h(x,t)$ and $u(x,t)$. In order to solve the Saint-Venant equations for complex hydraulic conditions, numerical method must be applied. In the paper, a finite difference method (FDM) was chosen to integrate the model equations in space and time. FDM schemes discretize continuous space and time into a grid system, and values of the variables are evaluated at separate nodes of the numerical grid. In simple FDM schemes, the first-order derivatives are approximated with either central, backward, or forward discretization, while the second–order derivatives are approximated with central discretization. After discretization of integration space, selected time level can be represented as time $t^n = n \cdot \Delta t$, and each point in space (along the channel length) defines the computing node $x_i = (i-1) \cdot \Delta x$, where Δt is the time increment and Δx is the size of an uniform mesh.

There are various FDM numerical schemes that can be used to solve the Saint-Venant equations. In order to ensure the second order accuracy of derivatives approximation in space and time, and keep calculations simple, two explicit schemes known as Lax-Wendroff and McCormack methods were applied in numerical solution.

The two stage Lax-Wendroff scheme (Potter 1973) is second order accurate in space and time. At the first stage, values of the $\boldsymbol{U}^{n+1/2}$ variables values are calculated using the method of Lax for half time step and at half step grid $x_{i+1/2}$:

$$\boldsymbol{U}_{i+1/2}^{n+1/2} = \frac{1}{2}\left(\boldsymbol{U}_{i+1}^{n} + \boldsymbol{U}_{i}^{n}\right) - \frac{\Delta t}{2\Delta x}\left(\boldsymbol{F}_{i+1}^{n} - \boldsymbol{F}_{i}^{n}\right) - \frac{1}{4}\Delta t\left(\boldsymbol{S}_{i+1}^{n} + \boldsymbol{S}_{i}^{n}\right). \tag{4}$$

Next, the fluxes $\boldsymbol{F}$ and source terms $\boldsymbol{S}$ are calculated at intermediate points of space and time as:

$$\boldsymbol{F}_{i+1/2}^{n+1/2} = \boldsymbol{F}\left(\boldsymbol{U}_{i+1/2}^{n+1/2}\right) \tag{5}$$

$$\boldsymbol{S}_{i+1/2}^{n+1/2} = \boldsymbol{S}\left(\boldsymbol{U}_{i+1/2}^{n+1/2}\right). \tag{6}$$

At the second stage, values of the midpoint variables values are used for final calculation:

$$\boldsymbol{U}_i^{n+1} = \boldsymbol{U}_i^n - \frac{\Delta t}{\Delta x}\left(\boldsymbol{F}_{i+1/2}^{n+1/2} - \boldsymbol{F}_{i-1/2}^{n+1/2}\right) - \frac{1}{2}\Delta t\left(\boldsymbol{S}_{i+1/2}^{n+1/2} + \boldsymbol{S}_{i-1/2}^{n+1/2}\right). \tag{7}$$

The Lax-Wendroff scheme is an explicit technique of FDM, so in order to be stable, it must satisfy the Courant-Friedrich-Lewy (CFL) criterion (Potter 1973) at each grid point i in order to be stable. The CLF criterion is defined as:

$$\mathrm{Cr} = \frac{|u| + c}{\Delta\ x/\Delta t} \le 1, \tag{8}$$

where Cr is the Courant number at point i and $c = \sqrt{gh}$ is a celerity.

To perform numerical simulation of unsteady flow in open channel it is necessary to specify additional solution conditions. According to the theory of solving partial differential equations they include the initial condition and boundary conditions (Cunge et al 1980). For the solution of unsteady flow equations, before the start of the calculation, the initial water surface profile and flow rate must be known and adopted along the channel. The boundary conditions depend on the time variability of flow parameters at the inflow and outflow cross-sections of the channel. The previously assumed calculations at the inlet section developed hydrogram of water inflow. At the outlet section of the channel time-varying water table position was forced. The Manning's formula was used as a known relationship between parameters of flow at the outflow cross-section.

