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Research Article

Time Delay Estimation in Two-Phase Flow Investigation Using the γ-Ray Attenuation Technique

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Time delay estimation is an important research question having many applications in a range of technologies. Measurement of a two-phase flow in a pipeline or an open channel using radioisotopes is an example of such application. For instance, the determination of velocity of dispersed phase in that case is based on estimation of the time delay between two stochastic signals provided by scintillation probes. The proper analysis of such signals, usually in presence of noise, requires the use of advanced statistical signal processing. In this paper, the simulation studies of time delay estimation were carried out with the use of the following differential methods: average magnitude difference function, and average square difference function and proposed combined methods comprising the above-mentioned differential and cross-correlation functions are presented. Attached simulations have been carried out for models of stochastic signals corresponding to the signals obtained in gamma-ray absorption measurements of gas-liquid flow in a horizontal pipeline. The standard uncertainties of time delay estimations have been determined for each of the methods. Improved metrological properties have been stated in the combined methods in comparison with the classical cross-correlation procedure.

1. Introduction

Determination of the time delay between signals is a significant issue in many fields such as radar and sonar technologies, seismology, communications, and medical diagnostics. Over the past few decades numerous methods and algorithms have been developed ranging from cross-correlation to the advanced blind channel identification techniques [1–6]. The scope of particular application method depends on the type of the analysed signals (random or determined, stationary or nonstationary) and their parameters (distribution of probability, as well as the signal-to-noise ratio).

Estimation of the time delay is also essential in contactless measurements of flow parameters, especially in the following two-phase mixtures transportation as liquid-gas, liquid-solids, or gas-solids. In these measurements, mutually delayed stochastic signals may be provided by sensors of such type as capacitance, optical, electrostatic, and thermometric or scintillation probes, situated on a wall of a pipeline or open channel [7–9]. In such case, the measurement of time delay allows us the mean velocity determination of the flow dispersed phase.

Two-phase flow measurements utilizing radioactive isotopes have been used for more than 50 years. Those measurements can be divided into two types: the tracer method and the absorption based on an analysis of a mixture flowing through a beam of photons emitted by a closed gamma-ray source [10–16]. The latter method is noninvasive and relatively safe in application.

The analysis of random signals provided by scintillation probes requires the usage of advanced statistical methods of signal processing in both time and frequency domain. Because these signals, after proper preprocessing, may be ergodic and Gaussian, such classical methods as the cross-correlation function (CCF) and the phase of cross-spectral density may be applied [1, 3, 9–12, 14, 16–22]. Recently, not so popular methods of the time delay estimation, as differential [3, 23–25], the cross-correlation with use of the Hilbert Transform [17, 26–29], and the method based on conditional averaging of the signals [30–33], have appeared. The usefulness of the last three in two-phase flow investigation by radioisotopes is not known in detail. They can be, in the first stage, examined using simulation.

The first part of the paper presents two mathematical models used in estimation of the time delay of random signals (Section 2). Then, the cross-correlation method is described together with the differential one as average magnitude difference function (AMDF), average square difference function (ASDF), and an original one combining the differential functions and the CCF (Section 3). Section 4 describes the basis of two-phase flow velocity evaluation by application of the gamma-ray absorption method. Consequently, the next section presents exemplary results of simulation studies of the metrological properties of the above functions, performed on the basis of mutually delayed stochastic signals simulation. The parameters of the above-mentioned models have been selected in a way which ensures the proper correspondence with real signals obtained in gamma-ray absorption measurements of a liquid-gas flow in a horizontal pipeline. The results of time delays and their standard uncertainties received in differential and combined methods have been compared with corresponding results of the classical cross-correlation procedure. Finally, Section 6 summarizes results and presents conclusions of the paper.

2. Signal Models in Time Delay Estimation of the Random Signals

In many time delay estimation issues, the relations between $x(t)$ and $y(t)$ signals received from two sensors can be expressed by the following formulas [17, 20]:

$$x(t) = s(t) + m(t), \tag{1}$$

$$y(t) = c \cdot s(t - \tau_0) + n(t), \tag{2}$$

where $s(t)$ is a stationary random signal with Gaussian $\mathbf{N}(0, \sigma_s)$ distribution of probability, frequency band B, and one-sided power spectral density:

$$G_{ss}(f) = \begin{cases} K & f \le B \\ 0 & f > B, \end{cases} \tag{3}$$

where c is a coefficient; τ_0 is the most probable transportation time delay; $m(t)$ and $n(t)$ are white noises with Gaussian $\mathbf{N}(0, \sigma_m)$, $\mathbf{N}(0, \sigma_n)$ distributions, not correlated with $s(t)$ and mutually not cross-correlated.

The autocorrelation function of the $s(t)$ signal can be expressed as

$$R_{ss}(\tau) = KB\left(\frac{\sin 2\pi B\tau}{2\pi B\tau}\right). \tag{4}$$

The following assumptions concerning the models of signals (1) can be stated:

$$\begin{aligned} \sigma_x^2 &= R_{xx}(0) = \sigma_s^2 + \sigma_m^2, \\ \sigma_y^2 &= R_{yy}(0) = c^2\sigma_s^2 + \sigma_n^2, \end{aligned} \tag{5}$$

where σ_x, and σ_y are standard deviations of $x(t)$ and $y(t)$ signals and $R_{xx}(\cdot)$, $R_{yy}(\cdot)$ are their autocorrelation functions.

