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Asynchronous Wide Area Multilateration System

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Abstract
A new method for a location service in the wide area multilateration (WAM) system is outlined. This method, which is called asynchronous WAM (AWAM), enables calculation of the geographical position of an aircraft without knowledge of relative time differences (RTDs) between measuring ground stations (sensors). The AWAM method is based on the measurement of round trip times (RTTs) between the aircraft and the serving ground station, and the solution of a nonlinear system of equations with ten variables. The elimination of the RTD parameters significantly simplifies the localization process in real-life WAM system. The proposed asynchronous method could be an alternative solution for the synchronous one as a backup method during a system fails or as an independent method. The paper concentrates on the description of the method, solving the nonlinear system of equations with ten variables and simulation results.

Keywords
Wide area multilateration system; wireless sensor networks; location service; radiolocation; radio navigation; TDOA; TOA

1. INTRODUCTION

The currently available aircraft radio navigation systems are based on two techniques: passive in which the aircraft navigates on received radio information only, and active in which the aircraft participates both as receiver and transmitter of information. The first group is counted mainly: automatic direction finder (ADF), very high frequency (VHF) omnidirectional radio range (VOR), instrument landing system (ILS), microwave landing system (MLS) and global position system (GPS). The distance measuring equipment (DME), the radio altimeter (RA) and the wide area multilateration (WAM) systems belong to the second group. This paper focuses on the multilateration (MLAT) solution. The MLAT is a technology for determining the position of an emitter (e.g., aircraft transponder) by measuring the time difference of arrival (TDOA) of a signal between several known and carefully surveyed observation points (e.g., MLAT sensors or ground stations) [1]. The MLAT employs a number of ground stations, which are placed in strategic locations around an airport that covers the larger surrounding airspace. Generally speaking, the multilateration system utilizes signals from the secondary surveillance radar (SSR) system. The SSR consists of a ground component (the radar) and an airborne component (transponder) onboard an aircraft. The radar emits a signal (at 1030 MHz) which triggers a response from the airborne transponder (at 1090 MHz). This response is the basic signal for the WAM system [2]. A critical aspect of a working WAM system is precise synchronization of the ground stations (sensors) amongst each other [3]. In order to calculate the position of an aircraft, it is necessary to know the time difference from a signal arriving at one antenna (sensor) in the system to the arrival of the signal at another antenna (sensor) in the system and relative time differences (RTDs) between measuring ground stations (sensors). The RTDs are compensated in the process of synchronization. The methods of synchronization complicate positioning architecture in the WAM system. When the synchronization procedures to estimate the position of the aircraft are not needed, the location service is cheaper.

In this paper, a new method, asynchronous WAM (AWAM), which enables the calculation of the geographical position of an aircraft without knowledge of relative time differences, is outlined. This method is supported by round trip time (RTT) and a system of nonlinear equations [4, 5]. With respect to this, the paper concentrates on description of new method and solving the nonlinear system of equations with ten variables. In this context, the new method was tested using software simulation.

2. DESCRIPTION OF THE AWAM METHOD

The proposed new method is as follows. In a separate area (WAM) are deployed at least five ground stations (sensors) and the secondary surveillance radar. The SSR is located in the same place as the selected measuring sensor – master (serving) sensor (Figure 1). The SSR transmits interrogation pulses on 1030 MHz. The target aircraft's transponder replies on 1090 MHz a signal containing the requested information, which is received by all ground stations (sensors). In the study case, ground sensors network works asynchronous – the sensors are not synchronized with each other. All sensors in the WAM perform measurements in the rhythm of their own clocks. Only the master sensors can measure true distance to the aircraft, because is synchronized to the SSR. Between the master sensor and neighbouring sensors occur the relative time differences (RTDs) in the synchronization. The proposed method is based on elimination of RTDs in the location service.

The classic TDOA method, in a three-dimensional plane for five measuring sensors (the case is illustrated in Figure 1)[1], is reduced to the solution of the following system of nonlinear equations [6]

$$\begin{aligned} c\cdot[t_i(n)-t_1(n)] &= \sqrt{[X_i-x(n)]^2+[Y_i-y(n)]^2+[Z_i-z(n)]^2} \\ &\quad -\sqrt{[X_1-x(n)]^2+[Y_1-y(n)]^2+[Z_1-z(n)]^2} \qquad \text{for} \quad i=2,...,5 \end{aligned}, \tag{1}$$

where $t_1(n)$ and $t_i(n)$ denote the measured signal of transfer times from the sensors S_1 to S_5 to the aircraft transponder (AT) at the same discrete time n, (X_1, Y_1, Z_1) and (X_i, Y_i, Z_i) represent the coordinates of the sensors, (($x(n)$, $y(n)$, $z(n)$)) are the coordinates of the aircraft transponder at the discrete time n and c is the speed of light. When the sensors work asynchronously, the master sensor measures observed time difference of arrivals (OTDOAs) $t_{i,1}(n)$ (for $i = 2,\ldots, 5$) using current data and measurement data from the auxiliaries sensors (slave sensors):

$$t_{i,1}(n)=t_i(n)+\Delta t_{i,1}-t_1(n)\,, \tag{2}$$

where $\Delta t_{i,1}$ describe the relative time differences between the serving sensor S_1 and auxiliaries S_i. However, it is only possible to solve this system of equations (1) when we know relative time differences, i.e. $\Delta t_{i,1}$ (normally, in the WAM system the RTDs are compensated by the synchronization procedures). In connection with this, a second system of nonlinear equations at the discrete time n+1 is proposed

$$\begin{aligned} c\cdot[t_i(n+1)-t_1(n+1)] &= \sqrt{[X_i-x(n+1)]^2+[Y_i-y(n+1)]^2+[Z_i-z(n+1)]^2} \\ &\quad -\sqrt{[X_1-x(n+1)]^2+[Y_1-y(n+1)]^2+[Z_1-z(n+1)]^2} \qquad \text{for} \quad i=2,...,5 \end{aligned}. \tag{3}$$

Analogous to the expression (2), at the discrete time n+1 the master sensor measures observed time difference of arrivals $t_{i,1}(n+1)$ (for $i = 2,\ldots, 5$):

$$t_{i,1}(n+1)=t_i(n+1)+\Delta t_{i,1}-t_1(n+1)\,. \tag{4}$$

[1] In a three-dimensional plan are needed only 4 sensors, but to explain the principle of the AWAM method at least five sensors are necessary.

