RADIATION OF HIGH INTENSITY SOUND BY CIRCULAR DISC BY MEANS OF THE NUMERICAL METHOD

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The main goal of this paper is to find sound pressure distribution radiated by the circular piezoelectric disc that vibrates with the finite amplitude. There has been presented the pressure distribution close to the radiating surface. Also it is shown the sound pressure distribution in the 3D form. The mathematic modeling was carried out on the base of the nonlinear acoustic equation with the proper boundary condition. The axial symmetry of radiation was assumed. The results have been shown in the form of pressure distribution at different distances from the source. The final results will be applied to the designing of the underwater sources of finite amplitude.

INTRODUCTION

The higher harmonics are generated as a result of interaction of nonlinear waves of high intensity with the environment, in the area directly adjacent to the transmitting transducer. Unlike the primary wave source, which is the surface source, the source generating the harmonics is the volume source. The higher harmonics are created as a result of distortion of the wave, and their amplitude increases as the growth of the distortion.

Near the source, within a distance that is equal to few or several wavelengths, the distortion of the radiated monochromatic wave is small. In this area, process of generation of harmonics begins. Small distortion is justified only when it is taken into account the description of the phenomenon of changes of the first or second harmonic.

1. GENERATION OF THE HARMONICS IN THE NEARFIELD OF THE SOURCE

In underwater acoustics, often as a primary source of waves the transducer in form of a circular piston is used. The nearfield of this transmitter have a complex spatial structure, similar to that shown in Figure 1.

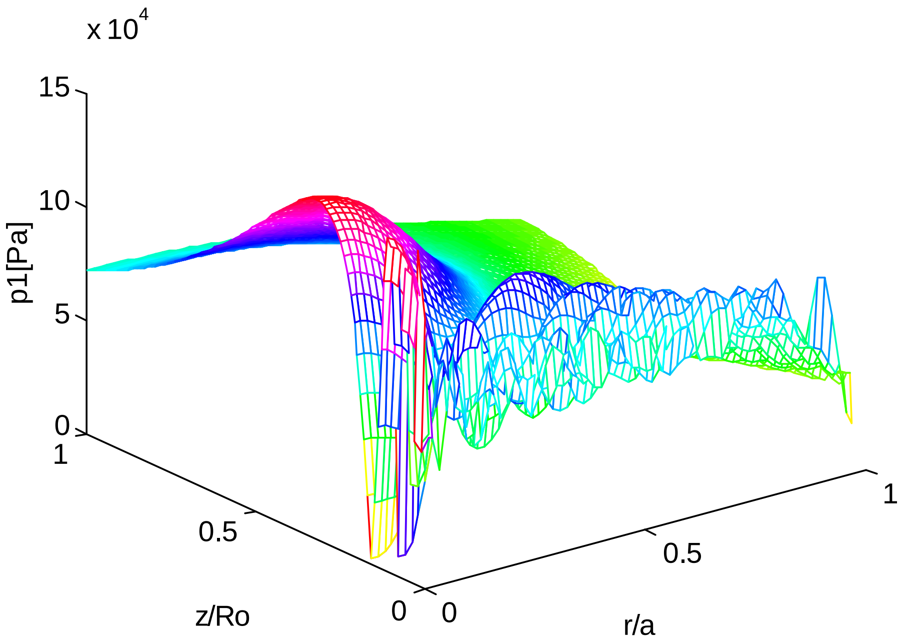

Fig. 1. Pressure distribution in the nearfield of circular piston of radius a=19 mm, f=2.25 MHz, p_o=100 kPa.

Generation of the second harmonic in the vicinity of the source can be traced on the basis of the solution of equation of Chocholow-Zabolotska-Kuznetsov (KZK equation) [3]

$$\frac{\partial}{\partial\tau}\left[\frac{\partial p'}{\partial z_1}-\frac{B/A+2}{2\rho_o c_o^3}p'\frac{\partial p'}{\partial\tau}-\frac{b}{2\rho_o c_o^3}\frac{\partial^2 p'}{\partial\tau^2}\right]=\frac{c_o}{2}\left(\frac{\partial^2 p'}{\partial x_1^2}+\frac{\partial^2 p'}{\partial y_1^2}\right) \tag{1}$$

where p' denotes acoustic pressure, c_0 – the speed of sound, ρ_0 – density of the undisturbed medium, b – attenuation coefficient, B/A – the nonlinearity parameter, x_1,y_1,z_1 - the Lagrange coordinate system, τ – time in the Lagrange coordinate system, by means of the method of the successive approximations [5] for the boundary conditions described by the relations (2).

$$\begin{aligned} p'(r,z=0) &= p_o \sin\omega t, \qquad r\le a \\ p'(r,z=0) &= 0, \qquad r>a \end{aligned} \tag{2}$$

