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## Observation value analysis – integral part of Bayesian diagnostics

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### Abstract

The decision making process, in general, is understood as a process of selecting one of the available solutions to the problem. One of possible approaches supporting the process is Bayesian statistical decision theory providing a mathematical model to make decisions of a technical nature in conditions of uncertainty. Regarding above, a detailed subject of the research is to analyze the value of the observation, which is a part of preposterior analysis. For the presented network, the main objective was to determine, conducting of which of three tests is the most valuable from the perspective of determining possible need or possibility to omission expensive technical expertise. The basis of verification, which test is the most valuable, is therefore the comparison of expected value of sample information (EVSI) for each of the three tests.

The main advantage of preposterior analysis is answering the question which of the considered experiments is potentially the best source of information and what cost needs to be incurred on its execution (price of new information). If the cost of such an examination is relatively small compared to the value of information on the state of nature, this implies a direct recommendation to conduct the experiment.

In conclusion, the construction of the decision-making model being a reflection of the state of nature allows to determine the ranking of decisions, including those regarding selection of the optimal test. It is noteworthy, that test result itself can contribute to an increase in the expected value of the decision involved, but on the other hand, the reduction of uncertainty may be considerably outweighed to the necessity to incur expenses for this examination.

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## 1. Expected value as the basis of decision making

Decision making process in general is understood as a process of selecting one of available decisions included in the decisions space and its graphic representation is decision tree. One of possible approaches supporting this process is Bayesian Decision Theory that provides mathematical model for making technical decisions under uncertainty [3]. The methodology of Bayesian Belief Networks was developed in the 1990's by a joint efforts of statistical and AI communities. The first practical applications of Bayesian Nets were in the field of medical diagnosis. Only recently the methodology was used to analyse environmental [7] and construction problems [10].

Assuming that uncertainties can be represented through probabilities, those can be established based on:

- relative frequency,
- conclusions based on mathematical models,
- conviction of certain distribution of states of nature based on engineer's experience.

As much as the first two methods can be verified in objective tests, engineer's conviction has to be treated as subjective measure. Its relative credibility depends on the expert's experience. Subjective measure of probability proves to be useful when we want to express the opinion of the entire team working on a given project, which is important especially in complex projects involving multiple areas [11]. Consideration of decisions under uncertainty is also crucial in everyday engineering practice. The advantage of Bayesian networks is the ability of combining objective results of conducted tests with subjective aspects that allow describing, for example, decision-maker's attitude, depending on benefits of certain decision (it is one of the frequently debated subjective elements of Bayesian Decision Networks, allowing however to express widely defined "political" aspects).

Results of actions taken are directly affected by factors, which state is often not fully known. In order to eliminate doubt regarding the state of certain factor, for example compressive strength of concrete, it is possible to conduct tests. A sample can be taken and its strength verified, by using compression machine, taking into account its reliability. In some cases using Schmidt hammer to assess the compression strength of concrete is an equally reliable and a lot more cost-effective experiment. It is important to note that such tests may not eliminate all doubts, but they still provide new information allowing for decreasing the level of uncertainty [12]. Considering the analysis of the value of observations, a part of preposterior analysis, we are considering the relation of cost of (potential) experiment to its benefit of higher value of the decision being result of the experiment. The test result may increase the value of expected decision, but on the other hand such increase may be disproportionate to the cost required to conduct the test.

The main task of the value of observation analysis is establishing conducting of which test is the most profitable with regards to the issue considered. The basis of this verification is the notion of expected value, defined as the sum of products of probability values and benefits of certain occurrence. The most efficient test is defined as the one that provides the highest value of the sample, being the difference between decision value resulting from this test and optimal value of the decision without conducting it. It is worth mentioning, that reliability of the expected value criterion is based on correctly applied measure of selection preference from analysed results i.e. it fulfills formal requirements, including transitivity criterion. Only then the maximum expected value is the logical base to select from potential decisions [3].

In this paper we are concerned with combining new observed information related to the "health" of civil engineering structures with expert subjective opinions about the nature of relationships among the physical and financial elements of construction projects. Definition of decision model that is reflection of reality and existing relations between factors occurring in nature (so called state of nature) and then conducting appropriate experiments allows establishing a ranking of decisions, including those concerning selection of optimal tests.

