

Hysteresis curves for some periodic and aperiodic perturbations in gases

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Summary

Evolution of sound in a medium whose properties irreversibly vary in the course of wave propagation, is studied. As example a gas, which is a particular case of Newtonian fluid, is considered. Hysteresis curves in the plane of thermodynamic states are plotted which are pictorial images of irreversible attenuation of the sound energy. The irreversible losses in internal energy are proportional to the total attenuation and depend on the intensity and shape of sound waveform. Curves and loops for some periodic (including the sawtooth wave) and aperiodic, impulse sound are discussed and compared.

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1 Introduction

A connection between acoustic pressure and excess acoustic density in a viscous fluid is not algebraic, but depends generally on the history of perturbations in a fluid. In many relaxing media, the connection is integral with some kernel which reflects the molecular properties of a fluid. In Newtonian fluids, e.g. gases, the relation between acoustic pressure and excess acoustic density includes a term proportional to the partial derivative of excess acoustic density with respect to time which differs from zero in the thermoconducting fluids [1]. To conclude about equation of state, that is, about connection of total perturbations of pressure and density in a medium, one requires knowledge not about only acoustic perturbations, but about an irreversible transfer of sound energy into the energy of the entropy mode. This transfer is followed by an isobaric increase in the temperature of a medium, and, as a consequence, in decrease of its density [1, 2]. There are many reasons for irreversibility in acoustics of fluids. Among them, thermal conductivity, molecular absorption, scattering and relaxation processes of different origin, may be listed [1, 3, 4]. Along with attenuation of different types which reflects the molecular properties of a medium, nonlinearity is the necessary condition for irreversible loss

in acoustic energy: in the linear flow, acoustic and non-wave motions of a fluid do not interact. The irreversible losses depend on the kind of attenuation in a fluid, on the intensity of the wave, but also on gradients of acoustic perturbations. Thermodynamic state of a fluid depends on a history of sound perturbations thereby causing this fluid hysteresis. Thermodynamic cycles in the strain-stress diagrams for solid materials with hysteresis nonlinearity are usually represented by loops [5, 6]. Not so pronounced, but similar loops are specific in Newtonian thermoconducting fluids. The pure nonlinear attenuation on the front of the shock wave is well-established. It leads also to irreversible loss in acoustic energy which may be represented by the hysteresis curves in the plane of thermodynamic state (total excess density \Leftrightarrow total excess pressure).

2 Dynamics of total excess density and pressure

The continuity, momentum and energy equations of a planar one-dimensional flow of a Newtonian gas take the forms:

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} &= 0, \\ \rho \left(\frac{\partial v}{\partial t} + v \frac{\partial \rho}{\partial x} \right) + \frac{\partial p}{\partial x} &= \left(\frac{4\mu}{3} + \eta \right) \frac{\partial^2 v}{\partial x^2} \\ T \left(\frac{\partial s}{\partial t} + v \frac{\partial s}{\partial x} \right) &= \frac{\chi \Delta T}{\rho} + \left(\frac{4\mu}{3\rho} + \frac{\eta}{\rho} \right) \left(\frac{\partial v}{\partial x} \right)^2, \end{aligned} \quad (1)$$

where ρ , p , v , s , T denote density, pressure, velocity of fluid, entropy per unit mass and temperature, respectively, x, t are spatial co-ordinate and time, and χ , μ , η mark thermal conductivity, shear and bulk viscosity, all assumed to be constants. The system (1) should be completed by the caloric and thermal equations of state. Internal energy e and temperature of an ideal gas are functions of ρ , p as follows (C_v marks heat capacity under constant volume per unit mass, and $\gamma = C_p/C_v$ denotes the ratio of specific heats):

$$e = C_v T = \frac{p}{(\gamma - 1)\rho}. \quad (2)$$

The types of viscous fluid motion of infinitely-small magnitude are well-established [7, 1, 4]. The linear classification applies in a weakly nonlinear flow. In one dimension, there exist two acoustic branches and the entropy mode. The details of analysis of weakly nonlinear dynamics and interaction of sound and the entropy mode may be found in [8, 9]. It applies not only to periodic sound and describes, among other, instantaneous decrease of excess density specifying the entropy mode in the field of the dominative sound (ρ_0 is an unperturbed density of a fluid, ρ_a , ρ_e denote excess quantities associated with acoustic and entropy modes, and Q_a is an acoustic source of the entropy mode),