In this study, the original and improved McCormack schemes were also investigated. The main advantage of the original scheme is an ability to calculate gradually and rapidly varied flow, what is needed to simulate water flow during flush floods. Moreover, the inclusion of the source terms is relatively simple and suitable for implementation in the explicit time-marching algorithm. The standard algorithm based on McCormack original scheme (McCormack 1971) involves a two stage procedure known as the predictor–corrector method and it can be presented as:

$$\boldsymbol{U}_i^{p} = \boldsymbol{U}_i^n - \frac{\Delta t}{\Delta x}\left(\boldsymbol{F}_{i+1}^{n} - \boldsymbol{F}_{i}^{n}\right) + \Delta t\boldsymbol{S}_i^n, \tag{9}$$

$$\boldsymbol{U}_i^{c} = \boldsymbol{U}_i^n - \frac{\Delta t}{\Delta x}\left(\boldsymbol{F}_{i}^{p} - \boldsymbol{F}_{i-1}^{p}\right) + \Delta t\boldsymbol{S}_i^p, \tag{10}$$

where the superscript $p(c)$ refers to the predictor (corrector) step and n is the time level.

The final updating formula, representing the solution at the next time level $n + 1$) has a form:

$$\boldsymbol{U}_i^{n+1} = \frac{1}{2}\left(\boldsymbol{U}_i^{p} + \boldsymbol{U}_i^{c}\right). \tag{11}$$

The numerical stability condition for the McCormack scheme is the same as for the Lax-Wendroff method due to the explicit nature of both schemes and it is defined by CFL condition (8). The initial and boundary conditions used for the numerical solution of Saint-Venant using McCormack were also the same as before.

In order to asses the quality of numerical solution based on standard Lax-Wendroff and McCormack schemes for rapidly varied flow, the classical problem of hydrodynamics known as the dam break test was simulated. The simulation was performed for water flow in a 200 m long horizontal and frictionless (n = 0.0 s/m$^{1/3}$) channel with rectangular cross-section of unit width. The channel was divided by the dimensionless wall (virtual dam) into two 100 m segments. Water depth in the first section was 2 m, while in the other 1 m. Initially, water in channel was at rest. At the start of the simulation (t = 0) s the dam failure is forced (the barrier that separated two different levels of water is suddenly removed). As a consequence, two water waves could be observed in the channel. The negative wave was moving upstream the channel and the positive one was travelling downstream. This hydraulic phenomena of rapidly varied flow was simulated using two standard numerical schemes. For the numerical calculation the channel was discretized into 201 nodes with the spatial step Dx = 1 m long. Simulations were carried out with the time step equal to Dt = 0.01 s, ensuring stability of the solution.

The initial condition representing the water surface profile before the crash of the virtual barrier is presented in Figure 1 as a dashed line. The comparison of the analytical and numerical solution of the Saint-Venant equations after 10 s of the flow simulation is shown in the same figure. The analytical results were acquired using Stoker (1957) solution of the Saint-Venant equations. It can be observed that, regardless of the method applied, an unphysical water surface oscillation have been occurred in the numerical solutions. However, the solutions obtained using both FDM schemes are stable. The highest oscillations are visible at the front of the positive (shock) wave. There are no oscillations in the vicinity of the negative wave, but the simulated water surface is slightly smeared due to numerical diffusion. The high discrepancy between analytical and both numerical solutions suggests that all tested methods are not adequate to simulate rapidly varied flow in open channel during extreme episodes like flush floods.

In order to eliminate or reduce the problem of the spurious oscillations in rapidly varied flow modelling with FDM schemes, an improvement for the method based on TVD theory can be implemented. The concept of TVD schemes was introduced by Harten and Hyman (Toro 1997). Generally, for certain types of equations TVD algorithms ensure that the total variation (TV) does not increase with time, that is:

$$\mathrm{TV}\left(\boldsymbol{U}^{n+1}\right) \leq \mathrm{TV}\left(\boldsymbol{U}^{n}\right) \quad \Rightarrow \quad \sum_{i}\left|\boldsymbol{U}_{i+1}^{n+1}-\boldsymbol{U}_{i}^{n+1}\right| \leq \sum_{i}\left|\boldsymbol{U}_{i+1}^{n}-\boldsymbol{U}_{i}^{n}\right|. \tag{12}$$

For the investigation how the standard FDM technique can be improved using TVD approach, the McCormack scheme was chosen for the analysis. The TVD improved McCormack scheme is an extension of the original method and it includes a shock-capturing technique capable of rendering the solution oscillation. The scheme