Suppose that the relative time differences $\Delta t_{i,1}$ are constants at the discrete time n and n+1. By transforming the nonlinear equations (1) to (4) (by transforming the TDOA system to a time of arrival (TOA) system), and using Figure 1, we can get a new nonlinear system of equations with ten variables:

$$\begin{cases} \sqrt{[X_1-x(n)]^2+[Y_1-y(n)]^2+[Z_1-z(n)]^2}-c\cdot t_1(n)=0 \\ \sqrt{[X_2-x(n)]^2+[Y_2-y(n)]^2+[Z_2-z(n)]^2}-c\cdot[t_1(n)+t_{2,1}(n)-\Delta t_{2,1}]=0 \\ \sqrt{[X_3-x(n)]^2+[Y_3-y(n)]^2+[Z_3-z(n)]^2}-c\cdot[t_1(n)+t_{3,1}(n)-\Delta t_{3,1}]=0 \\ \sqrt{[X_4-x(n)]^2+[Y_4-y(n)]^2+[Z_4-z(n)]^2}-c\cdot[t_1(n)+t_{4,1}(n)-\Delta t_{4,1}]=0 \\ \sqrt{[X_5-x(n)]^2+[Y_5-y(n)]^2+[Z_5-z(n)]^2}-c\cdot[t_1(n)+t_{5,1}(n)-\Delta t_{5,1}]=0 \\ \sqrt{[X_1-x(n+1)]^2+[Y_1-y(n+1)]^2+[Z_1-z(n+1)]^2}-c\cdot t_1(n+1)=0 \\ \sqrt{[X_2-x(n+1)]^2+[Y_2-y(n+1)]^2+[Z_2-z(n+1)]^2}-c\cdot[t_1(n+1)+t_{2,1}(n+1)-\Delta t_{2,1}]=0 \\ \sqrt{[X_3-x(n+1)]^2+[Y_3-y(n+1)]^2+[Z_3-z(n+1)]^2}-c\cdot[t_1(n+1)+t_{3,1}(n+1)-\Delta t_{3,1}]=0 \\ \sqrt{[X_4-x(n+1)]^2+[Y_4-y(n+1)]^2+[Z_4-z(n+1)]^2}-c\cdot[t_1(n+1)+t_{4,1}(n+1)-\Delta t_{4,1}]=0 \\ \sqrt{[X_5-x(n+1)]^2+[Y_5-y(n+1)]^2+[Z_5-z(n+1)]^2}-c\cdot[t_1(n+1)+t_{5,1}(n+1)-\Delta t_{5,1}]=0 \end{cases} . \quad (5)$$

These results can be used to calculate the geographical position of an aircraft without the knowledge of relative time differences. In the AWAM system, the variables of $t_1(n)$ and $t_1(n+1)$ can be calculated by measuring the round trip times (RTTs) in the master sensor, and the elements $t_{i,1}(n)$ and $t_{i,1}(n+1)$ (for i = 2,…, 5) by measuring observed time difference of arrivals.
As mentioned above, the proposed method without the synchronization procedures can be called the AWAM method. On the other hand, it is necessary to find the coordinates of an aircraft (($x(n)$, $y(n)$, $z(n)$)) and (($x(n+1)$, $y(n+1)$, $z(n+1)$)). However, we must assume that we know:

- the coordinates of sensors S_i (for i = 1,…, 5),
- the distances between S_1 and the aircraft ($c\cdot t_1(n)$ and $c\cdot t_1(n+1)$),
- the observed differences in distance between S_1, S_i and the aircraft ($c\cdot t_{i,1}(n)$ and $c\cdot t_{i,1}(n+1)$) (for i = 2,…, 5) at the discrete time n and n+1.

The distance, d_{min}, between two distinct positions of an aircraft at the discrete time n and n+1, depends on the frequency of transmission of requests from the SSR to the AT (e.g. for aircraft speed 900 km/h and frequency requests 1 second, $d_{\min}$ approximately equals 250 m). Consequently, the assumption of constant the relative time differences (RTDs) between discrete time n and n+1 (in the range of several seconds) is true.

3. CALCULATION OF AN AIRCRAFT POSITION

A nonlinear system of equations is defined as [7]

$$\mathbf{f}(\mathbf{x})=\begin{bmatrix} f_1(\mathbf{x}) \\ f_2(\mathbf{x}) \\ \vdots \\ f_n(\mathbf{x}) \end{bmatrix}, \quad (6)$$

where $\mathbf{x} = (x_1, x_2, \ldots, x_n)^T$ is the vector of n independent variables, $\mathbf{f}$ is the column vector of nonlinear functions f_j ($j = 1, 2, \ldots, n$). Finding a solution for a nonlinear system of equations $\mathbf{f}(\mathbf{x})$ involves finding a solution such that every equation in the nonlinear system is 0, i.e.

$$\begin{cases} f_1(x_1, x_2, \ldots, x_n) = 0 \\ f_2(x_1, x_2, \ldots, x_n) = 0 \\ \vdots \\ f_n(x_1, x_2, \ldots, x_n) = 0 \end{cases}. \tag{7}$$

The Jacobian matrix for $\mathbf{f}(\mathbf{x})$ is given by

$$\mathbf{J} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}. \tag{8}$$

Now the system of nonlinear equations (5) can be rewritten as follows:

$$\begin{cases} f_1(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2 - a_1 \cdot x_1 - a_2 \cdot x_2 - a_3 \cdot x_3 + a_4 = 0 \\ f_2(x_1, x_2, x_3, x_4) = x_1^2 + x_2^2 + x_3^2 - x_4^2 - a_5 \cdot x_1 - a_6 \cdot x_2 - a_7 \cdot x_3 + 2 \cdot a_8 \cdot x_4 + a_9 = 0 \\ f_3(x_1, x_2, x_3, x_5) = x_1^2 + x_2^2 + x_3^2 - x_5^2 - a_{10} \cdot x_1 - a_{11} \cdot x_2 - a_{12} \cdot x_3 + 2 \cdot a_{13} \cdot x_5 + a_{14} = 0 \\ f_4(x_1, x_2, x_3, x_6) = x_1^2 + x_2^2 + x_3^2 - x_6^2 - a_{15} \cdot x_1 - a_{16} \cdot x_2 - a_{17} \cdot x_3 + 2 \cdot a_{18} \cdot x_6 + a_{19} = 0 \\ f_5(x_1, x_2, x_3, x_7) = x_1^2 + x_2^2 + x_3^2 - x_7^2 - a_{20} \cdot x_1 - a_{21} \cdot x_2 - a_{22} \cdot x_3 + 2 \cdot a_{23} \cdot x_7 + a_{24} = 0 \\ f_6(x_8, x_9, x_{10}) = x_8^2 + x_9^2 + x_{10}^2 - a_1 \cdot x_8 - a_2 \cdot x_9 - a_3 \cdot x_{10} + a_{25} = 0 \\ f_7(x_4, x_8, x_9, x_{10}) = x_8^2 + x_9^2 + x_{10}^2 - x_4^2 - a_5 \cdot x_8 - a_6 \cdot x_9 - a_7 \cdot x_{10} + 2 \cdot a_{26} \cdot x_4 + a_{27} = 0 \\ f_8(x_5, x_8, x_9, x_{10}) = x_8^2 + x_9^2 + x_{10}^2 - x_5^2 - a_{10} \cdot x_8 - a_{11} \cdot x_9 - a_{12} \cdot x_{10} + 2 \cdot a_{28} \cdot x_5 + a_{29} = 0 \\ f_9(x_6, x_8, x_9, x_{10}) = x_8^2 + x_9^2 + x_{10}^2 - x_6^2 - a_{15} \cdot x_8 - a_{16} \cdot x_9 - a_{17} \cdot x_{10} + 2 \cdot a_{30} \cdot x_6 + a_{31} = 0 \\ f_{10}(x_7, x_8, x_9, x_{10}) = x_8^2 + x_9^2 + x_{10}^2 - x_7^2 - a_{20} \cdot x_8 - a_{21} \cdot x_9 - a_{22} \cdot x_{10} + 2 \cdot a_{32} \cdot x_7 + a_{33} = 0 \end{cases}, \tag{9a}$$