$$\begin{aligned} \frac{\partial \rho_e}{\partial t} - \frac{\chi}{\rho_0 C_p} \frac{\partial^2 \rho_e}{\partial x^2} &= -\frac{\chi(\gamma - 1)}{\rho_0^2 C_p} \rho_a \frac{\partial^2 \rho_a}{\partial x^2} - \left(\frac{(\gamma - 1)}{\rho_0} \left(\frac{4\mu}{3\rho_0} + \frac{\eta}{\rho_0} \right) + \frac{\gamma(\gamma - 1)}{\rho_0} \frac{\chi}{\rho_0 C_p} \right) \left(\frac{\partial \rho_a}{\partial x} \right)^2 - \\ &\frac{(\gamma - 1)(\gamma - 5)\chi}{4\rho_0^2 C_p} \frac{\partial^2 \rho_a^2}{\partial x^2} = \rho_0 Q_a. \end{aligned} \quad (3)$$

In the case of periodic sound, Eq.(3) may be rearranged in the leading order as

$$\frac{\partial \rho_e}{\partial t} - \frac{\chi}{\rho_0 C_p} \frac{\partial^2 \rho_e}{\partial x^2} = -\frac{(\gamma-1)b}{\rho_0} \left\langle \left(\frac{\partial \rho_a}{\partial x} \right)^2 \right\rangle = -\frac{(\gamma-1)b}{\rho_0 c_0^2} \left\langle \left(\frac{\partial \rho_a}{\partial t} \right)^2 \right\rangle, \quad (4)$$

with square brackets denoting averaging over integer number of sound periods, $c_0 = \sqrt{\gamma p_0 / \rho_0}$ is the linear sound velocity and b is the total attenuation,

$$b = \frac{4\mu}{3\rho_0} + \frac{\eta}{\rho_0} + \frac{(\gamma-1)\chi}{\rho_0 C_p}. \quad (5)$$

The above formula (4) applies only to the periodic sound and is averaged over the integer number of sound periods. It is well-established and widely used in medical and technical studies of nonlinear effects which follow propagation of ultrasound [2, 10]. The total excess pressure p' is a sum of an acoustic pressure and an excess pressure associated with the entropy mode, this last part equals zero: the entropy motion is isobaric. The total excess density consists also of two parts. The leading-order relation between total excess pressure and density takes the form

$$\frac{p'}{\rho_0 c_0^2} = \frac{p_a}{\rho_0 c_0^2} = \frac{\rho_a}{\rho_0} + \frac{\gamma-1}{2\rho_0^2} \rho_a^2 + \frac{(\gamma-1)\chi}{2c_0^2 \rho_0^2 C_p} \frac{\partial \rho_a}{\partial t} = \quad (6)$$

$$\frac{\rho' - \rho_e}{\rho_0} + \frac{\gamma-1}{2\rho_0^2} \rho'^2 + \frac{(\gamma-1)\chi}{2c_0^2 \rho_0^2 C_p} \frac{\partial \rho'}{\partial t} = \frac{\rho'}{\rho_0} + \frac{\gamma-1}{2\rho_0^2} \rho'^2 - \int^t Q_a dt + \frac{(\gamma-1)\chi}{2c_0^2 \rho_0^2 C_p} \frac{\partial \rho'}{\partial t}.$$

The lower limit of integration of Q_a should be chosen in accordance to the beginning of the sound transmission. More accurately, ρ_e should be evaluated as a solution of the diffusivity equation, Eq.(3), it equals $\int^t Q_a$ if the thermal conduction is zero. Eq.(6) does not take into account thermal conduction in the leading order; the left-hand side of Eq.(3) is replaced by the partial derivative of ρ_e with respect to time. That is valid for small ρ_e compared to ρ_a and weakly attenuating sound, when the second term in the left-hand side of Eq.(3) is much smaller than the first one.