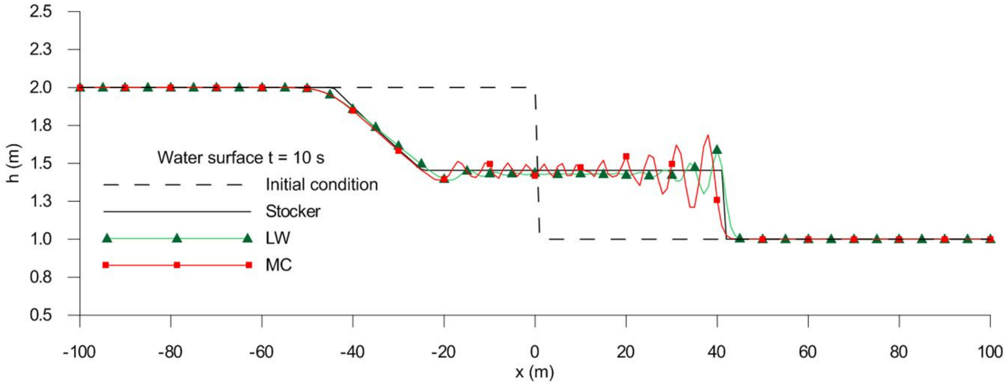

Fig. 1. The dam break problem. Initial condition (dashed line) and calculated water profile after 10 s of flow – analytical solution (solid line), Lax-Wendroff scheme (triangle marker) and McCormack scheme (rectangle marker)

is of second-order accuracy both in time and space in non-critical sections, but it switches the accuracy to the first-order at extreme points. The improved scheme involves an additional computational term in updating step of the original predictor corrector procedure (11) (Garcia-Navarro et al 1992), which can be written as:

$$\boldsymbol{U}_i^{n+1} = \frac{1}{2}\left(\boldsymbol{U}_i^p + \boldsymbol{U}_i^c\right) + \frac{1}{2}\frac{\Delta t}{\Delta x}\left(\boldsymbol{R}_{i+1/2}\boldsymbol{\Phi}_{i+1/2} - \boldsymbol{R}_{i-1/2}\Phi_{i-1/2}\right). \tag{13}$$

The second term in equation (13), which is calculated at intermediate states between grid points $i-1$, i and $i+1$, what is described later in this point, equips the McCormack scheme with TVD properties adding a numerical dissipation to the original method. Due to this modification, the scheme retains second-order accuracy in space and time for continuous regions and it is able to limit the solution oscillations near the extremes by reducing the accuracy to first-order in these sections.

To calculate the additional term in formula (13), the TVD improvement requires the quasi linear form of the Saint-Venant equations (1, 2). The original problem (1) can be transformed to the following form:

$$\frac{\partial \boldsymbol{U}}{\partial t} + \boldsymbol{A}\frac{\partial \boldsymbol{U}}{\partial x} = \boldsymbol{S}, \tag{14}$$

where **U** is the same as in equation (1) and the jacobian matrix $\boldsymbol{A}$ of $\boldsymbol{F}$ with respect to $\boldsymbol{U}$ can be written as:

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ c^2 - u^2 & 2u \end{bmatrix}, \tag{15}$$

where $c = \sqrt{gh}$ is a celerity. The jacobian matrix $\boldsymbol{A}$ is diagonalizable, so the following equation must be satisfied:

$$\boldsymbol{A} = \boldsymbol{R}\,\Lambda\,\boldsymbol{L}, \tag{16}$$

where Λ is a diagonal matrix containing the eigenvalues of matrix $\boldsymbol{A}$, whereas $\boldsymbol{R}$ and $\boldsymbol{L}$ contain associated right and left eigenvectors. The eigenvalues λ of matrix $\boldsymbol{A}$ can be evaluated by solution of the characteristic equation:

$$|\boldsymbol{A} - \lambda \boldsymbol{I}| = 0, \tag{17}$$

where $\boldsymbol{I}$ is the identity matrix. Considering jacobian matrix (15), the roots of (17) equal:

$$\lambda_1 = u - c, \quad \lambda_2 = u + c. \tag{18}$$

The matrix Λ and the corresponding right, used in the updating step of TVD McCormack scheme (13), and left eigenvector matrices for matrix $\boldsymbol{A}$ are defined as:

