

# Self-refraction of acoustic pulses with shock fronts in some non-equilibrium media

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## Summary

The non-linear self-refraction of acoustic pulsed beams which include shock fronts, is studied. The medium of sound propagation is a gas where thermodynamically non-equilibrium processes take place, such as exothermic chemical reaction or excitation of vibrational degrees of a molecule's freedom. Comparative analysis of the features of sound propagation over gases where pure nonlinear attenuation of the shock wave occurs, and gases with non-equilibrium and equilibrium processes, is performed.

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## 1 Introduction

There are different kinds of self-action of sound, which have been observed in experiments, among them nonlinear self-action and inertial types of self-action. The first kind reflects the nonlinear distortions of the waveform as wave propagates due to dependence of a local signal speed on the particles velocity. The last kind in fact includes two subspecies, one thermal self-action, and the second relating to the mean flow in the field of sound. These types associate with the nonlinear losses in acoustic energy and momentum, respectively. They arise due to generation of the non-wave types of motion, i.e. to appearance of the areas with larger temperature and mean flow, hence with the modified parameters of the background of sound propagation. It is well-established, that a beam is defocused in media for which the sound speed increases with temperature, while a beam is self-focused in media with negative temperature coefficient. In spite of the first theoretical papers concerning self-action of acoustic beams were reviewed in [1], this effect was observed later [2, 3]. Considerable attention was paid to the thermal self-action of quasi-harmonic sound waves because many interesting results obtained in nonlinear optics have their counterparts in acoustics [4, 5]. Unlike optic waves, dispersion of

sound is usually negligible. The harmonic plane wave acquires a "sawtooth" shape in a quadratic nonlinear medium. Nonlinear attenuation of the waves containing shock fronts enhances losses in energy and momentum of sound, and the inertial self-action occurs in principle differently as compared with quasi-harmonic optic waves. As usual, inertial self-action of initially harmonic beams are considered.

Self-action of pulsed acoustic beams is of interest in many applications of nonlinear acoustics. Nonlinear pulses are generated by laser radiation, explosions, electric discharges, they are important in many technical applications. Studies of powerful pulsed beams are stimulated by their importance in medicine, such as non-contact noninvasive destruction of tissues, remote ultrasonic elastometry [6]. In this connection, self-action of pulsed sound which propagates in media different from newtonian fluids is of increasing interest. In the non-equilibrium media, sound may enhance under some conditions. In contrast to newtonian fluids, variations in the sound magnitude in the thermodynamically non-equilibrium gases may occur due to transfer of energy of internal degrees of molecule's freedom into acoustic energy. The nonlinear effects of sound also reveal anomalous features. Apart from acoustic cooling, the mean flow which is induced in the field of sound, may be directed oppositely to the direction of acoustic streaming in a newtonian fluid. An anomalous behavior of sound and relative nonlinear phenomena are specific not only in vibrationally excited gases and chemically active fluids, but in all media, where thermodynamic equilibrium is disturbed, such as non-isothermal plasma, suspensions of microparticles in a gas, the interstellar gas and upper atmosphere [7, 8, 9]. Some important anomalies in a linear non-equilibrium gas flow have been reviewed in Ref.[8].

This study is devoted to the non-newtonian kind of self-action of sound beams in a gas where internal degrees of freedom are excited, or in a gas where exothermic chemical reaction takes place, namely to the self-refraction of pulsed signals containing shock fronts. Despite different physical reasons for possible instability, the equations which govern sound and relative nonlinear phenomena of sound in these gases, are similar. The phenomenon of self-refraction is related to a nonlinear variation in the shock propagation velocity, which enlarges with increase in pressure step in a shock wave. In newtonian fluids, this step is larger near the axis than in the periphery, and the "straightening" of a focused-wave front is observed [10].