## 2. Methodology of preposterior analysis

The starting point of analysis to select test being the best source of information (providing maximum value of the sample) is to conduct a posteriori analysis of every possible test-result pair. The result of each pair relates to the optimal decision and expected value  $u(e, z)$  related to it, where:

$$u(e, z) = \max \{E[u(e, z, d | e, z)]\} \quad (1)$$

$$E[u(e, z, d) | e, z] = \sum_i u(e, z, d, x) p(x | e, z) \quad (2)$$

$e$  – type of test (experiment)

$z$  – result of the test (experiment)

$u(e, z)$  – value of the most beneficial decision when test  $e$  is conducted and state  $z$  is observed

$E[u(e, z, d) | e, z]$  – a posteriori expected decision value when test  $e$  is conducted and state  $z$  is observed

$u(e, z, d, x)$  – value of consequences of making decision  $d$  and observing state  $x$  as a result of observing state  $z$  when conducting test  $e$ ; cost of the experiment

$p(x | e, z)$  – conditional a posteriori probability of occurrence of state  $x$  when conducting test  $e$  and observing state  $z$

The next step is to calculate expected value of every test and select the one that maximizes expected value [3].

$$E[u | (e)] = \sum_k p(z | e) u(e, z) \quad (3)$$

$E[u | (e)]$  – expected value of the decision as a result of conducting test  $e$

$p(z | e)$  – conditional a priori probability of observing state  $z$  when conducting test  $e$

To calculate expected value of each test it is necessary to establish probability of its different outcomes [3]:

$$p(z | e) = \sum_i p(z | e, x) p(x) \quad (4)$$

$p(z | e, x)$  – reliability of the sample, conditional probability of occurrence of state  $z$  for test  $e$  and state  $x$

The basis of calculations conducted in chapters 3.1, 3.2, 3.3 is the definition of expected value. The basis for verification which test (experiment) is the most beneficial is the comparison of expected value of sample information (EVSI), as defined in formula 5.

$$EVSI = EV | SI - EMV \quad (5)$$

$EV | SI$  – expected value of sample information as a result of conducting additional test;  $E[u | (e)]$

$EV | SI$  – for  $e_1, e_2, e_3$  ( $e_i$  – tests conducted in chapter 3 of this article) respectively:  $E[u | (e_1)]$ ,  $E[u | (e_2)]$  and  $E[u | (e_3)]$

$EMV$  – expected value of the decision without conducting additional tests; for action  $e_0$  (not conducting additional test)  $EMV = E[u | (e_0)] = u(e_0, z)$

## 3. Observation value analysis – case study

The network presented below, showing sample (simplified) decision problem, should help to answer the question: is it necessary to conduct technical examination, or maybe the previously conducted technical assessment

of the object is sufficient (Fig. 1). Technical assessment is defined here as a document based on visual inspection of the construction and potential analysis of available documentation. Assessment conducted this way is not always unequivocal as it is based on engineer's experience, which can only be considered a subjective judgment. Technical examination is an exercise that in addition to engineer's experience (subjective opinion) includes elements of objective tests of construction materials, measurements of deflection and structural movements, as well as required calculations. In this case the diagnosis is made reliable by objective tests that are fully verifiable [1].

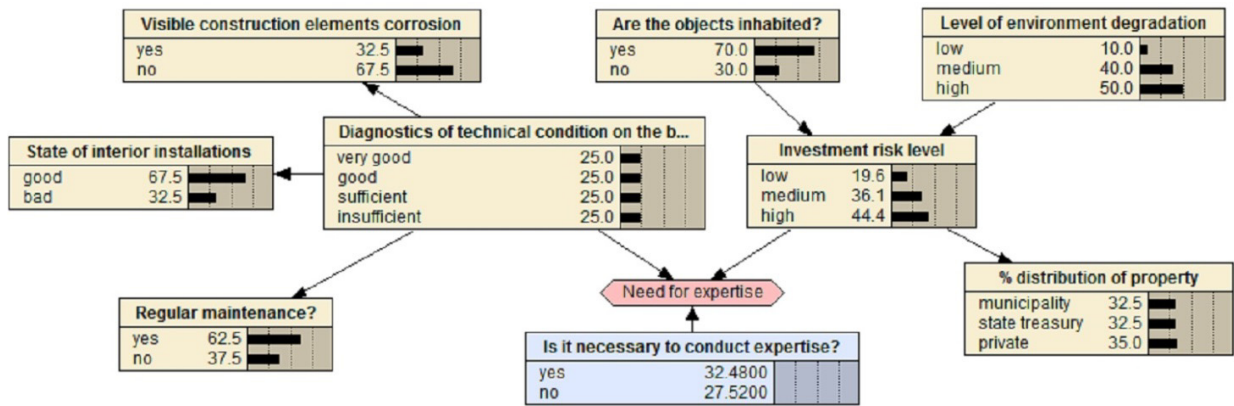


Fig. 1. An exemplary decision problem: "Is it necessary to conduct technical expertise?"