The increase of the pressure can be written as:

$$p' = p^1 + p^2 \tag{3}$$

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Where p(1), P(2) are respectively the first and second approximation of the solution. It can be assumed that they can be in the following form:

$$2p^{(1)} = p_1' e^{j\omega\tau} + p_1'^{*} e^{-j\omega\tau} \tag{4}$$

$$2p^{(2)} = p_2' e^{j2\omega\tau} + p_2'^{*} e^{-j2\omega\tau}$$

In these equations, p_1' and p_2' are the pressure amplitude of the primary wave and the second harmonic. The asterisk indicates the complex conjugate quantities. After substituting relations (4) to KZK equation (1), we obtain the following set of equations:

$$2jk\left(\frac{\partial p_1'}{\partial z} + \alpha_1 p_1'\right) = \Delta\perp\, p_1'$$

$$4jk\left(\frac{\partial p_2'}{\partial z} + \alpha_2 p_2'\right) = \Delta\perp\, p_2' - \frac{(B/A+2)k^2}{\rho_0 c_0^2} p_1'^2 \tag{5}$$

where:

$$\Delta\perp = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

$$\alpha_1 = \frac{b\omega^2}{2\rho_0 c_0^3}$$

$$\alpha_2 = \frac{2b\omega^2}{\rho_0 c_0^3}$$

$\alpha_{1,}\alpha_2$ denote attenuation coefficients of the wave of infinitesimal amplitude and angular frequency, respectively ω and 2ω. In order to simplify the form of equations (5) it can be made the following substitution:

$$p_1' = p_1(-\alpha_1 z)$$

$$p_2' = p_2(-\alpha_2 z) \tag{6}$$

The system of equations (5) then becomes:

$$2jk\frac{\partial p_1}{\partial z} = \Delta\perp\, p_1$$

$$4jk\frac{\partial p_2}{\partial z} = \Delta\perp\, p_2 - \frac{(B/A+2)k^2}{\rho_0 c_0^2} p_1^2 e^{\alpha z} \tag{7}$$

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where:

$\alpha = \alpha_2 - 2\alpha_1$

The solution of the system of equations (7) can be found by using the integral Hankel's transformation. The assumed uniform distribution of the amplitude of the first harmonic on the surface of the transducer, the first approximation of the solution has the form:

$$p_1(r,z) + p_0\alpha \int_0^\infty J_1\,(\lambda\alpha) J_0(\lambda r) exp\left(j\frac{\lambda^2 z}{2k}\right) d\lambda =$$

$$= - \sum_{m=0}^{\infty} \frac{p_0}{(m+1)!}\left(-j\frac{ka^2}{2z}\right)^{m+1} \mathrm{F}\left(-m, -m-1; 1; \frac{r^2}{a^2}\right) \quad (8)$$

Where F () is the hypergeometric function. Equation (8) for r = 0 describes the changes of the amplitude of the first harmonic in the beam axis:

$$p_1(0,z) = -\sum_{m=0}^{\infty} \frac{p_0}{(m+1)!}\left(-j\frac{ka^2}{2z}\right)^{m+1} = p_0\left(1 - e^{-\frac{jka^2}{2z}}\right) \quad (9)$$

and the modulus of the amplitude as a function of distance from the source is changed according to the relation:

$$|p_1(0,z)| = 2p_0\left|\sin\left(\frac{ka^2}{4z}\right)\right| \quad (10)$$

The relations (8), (9) and (10) describe the first harmonic field distribution using quasi-optical approximation. It is correct for the distance from the source that meets the condition (2). An example of the pressure distribution of the first harmonic along the beam axis is shown in Figure 2. The solid line indicates the distribution based on analytical solution for the wave of infinitely small amplitude, and the dotted line – the distribution obtained using quasi-optical approximation. Calculations were performed for a transmitter having a radius of α= 25 mm radiating to water the wave of frequency f = 323 kHz, $p_0 \approx 95$ kPa. Numerical model is proper [5] when the following condition is fulfilled:

$$z > 2(ka)^{-2/3} \quad (11)$$

In this case it is for z > 0,19 m.