where

$$\begin{gathered} x_1 = x(n),\ x_2 = y(n),\ x_3 = z(n), \\ x_4 = c \cdot \Delta t_{2,1},\ x_5 = c \cdot \Delta t_{3,1},\ x_6 = c \cdot \Delta t_{4,1},\ x_7 = c \cdot \Delta t_{5,1}, \\ x_8 = x(n+1),\ x_9 = y(n+1),\ x_{10} = z(n+1), \end{gathered} \tag{9b}$$

and

$$\begin{gathered} a_1 = 2 \cdot X_1,\ a_2 = 2 \cdot Y_1,\ a_3 = 2 \cdot Z_1,\ a_4 = X_1^2 + Y_1^2 + Z_1^2 - (c \cdot t_1(n))^2, \\ a_5 = 2 \cdot X_2,\ a_6 = 2 \cdot Y_2,\ a_7 = 2 \cdot Z_2,\ a_8 = c \cdot t_1(n) + c \cdot t_{2,1}(n),\ a_9 = X_2^2 + Y_2^2 + Z_2^2 - a_8^2, \\ a_{10} = 2 \cdot X_3,\ a_{11} = 2 \cdot Y_3,\ a_{12} = 2 \cdot Z_3,\ a_{13} = c \cdot t_1(n) + c \cdot t_{3,1}(n),\ a_{14} = X_3^2 + Y_3^2 + Z_3^2 - a_{13}^2, \\ a_{15} = 2 \cdot X_4,\ a_{16} = 2 \cdot Y_4,\ a_{17} = 2 \cdot Z_4,\ a_{18} = c \cdot t_1(n) + c \cdot t_{4,1}(n),\ a_{19} = X_4^2 + Y_4^2 + Z_4^2 - a_{18}^2, \\ a_{20} = 2 \cdot X_5,\ a_{21} = 2 \cdot Y_5,\ a_{22} = 2 \cdot Z_5,\ a_{23} = c \cdot t_1(n) + c \cdot t_{5,1}(n), \\ a_{24} = X_5^2 + Y_5^2 + Z_5^2 - a_{23}^2,\ a_{25} = X_1^2 + Y_1^2 + Z_1^2 - (c \cdot t_1(n+1))^2, \\ a_{26} = c \cdot t_1(n+1) + c \cdot t_{2,1}(n+1),\ a_{27} = X_2^2 + Y_2^2 + Z_2^2 - a_{26}^2,\ a_{28} = c \cdot t_1(n+1) + c \cdot t_{3,1}(n+1), \\ a_{29} = X_3^2 + Y_3^2 + Z_3^2 - a_{28}^2,\ a_{30} = c \cdot t_1(n+1) + c \cdot t_{4,1}(n+1),\ a_{31} = X_4^2 + Y_4^2 + Z_4^2 - a_{30}^2 \end{gathered} \tag{9c}$$

$$a_{32} = c \cdot t_1(n+1) + c \cdot t_{5,1}(n+1),\ a_{33} = X_5^2 + Y_5^2 + Z_5^2 - a_{32}^2 .$$

In our case, the Jacobian matrix is equal

$$\mathbf{J}_{10x10} = \begin{bmatrix} 2x_1 - a_1 & 2x_2 - a_2 & 2x_3 - a_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2x_1 - a_5 & 2x_2 - a_6 & 2x_3 - a_7 & -2x_4 + 2a_8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2x_1 - a_{10} & 2x_2 - a_{11} & 2x_3 - a_{12} & 0 & -2x_5 + 2a_{13} & 0 & 0 & 0 & 0 & 0 \\ 2x_1 - a_{15} & 2x_2 - a_{16} & 2x_3 - a_{17} & 0 & 0 & -2x_6 + 2a_{18} & 0 & 0 & 0 & 0 \\ 2x_1 - a_{20} & 2x_2 - a_{21} & 2x_3 - a_{22} & 0 & 0 & 0 & -2x_7 + 2a_{23} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2x_8 - a_1 & 2x_9 - a_2 & 2x_{10} - a_3 \\ 0 & 0 & 0 & -2x_4 + 2a_{26} & 0 & 0 & 0 & 2x_8 - a_5 & 2x_9 - a_6 & 2x_{10} - a_7 \\ 0 & 0 & 0 & 0 & -2x_5 + 2a_{28} & 0 & 0 & 2x_8 - a_{10} & 2x_9 - a_{11} & 2x_{10} - a_{12} \\ 0 & 0 & 0 & 0 & 0 & -2x_6 + 2a_{30} & 0 & 2x_8 - a_{15} & 2x_9 - a_{16} & 2x_{10} - a_{17} \\ 0 & 0 & 0 & 0 & 0 & 0 & -2x_7 + 2a_{32} & 2x_8 - a_{20} & 2x_9 - a_{21} & 2x_{10} - a_{22} \end{bmatrix} . \quad (10)$$

There are several ways to solve nonlinear equation systems (7). The most popular technique is probably Newton's method. The method (gradient method) is a generalized process to find an accurate root of system equations by approximating the residual function over an interval using other simpler functions that can be solved directly and easily. The residual functions are simply equations describing the process of interest, organized in such a way that the sum of the relevant factors equals zero. Newton's method uses the Jacobian matrix of the system of equations. In our case we consider the following methods for the solution of nonlinear equations (9):

– Newton's method [8],
– Broyden's method [9].

Next, the above methods will be introduced briefly. The nonlinear system of equations (7) can be written more concisely as

$$\mathbf{f}(\mathbf{x}) = 0 . \quad (11)$$

If $\mathbf{x}_i$ is the *i*-th approximation to the solution of (11) and $\mathbf{f}_i$ is written for $\mathbf{f}(\mathbf{x}_i)$, then Newton's method is defined by [8]

$$\mathbf{x}_{i+1} = \mathbf{x}_i - \mathbf{J}_i^{-1}\mathbf{f}_i , \quad (12)$$

where $\mathbf{J}_i^{-1}$ is the Jacobian matrix evaluated at $\mathbf{x}_i$. Let us denote the approximate Jacobian by $\mathbf{B}$. Then the *i*-th step $\delta\mathbf{x}_i$ is the solution of [10]

$$\mathbf{B}_i \cdot \delta\mathbf{x}_i = -\mathbf{f}_i \Rightarrow \delta\mathbf{x}_i = -\mathbf{B}_i^{-1}\mathbf{f}_i \,, \tag{13}$$

where $\delta\mathbf{x}_i = \mathbf{x}_{i+1} - \mathbf{x}_i$. The secant condition is that $\mathbf{B}_{i+1}$ satisfies

$$\mathbf{B}_{i+1} \cdot \delta\mathbf{x}_i = -\delta\mathbf{f}_i \,, \tag{14}$$

where $\delta\mathbf{f}_i = \mathbf{f}_{i+1} - \mathbf{f}_i$. Many different equations for calculating $\mathbf{B}_{i+1}$ have been explored, but one of the best-performing algorithms in practice results from Broyden's formula [9]

$$\mathbf{B}_{i+1} = \mathbf{B}_i + \frac{(\delta\mathbf{f}_i - \mathbf{B}_i\delta\mathbf{x}_i)\cdot\delta\mathbf{x}_i}{\delta\mathbf{x}_i^T \cdot \delta\mathbf{x}_i} \,, \tag{15}$$

Next, the results of numerical calculations are presented. These calculations were carried out in order to estimate the effectiveness of solving nonlinear equations for the new location method AWAM in the wide area multilateration system.

