

Bogdan Rozmarynowski¹, Marek Jasina²
Gdańsk University of Technology

Subsoil degradation effect in reliability analysis of the jack-up platform structure³

Sea waves, wind and sea current are typical examples of dynamic loadings which are transferred to the subsoil through a fixed offshore structure. To estimate behaviour of the soil-structure system, their dynamic state parameters should be determined. Furthermore, soil subsystem acts on structural behaviour through the foundation and vice-versa, therefore a soil-structure interaction problem emerges. The knowledge on different aspects of structural dynamics including geometrical and physical nonlinearities is well developed domain now. On the contrary, the description of the subsoil dynamic behaviour is more complicated, mainly due to a high dependency on laboratory test results. Performing laboratory tests of soil samples from the sea bed may be especially difficult and expensive.

The present paper follows the research [5], [10] related to the dynamic, sensitivity and reliability analysis of the Baltic drilling platform structure shown in Fig. 1. It belongs to the class of jack-up and movable type of structures, however crucial construction changes are planned to take place in order to rearrange and prepare it to perform various stationary functions. The soil-structure interaction model in the mentioned papers was based on the laboratory estimated penetration of the platform leg bottom ends (footings) into the sea bed, assuming constant values of the Young's modulus and Poisson's ratio. Such an approximation does not take into account highly non-linear and load dependent stress-strain relationships of the soil continuum. Due to this nonlinearity and the cyclic nature of wave loads, the contact area between the structure and subsoil shows degradation, expressed herein as a Young's modulus degradation. The evaluation method of a cyclic stress distribution in the subsoil producing Young's modulus variations is described in [11].

In the analysis to follow the soil-structure interaction model is idealised as a set of linear translational and rotational springs. The degradation form (assessed by laboratory tests) causes that the spring constants have to be modified iteratively.

Having defined a discrete model of the Baltic drilling platform [5], [6], the structural response to wave and wind loads generated by storm conditions [4], specifically, the reactions of the platform leg ends are of major concern. A failure event means here exceeding either an allowable reaction value or a cross-section load-carrying capacity due to bending, axial or shear. Water waves and wind loads are either uncorrelated or correlated variables. The first and second order time-invariant problems of structural system reliability analysis [8], [9] (FORM and SORM) are considered and computed. Two FORM methods have been used in the search of a design point, i.e. the method known as the HL-RF index method [7] and the directional Monte Carlo simulation method, of a failure probability in each sample computed conditionally in a given direction in the standard normal space [1], [8]. Second order reliability analysis (SORM) with the use of individual performance functions deals with the point-fitting method with the use of asymptotic Breitung formula [2], [8].

¹ dr B. Rozmarynowski, senior lecturer, Gdańsk University of Technology, Faculty of Ocean Engineering and Ship Technology, Department of Ship Design and Subsea Robotics.

² dr M. Jasina, assistant professor, Gdańsk University of Technology, Faculty of Civil and Environmental Engineering, Department of Structural Mechanics and Bridges.

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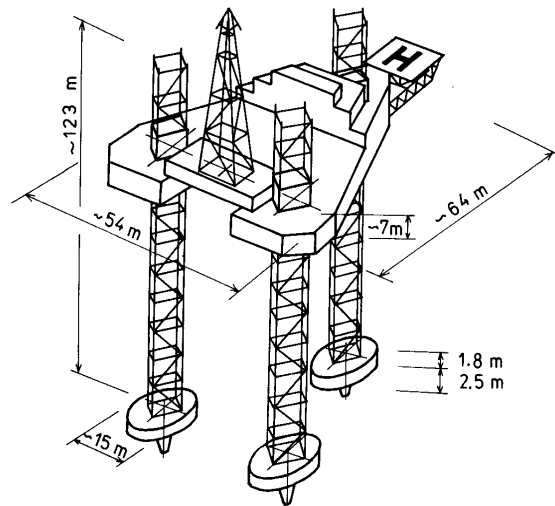


Fig. 1. General view of the Baltic drilling platform

Source: The material referred to [5] and [6]

Subsoil degradation due to cyclic loading

Three major stages of the procedure to estimate Young's modulus variations are:

1. Computing support reactions of the platform (i.e. forces acting on the subsoil) for the input values of the Young's modulus and Poisson's ratio, having predicted depth penetration of the platform leg bottom ends into the sea bed.
2. The FEM computation of shear and normal distributions of stress levels in a soil continuum modelled as an elastic half-space, due to computed reactions.
3. Finding a relationship between the Young's modulus and the number of load cycles for the stress levels in selected zones under a circular foundation of the platform structure, by means of interpolation of laboratory results.