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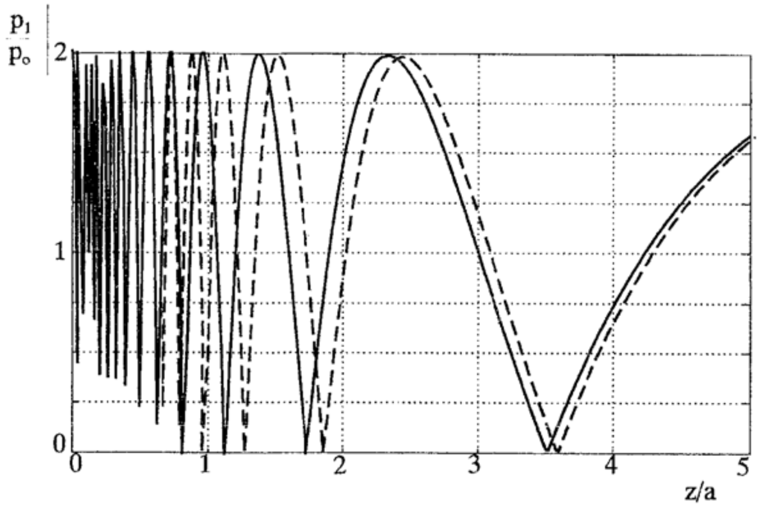

Fig. 2. The module of the first harmonic changes as a function of distance from the source: (------------------) linear approximation (- - - - - - -) quasi-optical approximation [based on [5]].

The solution of system of equations (7) describing the change of the second harmonic can be represented as:

$$p_2^{'}(r,z) = -\frac{(B\,/\,A+2)k}{2\rho_0 c_0^2} e^{-a_2 z} \int_0^z e^{-\frac{jkr^2}{z-z^{'}}+\alpha z} \frac{dz^{'}}{z-z^{'}} \int_0^\infty p_1^2\,(r^{'},z^{'}) e^{-\frac{jkr^2}{z-z^{'}}} J_0\left(\frac{2krr^{'}}{z-z^{'}}\right) r^{'} dr^{'} \tag{12}$$

After substituting to (12) integral expressions describing the distribution of the first harmonic (8), searching for the solution along the beam axis (r = 0), after a series of transformations it can be obtained:

$$p_2^{'}(0,z) = j\frac{(B\,/\,A+2)k}{4\rho_0 c_0^2} a^2 p_0^2 e^{-2a_1 z} \iint_0^\infty J_1\,(\lambda^{'}\alpha) J_1(\lambda^{''}\alpha) e^{\frac{jz(\lambda^{'2}+\lambda^{''2})}{2k}} d\lambda^{'} d\lambda^{''}$$

$$\int_0^z J_0\left(\lambda^{'}\lambda^{''}\frac{x}{k}\right) e^{-\frac{jx\left(\lambda^{'2}+\lambda^{''2}\right)}{4k}} e^{-\alpha z}\, dx \tag{13}$$

For small distances from the transmitter the formula (13) can be simplified to the form:

$$p_2^{'}(0,z) = j\frac{(B/A+2)k}{4\rho_0 c_0^2} e^{-2a_1 z} \int_0^z p_1^2\left(0, z-\frac{x}{2}\right) e^{-\alpha x} dx \tag{14}$$

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which using the formula (9) can be written as:

$$|p_2(0,z)| = \frac{(B/A+2)}{4\rho_0 c_0^2}\,(k\alpha)^2\, p_0^2 \ .$$

$$\left[\frac{z}{ka^2} + 4\left(\frac{z}{ka^2}\right)^2\left(2sin\frac{ka^2}{2z} - sin\frac{ka^2}{z} + \frac{1}{8}sin\frac{2ka^2}{2z}\right) + O\left(\left(\frac{z}{ka^2}\right)^2\right) + O(\alpha_2 z)\right] \qquad (15)$$

Determined on the basis of the relation (15) changes in the amplitude of the second harmonic as a function of the distance to the source, which the first harmonic is shown in Fig. 2., are shown in Fig. 3. For comparison is shown a solution indicated by a dotted line for a plane wave approximation.

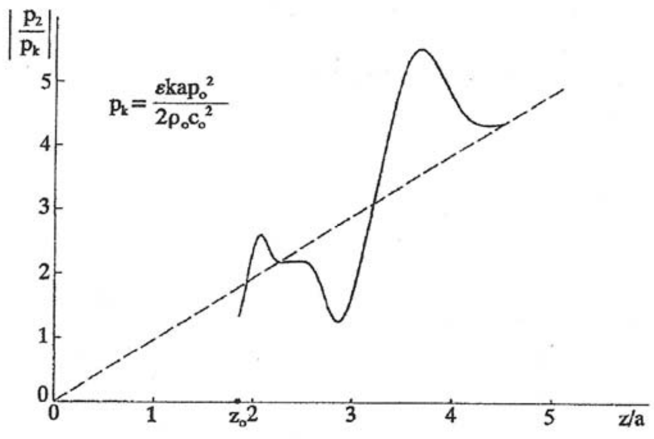

Fig. 3. The changes of the module of the second harmonic as a function of distance from the source: solid line: quasi-optical approximation, dotted line: plane wave approximation [based on [5]].

Presented above the analytical solution of the problem of generating harmonics in the field of the sources radiating wave of finite amplitude describes the experimentally observed spatial fluctuations of the amplitude of the second harmonic near the radiating surface.