The signal-to-noise ratio (SNR) for (1) and (2) can be defined correspondingly as $\text{SNR}_x = (\sigma_s/\sigma_m)^2$ for $x(t)$ and $\text{SNR}_y = (\sigma_s/\sigma_n)^2$ for $y(t)$.

Depending on presence of the noise in one or both signals, three models can be considered [32], but in practice only two cases are worthwhile of deliberation:

(i) Model I: $\sigma_m = 0$, $\sigma_n = \sigma_z \neq 0$ and $\text{SNR}_y = \text{SNR}$; then

$$y(t) = c \cdot s(t - \tau_0) + z(t) = c \cdot x(t - \tau_0) + z(t), \tag{6}$$

$$\text{SNR} = \left(\frac{\sigma_s}{\sigma_z}\right)^2 = \left(\frac{\sigma_x}{\sigma_z}\right)^2, \tag{7}$$

(ii) Model II: $\sigma_m = \sigma_n = \sigma_z \neq 0$ and $\text{SNR}_x = \text{SNR}_y = \text{SNR}$; then

$$\begin{aligned} x(t) &= s(t) + z_1(t), \\ y(t) &= c \cdot s(t - \tau_0) + z_2(t), \end{aligned} \tag{8}$$

$$\text{SNR} = \left(\frac{\sigma_s}{\sigma_z}\right)^2. \tag{9}$$

The disturbing signals in both channels have identical distributions $\mathbf{N}(0, \sigma_z)$ but have distinct, not cross-correlated realisations.

3. Selected Statistical Methods of Time Delay Estimation

3.1. Cross-Correlation. A cross-correlation function of $x(t)$ and $y(t)$ ergodic signals is defined by the following formula:

$$R_{\text{CCF}}(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_0^T x(t)\, y(t+\tau)\, dt, \tag{10}$$

where T is the averaging time and τ is time delay [17].

The τ_0 transportation time delay is determined on position of the main maximum of the CCF. The typical waveform of the cross-correlation function is shown in Figure 1.

The normalized CCF value for $\tau = \tau_0$ can be expressed in the following form [17]:

$$\rho_{\text{CCF}}(\tau_0) = \frac{R_{\text{CCF}}(\tau_0)}{\sqrt{R_{xx}(0)\, R_{yy}(0)}} = \frac{cR_{ss}(0)}{\sigma_x\sigma_y} = \frac{c\sigma_s^2}{\sigma_x\sigma_y}. \tag{11}$$

When we substitute in (11) the formulas (5), as well as $c = 1$, then for $\rho_{\text{CCF}}(\tau_0) = f(\text{SNR})$ the following approach can be made.

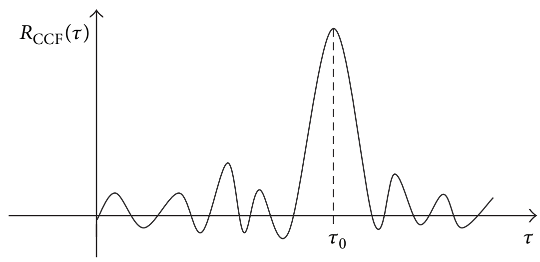


FIGURE 1: Typical graphical representation of a CCF function.

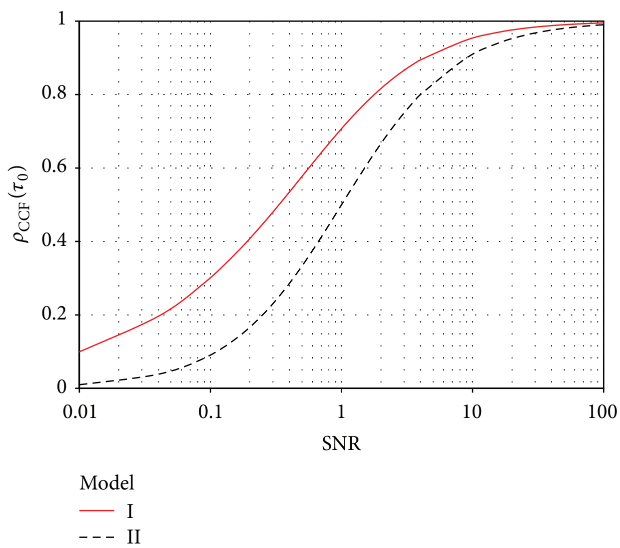


FIGURE 2: View of $\rho_{CCF}(\tau_0) = f(\text{SNR})$ expression for models I and II.

(i) Model I, for (6) signal:

$$\rho_{CCF}(\tau_0) = \left[1 + \left(\frac{\sigma_z^2}{\sigma_s^2}\right)^2\right]^{-1/2} = \left[1 + \frac{1}{\text{SNR}}\right]^{-1/2}, \tag{12}$$

(ii) Model II, for (8) signals:

$$\rho_{CCF}(\tau_0) = \left[\left(1 + \frac{\sigma_z^2}{\sigma_s^2}\right)^2\right]^{-1/2} = \left[1 + \frac{1}{\text{SNR}}\right]^{-1}. \tag{13}$$

The graphical representations of (12) and (13) are shown in Figure 2.

Formulas (12) and (13) can be applied in the determination of SNR on the base of the normalised cross-correlation of the recorded signals.