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}, \quad \boldsymbol{R} = \begin{bmatrix} 1 & 1 \\ \lambda_1 & \lambda_2 \end{bmatrix}, \quad \boldsymbol{L} = \frac{1}{2c}\begin{bmatrix} -\lambda_2 & 1 \\ \lambda_1 & -1 \end{bmatrix}. \tag{19}$$

The two components of vector $\boldsymbol{\Phi}_i$ in Eq. (13), that are evaluated at the intermediate state between grid points i and $i+1$, are defined as:

$$\Phi^k_{i+1/2} = \Psi\left(\lambda^k_{i+1/2}\right)\left(1 - \frac{\Delta t}{\Delta x}\left|\lambda^k_{i+1/2}\right|\right)\left(1 - \varphi\left(r^k_{i+1/2}\right)\right)\alpha^k_{i+1/2} \qquad (k = 1, 2)\,. \tag{20}$$

The function Ψ is an entropy correction to the eigenvalues preventing the appearance of unphysical flow discontinuities, those in which energy increases across the shock. In the simplest form it can be written as (Garcia-Navarro et al 1992):

$$\Psi(\lambda) = \begin{cases} |\lambda| & \text{if } |\lambda| \geq \varepsilon, \\ \varepsilon & \text{if } |\lambda| < \varepsilon, \end{cases} \tag{21}$$

where ε is a small positive number (from 0.1 to 0.3), which value must be determined for each individual problem. Formulas for the ε evaluation and other forms of the entropy correction were proposed by Harten and Hyman (Toro 1997).

The characteristic variable α in formula (20) is defined as:

$$\alpha_{i+1/2} = \frac{1}{2c_{i+1/2}}\begin{bmatrix} -\lambda_2 & 1 \\ \lambda_1 & -1 \end{bmatrix}_{i+1/2}\begin{bmatrix} h_{i+1} - h_i \\ (uh)_{i+1} - (uh)_i \end{bmatrix}. \tag{22}$$

In order to calculate the mean values of flow parameters in (22), that need to be determined at the intermediate point $i + 1/2$, the averaging procedure proposed by Roe (1981) can be applied. The discrete approximations of the local water velocity and wave celerity can be presented as:

$$u_{i+1/2} = \frac{u_{i+1}\sqrt{h_{i+1}} + u_i\sqrt{h_i}}{\sqrt{h_{i+1}} + \sqrt{h_i}}, \qquad c_{i+1/2} = \frac{c_i + c_{i+1}}{2}. \tag{23}$$

For obtaining non-oscillatory solutions in regions where some flow discontinuities like hydraulic jumps or bores exist, the limiter parameter has to be incorporated into the solution procedure. In equation (20) the function φ is a limiter parameter and it is responsible for adding artificial dissipation to the numerical solution in a regions of steep gradients. The numerical dissipation makes the solution monotone at extreme points. In the continuous regions of smooth variation little or no dissipation is added. Many forms of the limiting function can be found in the literature. Their review in terms of the water flow problem was presented by Toro (1997). Following Tseng (2003), the minmod limiter was used to simulate rapidly varied open channel flow. This function can be written as:

$$\varphi\left(r_{i+1/2}^{k}\right)=\begin{cases}\min\left(\left|r_{i+1/2}^{k}\right|,\ 1\right) & \text{if } r_{i+1/2}^{k}>0,\\ 0 & \text{if } r_{i+1/2}^{k}\le 0,\end{cases} \tag{24}$$

where r is the ratio of characteristic variables estimated as follows:

$$r_{i+1/2}^{k}=\frac{\alpha_{i+1/2-s}^{k}}{\alpha_{i+1/2}^{k}}\qquad s=\operatorname{sign}\left(\alpha_{i+1/2}^{k}\right). \tag{25}$$