We consider propagation of an axially symmetric sound beam over a relaxing gas. The simplified system of equations includes the analogue of Khokhlov-Zabolotskaya equation supplemented by the term responsible for attenuation in a non-newtonian gas. The mathematical content of further solution is very close to that one which has been developed by Rudenko et al. in studies of self-action of sound beams in a newtonian fluid [11]. In the subsections 1.1 and 1.2, the acoustic increment (or decrement) of sound planar wave,  $B$ , is determined for two examples of thermodynamical processes in gases, excitation of vibrational degrees of molecules' freedom and exothermic chemical reactions.

## 1.1 Gases with excited vibrational degrees of molecule's freedom

The first example of fluid where equilibrium or non-equilibrium thermodynamic processes take place, is a gas whose steady but non-equilibrium state is maintained by pumping energy into the vibrational degrees of freedom by power  $I$  ( $I$  refers to a unit mass). The relaxation equation for the vibrational energy  $\varepsilon$  per unit mass has the form:

$$\frac{d\varepsilon}{dt} = -\frac{\varepsilon - \varepsilon_{eq}(T)}{\tau} + I. \quad (1)$$

The equilibrium value of the vibrational energy at given temperature  $T$  is denoted by  $\varepsilon_{eq}(T)$ , and  $\tau(\rho, T)$  marks the vibrational relaxation time. The quantity  $\varepsilon_{eq}(T)$  equals in the case of a system of harmonic oscillators:

$$\varepsilon_{eq}(T) = \frac{\hbar\Omega}{m(\exp(\hbar\Omega/k_B T) - 1)}, \quad (2)$$

where  $m$  is the mass of a molecule,  $\hbar\Omega$  is the magnitude of the vibrational quantum,  $k_B$  is the Boltzmann constant. Eq.(2) is valid over the temperatures, where one can neglect anharmonic effects, i.e., below the characteristic temperatures, which are fairly high for most molecules [8, 12]. The quantity

$$B = -\frac{(\gamma - 1)^2 T_0}{2c_0^3} \left( \frac{C_v}{\tau} + \frac{\varepsilon - \varepsilon_{eq}}{\tau^2} \frac{d\tau}{dT} \right)_0 \quad (3)$$

is increment, if positive in the non-equilibrium regime of excitation of internal degrees of molecule's freedom (or decrement, if negative in the equilibrium regime) of the sound planar wave. It is the quantity evaluated at unperturbed pressure  $p_0$ , temperature  $T_0$ , and  $C_v = d\varepsilon_{eq}/dT$ . The non-equilibrium excitation is possible in principle due to negative  $d\tau/dT$ . The relaxation time in the most important cases may be thought as a function of temperature accordingly to Landau and Teller with some positive constants  $\tilde{A}$  and  $\tilde{B}$ ,  $\tau(T) = \tilde{A} \exp(\tilde{B}T^{-1/3})$  [8, 13, 14]. There exists the threshold quantity of pumping magnitude  $I$  starting from which the excitation is non-equilibrium, since  $\varepsilon - \varepsilon_{eq} \approx I\tau$ .

## 1.2 Gases in which exothermic chemical reactions occur

For this kind of processes in a gas [15],

$$B = \frac{Q_0(\gamma - 1)(Q_\rho + (\gamma - 1)Q_T)}{2c_0^2 m} \quad (4)$$

is the quantity evaluated at unperturbed  $p_0$ ,  $T_0$ ,  $Y_0$ , where  $Y$  denotes mass fraction of a reagent  $A^*$  in  $A^* \rightarrow B^*$  reactions;  $Q$  is the heat produced in a medium per one molecule due to a chemical reaction,  $Q_0 = Q(T_0, \rho_0, Y_0)$ . The dimensionless quantities  $Q_T$ ,  $Q_\rho$  are conditioned by dependence of the heat produced due to a chemical reaction on temperature and density of the mixture:

$$Q_T = \frac{T_0}{Q_0} \left( \frac{\partial Q}{\partial T} \right)_{T_0, \rho_0, Y_0}, \quad Q_\rho = \frac{\rho_0}{Q_0} \left( \frac{\partial Q}{\partial \rho} \right)_{T_0, \rho_0, Y_0}. \quad (5)$$