4. NUMERICAL RESULTS

To test the new method for the wide area multilateration system considers the particular case shown in Figure 2. The figure presents a hypothetical WAM system for the territory of Poland. There are four slave sensors and the master sensor. Additionally, the master sensor cooperates with the secondary surveillance radar SSR. Next to each sensor given its two types coordinates: geodetic coordinates (latitude, longitude and altitude) and Cartesian coordinates (X, Y, Z). In the area under consideration is a plane at an altitude of 10 kilometres and is moving at a speed 900 km/h. The SSR sends test signals in the direction of the aircraft at about 1 second. In numerical calculations, m = 10 000 the initializing vectors for the iterative process were considered. Using the two methods outlined above (Newton's method and Broyden's method), the aircraft position in Figure 2 was estimated. The following parameters were thus chosen as the measure of the effectiveness of methods for solving a particular problem:

– the mean value of the number of iteration M, where M is given by

$$M = \frac{1}{m}\sum_{k=1}^{m} I_k \,, \tag{16}$$

and where I_k is the total number of function evaluations during an iteration process,

– the mean convergence rate V, where V equals

$$V = \frac{1}{m}\sum_{k=1}^{m} v_k \,, \tag{17}$$

and where v_k is given by [9]

$$v_k = \frac{1}{I_k}\ln\frac{N_k^i}{N_k^f} \,, \tag{18}$$

N_k^i, N_k^f are initial and final Euclidean norms of $\mathbf{f}$ for k-th calculation,

– the percentage P of estimated position of the aircraft, for which the absolute position error Δd, expressed in the following equation, is smaller than 1 m

$$\Delta d = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2} \,, \tag{19}$$

where x, y and z represent the real coordinates of an aircraft and x_0, y_0 and z_0 denote the estimated coordinates of an aircraft.

In order to compare the performance of the various methods, each problem was started from the same initial conditions. Each initial condition was located inside a sphere with radius *R* and at the centre of the real coordinates of the aircraft. In each case the iteration was terminated as soon as the Euclidean norm of the vector **f** became less than a tolerance arbitrarily chosen to be 1. The performance of the methods is summarized in Table 1 and Table 2, which give the size of radius *R*, the mean value of the number of iteration *M*, the mean convergence rate *V* and the percentage *P* of the estimated position of the aircraft, for which the absolute errors Δd at the discrete time *n* and *n*+1 were smaller than 1 m.

Next, in the Table 3 partial results of the iterative process of solving nonlinear equations for the case illustrated in Figure 2 are introduced. The exemplary initializing vector for the iterative process was as follows:

$$\mathbf{x}_0 = (3565954, 1165634, 5140435, 0, 0, 0, 0, 3566054, 1165734, 5140535)^T . \quad (20)$$

The initial value of variables x_1, x_2, x_3 are equal to $(X_1+X_2+X_3+X_4+X_5)/5$, $(Y_1+Y_2+Y_3+Y_4+Y_5)/5$ and $(Z_1+Z_2+Z_3+Z_4+Z_5)/5$ respectively. The initial value of variables x_8, x_9 and x_{10} are equal to x_1+100, x_2+100 and x_3+100. The real values of variables x_4, x_5, x_6 and x_7 which represent the products $\Delta t_{2,1}\cdot c$, $\Delta t_{3,1}\cdot c$, $\Delta t_{4,1}\cdot c$ and $\Delta t_{5,1}\cdot c$ are equal to 5235 m, 6251 m, 3736 m and 6854 m respectively.

On the basis of the above results, the following conclusions are put forward. Broyden's method gave the best results of solving nonlinear equations, i.e. a relatively small number of iterations and almost one hundred percent efficiency in the search for an appropriate solution. The number of correct solutions for all considered methods practically does not depend on the values of the initializing vector in the iterative process. Thereafter the impact of errors on the accuracy of the aircraft position estimation in the AWAM system was analyzed.

5. ERRORS IN AWAM SYSTEM

Statistical model of AWAM errors is very similar to the MLAT errors and can distinguish three basic error categories [11]:

- systematic errors,
- correlated errors,
- random errors.

Systematic errors are associated with the local troposphere changes, imperfect antennas position measurement and varying hardware parameters as a function of temperature. This type of errors can be significantly reduced by continuous calibration process. Correlated errors are caused among others by the multipath signal propagation. These errors are difficult to eliminate. The random errors have white noise character and are produced by additive external and/or internal noise at received signals, by quantization effects due to the digitalization of the measurements and by clock jitter between the master sensor and the aircraft transponder and between sensors (master – slaves) in the asynchronous mode.

Jitter of the leading edge of the pulse, caused by noise is equal [11]:

$$\sigma_{SNR} = \frac{t_r}{\sqrt{2 \cdot SNR}} , \quad (21)$$

where σ_{SNR} is the standard deviation of the above jitter, t_r represents the length of the pulse leading edge and SNR is the signal to noise ratio. For the SSR pulse with typically t_r = 70 ns and SNR = 18 dB, the σ_{SNR} = 6.2 ns. The quantization error σ_q is typically equal

3.6 ns [11]. The RTT measurement parameter in the new method AWAM is vitiated by a clock drift effect. Therefore the master sensor can only estimate the propagation delay $\hat{\tau}_p$ to the AT [12]:

$$\hat{\tau}_p = \tau_p \cdot (1+\delta_{S_1}) + \frac{\varepsilon \cdot \tau_{rd}}{2 \cdot (1+\delta_{S_1} - \varepsilon)}, \tag{22}$$

where τ_p is the true propagation delay, δ_{S_1} is the clock drift of the master sensor, $\varepsilon = \delta_{S_1} - \delta_{AT}$ is the relative clock offset, δ_{AT} is the clock drift of the aircraft transponder and τ_{rd} represents the effective response delay generated by the AT. The ε parameter ranges from 10^{-7} to 10^{-5}. The time delay τ_{rd} has been specified by the International Civil Aviation Organization to be 3 μs ± 0.5 μs [13]. The ± 0.5 μs time spread makes a relatively large distance estimation inaccuracy. In addition, a crystal oscillator used in the sensor is also a source of errors in the AWAM solution. This type of errors can be described by total root mean square (RMS) jitter J_i (i = 1, …, 5) in the time domain, corresponding to phase noise in the frequency domain. The typical jitter for a high quality signal generator equals 1 ps [14]. Let's assume that clock errors are statistically independent for pairs of sensors and therefore

$$J_{1,p} = \sqrt{J_1^2 + J_p^2} \quad \text{for} \quad p = 2, \dots, 5. \tag{23}$$