In general, when we want to solve the problem in question, first step is to determine its nature and confirm whether probabilistic network is the right modelling methodology in this situation. In order to do that a number of criteria need to be considered, including [11]:

- well defined variables,
- highly organized problem with identifiable cause and effect relations,
- uncertainty regarding the abovementioned relations,
- solving repetitive problems or issues related to high implementation costs.

In relation to abovementioned criteria, the decision problem, related to potential necessity of conducting technical examination, is presented in the context of preparation for urban regeneration project. Urban regeneration includes multitude of actions from such areas as construction, spatial planning, economy and social politics, aimed at reviving, functionality improvements, aesthetic and comfort improvement, as well as improving the quality of life of inhabitants of a certain area. Enterprises included in urban regeneration projects are comprised of tasks of investment (construction works in the area, as well as widely defined infrastructure) and non-investment (so called soft project elements) nature. Despite the important role of non-investment (social) tasks the research proved that majority of financial means in this type of projects is allocated to investment construction tasks [2].

Assuming then, that the success of urban regeneration depends, among other things, on minimization of additional costs and those are mainly dependent on technical state of the constructions; decision whether technical examination is required seems to be key to success of the investment part of regeneration projects. Keeping in mind extraordinary complexity of described enterprise (complex and social character), scale of its costs and uncertainty regarding the state of nature, making a decisions whether examination should be conducted is very difficult using traditional inference. Bayesian network allows to make decisions under conditions of limited information (lack of historical data, statistics), and it allows to avoid expensive tests of environment [4] that can be modelled based on experience [6].

The example discussed here is related to a situation where decision needs to be made based on available (limited) information regarding the modelled state of nature.

Decision, whether examination should be conducted, depends on the level of urban regeneration investment risk and diagnosis of technical state based on conducted technical assessment. Occurrence of any of the states of those values is uncertain; therefore the state of nature has been described through the pair of values: level of investment risk – diagnosis of technical state. Those pairs can be treated as bimodal states  $x_{ij}$ , presenting combined occurrence of investment risk level and certain diagnosis of technical state. Decision tree of the case described is presented on Fig. 3. For each of technical states and risk levels independently a discrete probability distribution has been defined, based on calculations of the model built in Netica application [11].

