Annals of Warsaw University of Life Sciences - SGGW
Forestry and Wood Technology № 85, 2014: 31-35
(Ann. WULS - SGGW, For. and Wood Technol. 85, 2014)

Methods of solving the Atkins equation determine shear angle with taking into consideration a modern fracture mechanics

CHUCHALA DANIEL[1], CHUCHALA RADOSŁAW[3], OCHRYMIUK TOMASZ[2], ORLOWSKI KAZIMIERZ[1],
[1]Gdansk University of Technology, Faculty of Mechanical Engineering, Department of Manufacturing Engineering and Automation, Gdansk
[2]Department of Numerical Methods and Transonic Flow, Polish Academy of Sciences, Szewalski Institute of Fluid-Flow
Machinery, Gdansk
[3]Gdansk University of Technology, Faculty of Electronics, Telecommunications and Informatics, Department of Radiocommunication Systems and Networks, Gdansk

Abstract: *Methods of solving the Atkins equation determine shear angle with taking into consideration a modern fracture mechanics.* In the paper are presented methods of solving nonlinear Atkins equation . The Atkins equation describe shear angle with taking into account properties of material cutting. To solve Atkins equation has been used iterative methods: Newton method and simplified method of simple iteration. Method of simple iteration is presented in the form of Java application.

Keywords: shear angle, modern fracture mechanics, Newton method, Atkins equation

INTRODUCTION

The shear angle Φ_c, which determines position of the shear plane with considering the surface workpiece and the tool line of action [1, 2, 6, 8, 9], is an essential parameter to characterize the process of sawing (cutting) wood. The shear angle Φ_c is calculated in Merchant equation (equation 1) for larger values of feed per tooth f_z. Because shear angle is set to a fixed for large values of the thickness of the cut layer (Φ_c = const.) [2, 3]:

$$\Phi_c = (\pi/4) - (1/2)(\beta_\mu - \gamma_f) \tag{1}$$

where:
β_μ – angle of friction, which is defined as $\tan^{-1}\mu = \beta_\mu$,
γ_f – rake angle [°].

The values of shear angle Φ_c, which is determined from Merchant model (equation (1)), are not equal to real value, because Marchant model do not take into account the relationship between fracture toughness and the yield strength of the workpiece at the shear plane R/τ_y [2, 3]. The values of shear angle Φ_c from Marchant model are always greater than the value of Φ_c experimentally determined [2, 3, 9]. Atkins [2] showed that for the least values of force F_c, shear angle Φ_c satisfies the following relationship:

$$\begin{aligned}&\left[1-\frac{\sin\beta_\mu \sin\Phi_c}{\cos(\beta_\mu-\gamma_f)\cdot\cos(\Phi_c-\gamma_f)}\right]\cdot\left[\frac{1}{\cos^2(\Phi_c-\gamma_f)}-\frac{1}{\sin^2\Phi_c}\right]=\\&=-\left[\cot\Phi_c+\tan(\Phi_c-\gamma_f)+Z\right]\cdot\left[\frac{\sin\beta_\mu}{\cos(\beta_\mu-\gamma_f)}\left\{\frac{\cos\Phi_c}{\cos(\Phi_c-\gamma_f)}+\frac{\sin\Phi_c\sin(\Phi_c-\gamma_f)}{\cos^2(\Phi_c-\gamma_f)}\right\}\right]\end{aligned} \tag{2}$$

The constant $Z = \frac{R}{\tau_\gamma \cdot f_z}$ makes that value shear angle Φ_c take into account properties of material cutting (τ_γ – the yield strength; R – fracture toughness).
Equation (2) is solved numerically and allows the determination of shear angle values for small values for the thickness of the cut layer [7], and also allows to increase the precision modeling of cutting forces [2], [3].

NUMERICALLY METHODS
Equation (2) is a non-linear equation and can be solved numerically by using iterative methods. Newton's method is one of the iterative methods, which can find the roots of non-linear equations. The roots are found in an approximate way, which is a characteristic feature of all iterative methods. Newton's method is also known as the method of tangents.
The basic conditions that must be fulfilled the given interval *[a, b]* by the function *F(t)* are [4, 5, 10]:

- function is continuous and its first derivative is continuous,
  $F(t) \in C^0 \cap C^1$ (3)
- characters function values within the ranges are different,
  $F(a)F(b) < 0$ (4)
- character of the first and second derivative at the interval does not change, which means no extremes of the function *F(t)* and changes in the convexity,
  $F'(a)F'(b) > 0$, $F''(a)F''(b) > 0$ (5)
- in addition, the range should not be wider than a preset value,
  $|a - b| \le d$. (6)