Using the above random errors description the analysis of their effect on the accuracy of estimating the position of the aircraft for the case illustrated in Figure 2 was carried out. The AWAM system simulator in the MATLAB environment was written. During the testing, the errors of OTDOAs ($\Delta\varepsilon_{total}$) and the errors of signal of transfer times $t_1(n)$ and $t_1(n+1)$ from the sensor S_1 to the aircraft transponder ($\Delta\varepsilon_\tau$) were considered. For modelling the above errors the MATLAB function of normally distributed random numbers (*randn*) was used:

$$\Delta\varepsilon_{total} = \varepsilon_{total} \cdot randn \quad \text{for} \quad \varepsilon_{total} = 10^{-9}s \quad \text{or} \quad \varepsilon_{total} = 10^{-10}s \tag{24a}$$

$$\Delta\varepsilon_\tau = \varepsilon_\tau \cdot randn \quad \text{for} \quad \varepsilon_\tau = 10^{-8}s \quad \text{or} \quad \varepsilon_\tau = 10^{-9}s \quad \text{or} \quad \varepsilon_\tau = 10^{-10}s \tag{24b}$$

In the same way the spread of time delay was modelled.

$$\Delta\tau_{rd} = \tau_{rd} \cdot randn \quad \text{for} \quad \tau_{rd} = 10^{-7}s \quad \text{or} \quad \tau_{rd} = 10^{-8}s \tag{24c}$$

The simulation results for case described on the Figure 2 are shown in Figure 3 and Figure 4. For the solution of equations (9) the Broyden's method was used. The best results were obtained for high-precision oscillators in the measuring sensors. For the results have a significant impact the spread of time delay $\Delta\tau_{rd}$ which depends on the practical implementation of the aircraft transponder. The simulation studies were carried out in each case for the same initial vector described by the equation (20). Of course, in a real AWAM system the estimated position of the aircraft at the current time will be used in the process of calculating the position of the aircraft at the next time. Certainly, this will increase the accuracy of the determination of the actual aircraft position.

6. SIMULATION RESULTS

The experiments were carried out by using of the simulation model shown in Figure 5 [15]. It consists of an aircraft that performs a left turn after a straight segment which is continued by another straight segment. It goes from the west to the east at a constant velocity of

250 m/s. There are four slave sensors and the master sensor, which cooperates with the SSR. The SSR sends test signal at about 4 second – it is a typical value for Gdansk airport in Poland (International Civil Aviation Organization airport code – EPGD). The altitude aircraft was constant at 10 km. The altitude sensors were equal about 100 m. The cumulative probability distribution function (CDF) of the absolute position error (19) was obtain from the simulation investigations. The simulation results are shown in Figure 6. During the simulation process the particular parameters were equal: $\varepsilon_{total} = 10^{-10}$, $\varepsilon_{\tau} = 10^{-10}$ and $\tau_{rd} = 10^{-8}$. In our case, the aircraft is located within 425 m in 60 % of the time. As we can see, the proposed method gives quite good results and significantly simplifies the localization process in the wide area multilateration system.

7. CONCLUSIONS

A new approach for the aircraft location in the WAM system is proposed. As a result, a new method for calculating the geographical position of an aircraft without the knowledge of relative time differences, called AWAM, is suggested. The new method requires co-operation of one of the sensors with SSR to determine the round trip time parameter. Implementation of the classic TDOA method for WAM system is rather expensive. Taking into account the obtained results, the proposed new method for calculating the geographical position of an aircraft is an exemplary low cost alternative to the classic one.
At the end it is worth mentioning that the new method can be also successfully used during synchronization failure in the classical implementation of WAM system.

REFERENCES

[1] H. Niles, R. S. Conker, M. Bakry El-Arini, D. G. O'Laughlin, D. V. Baraban, Wide Area Multilateration for Alternate Position, Navigation, and Timing (APNT), Federal Aviation Administration, United State of America.

[2] W. H. L. Neven, T. J. Quilter, R. Weedon, R. A. Hogendoorn, Wide Area Multilateration, Report on EATMP TRS 131/04, ver. 1.1, National Aerospace Laboratory NLR, 2005.

[3] M. Pelant, V. Stejskal, Multilateration System Time Synchronization via Over-Determination of TDOA Measurements, IEEE Workshop on Digital Communications - Enhanced Surveillance of Aircraft and Vehicles, 2011 Tyrrhenian International, 2011, pp. 179 – 183.

[4] J. Stefanski, Simplified Algorithm for Location Service for the UMTS, Proceedings of IEEE Vehicular Technology Conference, vol. 4, 2005, pp. 2741-2744.

[5] J. Stefanski, Low Cost Method for Location Service in the WCDMA System, Nonlinear Analysis: Real World Applications, vol. 14, iss. 1, 2013, pp. 626–634.

[6] Y. T. Chan, K. C. Ho, A Simple and Efficient Estimator for Hyperbolic Location, IEEE Transactions on Signal Processing, vol. 42, no. 8, 1994, pp. 1905-1915.

[7] C. Grosan, A. Abraham, A New Approach for Solving Nonlinear Equations Systems, IEEE Transactions on Systems, Man, and Cybernetics – Part A: Systems and Humans, vol. 38, no. 3, 2008, pp. 698-714.

[8] J. E. Denis, On Newton's Method and Nonlinear Simultaneous Replacements, SIAM Journal of Numerical Analysis, vol. 4, 1967, pp. 103-108.

[9] C. G. Broyden, A Class of Methods for Solving Nonlinear Simultaneous Equations, Mathematics of Computation (American Mathematical Society), vol. 19, no. 92, 1965, pp. 577–593.

[10] C. L. Karr, B. Weck, L. M. Freeman, Solutions to Systems of Nonlinear Equations via a Genetic Algorithm, Engineering Applications of Artificial Intelligence, vol. 11, no. 3, 1998, pp. 369-375.

[11] Y. Trofimova, Multilateration Error Investigation and Classification. Error Estimation, Transport and Telecommunication, vol. 8, no. 2, 2007, pp. 28-37.

[12] D. Dardari, U. Ferner, M. Z. Win, Ranging with Ultrawide Bandwidth Signals in Multipath Environments, Proceedings of the IEEE, vol. 97, no. 2, 2009, pp. 404-426.

[13] N. L. R. Petrochilos, Algorithms for Separation of Secondary Surveillance Radar Replies, DUP Science, Netherlands, 2002.

[14] W. Kester, Converting Oscillator Phase Noise to Time Jitter, Analog Devices, 2009.

[15] J. Garcia, A. Sato, G. de Miguel, J. Besada, P. Tarrio, Trajectory Reconstruction Techniques for Evaluation of ACT Systems, International Workshop on Digital Communications – Enhanced Surveillance of Aircraft and Vehicles, Italy, 2008, pp.1-6.

Table 1. Results for Newton's method.

R **[m]**	**100**	**200**	**500**	**1000**	**2000**	**5000**
M	534 520	573 450	622 810	661 530	699 520	747 170
V	$3.1365 \cdot 10^{-5}$	$3.0216 \cdot 10^{-5}$	$2.9377 \cdot 10^{-5}$	$2{,}86326 \cdot 10^{-5}$	$2.8081 \cdot 10^{-5}$	$7.7568 \cdot 10^{-5}$
P	100	100	100	100	100	100

Table 1

Table 2. Results for Broyden's method.