## 2 The governing equations and starting points

The equation which describes evolution of an acoustic pressure  $p$  without account for thermal and inertial self-action in media with unusual attenuation, takes the form

$$\frac{\partial}{\partial \eta} \left( \frac{\partial p}{\partial x} - \frac{\varepsilon}{c_0^3 \rho_0} \left( p - \frac{a}{2} \right) \frac{\partial p}{\partial \eta} - Bp \right) = \frac{c_0}{2} \Delta_{\perp} p, \quad (6)$$

were  $x$  and  $r$  are cylindrical coordinates, the axis  $Ox$  coincides with the axis of a beam,  $c_0 = \sqrt{\frac{\gamma p_0}{\rho_0}}$  denotes the infinitely small-signal sound speed in a perfect uniform gas ( $\gamma$  is the ratio of specific heats,  $\rho_0$  denotes unperturbed density),  $\eta = t - x/c_0$  is the retarded time in the

reference frame which moves with the sound speed  $c_0$ ,  $\Delta_{\perp}$  is the Laplacian with respect to the radial coordinate  $r$ ,  $\varepsilon = (\gamma + 1)/2$  is the parameter of nonlinearity, and  $B$  is responsible for attenuation or amplification of sound in a gas different from that in the newtonian fluids. Eq.(6) accounts for a nonlinear variation  $\Delta c$  in the shock propagation velocity, which increases with enlargement of pressure step in a triangular at a transducer shock wave,  $a$ :

$$\Delta c = \frac{\varepsilon a}{2c_0\rho_0}. \quad (7)$$

Since values of  $a(x, r)$  differ at the axis of a beam and on its periphery, the change of an impulse profile takes place. This phenomenon is called self-refraction. Eq.(6) without the dispersive term,  $-Bp$ , is well-studied in the theory of nonlinear acoustics [17, 16]. In this study, we consider new peculiarities of beams' self-refraction which reflects anomalous or normal dispersion in a gas. An acoustic pressure may be found in the form which follows from the theory of geometrical acoustics [16],

$$p = p(x, r, \theta = \eta - \psi(x, r)/c_0). \quad (8)$$

The form (8) is suitable in the cases where acoustic nonlinearity is essential and description of the entire nonlinearly distorted waveform is required [18]. This leads to the equations for unknown eikonal  $\psi$  and acoustic pressure  $p$ ,

$$\frac{\partial p}{\partial x} - \frac{\varepsilon}{c_0^3\rho_0} \left(p - \frac{a}{2}\right) \frac{\partial p}{\partial \theta} - Bp + \frac{\partial \psi}{\partial r} \frac{\partial p}{\partial r} + \frac{\Delta_{\perp} \psi}{2} p = 0, \quad (9)$$

$$\frac{\partial \psi}{\partial x} + \frac{1}{2} \left(\frac{\partial \psi}{\partial r}\right)^2 = -\frac{\varepsilon}{2c_0^2\rho_0} a. \quad (10)$$

Eqs (9),(10) are valid at the limit of short wavelengths, small in comparison with the scales of thermal and hydrodynamical inhomogeneities. In the new variables

$$P = \exp(-Bx)p, \quad \Psi = \exp(-Bx)\psi, \quad X = (\exp(Bx) - 1)/B, \quad A = \exp(-Bx)a, \quad (11)$$

they may be readily rearranged into the set

$$\frac{\partial P}{\partial X} - \frac{\varepsilon}{c_0^3\rho_0} \left(P - \frac{A}{2}\right) \frac{\partial P}{\partial \theta} + \frac{\partial \Psi}{\partial r} \frac{\partial P}{\partial r} + \frac{\Delta_{\perp} \Psi}{2} P = 0, \quad (12)$$

$$\frac{\partial}{\partial X} ((BX + 1)\Psi) + \frac{BX + 1}{2} \left(\frac{\partial \Psi}{\partial r}\right)^2 = -\frac{\varepsilon}{2c_0^2\rho_0} A. \quad (13)$$