There are acoustic losses in energy due to the first and second viscosity, described by nonlinear terms, and losses due to thermal conduction, which is described by the term proportional to the Laplacian of temperature (the right-hand side of the last equation in the set Eqs(1)). Account for this last term results in the third term in the link between acoustic pressure and acoustic density in Eq.(6), which is proportional to the derivative of excess acoustic density with respect to time. In the study [11], this term is considered erroneously as proportional to the total attenuation. Also, it seems inappropriate to conclude about irreversible variations in internal energy of a medium on the assumption of the isentropic energy balance exclusively for sound perturbations. The statement that the acoustic work over the period equals variation in the internal energy of a medium, is incorrect. Thermodynamic state of a fluid is described by the total excess quantities, so that the hysteresis images should be plotted the terms of the total excess density, $\rho' = \rho_a + \rho_e$ and pressure, $p' = p_a$. The analysis below concerns harmonic sound, sawtooth sound and some impulses.

3 Hysteresis curves for some periodic sound

3.1 Harmonic acoustic pressure

If excess pressure is periodic harmonic function of time for any distance from a transducer x ,

$$P = \frac{p'}{Mc_0^2\rho_0} = \sin(\omega t + \varphi), \quad (7)$$

the total excess density in accordance to Eq.(6), is described by

$$R = \frac{\rho'}{M\rho_0} = \sin(\omega t + \varphi) - \xi \cos(\omega t + \varphi) - \frac{1}{2}M(\gamma - 1) \sin^2(\omega t + \varphi) - \frac{Mb\omega(\gamma - 1)}{c_0^2} \left(\frac{1}{4} \sin(2\omega t + 2\varphi) - \frac{1}{4} \sin 2\varphi + \frac{\omega t}{2} \right),$$

where

$$\xi = \frac{(\gamma - 1)\omega\chi}{2\rho_0c_0^2C_P}, \quad (8)$$

ω is the sound frequency and M denotes the acoustic Mach number, P and R denote dimensionless total excess pressure and density. Eqs (7) determine dependence of the total excess pressure p' on the total excess density ρ' in the parametric form. Two cycles of this hysteresis curve for $\varphi = 0$, $\gamma = 1.4$, $M = 0.5$, $\omega b/c_0^2 = 0.2$, and $\xi = 0$ or $\xi = 0.05$ are plotted in the Fig.1. The starting time of transmission is $t = 0$.

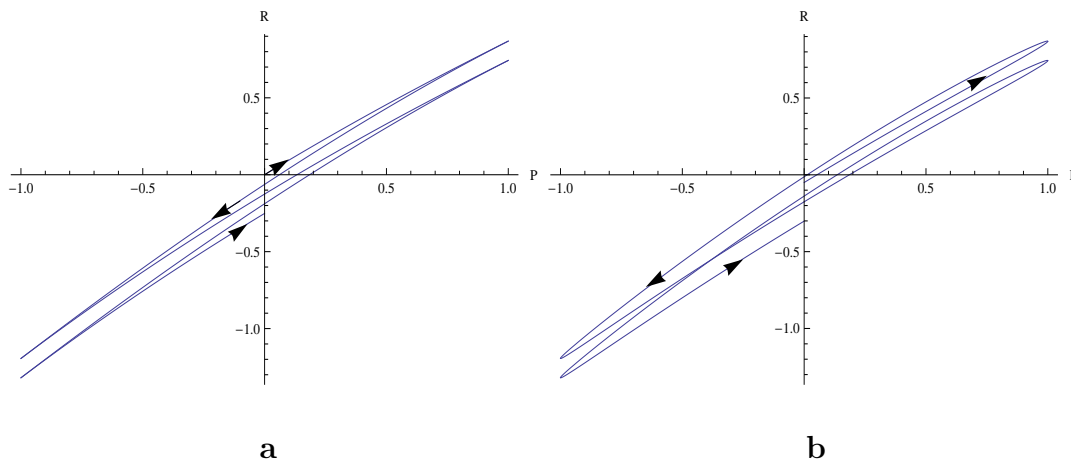


Figure 1. *Dependence of the total excess density on the total excess pressure in a Newtonian gas without thermal conduction (a) and with thermal conduction (b). Case of the periodic harmonic sound.*

The curve which relates to the case without thermal conduction, starts from the point $(0, 0)$ in the plane RP , and this one accounting for thermal conductivity, starts from the point $(0, -\xi)$. The direction of turn in the positive quarter is different in the both cases. The turn point in the second case is smooth, and the curve contains loops. In the both cases, the total excess density gets smaller after the whole period. That reflects the nonlinear isobaric growth of the entropy mode temperature, which is proportional to the total attenuation of a fluid.