The discrete estimator of CCF can be determined from the following equation:

$$\widehat{R}_{CCF}(l) = \frac{1}{N}\sum_{n=0}^{N-1} x(n)\, y(n+l), \tag{14}$$

where N is the number of discrete values of $x(n)$ and $y(n)$ signals, $n = t/\Delta t$, $l = \tau/\Delta t$, and Δt is the sampling interval [34].

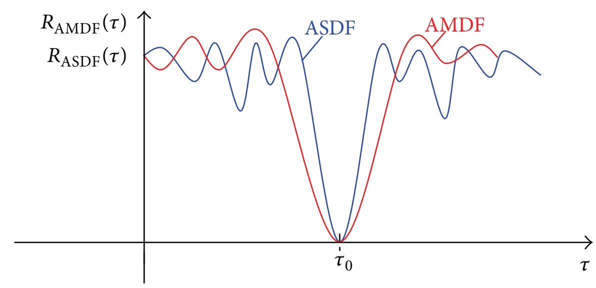


FIGURE 3: Exemplary view of AMDF and ASDF differential functions.

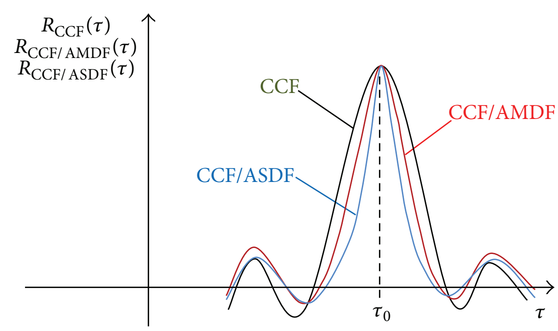


FIGURE 4: Exemplary view of combined CCF/AMDF and CCF/ASDF functions in comparison with CCF.

3.2. Differential Methods. The discrete estimators for AMDF and ASDF can be represented by the following formulas:

$$\widehat{R}_{AMDF}(l) = \frac{1}{N}\sum_{n=0}^{N-1} \left|x(n) - y(n+l)\right|,$$
$$\widehat{R}_{ASDF}(l) = \frac{1}{N}\sum_{n=0}^{N-1} \left[x(n) - y(n+l)\right]^2. \tag{15}$$

In both of the above differential methods the determination of transportation time delay consists of localization of the position of main minimum of the corresponding function [23].

Examples of graphical representations of AMDF and ASDF are presented in Figure 3.

3.3. Combined Methods. Improvement of signal processing can be obtained due to the use of such combined methods as a quotient of CCF and differential AMDF or ASDF functions:

$$\widehat{R}_{CCF/AMDF}(l) = \frac{\widehat{R}_{CCF}(l)}{\widehat{R}_{AMDF}(l) + \varepsilon},$$
$$\widehat{R}_{CCF/ASDF}(l) = \frac{\widehat{R}_{CCF}(l)}{\widehat{R}_{ASDF}(l) + \varepsilon}, \tag{16}$$

where ε is a small positive value.

The ε has been introduced in order to prevent the division by zero, because the AMDF and ASDF functions in an ideal case of a lack of noise can assume a null for $\tau = \tau_0$. For

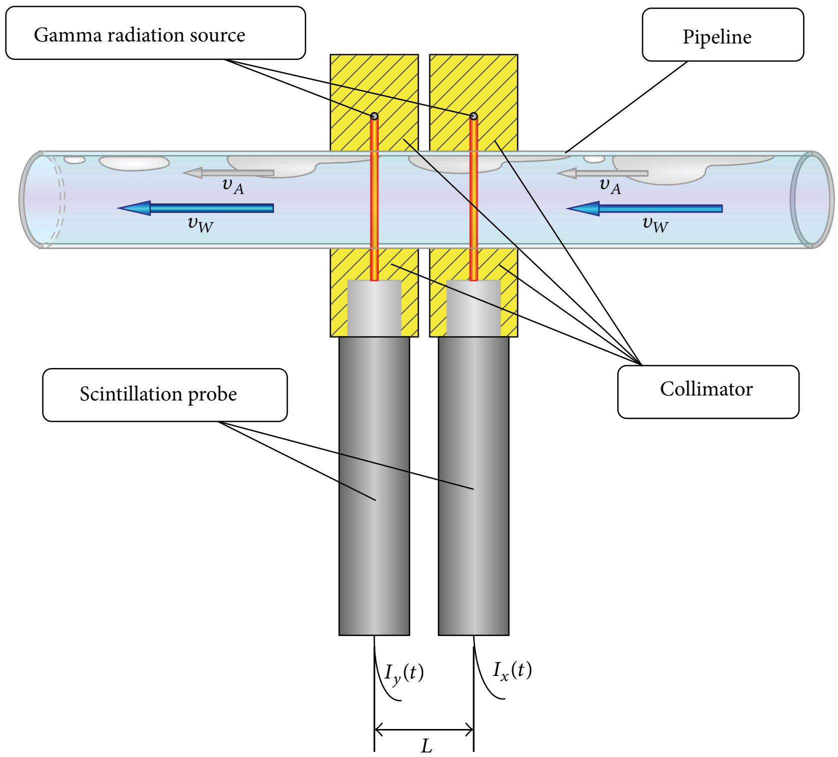


Figure 5: The principle of gamma-absorption measurement of two-phase flow.