It is assumed that the wavefront is parabolic [18]:

$$\Psi(X, r, t) = \Psi_0(X, t) + \frac{r^2}{2} \frac{\partial}{\partial X} \ln F(X). \quad (14)$$

This implies that distance from the axis of a beam  $r$  is small compared to its initial width  $h$ ,  $r^2/h^2 \ll 1$ . Applying the transformation of variables,

$$\bar{P} = F(X)P, \quad \bar{A} = F(X)A, \quad \zeta = r/(hF), \quad \xi = \int_0^X \frac{dX'}{F(X')}, \quad (15)$$

we reduce the transfer equation (12) to the readily solvable equation

$$\frac{\partial \bar{P}}{\partial X} - \frac{\varepsilon}{c_0^3 \rho_0} \left( \bar{P} - \frac{\bar{A}}{2} \right) \frac{\partial \bar{P}}{\partial \theta} = 0, \quad (16)$$

which is similar to the equation for a simple wave. Its solution yields the peak pressure

$$A(X, r) = \frac{P_0}{F} \Phi \left( \frac{r}{hF} \right) \left[ 1 + \frac{1}{2X_s} \Phi \left( \frac{r}{hF} \right) \int_{X_1}^X \frac{dX'}{F(X')} \right]^{-1/2} \quad (17)$$

in a single pulse of duration  $2S$ , which has the form of an isosceles triangle at a transducer,  $P_0$  is the maximum acoustic pressure at a transducer;  $X_s = c_0^3 \rho_0 S / \varepsilon P_0$  is the distance at which the shock arises in the planar wave,  $X_1 = R(1 - \exp(-X_s/R))$  is a distance of the shock forming in a focused wave with curvature at a transducer  $R^{-1}$ . Expanding (17) in the series in the vicinity of axis of initially Gaussian beam with  $\Phi \left( \frac{r}{hF} \right) = \exp \left( - \left( \frac{r}{hF} \right)^2 \right)$ , and substituting it into eikonal equation (13), we arrive finally at

$$F^2 \left( \frac{\partial^2 F}{\partial X^2} + \frac{B}{BX+1} \frac{\partial F}{\partial X} \right) = \frac{1}{2X_d X_s (BX+1)} \left( 1 + \frac{1}{4X_s} \int_{X_1}^X \frac{dX'}{F(X')} \right) \left( 1 + \frac{1}{2X_s} \int_{X_1}^X \frac{dX'}{F(X')} \right)^{-3/2}, \quad (18)$$

where  $X_d = h^2 / (2c_0 S)$  is the typical diffraction length. The boundary conditions take the form

$$F \Big|_{X=X_1} = 1 - \frac{X_1}{R}, \quad \frac{dF}{dX} \Big|_{X=X_1} = -\frac{1}{R}. \quad (19)$$

Eq.(18) tends to the analogous equation which describes newtonian fluids when  $B \rightarrow 0$ . Returning to the co-ordinate  $x$ , one may readily rearrange Eq.(18) into the following one:

$$F^2 \frac{\partial^2 F}{\partial x^2} = \frac{\exp(Bx)}{2X_d X_s} \left( 1 + \frac{1}{4X_s} \int_{x_1}^x \frac{\exp(Bx) dx'}{F(x')} \right) \left( 1 + \frac{1}{2X_s} \int_{x_1}^x \frac{\exp(Bx) dx'}{F(x')} \right)^{-3/2}, \quad (20)$$

where

$$x_1 = \ln[BX_1 + 1] / B. \quad (21)$$

Note, that shock front does not form at all for  $B < -1/X_1$  (that is, large attenuation) and always appears in the non-equilibrium regime, when  $B$  is positive. By use of the new variable  $z = x/R$ , and constants  $\Pi = R/X_s$ ,  $D = R/X_d$ ,  $\beta = RB$ , Eq.(20) takes the form