Increase of internal energy of a medium during a period of sound is determined by dynamics of the entropy mode. If the term proportional to the thermal conduction in the diffusivity equation, Eq.(4), may be neglected, relative variation in the internal energy U over the period is positive and equals

$$\frac{\omega}{2\pi} \frac{\delta U}{U_0} = - \left\langle \frac{1}{\rho_0} \frac{\partial \rho_e}{\partial t} \right\rangle = \frac{(\gamma - 1)b}{\rho_0^2 c_0^2} \left\langle \left(\frac{\partial \rho_a}{\partial t} \right)^2 \right\rangle. \quad (9)$$

For the periodic harmonic wave, that yields in the leading order

$$\frac{\omega}{2\pi} \frac{\delta U}{U_0} = \frac{(\gamma - 1)\omega^2 b M^2}{2c_0^2}. \quad (10)$$

3.2 Sawtooth wave

In this example, wave is strongly distorted by nonlinearity and has a sawtooth shape. Its one period is described by the Khokhlov's solution,

$$P = -\frac{\omega\tau}{\pi} + \tanh\left(\frac{\tau}{\Theta}\right), \quad -\pi < \omega\tau < \pi, \quad (11)$$

where $\tau = t - x/c_0$ denotes the retarded time, and $\Theta = \frac{2b}{M(\gamma+1)c_0^2}$ is the characteristic width of the shock front. In the limit of this solution when viscosity is low and the Reynolds acoustic number is high, the duration of shock front is small as compared with the sound period ($\omega\Theta \ll 1$), making the waveform take the sawtooth shape. In this case, relative variation in internal energy during a period takes the form

$$\frac{\omega}{2\pi} \frac{\delta U}{U_0} = \frac{(\gamma - 1)(\gamma + 1)M^3\omega}{3\pi}. \quad (12)$$

The integral in the right-hand side of the leading-order equality, Eq.(6), may be readily evaluated for the dimensionless acoustic pressure in the form (11). That gives a possibility to plot a curve $R(P)$ in the parametric form, or to derive leading order relations, recalling, that during the first half-periods, if $-\pi + 2\pi n < \omega\tau < 2\pi n$ (where n is an integer number), the first term in the right-hand side of Eq.(11) determines P , and in the vicinity of $\omega\tau = 2\pi n$ it almost equal the second term of this equation. During the first half-periods,

$$P = -1 + 2n - \frac{\omega\tau}{\pi}. \quad (13)$$

During the second half-periods, where $2\pi n < \omega\tau < \pi + 2\pi n$, an acoustic pressure is determined as

$$P = 1 + 2n - \frac{\omega\tau}{\pi}. \quad (14)$$

In the both cases, the total excess density takes the form

$$R = P - M \frac{\gamma - 1}{2} P^2 + \frac{\xi}{\pi} + \int_{-\pi/\omega}^{\tau} Q_a d\tau. \quad (15)$$

In the vicinity of $\omega\tau = 2\pi n$, an acoustic pressure equals

$$P = \tanh\left(\frac{\omega\tau - 2\pi n}{\omega\Theta}\right), \quad (16)$$

and the leading-order relation between P and R takes the form

$$R = P - M \frac{\gamma - 1}{2} P^2 - M \frac{\xi(\gamma + 1)}{4b} (1 - P^2) + \int_{-\pi/\omega}^{\tau} Q_a d\tau. \quad (17)$$

So that, evaluation of $R(P)$ requires calculation of $\int_{-\pi/\omega}^{\tau} Q_a d\tau$ and replacing of τ in terms of P in the different domains of $\omega\tau$. The constant decrease in the total excess density is caused by negative Q_a and, correspondingly, negative integral which absolute value increases in the course of time. Fig.2 shows the beginning of curves without thermal conduction (a) and with account for thermal conductivity (b). In the case a, the curve starts from the point $(0, 0)$, in the case b, it starts from the point $(0, \frac{\xi}{\pi})$. In evaluations, the parameters are as follows: $\gamma = 1.4$, $M = 0.5$, $\omega b/c_0^2 = 0.001$, and $\xi = 0$ or $\xi = 0.0003$.