These conditions can be applied to all iterative methods for the determination of roots non-linear equations. A choice of the starting point is very important and is associated with the formula (6). This formula (6) determines the length of the interval in which search zero of function. The base for Newton's method is distribution of function *F(t)* to series of Taylor for the specified argument values t_0:

$$F(t) = F(t_0) + \left.\frac{dF}{dt}\right|_{t_0} (t - t_0) + r(t, t_0), t \in R, \tag{7}$$

where $r(t,t_0)$ is the rest. The rest consist with the other derivatives of the function. In our case, the rest is negligible, because it is not taken into consideration when searching. The problem is reduced by function approximation in form of straight line. The straight line is a tangent to function at the point t_0. The algorithm is based on use of equation solutions:

$$F(t_0) + \left.\frac{dF}{dt}\right|_{t_0} (t - t_0) = 0, \tag{8}$$

in the form of

$$t_k = t_{k-1} - \frac{F(t_{k-1})}{F'(t_{k-1})}, \text{ for k = 1, 2, 3,…m,} \tag{9}$$

where m is the maximum allowed number of iterations. For this number of iterations are approximated to value the argument which satisfying the equation

$$F(t_k) = 0. \tag{10}$$

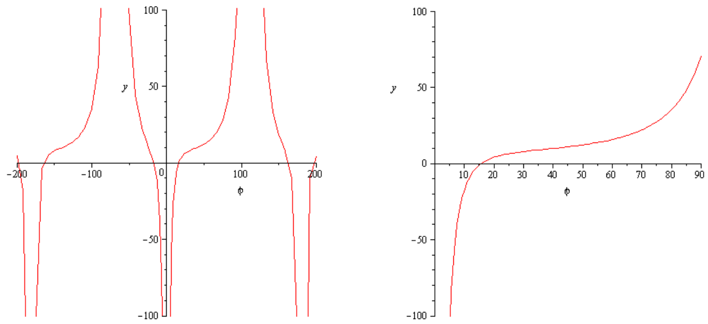

Figure 1. The course of Atkins equation (2) which describe the shear angle.

An effect by the choice at the starting point in the optimal finding the solutions of the Atkins equation (2) is shown in figure 2.

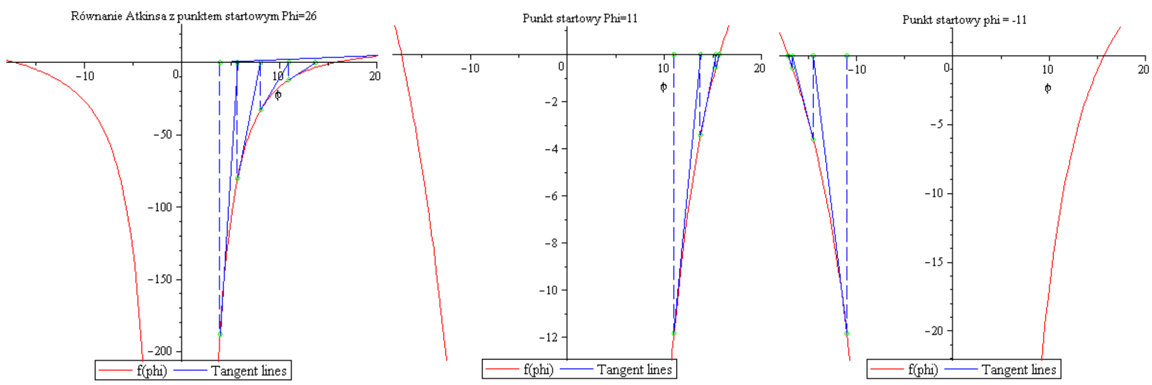

Figure 2. An effect of choice the starting point for the exploration of zeros for Atkins equation (Equation 2)

In each iteration method are taken into account the basic conditions of stopping:

- if the equation (10) is satisfied, then search are finished unambiguously,
- if for the k-th iteration, a absolute value of functions is less than the desired parameter,
  $$|F(t_k)| < \delta\,, \tag{11}$$
- if for (k) and (k-1) iteration, the distance between the arguments of the function is less than the predetermined parameter,
  $$|t_k - t_{k-1}| < \varepsilon\,.$$

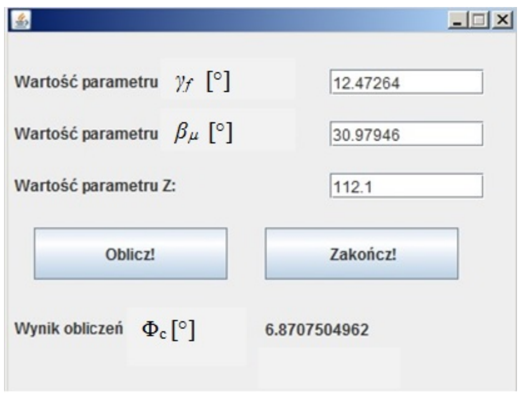

Figure 3. Java Application which is solving the Atkins equation (equation 2) with using the modified method of simple iteration. Sample data inserted in the window of application refer to the beech wood dried in air ($R = 4514.075$ J/m^2; $\tau_\gamma = 40.267$ MPa; $f_z = 0.001$ mm).