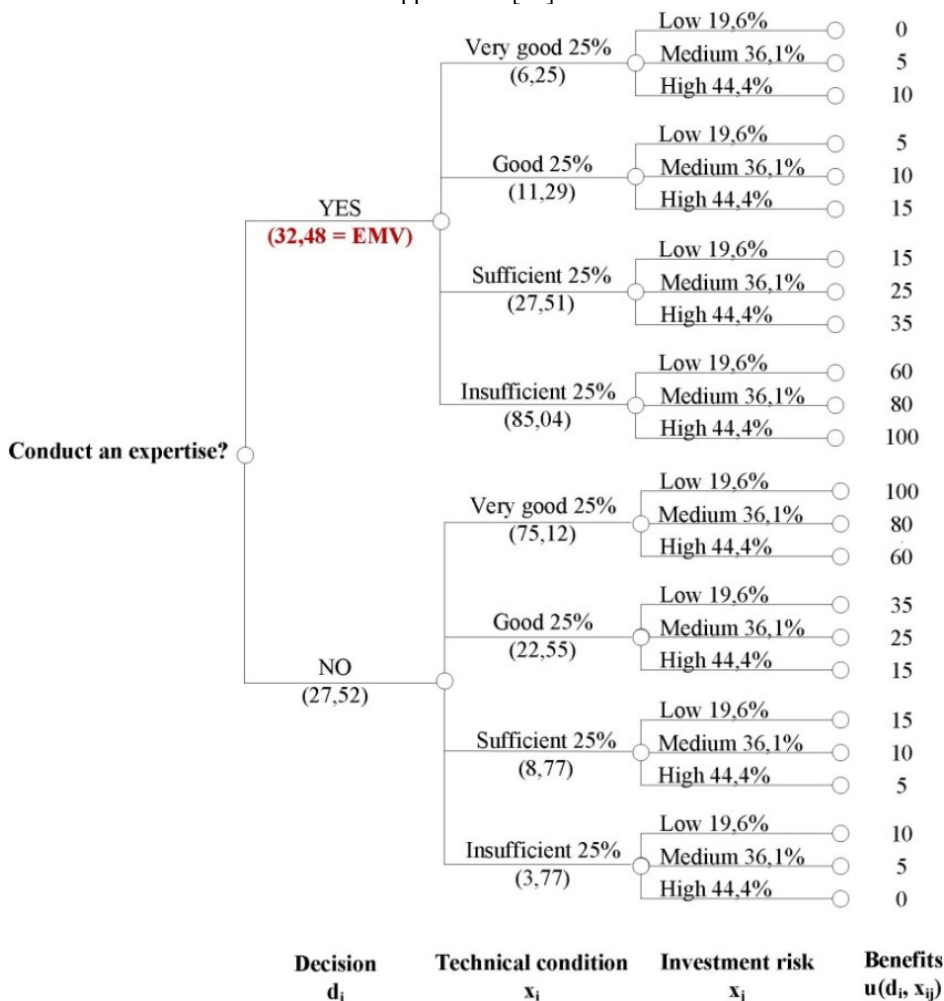


Fig. 2. Decision tree: level of investment risk – diagnosis of technical state.

The basis of probabilistic network analysis, aside of the necessity of creating a model, meaning the network structure, is definition of parameters presented as real numbers in the conditional probability tables and utility tables. First step is to identify variables, as well as causal, functional and information relations between them, providing this way the qualitative part of the model. Next, during iterative process, parameter values are defined through testing variables and conditional probabilities, as well as through verification of directionality of connections [8].

Conditional probability tables (CPT), described in [1], allow for quantitative representation of cause and effect relations between the model variables. What is more they allow describing expert knowledge, which introduced to the network as observations, using backward propagation of information, helps to reduce uncertainty of data and to improve model results. Utility tables are used to determine decision-maker's preferences. In the context of our example they serve to establish utility of conducting examination, so to express benefit of making specific decision for specific combination of factors, such as: positive or negative decision regarding conducting the examination, diagnosis of technical state of the object based on technical assessment and result of investment risk assessment. In general, preference allows representing the value of cost or benefit resulting from specific state of nature as numeric value.

In the analyzed example „utility”, as the benefits of conducting the examination is represented in sample absolute values, fulfilling the transitivity criteria.

The goal of this analysis is to verify conducting of which test (verification of visibility of construction elements corrosion, examination of the state of internal installations, verification of technical documentation of the object regarding ongoing maintenance) is the most profitable from the standpoint of determining potential necessity or exclusion of expensive technical examination. Taking into account the cost of the potential test, benefits of increased certainty of the decision taken, can be disproportionate to increased expected value.

### 3.1. Test no 1 – verification of visible construction elements corrosion ( $e_1$ )

$$EV | SI_1 = p(z_1 | e_1)u(e_1, z_1) + [1 - p(z_1 | e_1)]u(e_1, z_2)$$

where:

$e_1$  – verification of visibility of construction elements corrosion (experiment 1)

$z_1$  – construction elements corrosion is visible

$z_2$  – construction elements corrosion is not visible

$p(z_1|e_1)$  – conditional a priori probability of observing result  $z_1$  when conducting test  $e_1$

$$p(z_1|e_1) = 0,325$$

$u(e_1, z_1)$  – value of the most beneficial decision when conducting test  $e_1$  and observing state  $z_1$

$$u(e_1, z_1) = 49,877 \text{ (optimal decision value resulting from the simulation conducted in Netica)}$$

$u(e_1, z_2)$  – value of the most beneficial decision when conducting test  $e_1$  and observing state  $z_2$

$$u(e_1, z_2) = 34,190 \text{ (optimal decision value resulting from the simulation conducted in Netica)}$$

$$EV|SI_1 = 0,325*49,877 + (1-0,325)*34,190 = 39,288 \quad \text{– the decision value based on the test}$$

$$EMV = 32,480 \text{ – optimal decision value without testing (for } e_0; \text{ results of calculation conducted in Netica)}$$

Sample value:

$$EVSI_1 = EV|SI_1 - EMV = 39,288 - 32,480 = 6,808 \text{ [-]}$$

### 3.2. Test no 2 – examination of interior installations ( $e_2$ )

$$EV | SI_2 = p(z_1 | e_2)u(e_2, z_1) + [1 - p(z_1 | e_2)]u(e_2, z_2)$$

where:

$e_2$  – examination of interior installations (experiment 2)

$z_1$  – state of interior installations determined to be good

$z_2$  – state of interior installations determined to be bad

$p(z_1|e_2)$  – conditional a priori probability of observing result  $z_1$  when conducting test  $e_2$

$$p(z_1|e_2) = 0,675$$

$u(e_2, z_1)$  – value of the most beneficial decision when conducting test  $e_2$  and observing state  $z_1$

$$u(e_2, z_1) = 34,190 \text{ (optimal decision value resulting from the simulation conducted in Netica)}$$