$\sigma_z \neq 0$, which usually takes place in measurements, the addition of ε in the denominator of (16) is not necessary. Exemplary graphical representations of CCF/AMDF and CCF/ASDF functions and their comparison with CCF are shown in Figure 4.

The combination of CCF and AMDF functions has already been employed in the paper [25] for an acoustic signal processing. However the combination of CCF and ASDF is an original proposal of the authors.

4. Gamma-Ray Absorption Method in Two-Phase Flow Investigation

Nuclear methods have been in use for many years in measurement of two-phase flows in pipelines and open channels [10–16, 30, 35, 36]. The gamma-ray absorption is relatively simple in application but requires conformance with strict safety requirements for staff and the environment protection from effects of ionizing radiation and radioactive pollution.

The principle of measuring a flow velocity of gas phase transported by liquid in a horizontal pipeline is presented in Figure 5. Two beams of gamma ray emitted by sealed radioactive sources and formed by collimators are partially absorbed by the flow of a medium. Impulses $I_x(t)$ and $I_y(t)$ are received on the outputs of scintillation probes, situated at the L distance on the opposite side of the pipeline, and then counted down within Δt sampling intervals, giving the mutually delayed discrete stochastic signals $x(n)$ and $y(n)$. These signals describe the instantaneous state of flow in the studied section of the pipe [10–14].

Based on the τ_0 time delay of these signals one can calculate the average velocity of the gas phase $v_A = L/\tau_0$.

Signals from the probes, after conditioning (centring, filtration), are ergodic and have normal probability distributions [30]. Figure 6 presents a normalized cross-correlation function obtained for signals received in the BUB005, BUB006, and BUB010 experiments for water-air flow with the following parameters: $N = 300000$, $\Delta t = 1$ ms, and $L = 97$ mm. The velocity of gas flow through the tested pipeline section of 30 mm inner diameter has been equal to 0.90 m/s (BUB005), 0.76 m/s (BUB010), and 0.71 m/s (BUB006), respectively.

A closed ^{241}Am source of 59.5 keV energy gamma ray has been used in the experiments mentioned above together with scintillation detectors based on NaI(Tl) crystals. The laboratory stand and geometry of a requisite absorption set are described in detail in papers [14, 30, 35].

5. Exemplary Results of Simulation Studies

During flow measurements signals provided by scintillation probes contain not only statistical information about the studied mixture but also disturbing signals caused by the gamma radiation background, electronic noise, and fluctuations of the gamma ray decay. The proposed modelling of such signals can be performed with use of I and II models described in Section 2, with properly selected parameters. Model II, defined by (8), has been used in the reported study. In result of the simulation, white noise signal was processed by a low-pass Butterworth filter with parameters