(12)

Parameters ε and δ from formulas (11) and (12) determine the level of accuracy with which is determined zero of the function $F(t)$. These parameters, together with the parameter m from equation (9), determine time of solving equation.
Figure 1 is showing the function behavior of the solution of Atkins equation for different ranges of angles Φ_c solved using Newton's iterative method.

Method of simple iteration is implemented in the form of a Java application. This application allows the solution of the Atkins equation (2) at one point. This application does not allow visualization of function in form of graph. From application only the value of shear angle Φ_c is given (figure 3).

CONCLUSIONS

The iterative methods give possibility to solve nonlinear equations. The duration of solutions can be quite short, if used a suitable combination of parameters (ε and δ) (equations 11 and 12) and the determination of the optimal starting point t_0. A suitable combination of parameters allowed us to obtain a compromise between accuracy and number of iterations method.
The iterative method in the form of Java application makes it possible to quickly solve a equation at that point. However, this application does not allow any visualization of function in graphical form, which is quite useful for further analyzing in the cutting process.

REFERENCES

1. ASTAKHOV V.P., *On inadequacy of single-shear plane model of chip formation*. International Journal of Mechanical Sciences 47(2005): 1649–1672.
2. ATKINS A.G., *Modelling metal cutting using modern ductile fracture mechanics: quantitative explanations for some longstanding problems*. International Journal of Mechanical Sciences, 45(2003), 373–396.
3. ATKINS, A.G., *The science and engineering of cutting. The mechanics and process of separating, scratching and puncturing biomaterials, metals and non-metals*. Butterworth-Heinemann is an imprint of Elsevier, Oxford, 2009, 413 p.
4. BJARCK A., DAHLQUIST G., *Metody numeryczne*, Warszawa, PWN, 1987
5. FORTUNA Z., MACUKOW B., WĄSOWSKI J., *Metody numeryczne*, WNT, Warszawa, 1995
6. MANŽOS F.M., *Derevorežušie Stanki*. Izdatel'stvo "Lesnaâ promyšlennost'", Moskva, 1974.
7. ORŁOWSKI K.A., OCHRYMIUK T., *Sposób prognozowania kąta ścinania w strefie skrawania przy*
8. *przecinaniu drewna*, W: Obróbka Skrawaniem: Współczesne problemy, Szkoła Obróbki Skrawaniem, 4,
9. B. Kruszyński (red.). Łódź, Politechnika Łódzka (2010), 449–456.
10. ORŁOWSKI K.A., OCHRYMIUK T., CHUCHAŁA D., *Prognozowanie mocy skrawania przy przecinaniu polskiego drewna sosnowego na pilarkach tarczowych*. Mechanik, 2012, nr 8–9, 661–663 (Artykuł na płycie CD dołączonej do czasopisma).
11. ORLOWSKI K.A., OCHRYMIUK T., ATKINS A., CHUCHALA D., *Application of fracture mechanics for energetic effects predictions while wood sawing*. Wood Sci Technol, 47(2013)5, 949–963 (Open access).
12. WANAT K., *Algorytmy numeryczne*, Gliwice, Helion, 1994

Streszczenie: *Sposoby rozwiązywania równania Atkins'a wyznaczającego kąt ścinania, które uwzględnia nowoczesną mechanikę pękania.* W artykule przedstawiono sposoby rozwiązywania nieliniowego równania Atkins'a, które opisuje kąt ścinania z uwzględnienie własności materiału obrabianego. Do rozwiązania tego równania użyto metod iteracyjnych: metody Newtona i uproszczonej metody prostej iteracji. Uproszczoną metodę zawarto w postaci aplikacji w środowisku Java.

Corresponding authors:

Kazimierz Orlowski
Daniel Chuchala
Gdansk University of Technology,
Faculty of Mechanical Engineering,
Department of Manufacturing Engineering and Automation,
Narutowicza 11/12, 80-233 Gdansk, Poland,
e-mail: korlowsk@pg.gda.pl, daniel.chuchala@gmail.com