$$F^2 \frac{\partial^2 F}{\partial z^2} = \frac{\Pi D}{2} \exp(\beta z) \left( 1 + \frac{\Pi}{4} \int_{z_1}^z \frac{\exp(\beta z') dz'}{F(z')} \right) \left( 1 + \frac{\Pi}{2} \int_{z_1}^z \frac{\exp(\beta z') dz'}{F(z')} \right)^{-3/2}, \quad (22)$$

where

$$z_1 = \ln[\beta + 1 - \beta \exp(-1/\Pi)] / \beta. \quad (23)$$

Eq.(22) is solved numerically in the Sec.3 with the boundary conditions

$$F|_{z=z_1} = \exp(-1/\Pi), \quad \frac{dF}{dz} \Big|_{z=z_1} = -\exp(\beta z_1). \quad (24)$$

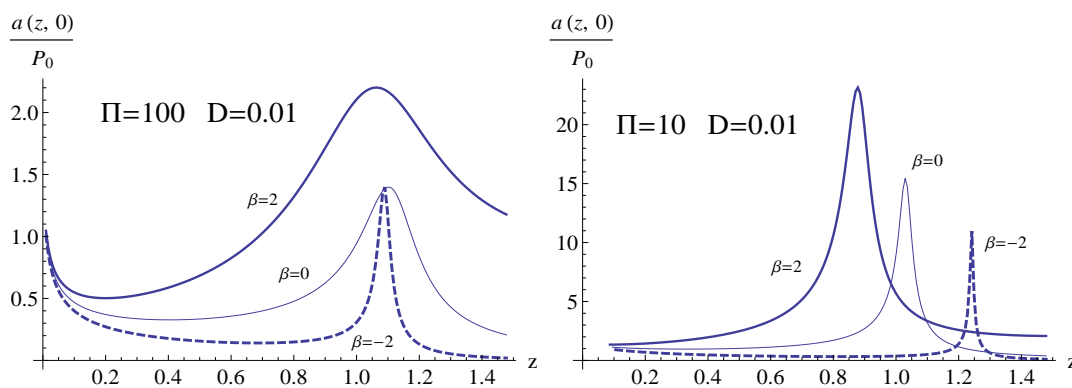
The increase in the pressure step at the axis of a beam, is evaluated as

$$a(z, 0) = \frac{P_0}{F} \exp(\beta z) \left[ 1 + \frac{\Pi}{2} \int_{z_1}^z \frac{\exp(\beta z') dz'}{F(z')} \right]^{-1/2}. \quad (25)$$

### 3 Self-refraction of weak shock waves

To review briefly the knowledge about nonlinear self-refraction of acoustic beams with discontinuities, it would be useful to remind the following. It has been established, that the quadratic nonlinearity alone can not lead to beam bending or self-focusing [11]. In the presence of diffraction, the wave acquires a frequency-dependent phase shift. That leads to the waveform with compression region which becomes higher and sharper and with the smooth rarefaction region [18]. The fact that the positive peak pressure exceeds negative one yields to acceleration of the shock front, it is flattened because of supersonic propagation near the axis as a beam approaches focus and due to larger attenuation at the axis of beam propagation. This effect is much pronounced in the case of monopolar pulses. The waist size enlarges, and focus becomes displaced due to this kind of nonlinear self-action. The numerical investigations revealed some important features of the self-refraction of sound pulses in newtonian fluids, such as decrease in the maximum peak pressure  $a_{max}$  with the growth of  $P_0$ . The longitudinal size of the focal area increases, and the nonlinear focus is formed at distances greater than the linear one. The amplitude of the pulse at the focus is practically independent on  $P_0$  at enough large values of  $P_0$ , that is, nonlinear saturation takes place [17]. For small  $\Pi$ , the peak pressure maximum is reached at the geometrical focus  $x = R$  and amounts to  $P_0/D$ .