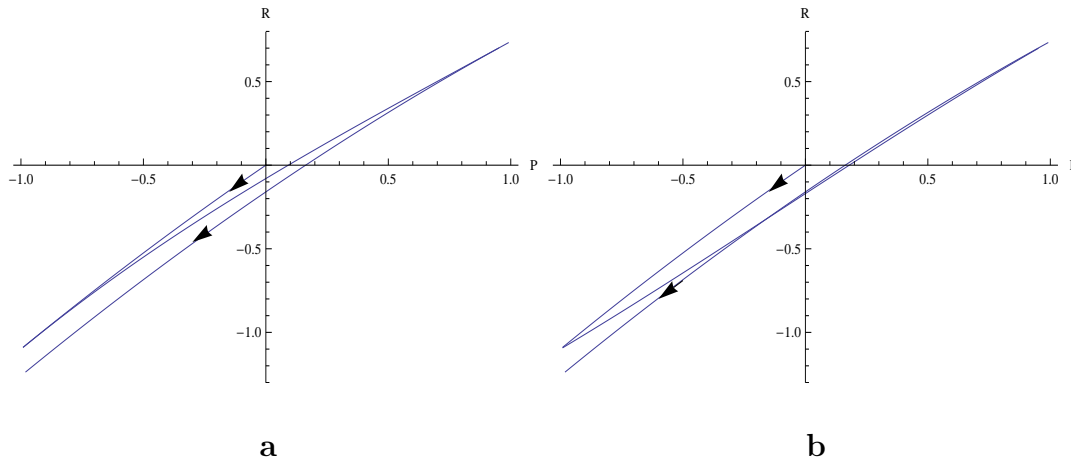


Figure 2. *Dependence of the total excess density on the total excess pressure in a Newtonian gas without thermal conduction (a) and with thermal conduction (b). Case of the sawtooth wave.*

4 Hysteresis curves for some aperiodic sound

In many cases, a solution of linear wave equation without attenuation may be useful in evaluations of relation between the total excess pressure and density and in a graphing of thermodynamic states of a medium.

4.1 The Gaussian impulse

If a signal may be approximately considered as Gaussian,

$$P = \exp(-(\omega\tau)^2), \quad (18)$$

where ω denotes the characteristic inverse duration of an impulse, Eq.(6) rearranges into

$$R = P - M \frac{\gamma - 1}{2} P^2 + 2\xi(\omega\tau)P + \int_{-\infty}^{\tau} Q_a d\tau, \quad (19)$$

where

$$\tau = \begin{cases} -\frac{\sqrt{-\ln P}}{\omega}, & \text{if } P \text{ enlarges,} \\ \frac{\sqrt{-\ln P}}{\omega}, & \text{if } P \text{ decreases.} \end{cases} \quad (20)$$