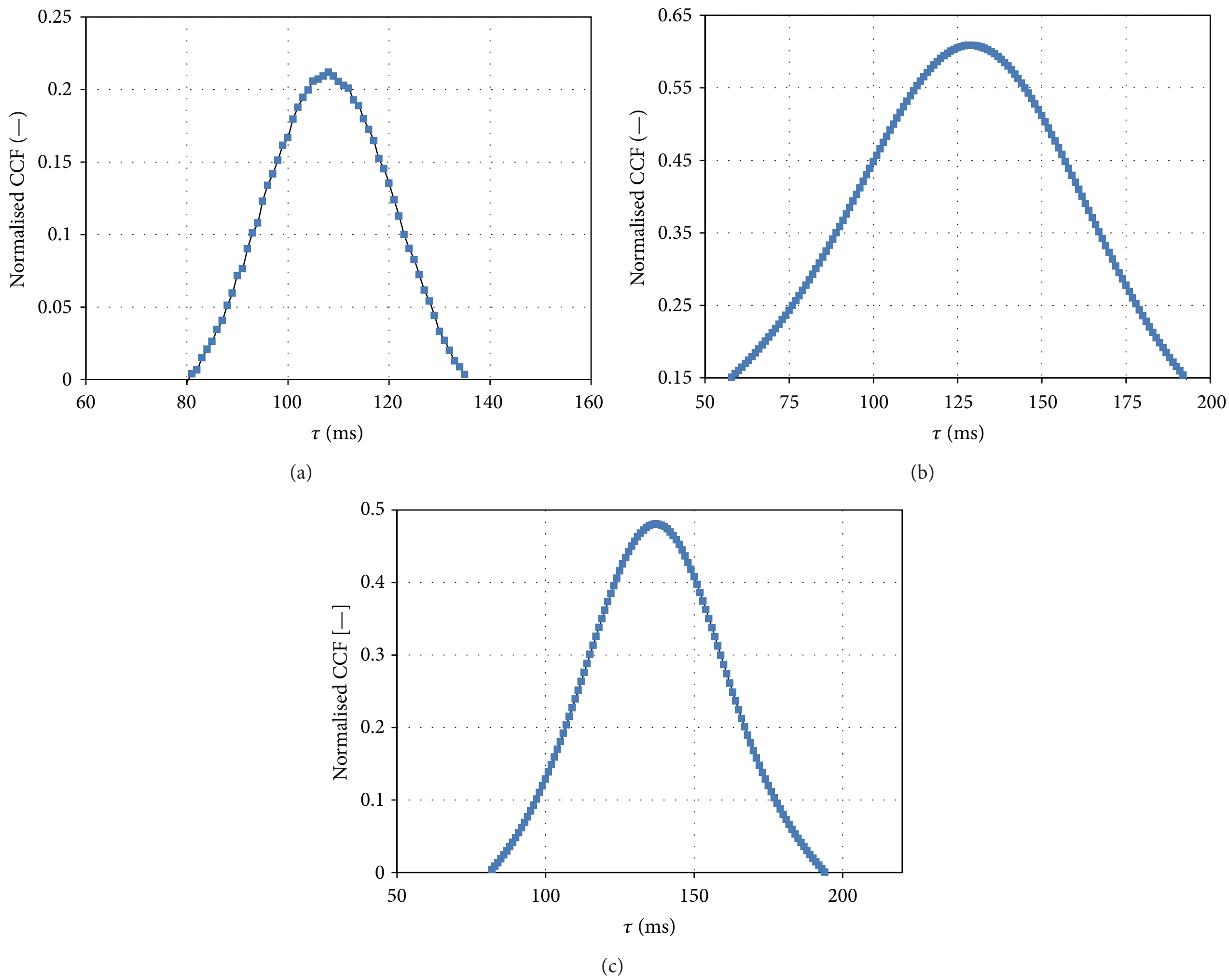


FIGURE 6: Exemplary view of the normalised CCF obtained in the experiment (a) BUB005, (b) BUB010, and (c) BUB006.

TABLE 1: Parameters of simulation approach.

Model of signal for experiment	τ_0 [ms]	N	SNR	Filter order	Filter relative cut-off frequency
BUB005	108	300000	0.27	3rd	0.017
BUB010	128	300000	1.56	2nd	0.005
BUB006	137	300000	0.94	3rd	0.009

given in Table 1 which gives $x(n)$ and $y(n)$ signals and ensures similarity of the shape and amplitude of the normalized cross-correlation functions to the $\rho_{\mathrm{CCF}}(\tau)$ functions shown in Figure 6. The $z_1(n)$ and $z_2(n)$ disturbing signals have been Gaussian white noises with $\mathbf{N}(0, \sigma_z)$ distributions, not correlated mutually and with the recorded signals. Figure 7 presents the normalized cross-correlation functions obtained by modelling with parameters given in Table 1.

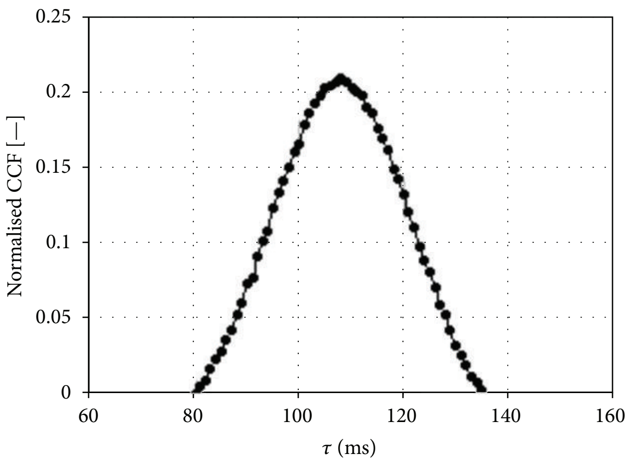
A high level of similarity is observed between CCF functions from Figure 6 and the one in Figure 7 in vicinity of the maximum point, which is essential in estimation of the time delay.

Due to the above procedure, the models of $x(n)$ and $y(n)$ signals allow determination of the functions (14)–(16) and estimation of the transportation time delay and its standard uncertainty. Figures 8 and 9 present exemplary of the ASDF and AMDF differential functions (Figure 8) as well as CCF/ASDF and CCF/AMDF combined functions (Figure 9) obtained for models corresponding to the BUB006 experiment. Furthermore Figure 9 additionally shows the CCF function. For easier comparison of all functions, their normalization in relation to the maximum value was applied.

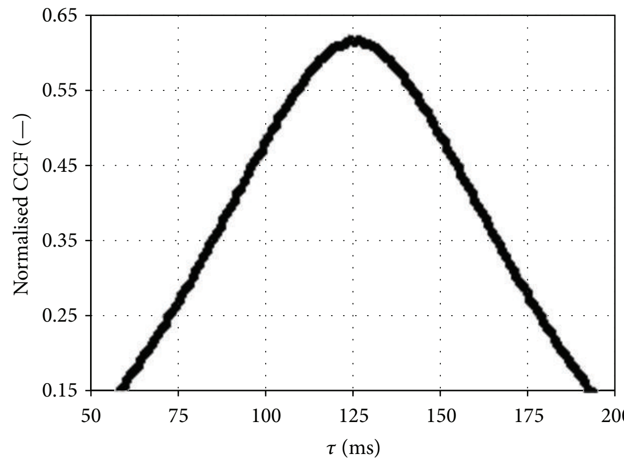
Determination of the transportation time delay for all examined functions consists in estimation of the extreme point position. In the paper, this step was achieved by interpolation of the selected part of a given characteristic by the Gauss function:

$$p(\tau) = p_0 + \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(\tau - \hat{\tau}_0)^2}{2\sigma^2}\right), \tag{17}$$

where p_0 is a normalization level of the Gauss function and σ is a standard deviation of its distribution (Figure 10).