In this study, we consider gases where unusual amplification of sound may take place due to thermodynamical processes connected with non-equilibrium phenomena in a gas. That corresponds to positive  $B$ . Hence, the sawtooth impulse enhances due to convergence of a beam and due to thermodynamical processes in a gas, but alters the nonlinear attenuation as it propagates towards the focus. In the planar wave which propagates in the non-equilibrium gas, the peak acoustic pressure does not tend to zero with enlargement of distance from a transducer, but to the value  $0.5c_0BSP_0/\varepsilon$ . The unusual behavior of a sound beam leads to the shift of the focus towards the transducer as compared to the focus in a gas without relaxation. Figure 1(a,b) shows the dimensionless acoustic peak pressure at the axis of beam propagation,  $a(z, 0)/P_0$ , as a function of  $z$  for the set of parameters  $\Pi = 100$  (a) and  $\Pi = 10$  (b), and  $D = 0.01$ . The thin solid line reproduces the results of numerical calculations by Musatov et al. [17] (formally this corresponds to  $\beta = 0$ ), the bold line represents the results of numerical simulations relating to a gas with the non-equilibrium processes ( $\beta > 0$ ), and the dotted line to a gas with equilibrium processes ( $\beta < 0$ ). As for the equilibrium case, the curve becomes sharper, the peak amplitude is smaller and the focal length increases as compared to the case of pure nonlinear attenuation (at least in the case of weaker nonlinearity,  $\Pi = 10$ ). In the non-equilibrium case, vice versa, the curve is flat, the peak amplitude is comparatively large and the focal length small.



a

b

Fig.1 The dimensionless peak amplitude of the shock pulse at the axis of beam propagation in equilibrium gas ( $\beta < 0$ ), non-equilibrium acoustically active gas ( $\beta > 0$ ), and in a gas where the pure nonlinear attenuation occurs ( $\beta = 0$ );  $z$  denotes the dimensionless distance from a transducer.

The difference is more pronounced in the case of small  $\Pi$ , in the weakly nonlinear regime. The shift of the focus towards transducer in the non-equilibrium regime may be explained by comparative decrease in the speed of shock front of an impulse due to non-linear change of co-ordinate. That may be simply evaluated for the planar wave with discontinuity. If  $\beta$  differs from zero, speed of the discontinuity equals  $c_0 + \frac{P_0 \exp(\beta z)}{c_0 \rho_0 \sqrt{1 + (\exp(\beta z) - 1) \Pi / \beta}}$ , and for  $\beta = 0$ , it equals  $c_0 + \frac{P_0}{c_0 \rho_0 \sqrt{1 + \Pi z}} = c_0 \left(1 + \frac{\Pi S c_0}{R \varepsilon \sqrt{1 + \Pi z}}\right)$ . Fig.2 shows the additional speed of the wavefront for different values of  $\beta$ . The joint influence of the difference in the local speeds of various parts of the wavefront and their nonlinear attenuation which is followed by unusual increase in the peak pressure, is the reason for shift of the focus towards transducer.

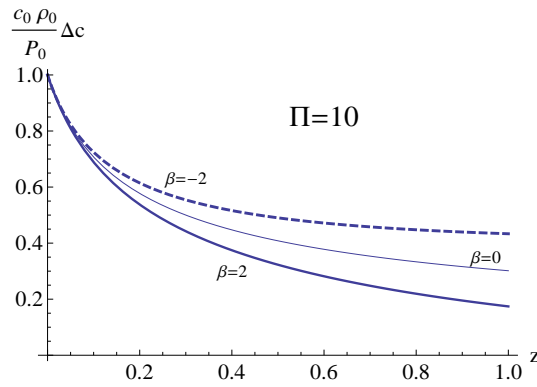


Fig.2 Variations in the speed of the shock front in the planar wave with discontinuity.