2. THE SOLUTION OF EQUATION OF NONLINEAR ACOUSTICS BY THE MEANS OF FINITE ELEMENTS

The research of the field distribution near the vibration surface cause quite a lot of problems. A widely used method of solving the KZK equation in domain of frequency is very effective. However, it does not give information about the course of phenomenon in the area close to the source. For example for the source of waves, the circular piston-like transducer is used in experimental research of the radius α = 23 mm vibrating with a frequency of f = 1 MHz at p_0 ≈157 kPa, it describes correctly, according to the condition (2), wave propagation over the distance of more than 90 mm from the radiating surface .

The above discussed method of successive approximations is effective only in case of weak distortion, ie where Re_a < 1 or at a higher amplitude of the radiated wave - in the initial part of the rise of non-linear distortion.

Seeking opportunities of obtaining data of field distributon in close proximity to the source of the wave of high intensity was one of the main reasons for the development of a

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numerical model based on the full form of nonlinear acoustics equation [2]. For the numerical solution of the equation the finite element method is used.

Using this model, the studies were performed for the initial conditions adequate to the conditions of experimental studies, it means it was assumed that the circular transmitter with a radius of 23 mm (piston) vibrate at a frequency of 1 MHz. The research focused on the effect of pressure distribution near the surface radiating in the spatial field distribution obtained as a result of modeling the phenomenon. Different cases of the boundary conditions were considered, a uniform pressure distribution near the transmitter and distributions approximating the real distribution, as shown in Fig. 4.

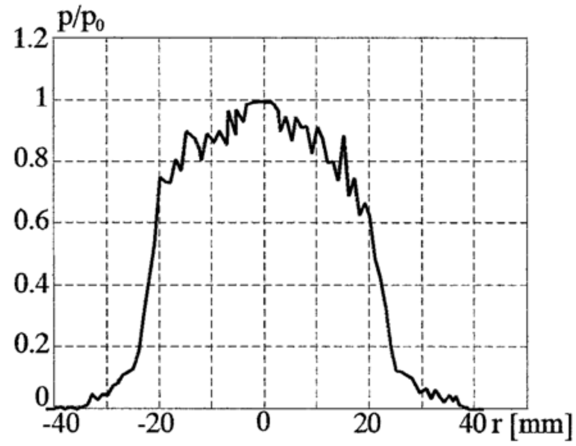

Fig. 4. The distribution of the pressure measured at a distance of 1 mm from the surface of the transducer with a radius of α= 23 mm vibrating at frequency 1 MHz.

Influence of the pressure distribution near the source on the spatial distribution of the field was examined on the example of two curves, approximating the real distribution, as shown in Figure 5.

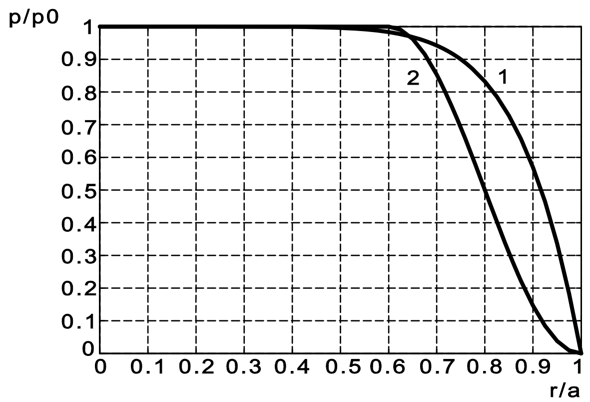

Fig. 5. The boundary conditions that were applied in the calculation

The pressure distribution in the beam cross section at a small distance from the transducer, calculated for both the approximating curves illustrate graphs in Figures 6 and 7. It is shown a pressure change as a function of the distance from the beam axis in planes perpendicular to the axis. The results of the studies indicate a large impact of the boundary conditions on the designated numerically the field distribution. They confirm the need for mapping as accurately as possible the real conditions in numerical models.

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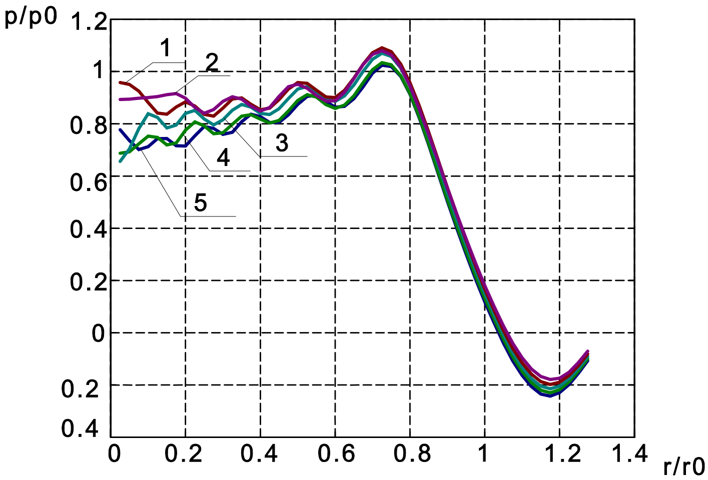

Fig. 6. The transverse distribution of the pressure calculated for the boundary conditions described by the curve 1 at varying distances from the transmitter 1 - 28.9 mm; 2 - 30.3 mm; 3-31,7 mm; 4 - 33.1 mm, 5 - 34,5 mm [3].