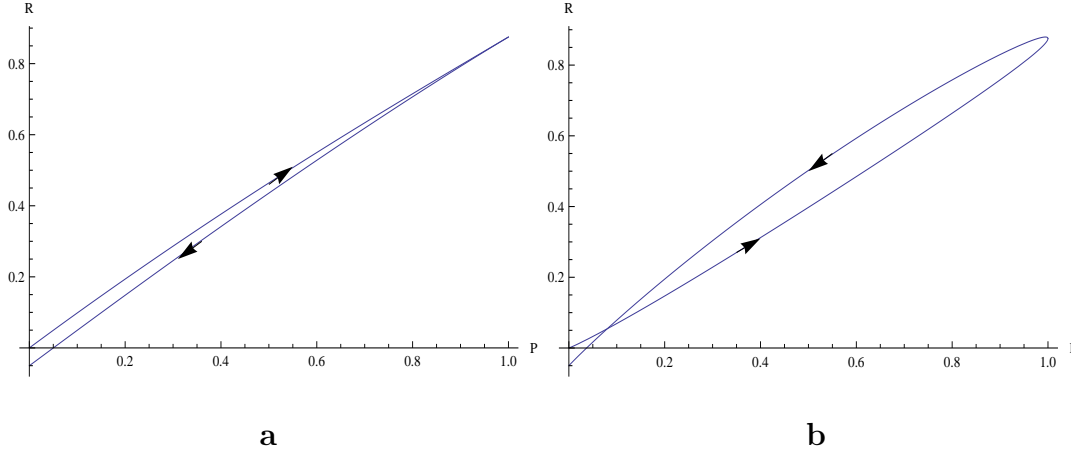


Figure 3. *The $R \Leftrightarrow P$ diagrams without thermal conduction (a) and with thermal conduction (b) for the Gaussian impulse.*

Fig.3 shows the curves for a thermoconducting fluid and a fluid without thermal conduction. The parameters used in the evaluations, are as follows: $\gamma = 1.4$, $M = 0.5$, $\xi = 0.1$ (in the case b) and $\omega b/c_0^2 = 0.2$. The total relative increase in the internal energy which associates with irreversible loss in acoustic energy, is

$$\frac{\delta U}{U_0} = - \int_{-\infty}^{\infty} Q_a d\tau = (\gamma - 1) \sqrt{\frac{\pi}{2}} \frac{\omega b M^2}{c_0^2}. \quad (21)$$

4.2 The bell-like impulse

Another example of a monopolar impulse is the bell-like one:

$$P = (1 + (\omega\tau)^2)^{-1}, \quad (22)$$

with ω denoting the characteristic inverse duration of an impulse, as well as in the previous subsection. Eq.(6) transforms into

$$R = P - M \frac{\gamma - 1}{2} P^2 + 2\xi(\omega\tau)P^2 + \int_{-\infty}^{\tau} Q_a d\tau, \quad (23)$$

where

$$\tau = \begin{cases} -\omega^{-1} \sqrt{\frac{1-P}{P}}, & \text{if } P \text{ enlarges,} \\ \omega^{-1} \sqrt{\frac{1-P}{P}}, & \text{if } P \text{ decreases.} \end{cases} \quad (24)$$

The curves in the Fig.4 are plotted accordingly to the same set of parameters as in the previous subsection.

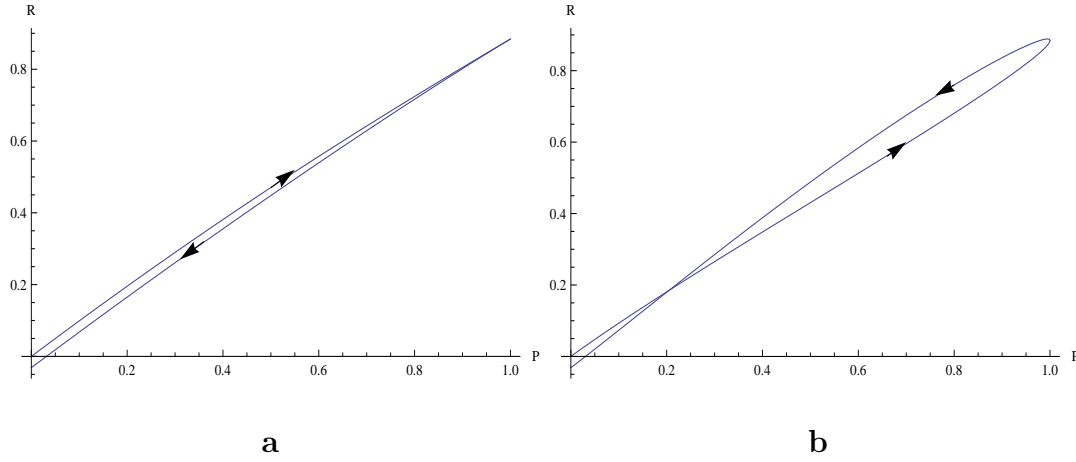


Figure 4. The $R \Leftrightarrow P$ diagrams without thermal conduction (a) and with thermal conduction (b) for the bell-like impulse.

The total relative increase in the internal energy which associates with irreversible loss in acoustic energy, in this case equals

$$\frac{\delta U}{U_0} = - \int_{-\infty}^{\infty} Q_a d\tau = (\gamma - 1) \frac{\pi \omega b M^2}{4 c_0^2}. \quad (25)$$

An efficiency of a medium heating due do nonlinear attenuation is almost 1.6 times smaller in the case of the bell-like impulse as compared to the Gaussian one.