$u(e_2, z_2)$  – value of the most beneficial decision when conducting test  $e_2$  and observing state  $z_2$   
 $u(e_2, z_2) = 49,877$  (optimal decision value resulting from the simulation conducted in Netica)

$$EV|SI_2 = 0,675*34,190 + (1-0,675)*49,877 = 39,288$$

Sample value:

$$EVSI_2 = EV|SI_2 - EMV = 39,288 - 32,480 = 6,808 [-]$$

### 3.3. Test no 3 – verification of technical documentation of the building object regarding ongoing maintenance ( $e_3$ )

$$EV | SI_3 = p(z_1 | e_3)u(e_3, z_1) + [1 - p(z_1 | e_3)]u(e_3, z_2)$$

where:

$e_3$  – verification of the technical documentation of the building object regarding ongoing maintenance (experiment 3)

$z_1$  – object has been undergoing regular maintenance

$z_2$  – object has not been undergoing regular maintenance

$p(z_1|e_3)$  – conditional a priori probability of observing result  $z_1$  when conducting test  $e_3$

$$p(z_1|e_3) = 0,625$$

$u(e_2, z_1)$  – value of the most beneficial decision when conducting test  $e_3$  and observing state  $z_1$

$$u(e_3, z_1) = 33,773 \text{ (optimal decision value resulting from the simulation conducted in Netica)}$$

$u(e_2, z_2)$  – value of the most beneficial decision when conducting test  $e_3$  and observing state  $z_2$

$$u(e_3, z_2) = 49,307 \text{ (optimal decision value resulting from the simulation conducted in Netica)}$$

$$EV|SI_3 = 0,625*33,773 + (1-0,625)*49,307 = 39,598$$

Sample value:

$$EVSI_3 = EV|SI_3 - EMV = 39,598 - 32,480 = 7,118 [-]$$

## 4. Summary

### 4.1. Advantages of preposterior analysis

The basis of preposterior analysis is to answer the question which of considered tests is the best potential source of information to maximize sample value and what will be the cost of conducting this test (price of new information). If the cost of such test is relatively low when compared to the value of information about the state of nature, it implies direct recommendation to conduct such test. As such using preposterior analysis especially when applying all currently available IT tools, gives wider possibilities to consider decision problems related to uncertainty, which is immensely important in everyday engineering practice.

### 4.2. Potential flaws of Bayesian networks (subjectivism)

Apart from indisputable advantages of presented procedure, subjectivism of expected value and benefit definition is sometimes considered as potential flaw of this theory.

The process of establishing preference (values of consequences or benefits being result of certain state of nature) conducted, for example, as an interview with experts in certain area (expert panel), is usually executed in situations that do not require actual risk taking. Such assessment reflects only hypothetical attitudes of the experts when faced with specific situations. It is worth noting, that individual character of every decision and reluctance to risk taking are elements that significantly affect the decision and individual preference description for specific results of considered situations. “Depending on circumstances such values may vary for different people and sometimes can





ever vary for the same person” [3]. The values therefore describe decision maker’s preference in a given moment. Because of the criteria verifying accuracy of expected value it is not required to repeat samples, as majority of engineering projects is by default unique. Despite problems with establishing benefits (consequences) in practice, requiring involvement and in-depth analysis of the situation, this concept is crucial in making rational decisions under uncertainty.

Authors of the paper [5] point to some limitation of Bayesian networks, related to the difficulty of their practical application. They also mention that the networks described are by default acyclic graphs, which may cause issues when modelling feedback. Contrary to those assertions, authors of the paper [11] claim that it is not a significant limitation, as it only requires more care when constructing the network, not limiting the possibility of modelling.

#### 4.3. Conclusions of presented tests

Due to small differences in probability distribution of occurrence of certain test outcomes, sample values of the tests are similar, however they still point to test  $e_3$  ahead of the remaining two. Keeping in mind conducted analysis, including checking which of the tests is the most beneficial from the perspective of establishing the need of conducting expensive technical examination, it has been determined that the most beneficial is verification of technical documentation of the object regarding ongoing maintenance. The value of this sample equal to  $EVSI_3 = 7,118$  [-], is slightly higher than the value of testing internal installations and the value of checking visibility of construction elements corrosion ( $EVSI_1 = EVSI_2 = 6,808$  [-]). In case of similar or equal results, the deciding criteria may be the cost analysis of each test.

It should be noted that criterion of expected value is used only in order to organize the space of potential decisions. It is incorrect to consider by how many percentage point one decision is better than the other. Due to some already mentioned flexibility in establishing benefits (utility value), expected value may have different ratios, but they will always lead to the same conclusions regarding which decision is better.

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