(a)



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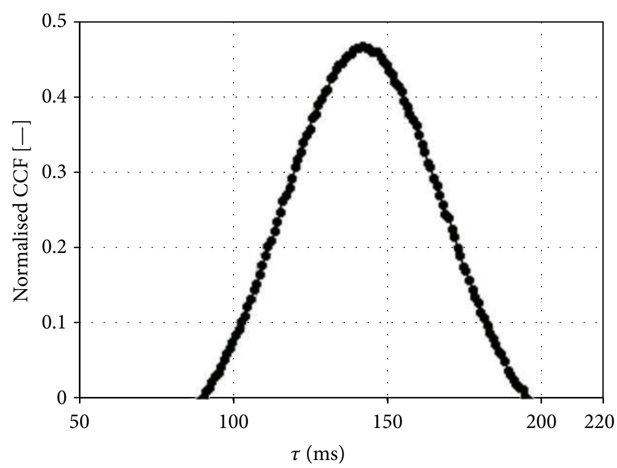

(c)

FIGURE 7: Exemplary view of normalized $\rho_{CCF}(\tau)$ functions obtained by modelling for (a) BUB005, (b) BUB010, and (c) BUB006 experiments.

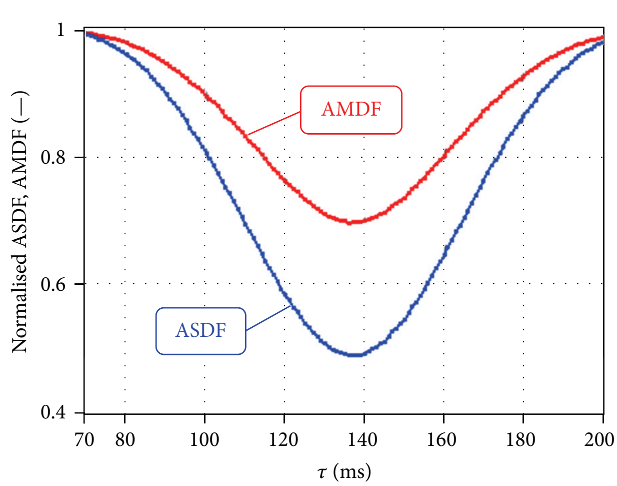

FIGURE 8: Graphical representation of AMDF and ASDF functions obtained by modelling.

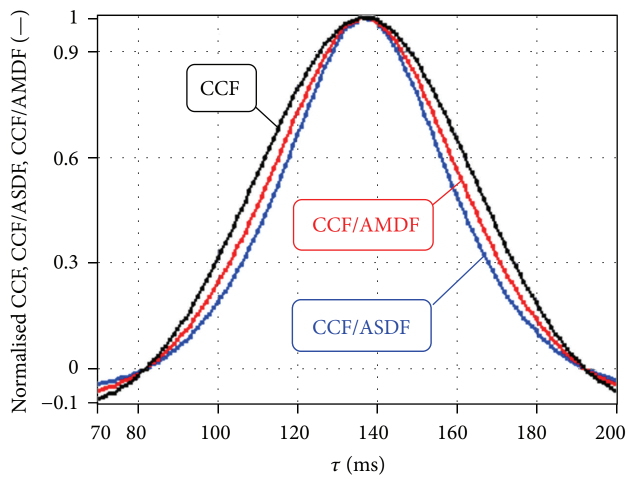

FIGURE 9: Comparison of the CCF/ASDF, CCF/AMDF, and CCF functions obtained by modelling.

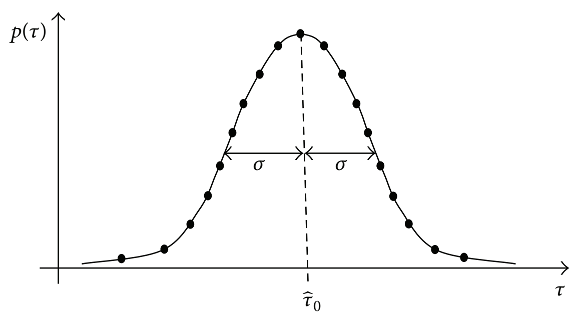

FIGURE 10: Explanation of fitting the Gauss function (-) to the estimated time delay distribution (•).

Then the $\hat{\tau}_0$ transportation time delay estimator is determined as the first moment of the fitted normal distribution [10, 30], while $u(\hat{\tau}_0)$ is its standard uncertainty [37], and in this case it is equal to the standard deviation of the mean value:

$$u\left(\hat{\tau}_0\right) = \frac{\sigma}{\sqrt{k}}, \tag{18}$$

where k is a number of points used in the interpolation procedure.

For all functions described above the similar fitting procedure has been used. The results obtained in this way are listed in Table 2.

For example, fitting a Gaussian curve to CCF, AMDF, and CCF/ASDF functions for models corresponding to the BUB006 experiment is presented in Figure 11.

TABLE 2: Exemplary results of simulation for selected experiments.