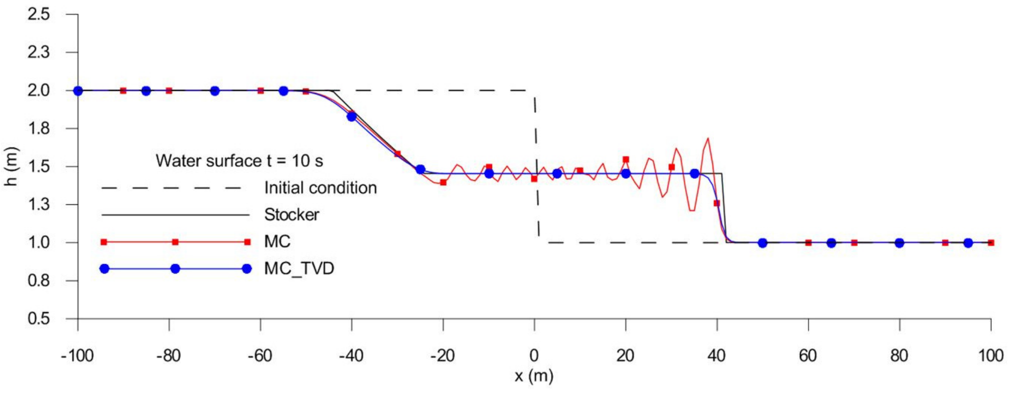

Fig. 2. The dam break problem. Initial condition (dashed line) and calculated water profile after 10 s of flow – analytical solution (solid line), McCormack scheme (rectangle marker) and TVD McCormack scheme (circle marker)

The results of the dam break flow simulation using standard and improved McCormack schemes are presented in Figure 2. The graph shows the shape of the water surface after the same time period like in Fig. 1 (t = 10 s). It can be observed that water level along the channel and speed of the waves fronts simulated by the TVD scheme seem to be in a good agreement with the analytical solution. It can be also seen that the results obtained with classic and improved McCormack schemes differ. The standard scheme produces the spurious oscillations near steep water level front, while the improved version ensures the solution to be quite smooth. However, the front of the positive wave is more smeared in case of the TVD than for standard scheme. It is a result of the scheme accuracy reduction in the vicinity of the surface discontinuity (wave front). The influence of the numerical diffusion is also visible near the negative wave front, but the shapes of the surface obtained using standard and improved McCormack schemes are almost the same.

In order to assess the quality of numerical solutions of the Saint-Venant model obtained from the presented FDM schemes for unsteady gradually varied flow, the simulation of simple flood wave propagation in open channel was performed. The test was prepared using data defined for the numerical simulation presented by Szymkiewicz (1995). The problem was solved for flow equations in an open prismatic channel with rectangular cross-section and bottom slope equal to 0.0001. The considered channel was 4 m wide and 50 km long. In the numerical calculation, the following parameters were used:

– spatial step $\Delta x = 1000$ m,
– Manning's coefficient of roughness $n = 0.02$ s/m$^{1/3}$,
– integration time step $\Delta t = 100$ s,
– initial flow $Q_0 = 49,646$ m^3/s,
– initial depth in the channel $H_0 = 2$ m.

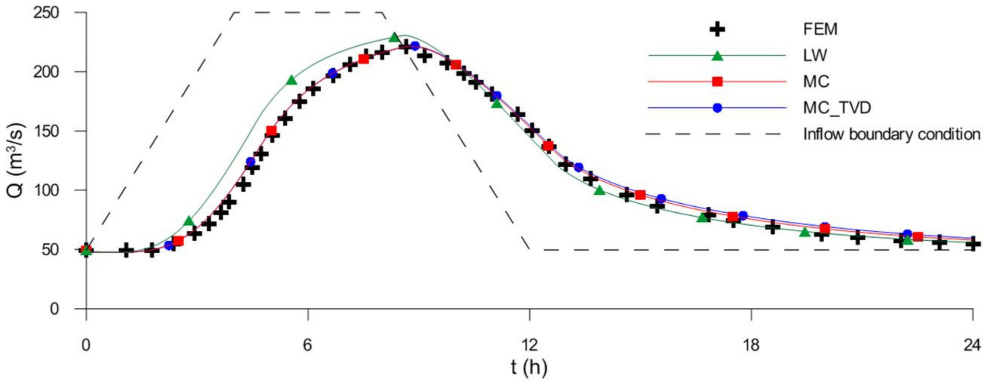

Fig. 3. The flood wave propagation problem. Boundary condition (dashed line) and FEM solution (cross marker) obtained by Szymkiewicz (1995), Lax-Wendroff scheme (triangle marker), McCormack scheme (rectangle marker) and TVD McCormack scheme (circle marker) for cross-section located at $x = 20$ km

Results of own calculations using the Lax-Wendroff scheme, as well as standard and improved McCormack schemes, compared to the results presented in the original article calculated using modified scheme of the finite element method, are presented in Figures 3 and 4. The FEM solution is digitized directly from the paper version of the source article, so precision of its display is limited. The hydrographs representing flow discharge varying in time at two analyzed cross-sections are presented in the

Figures. The boundary condition, forced at the inflow cross-section, was defined as an initial trapezoidal form of the wave and it is shown as a dashed line in the graphs.