4.3 An example of bipolar impulse

The impulse in the form

$$P = 2\omega\tau(1 + (\omega\tau)^2)^{-1} \quad (26)$$

is asymmetric. Eq.(6) takes the following form:

$$R = P - M \frac{\gamma - 1}{2} P^2 + 2\xi(\omega\tau)P^2 - \frac{\xi}{1 + (\omega\tau)^2} + \int_{-\infty}^{\tau} Q_a d\tau, \quad (27)$$

where

$$\tau = \begin{cases} \frac{1 - \sqrt{1 - P^2}}{\omega P}, & \text{if } P \text{ enlarges,} \\ \frac{1 + \sqrt{1 - P^2}}{\omega P}, & \text{if } P \text{ decreases.} \end{cases} \quad (28)$$

The hysteresis curves in the Fig.5 are plotted accordingly to the same set of parameters as in the previous subsections.

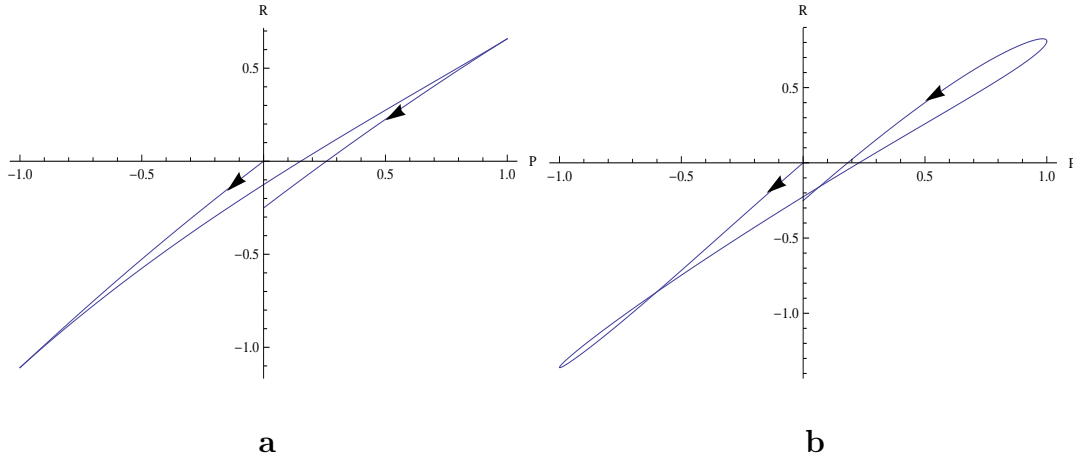


Figure 5. The $R \Leftrightarrow P$ diagrams without thermal conduction (a) and with thermal conduction (b) for the bipolar impulse.

The total relative increase in the internal energy which associates with irreversible loss in acoustic energy, is

$$\frac{\delta U}{U_0} = - \int_{-\infty}^{\infty} Q_a d\tau = (\gamma - 1)\pi \frac{\omega b M^2}{c_0^2}. \quad (29)$$

An efficiency of a medium heating due do nonlinear attenuation is almost 2.5 times larger in the case of the bipolar impulse as compared to the Gaussian one.

5 Concluding remarks

The nonlinear propagation of sound in Newtonian fluids, e.g. gases, is always followed by the irreversible loss in acoustic energy. The macroscopic wave energy transfers into the thermal energy of chaotic motion of molecules. The correspondent enlargement of a medium temperature and decrease in its density is isobaric. In the plane of the thermodynamic state (ρ', p') , that means that the total density gets smaller by the nature of the case. Even if sound is periodic, the point in the plane of state after the full period becomes shifted towards smaller density. In some parts of the curve $\rho'(p')$, density may temporarily additionally grow due to the thermal conductivity. That may happen in the domains where pressure decreases with time and thermal conduction is enough large.