Method	$\hat{\tau}_0$ [ms]	$u(\hat{\tau}_0)$ [ms]	$u(\hat{\tau}_0)/u(\hat{\tau}_0)_{CCF}$ [—]	Experiment
CCF	107.93	1.83	1.00	BUB005 ($k = 56$)
AMDF	107.96	1.78	0.97	
ASDF	107.94	1.83	1.00	
CCF/AMDF	107.95	1.62	0.94	
CCF/ASDF	107.94	1.71	0.88	
CCF	128.06	3.16	1.00	BUB010 ($k = 112$)
AMDF	128.11	2.79	0.88	
ASDF	128.07	3.16	1.00	
CCF/AMDF	128.07	2.44	0.77	
CCF/ASDF	128.07	2.10	0.66	
CCF	137.01	2.75	1.00	BUB006 ($k = 105$)
AMDF	137.02	2.49	0.91	
ASDF	137.02	2.75	1.00	
CCF/AMDF	137.01	2.26	0.82	
CCF/ASDF	137.01	1.97	0.72	

6. Conclusions

This paper proposes the use of the AMDF and ASDF differential functions as well as CCF/ASDF and CCF/ASMF combined methods for time delay estimation in a radioisotope investigation of two-phase flows. Intentionally selected examples allow one to compare the proposed methods with the CCF cross-correlation distribution of signals delivered by measurements. In this task simulations have been performed for computer-generated models of stochastic signals, corresponding to one received from the scintillation probes in gamma-ray absorption measurements of liquid-gas flows in a horizontal pipeline. The advantage of the simulation method

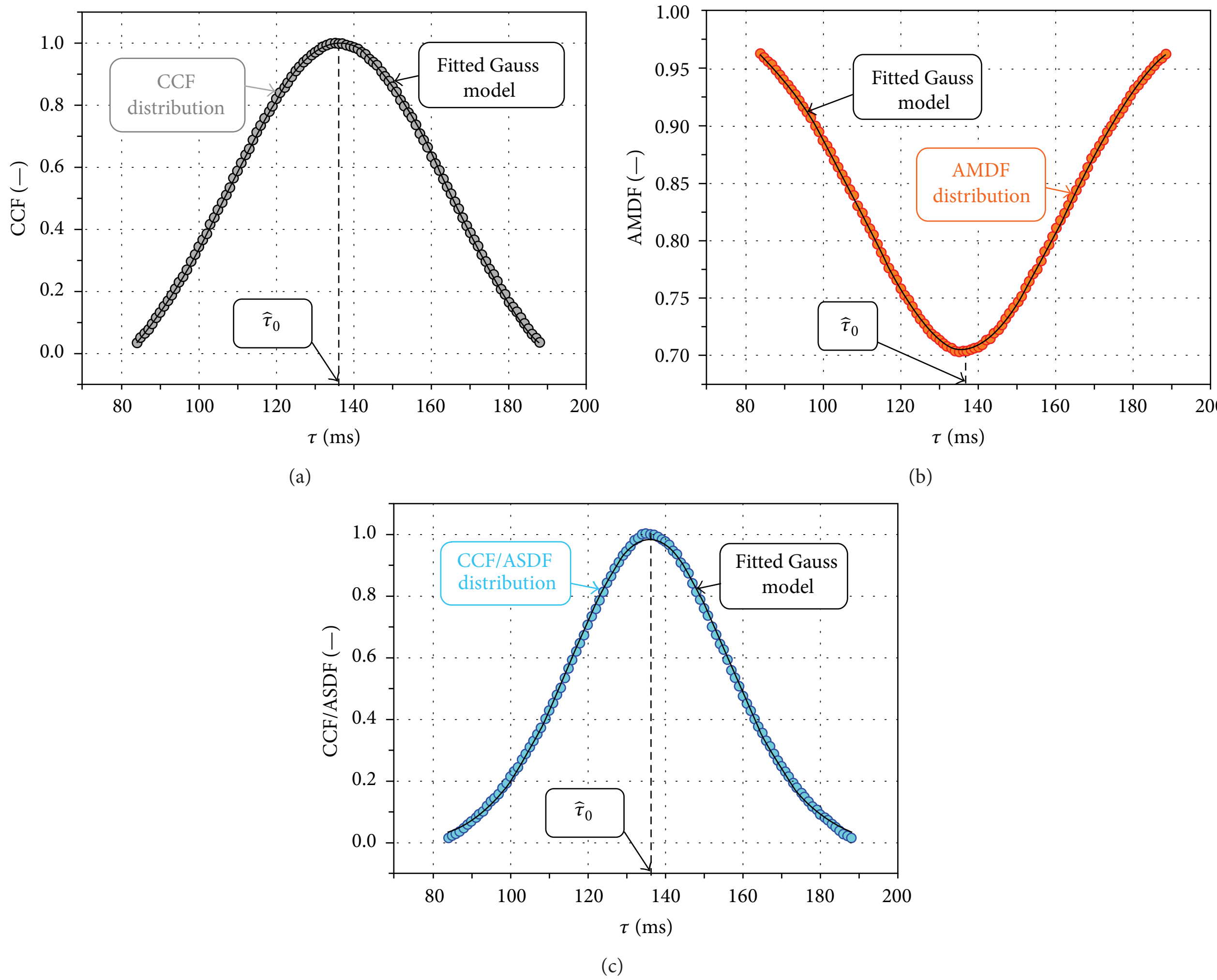

FIGURE 11: Exemplary results of interpolation: (a) CCF, (b) AMDF, and (c) CCF/ASDF.

is the possibility of SNR estimation in measurement signals based on adopted signal model and normalized CCF value. The signals which were used to conduct the calculations are characterized by SNR values ranging from 0.27 to 1.56. The studies have consisted in determination of time delays $\hat{\tau}_0$ and standard uncertainties $u(\hat{\tau}_0)$ of each function.