R [m]	100	200	500	1000	2000	5000
M	260	333	437	500	595	835
V	0.0724	0.0735	0.0678	0.0533	0.0373	0.0326
P	100	100	100	100	99	98

Table 2

Table 3. Partial results during the process of solving nonlinear equations for the case illustrated in Figure 2.

Iteration	Newton's method			Broyden's method		
	$\|\|\mathbf{f}\|\|$	$\Delta d(n)$ [m]	$\Delta d(n+1)$ [m]	$\|\|\mathbf{f}\|\|$	$\Delta d(n)$ [m]	$\Delta d(n+1)$ [m]
1	$4.4290 \cdot 10^9$	$1.4965 \cdot 10^4$	$1.4984 \cdot 10^4$	$4.4290 \cdot 10^9$	$1.4965 \cdot 10^4$	$1.4984 \cdot 10^4$
2	$7.9006 \cdot 10^8$	$2.5034 \cdot 10^4$	$2.4776 \cdot 10^4$	$9.5469 \cdot 10^7$	$1.3275 \cdot 10^4$	$1.3204 \cdot 10^4$
…	…	…	…	…	…	…
7747	$2.2694 \cdot 10^6$	$1.1834 \cdot 10^4$	$1.1463 \cdot 10^4$	1.0549	0.0023	0.0023
7748	$2.2693 \cdot 10^6$	$1.1834 \cdot 10^4$	$1.1462 \cdot 10^4$	0.9007	0.0023	0.0023
7749	$2.2692 \cdot 10^6$	$1.1833 \cdot 10^4$	$1.1462 \cdot 10^4$	**STOP**	**STOP**	**STOP**
…	…	…	…	---	---	---
808810	1.1018	0.0061	0.0060	---	---	---
808811	0.9927	0.0060	0.0059	---	---	---
808812	**STOP**	**STOP**	**STOP**	---	---	---

Table 3

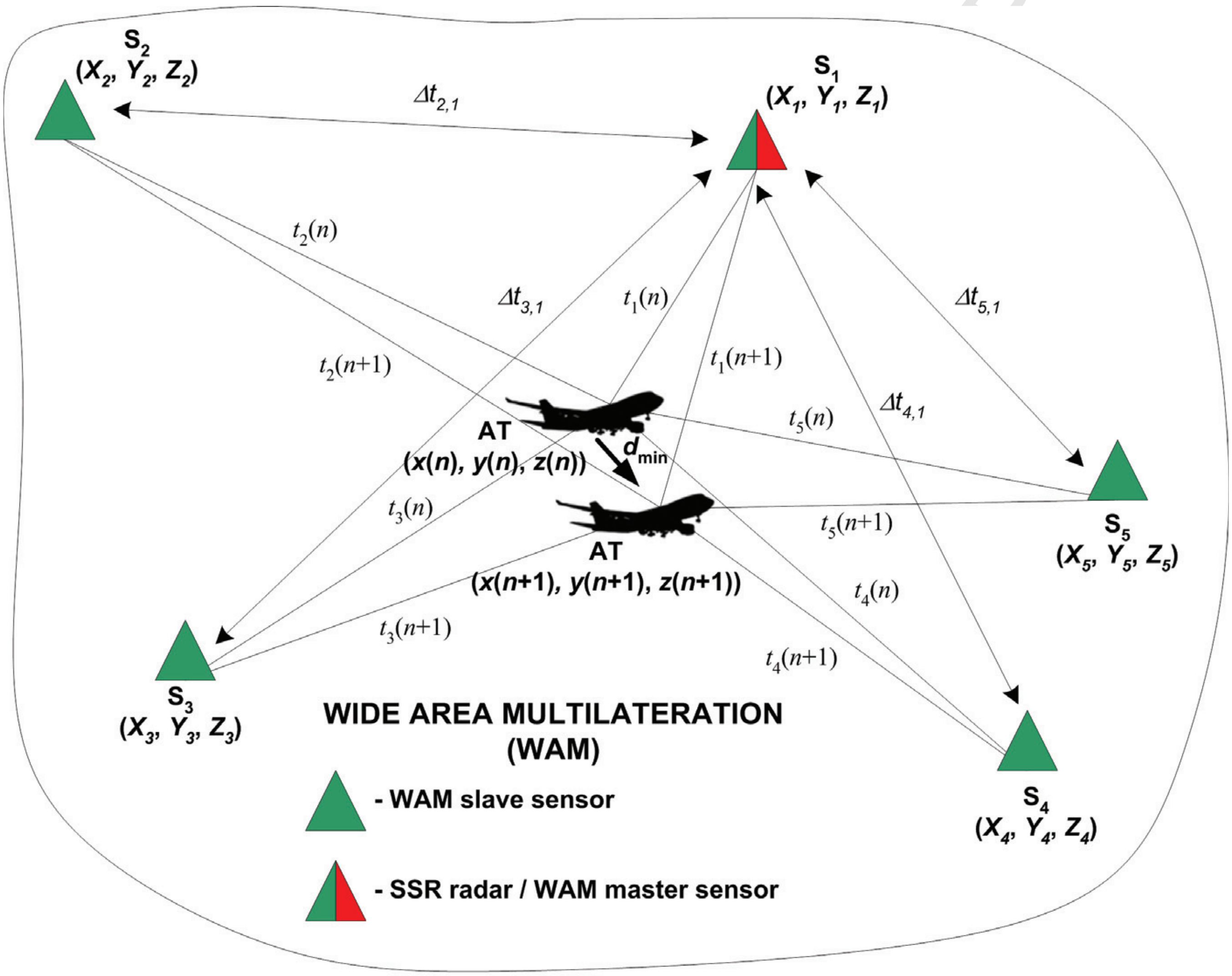

Figure 1. Graphical representation of the problem under consideration.