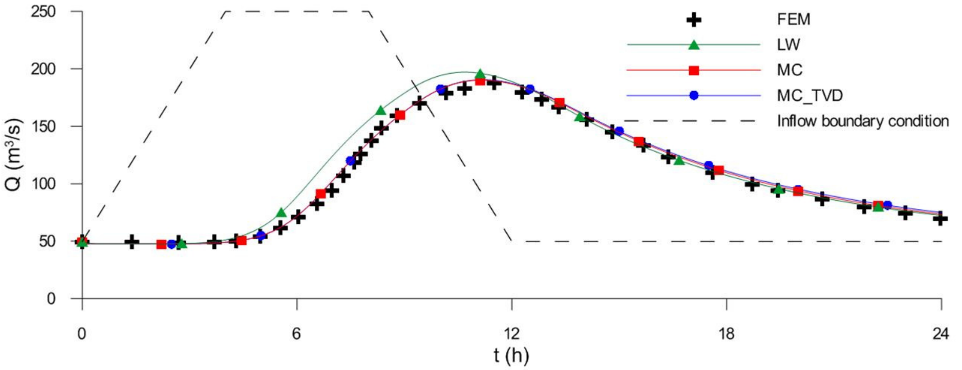

Fig. 4. The flood wave propagation problem. Boundary condition (dashed line) and FEM solution (cross marker) obtained by Szymkiewicz (1995), Lax-Wendroff scheme (triangle marker), McCormack scheme (rectangle marker) and TVD McCormack scheme (circle marker) for cross-section located at x = 40 km

The hydrographs calculated for x = 20 km cross-section (Fig. 3) have similar shape and agreement of the FEM and the FDM results is satisfactory. The results indicate the correctness of the FDM computational schemes adopted in the simulation and the appropriate solution with the equations of unsteady gradually varied flow. However, the solutions obtained using both McCormack schemes better fit the FEM simulation results. The Lax-Wendroff scheme is overestimated for increasing part of the hydrograph. The same effect can be seen in case of the solution obtained for the cross-section located at x = 40 km (Fig. 4). It can also be observed that the numerical solutions obtained using original and improved McCormack schemes are almost the same for the gradually flow simulations.

4. Conclusions

The numerical solution of the Saint-Venant equations for one–dimensional flow based on the some FDM schemes was analyzed in this paper. In particular, the Lax-Wendroff, as well as standard and improved McCormack scheme were investigated. The improvement of the original scheme was based on the theory of total variation diminishing schemes. The results of numerical simulations of open channel flow were presented in adequate Figures and analyzed. In the first numerical test there was a rapidly varied flow, but in the second numerical test there was a gradually varied flow. In first test of dam break flow type both standard schemes (Lax-Wendroff and McCormack) have produced unphysical results because the spurious oscillations near the wave steep front. The numerical results of the improved McCormack scheme better fit the analytical solution. The second numerical test has also proved that both McCormack schemes simulate the gradually varied flow more precisely than Lax-Wendroff method.

The following final conclusions can be drawn from the conducted research:

– The spurious oscillations of the calculated results obtained using standard Lax-Wendroff and McCormack schemes make the solution of rapidly varied flow unphysical. The improved McCormack scheme is capable of capturing sharp fronts without generating oscillations. The modification can be easily introduced into the standard McCormack scheme algorithm.
– The improved McCormack scheme allows to model rapidly as well gradually varied flow in open channels. It quite accurately describes main flow features, such as positive and negative open channel waves. The improved method better predicts the flow parameters than the standard algorithms.
– Results of numerical simulations compared to the arbitrary solutions show that the overall performance of the TVD McCormack method can qualify the method as a very good candidate for modeling of the open channel flow during storms and flush floods. It seems that the improved McCormack scheme can be incorporated into the integrated model of urban flooding.

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