It is stated that the smallest values of standard uncertainties for time delays are obtained, in descending order, for the following methods: CCF/ASDF, CCF/AMDF, AMDF, CCF, and ASDF. In the case of combined methods decreasing by 12–34 percent (for CCF/ASDF) and 6–23 percent (for CCF/AMDF) the standard uncertainties of time delay in comparison to the CCF values was obtained. During the study it was stated out that the advantages of combined methods are especially visible in the case of a wide peak of the cross-correlation function where significant reduction of the uncertainty was achieved. The unnormalized CCF/ASDF and CCF/AMDF functions have greater amplitude than the unnormalized cross-correlation for the same SNR values, which simplifies the determination of time delay with the use of combined methods. The AMDF method allowed for the decrease of standard uncertainty of time delay from 3 to 12 percent in comparison to CCF. For the AMDF method, the same $u(\hat{\tau}_0)$ values were achieved as for the cross-correlation for all cases of simulated signals.

The undertaken studies relate to practical implementation of the differential and combined methods in gamma densitometry used in measurements of two-phase flow parameters of liquid-gas or liquid-solid particle mixture transported through pipelines [35, 36] and open channels. In the authors' opinion the presented methods can be employed not only in gamma densitometry examinations of two-phase flows, but also in other measurements of time delay using cross-correlation.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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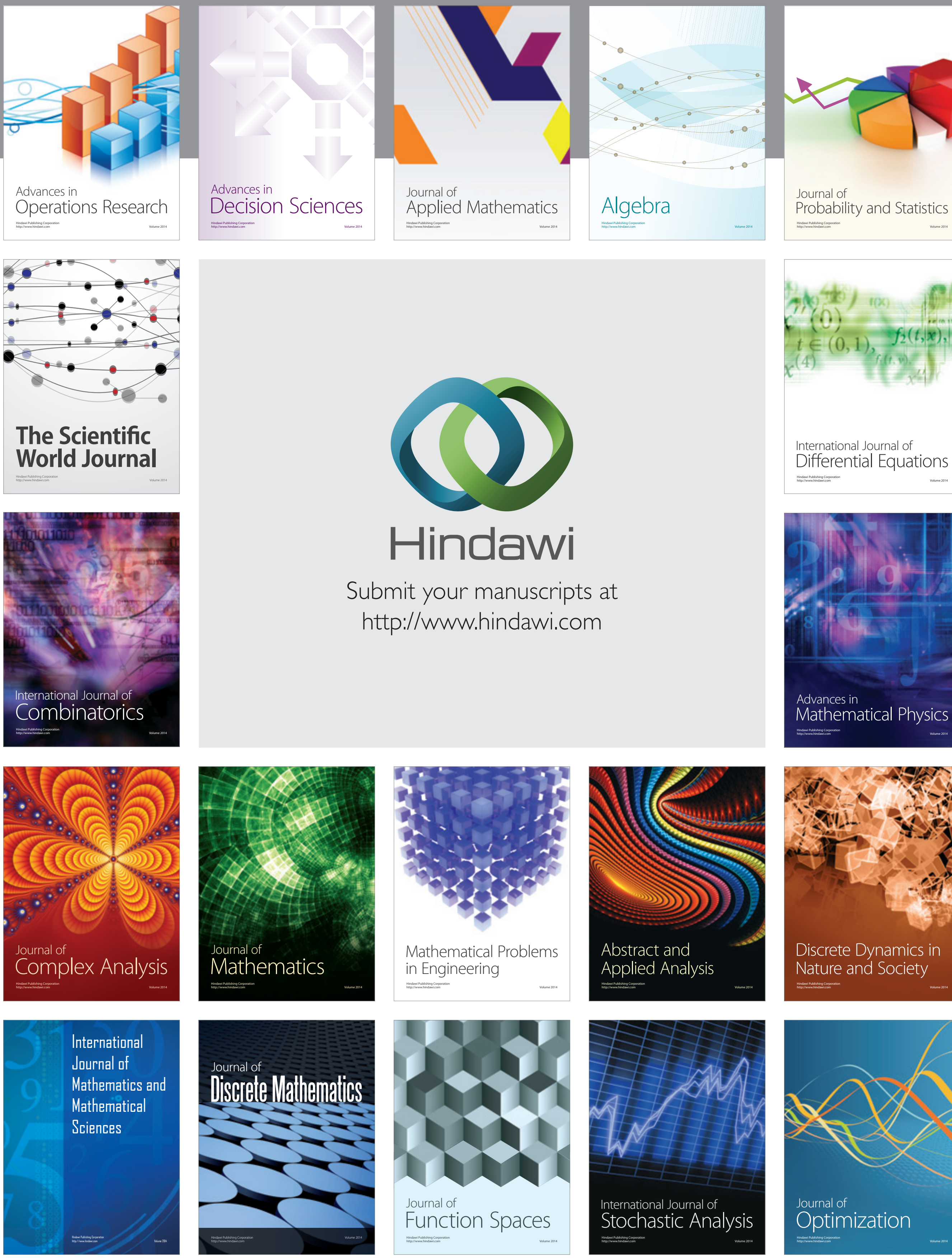
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