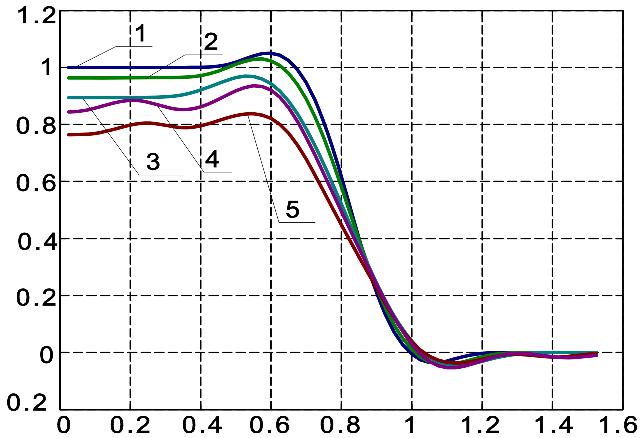

Fig. 7. The transverse distribution of the pressure calculated for the boundary conditions described by the curve 2 at different distances from the transmitter: 1 - 13.0 mm; 2 - 19.9 mm; 3 - 26.8 mm; 4 - 32.4 mm; 5 - 39.4 mm [3].

A model based on the solution of non-linear acoustics equation in full form was also used to perform studies of time and distance changes of the pressure distribution in the beam cross-section. The differences in the temporary pressure distribution within the beam are shown in Figure 8. All the pressure distributions that are shown in this Figure were determined for the same time but at different distances from the source of the wave. Upper and middle graphs show the amplitude distribution of pressure in two planes perpendicular to the beam axis, which distance is equal to the multiple of the wavelength. And the lower figure shows the pressure distribution in a plane parallel to the previous, where the distance to the others is not equal to a multiple of the wavelength.

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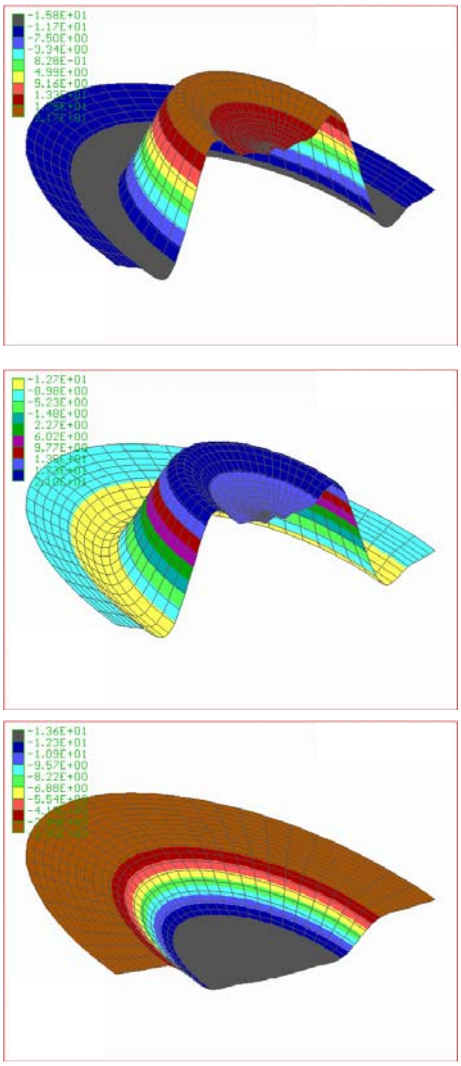

z_1

$z_2 \approx z_1 + n\lambda$

$z_3 = z_1 + n\lambda + x$

Fig. 8. The temporary pressure distribution in three planes perpendicular to the axis of a beam of a circular transducer at the same time.

In Fig. 9. it can be seen the temporary pressure distributions at the same cross section of the beam at different times. The time differences between the presented distributions are included in one period of vibration.

Presented data show how diverse the pressure distributions within the beam at a predetermined time depending on the distance from the source, or in a fixed beam cross-section depending on the phase.