Figure 1

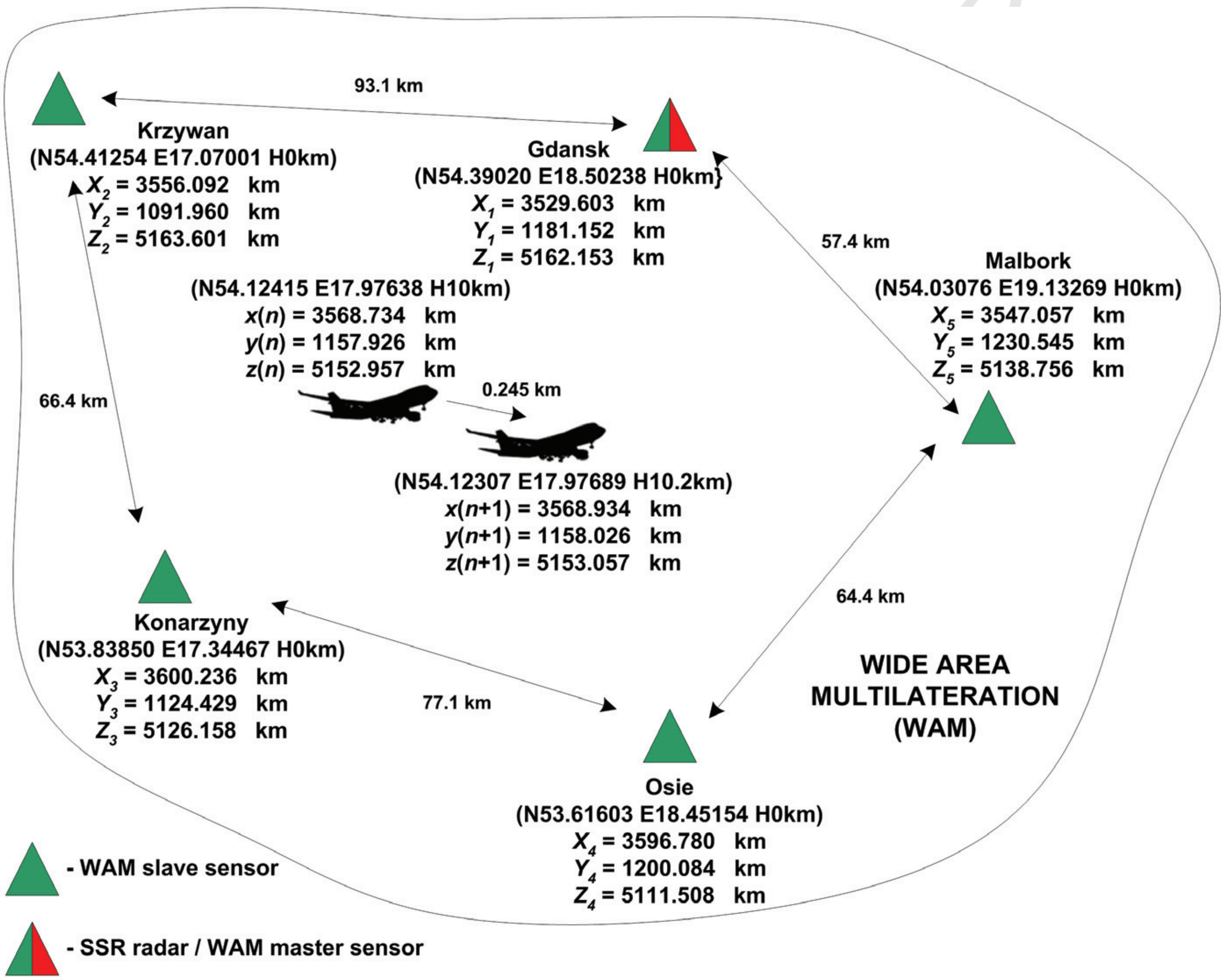

Figure 2. Illustration of the particular case.

Figure 2

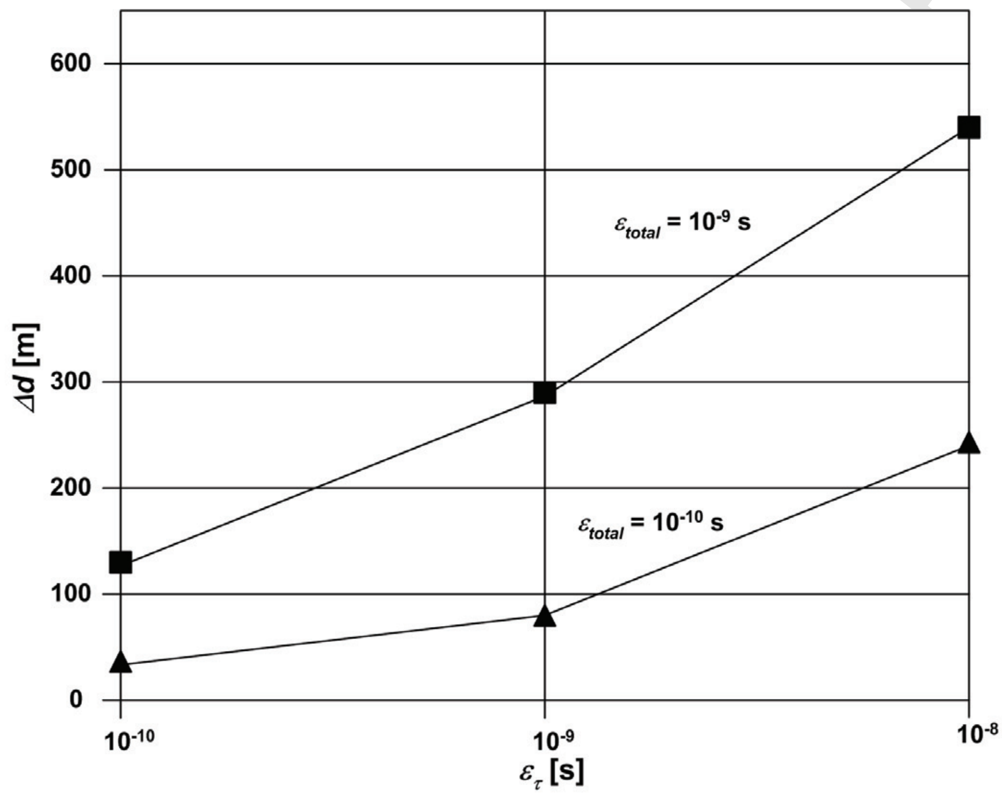

Figure 3. The absolute error Δd as a function of ε_τ for two values of the ε_{total} - parameter is the spread of time delay $\Delta\tau_{rd}$ = 10^{-8}.

Figure 3

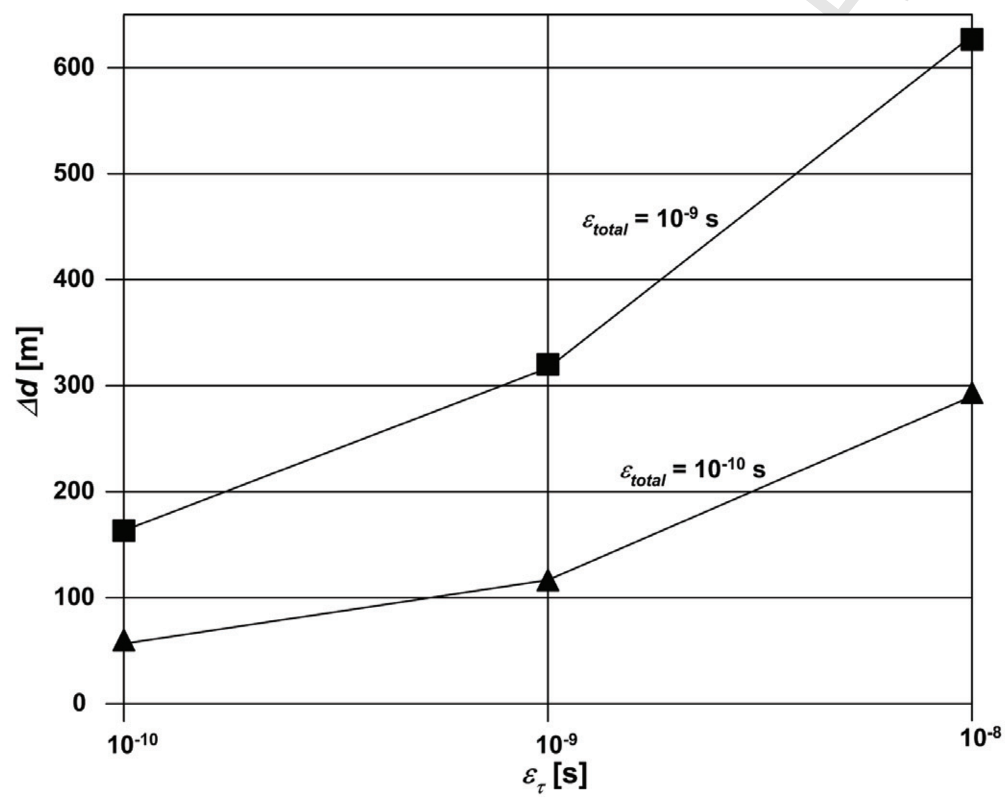

Figure 4. The absolute error Δd as a function of ε_τ for two values of the ε_{total} - parameter is the spread of time delay $\Delta\tau_{rd} = 10^{-7}$.