It is remarkable, that in the case of strong nonlinearity, all curves in Fig.1(a) behave similarly in the vicinity of transducer. At first, the nonlinear damping occurs, and, despite focusing, the peak pressure decreases. At the larger distances, the peak pressure increases and reaches a maximum at some point  $z > 1$ . Because of unusual acceleration of sound in the non-equilibrium regime, the amplitude of a beam is the largest as compared to the cases  $\beta \leq 0$ , and the curve is the most sloping. As for the transversal distribution of the peak pressure, there is a difference also, in accordance to the Fig.3(a,b,c). When  $\beta$  is less than zero, a beam rapidly converges, but before the focus the transverse structure becomes sharp. As for  $\beta > 0$ , the cross-section is slowly smoothed, and becomes slowly sharper as distance from a transducer increases.

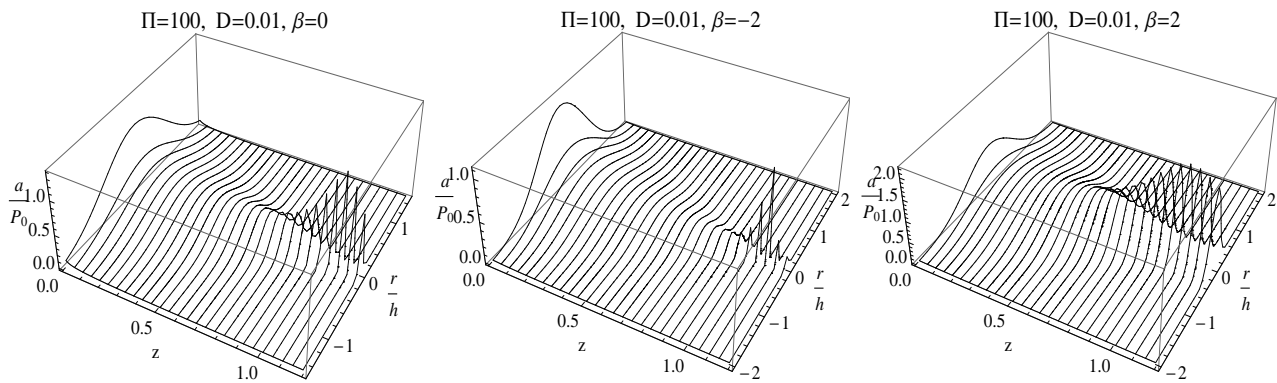


Fig.3 Transversal distribution of the dimensionless peak acoustic pressure in a shock wave.

## 4 Concluding remarks

The conclusions concerning self-refraction of a pulsed beam with discontinuities due to pure nonlinear attenuation, are well-understood. It has been established, that the flattening of a the focused wave front is observed due to increase in the nonlinear compound of the shock wave velocity,  $\Delta c$ . The nonlinear absorption, which occurs simultaneously with self-refraction, makes distribution of the peak pressure over the front more uniform. Evidently, both processes shift the nonlinear focus with respect to the geometric one and enlarge the beam waist. These effects were described in the review [19]. They have been observed experimentally.

In this study, the peculiarities of a pulsed shock beam which propagates in relaxing media, which may be thermodynamically non-equilibrium, are studied. Anomalous increase in the sound amplitude as a beam propagates, along with the nonlinear attenuation, result in the anomalous self-focusing and self-refraction of a beam in a non-equilibrium gas. The nonlinear focus shifts toward transducer, the distribution of the peak acoustic pressure along axis of a beam propagation is more acclivous as compared to a gas with pure nonlinear attenuation of a shock pulse. The maximum peak amplitude enlarges. Vice versa, in the equilibrium regime, the peak acoustic pressure is comparatively small, and the longitudinal distribution is sharp.

The nonlinear effects of sound in the non-equilibrium acoustically active media also behave atypically. The theory of anomalous cooling of the medium (instead of acoustic heating) and streaming (with streamlines inverted as compared with direction of sound propagation) has been recently developed in reference to aperiodic and periodic in time sound beams, including beams with discontinuities [20, 21, 22].



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