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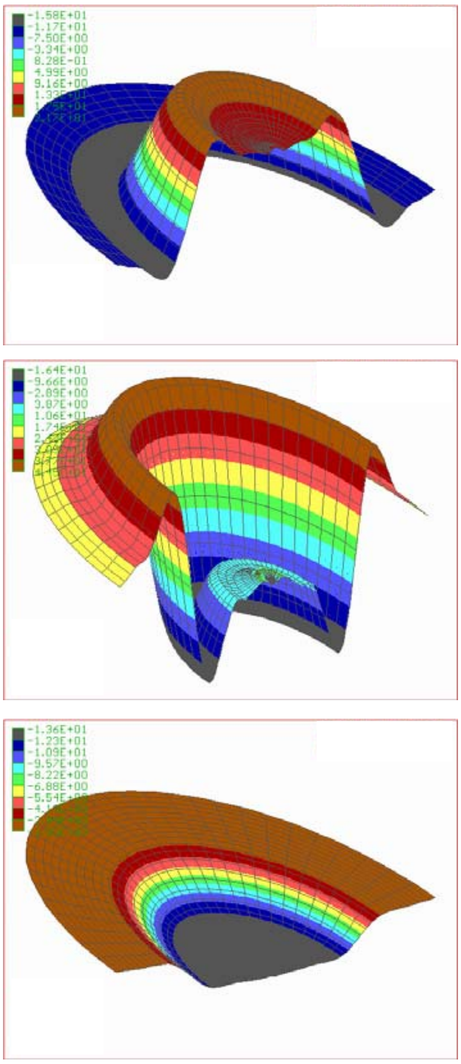

ϕ_1

$\phi_2 = \phi_1 + 48^o$

$\phi_3 = \phi_1 + 192^o$

Fig. 9. The temporary pressure distribution on the same plane perpendicular to the beam axis of the circular transducer at different times.

Presented data confirm the value of the model of the wave propagation of high intensity developed and based on the solution of full nonlinear acoustics equations by the method of the finite elements as a tool for obtaining valuable information about the course of phenomenon in the area directly adjacent to the radiating surface, it means in the area, for which the results using other commonly used models cannot be obtained.

This model does not have limitations due to the boundary conditions, so it can be used for prediction of field distribution of real sources of waves of high intensity.

The most important disadvantage of this model is the calculation time, much longer than, for example by using the Bergen's code model.

3. THE EFFECT OF PRESSURE DISTRIBUTION NEAR THE VIBRATING SURFACE ON THE FIELD DISTRIBUTION

The pressure distribution near the surface of the radiating piston planar source in not always uniform. The impact of this factor on the spatial distribution of the fields mentioned in

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the previous section, is illustrated in Figure 10 in which are compared changes in pressure distribution in the beam axis of the first three harmonic waves radiated assuming various forms of primary wave pressure distribution near the radiating surface.

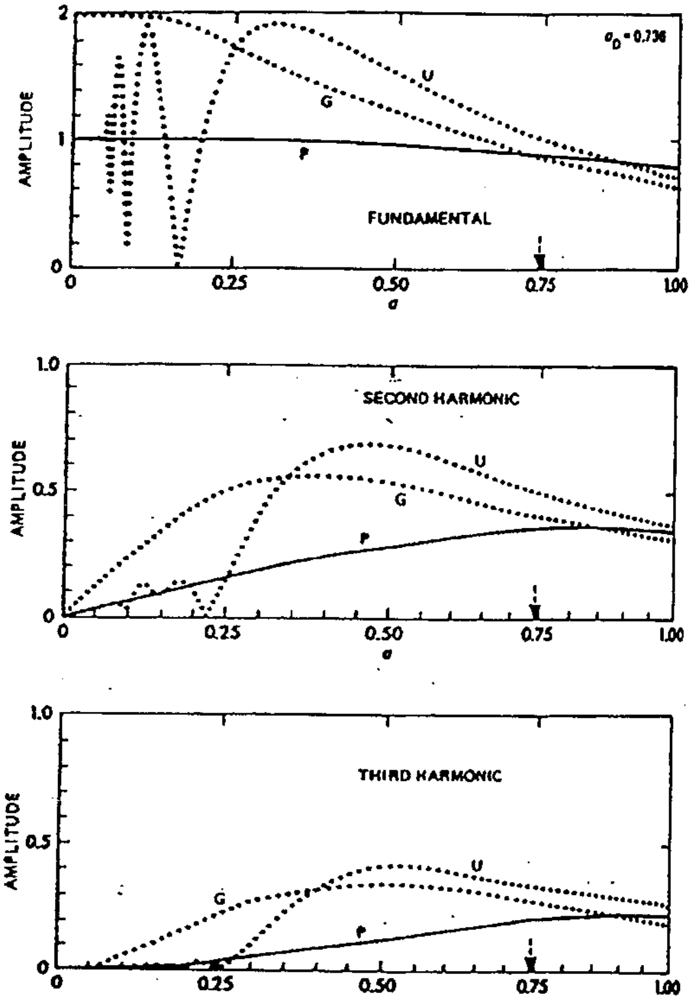

Fig. 10. The pressure distribution of the first three harmonics of pressure as a function of normalized relative Rayleigh's length: (U) - a circular transducer with uniform distribution, (G) - a circular transducer with Gauss' distribution, (P) - the source of a plane wave [1].