This enlargement does not contradict with the permanent production of entropy in the moving volume of a fluid. The last equation from the set (1) describes variations in the entropy of the moving volume which exchanges energy with the surrounding volumes due to difference in temperature of this volume and its environment. The term $\chi \Delta T$ takes into account a flux of energy \vec{q} through the surface of the volume, assuming that it is proportional to the temperature gradient, $\vec{q} = -\chi \vec{\nabla} T$. In view of that there is a thermal flux through the boundaries of the moving element, variations in its entropy may be negative or positive with time in accordance to the sign of $\chi \Delta T$. With account for the flux of entropy through boundaries of a moving fluid volume, $\frac{\dot{q}}{T}$, production of entropy in the unit volume is always positive [7]:

$$\dot{S} = \frac{\chi}{T^2} \left(\frac{\partial T}{\partial x} \right)^2 + \frac{1}{T} \left(\frac{4\mu}{3} + \eta \right) \left(\frac{\partial v}{\partial x} \right)^2. \quad (30)$$

Over temporal and spatial domains, where the entropy motion is weak as compared to sound, production of entropy takes the form

$$\dot{S} = \frac{(\gamma - 1)C_p b}{\rho_0 c_0^4} \left(\frac{\partial p_a}{\partial x} \right)^2. \quad (31)$$

Both production of entropy and variations in the internal energy of a medium, δU , are determined only by the total attenuation, shape and intensity of sound. The erroneous conclusions about enlargement of the chaotic energy of the molecules, which does not account for irreversible losses in the entropy equation, and improper linear link of perturbations of acoustic pressure and excess density, would lead to the incorrect expression for loss in internal energy during the sound period. Loss in internal energy over period of periodic sound (Eq.(5) from [11]), is in fact γ times greater than that given by Eqs (9),(12). Actually, the paper [11] is the first study which attracts attention to the pressure-volume diagrams, hysteresis curves and loops, and physical distinction between different irreversible processes accompanying the nonlinear propagation of intense sound in fluids. It considers loops in the plane of acoustic perturbations for periodic harmonic and sawtooth sound in a Newtonian gas and in a fluid with relaxation and includes important conclusions about dissipative and hysteresis processes in linear and nonlinear acoustic fields.

Loops of the curve $\rho'(p')$ may form only during the domains, where pressure decreases with time and thermal conduction is considerable as compared with the mechanic viscosity. In this study, we do not account for thermal conduction in the diffusivity equation for an excess density of the entropy motion, Eq.(3), evaluating it simply by integration of the acoustic source Q_a over time. This inaccuracy may be justified by the smallness of ρ_e : its magnitude is $M(\omega b/c_0^2)$ times larger than magnitude of acoustic excess density, ρ_a . The neglected term is proportional to b^2 that exceeds accuracy of evaluations in this study. For more precise evaluations, the diffusion equation Eq.(3) with an acoustic source should be solved. Along with Eq.(6), it describes dynamics of the thermodynamic state $\rho'(p')$ due to losses in energy of periodic and aperiodic sound.

An efficiency of the irreversible heating of a medium depends strongly on the shape of a signal. Among considered impulses, a bipolar impulse produces the most comparative heating of a medium. Impulses may propagate due to a sudden external action on a medium. They are widely used in the medical and technical applications, where accurate estimates of the thermodynamic state of a fluid are important. Equations which describe irreversible increase in the internal energy are applicable not only to planar waves, but to any one-dimensional wave. They may describe spherical and cylindrical waves and waves in horns. The expression for the acoustic Mach number in dependence on the coordinate, $M(x)$, should be established in any particular case. The results are also valid in the quasi-planar geometry of slowly divergent sound beams. The curves in the plane of thermodynamic states may be useful in reconstruction of dispersive and viscous properties of a medium [1, 11]. Plotted for different frequencies of a sound, these pictorial images make possible to evaluate the characteristic time of relaxation in a media: irreversible attenuation is the most pronounced when the sound frequency equals the inverse time of relaxation. The analogous problems in optics are usually solved by laser spectroscopy [12]. The kernels in integral relations of wave perturbations, which reflect dispersion, may be reconstructed by use of these experimental data. Vice versa, for a medium with known thermodynamic properties, the remote acoustic source may be reconstructed. Ultrasonic absorption makes possible to heat a medium remotely. The type of sound may be selected in order to produce lowest or highest heating at the same intensity of the initial wave.

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