Figure 4

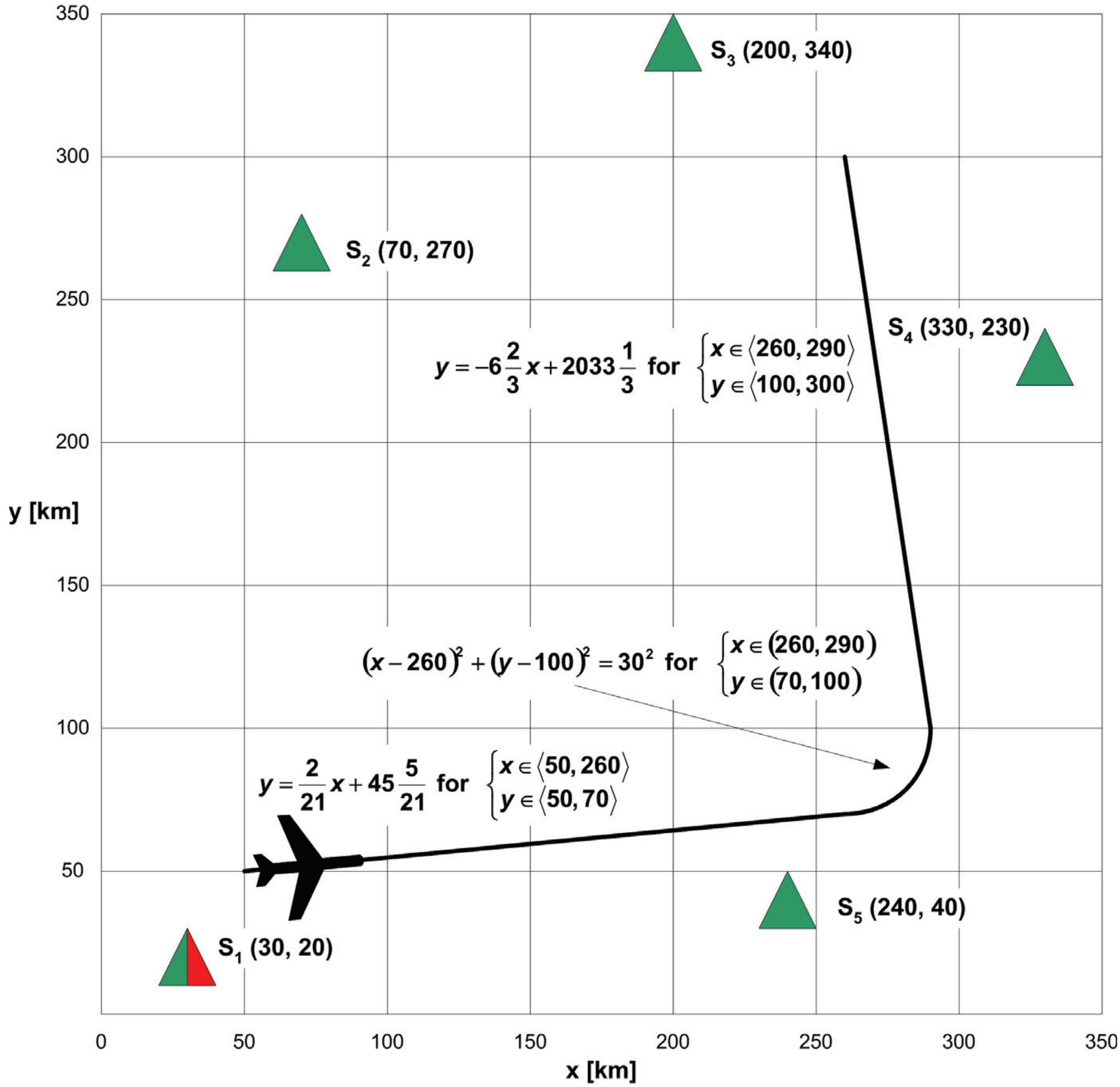

Figure 5. Simulation model.

Figure 5

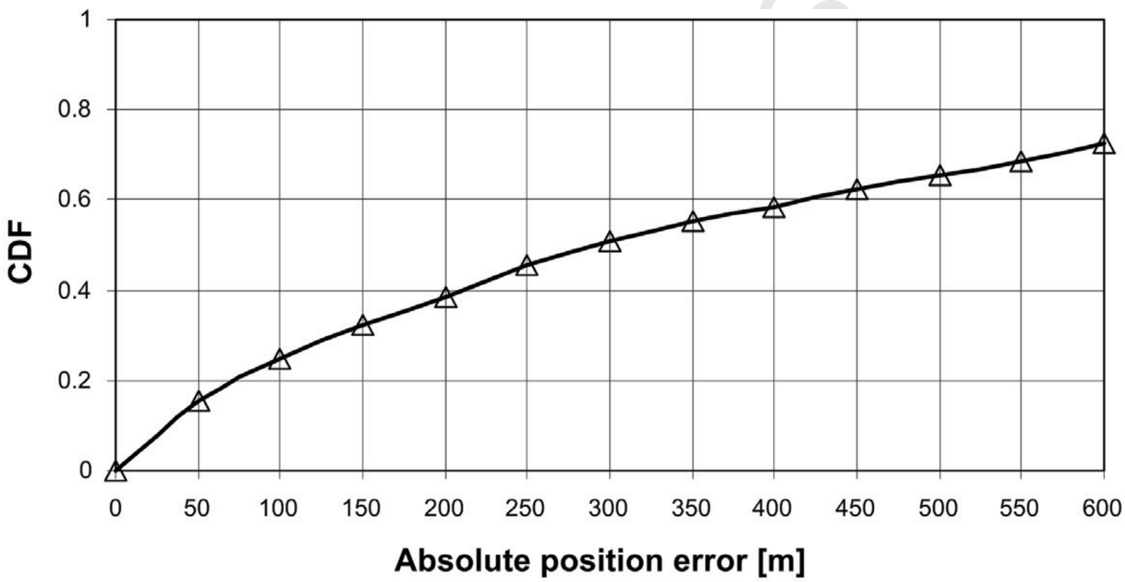

Figure 6. The CDF of absolute position error for the proposed method for WAM system.

Figure 6