These distributions are determined theoretically in paper [1] for three different pressure distributions near the radiating surface. Other parameters of the source and the medium were the same in all cases. The calculations were made for the source of uniform distribution (U), a source with Gauss' distribution (G) and the plane source (P). The graphs show the amplitude of the pressure as a function of distance normalized by Rayleigh's wave, determined for the primary wave. The distance of the loss of continuity of the plane wave $z_N \approx 0.736\, R_0$. The curves illustrated the individual harmonic distributions approach each other in the area in which the distance from the source normalized by R_0, is close to the unity.

In the case of real sources of waves assumption a priori of the uniform distribution of vibration on the surface of the circular transducer, particularly at high values of the ratio ka, is not always correct. Figures 11 and 12 show the difference in pressure distributions determined theoretically for the source used in the measurement of the frequency of 1 MHz and a radius α= 23 mm, assuming that the pressure distribution near the radiating surface is uniform or approximated by the curve:

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$$p'(r, z=0) = p_0\left(1-\left(\frac{r}{a}\right)^4\right)^2 \qquad r \le \alpha$$

$$p'(r, z=0) = 0 \qquad r > \alpha \tag{16}$$

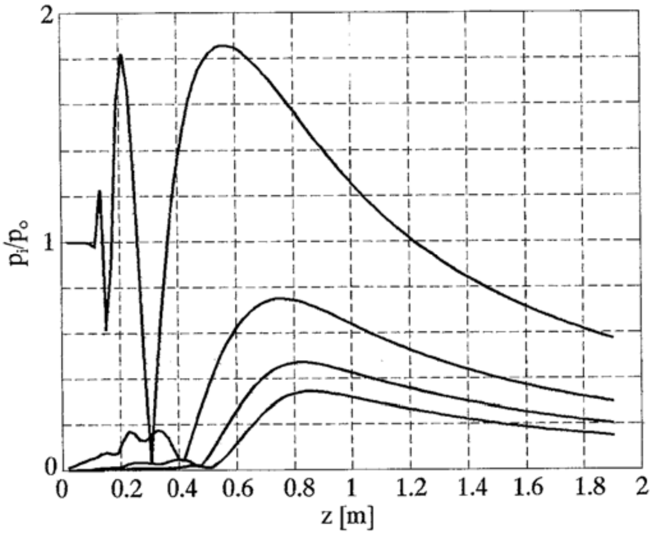

Fig. 11. The changes in the first, second, third and fourth harmonic of the pressure in the beam axis determined for uniform pressure distribution near the source.

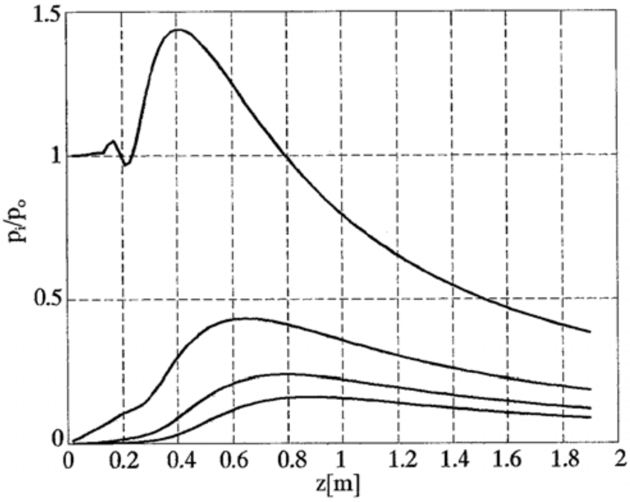

Fig. 12. The changes in the first, second, third and fourth harmonic pressure in the beam axis determined for the pressure distribution that approximates the real distribution close to the source.

The amplitude changes of the experimentally determined harmonics in the beam axis in comparision to the numerical studies results are shown in Figure 13. These results confirm the need of consideration the real distribution of the pressure of the radiating wave close to the source in the theoretical research, in the whole area of rising the deformation of the non-linear wave.

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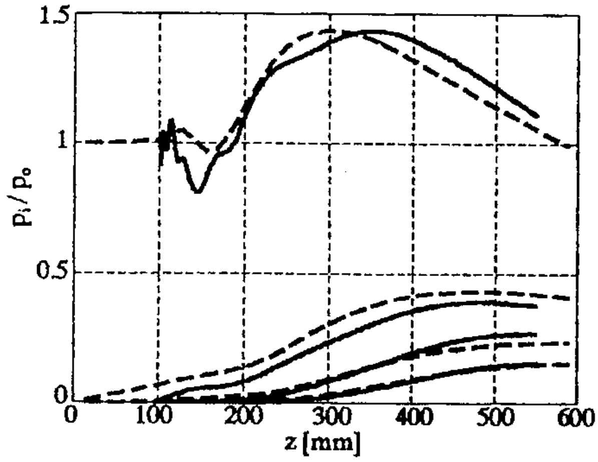

Fig. 13. The distribution of the pressure amplitudes of the first four harmonics in the beam axis experimentally determined: (-------------) and numerically based on the KZK equation with the boundary condition (16): (- - - - - - -).

The effect of the amplitude distribution of the radiated wave near the source on the pressure field within the beam is also seen in cross-sections in planes perpendicular to the beam axis. For example, the changes in the amplitudes of the first three harmonics of the pressure in the beam cross-section in characteristic areas of the field are shown in Figure 14.

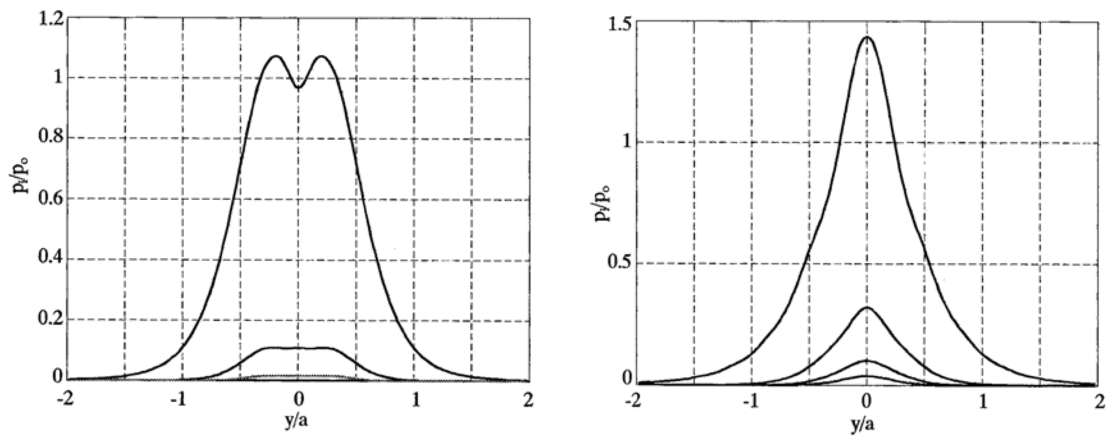

Fig. 14. The amplitude changes in the first three harmonics of the wave radiated by a plane source with a distribution (16) in the beam cross-section at a distance corresponding to the last minimum (left) and the last maximum (right) in the pressure distribution of the first harmonic in the beam axis.

CONCLUSIONS

The paper presents results of numerical investigations of pressure distribution in the nearfield of sources radiated waves of finite amplitude. The model elaborated for this purpose bases on equation of nonlinear acoustics in full form. It allows do predict the pressure

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distribution beginning from the plane located directly to the radiating surface and does not have the restrictions on the boundary conditions. Therefore it is very useful in investigation natural sources [4]. Another models, especially the widely used model based on solving the KZK equation does not give information about the distribution of the acoustic field close to the source of waves.

Numerical results were compared to experimental ones and shown a good agreement.

Presented data indicate the importance of impact of pressure distribution at the radiating surface on the acoustical field of the source.

ACKNOWLEDGMENTS

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REFERENCES

[1] S. I. Aanonsen, T. Barkve, J. N. Tjøtta, S. Tjøtta, Distortion and harmonic generation in the nearfield of a finite amplitude sound beam, J. Acoust. Soc. Am., 75, 3, 749-768, 1984.

[2] S. Dobrociński, E. Kozaczka, G. Grelowska, Numerical investigation of the finite amplitude wave pressure distribution radiated by a circular transducer, Proceedings of the International Symposium on Hydroacoustics and Ultrasonics, 71-74, Jurata, 1997.

[3] E. Kozaczka, G. Grelowska, An experimental investigation of the finite amplitude wave, Archives of Acoustics, 24, 1, 75-84, 1999.

[4] E. Kozaczka, G. Grelowska, S. Kozaczka, W. Szymczak, Detection of objects buried in the sea bottom with the use of parametric echosounder, Archives on Acoustics, Vol. 38, No 1, 99-104, 2013.

[5] Kunicyn V. E., Rudenko O. V., Generacja vtoroj garmoniki v pole porsnevogo izlucatela, Ak. Zurnal, 24, 549-555, 1978.

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