Cyclic load imply constant and cyclic stress components. The constant component is due to wind, wave current, dead and live loads, whereas the cyclic part is provided by wave loads. It can be presented as follows [11]

$$\begin{aligned}\sigma_1 &= \delta\sigma_1 + \sigma_{10} \\ \sigma_3 &= \delta\sigma_3 + \sigma_{30}\end{aligned}\quad (1)$$

where σ_1, σ_3 are principal stresses calculated in the centre of each subsoil element for the state components $\sigma_x, \sigma_y, \sigma_z, \tau_{xz}$; $\delta\sigma_i$ $i=1,3$, denotes stress components due to wave loads (cyclic), whereas σ_{i0} , $i=1,3$, stands for dead, wind and current loads. The variation of Young's modulus (of the secant type) with the number of load cycles has been evaluated on the basis of the ratio

$$\gamma = \frac{\sigma_d}{2\sigma_{c0}} \quad (2)$$

where σ_d denotes the cyclic stress deviator component, whereas σ_{c0} stands for the average isotropic pressure.

Due to the obtained stress level distributions isoclinic lines were drawn for all considered load cases acting on a platform structure. A number of values of γ factor appeared in the contact foundation-subsoil area, but the following: $\gamma = 0.05, 0.10, 0.15, 0.20$ deserved greater attention. The stress levels corresponding to these cases proved to be decisive due to the Young's modulus degradation by means

of cyclic loading generated by wave oscillations. The next step was the evaluation of the approximate formula for Young's modulus E incorporating the number of cyclic changes.

$$E = E_0 \left(1 - 0.9 \frac{\log N}{\log N_L} \right) \quad (3)$$

where E_0 is the initial value, N expresses the number of cycles, N_L is the number of cycles for the soil limit state.

The cyclic changes were generated in the tri-axial soil-testing machine where the lateral stresses were subjected to cyclic changes (half of the deviator component, i.e. $\pm \sigma_d/2$).

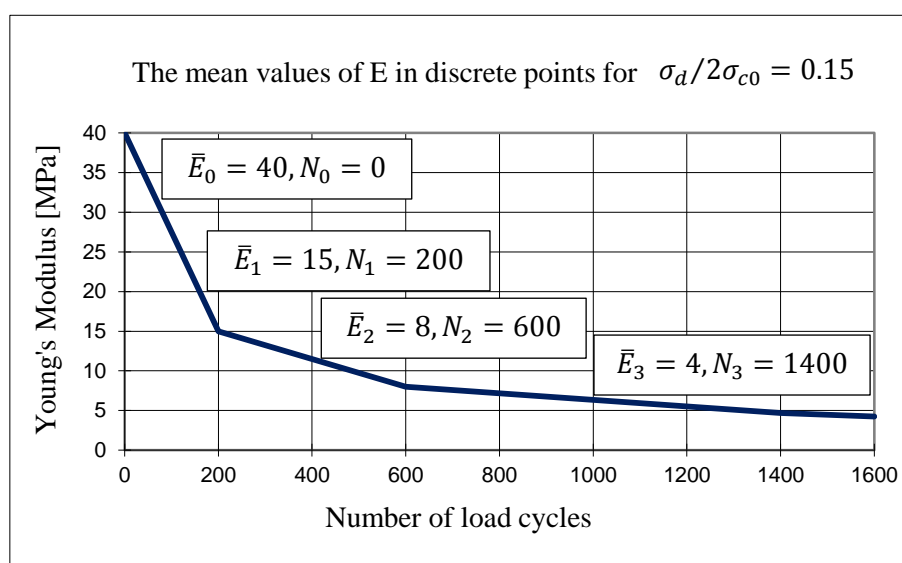


Fig. 2. The subsoil Young's modulus related to load cycles

Source: The Author's material

The analysis was carried out for the following three initial values of Young's modulus and Poisson's ratio, i.e. $E_{0i} = 10, 25, 40 \text{ MPa}$, $\nu_i = 0.25$; $i = 1, 2, 3$. Variation of Young's modulus $E_0 = 40 \text{ MPa}$ with numbers of cycles (Eq. 3 in no-logarithmic scale) by the linear interpolation is presented in Fig. 2.

Structural and load models

In order to present the application of structural reliability methods to the analysis of jack-up platforms, the Baltic drilling platform is considered prior to its construction changes undertaken. The structural parameters were defined in [4], [5] and [10]. A structural discretized space model is defined on the basis of the finite element method. In the present reliability analysis the space of basic random variables is composed of load parameters and support spring constants. Loads are assumed to act horizontally (X or Y Cartesian direction axes) and Z direction (gravity loads) modelled by point forces of random intensities acting in the nodes of a discrete model.

Statistical parameters, i.e. estimators of mean value and standard deviation of the load intensity distribution, are defined on the basis of static solution and spectral properties of the load process [10]. Wind load is modelled as extreme type Gumbel variable whereas wave load is a log-normal variable. The support conditions depend on the penetration depth of platform leg ends, modelled either as translational and rotational springs (six springs attached to each end) or pinned or fixed supports. The mean values and variances of the spring stiffnesses were taken according to [5], having horizontal, vertical, rock



and torsion components of spring stiffnesses evaluated, It is proved that the spring coefficients follow the Gaussian distribution.

Reliability analysis

A basic task within structural reliability is to evaluate the probability of failure as follows

$$P_f = P[g(\mathbf{x}) \leq 0] = \int_{g(\mathbf{x}) \leq 0} f(\mathbf{x}) d\mathbf{x} \quad (4)$$

in which $\mathbf{x} \in \mathbf{X}$ is a random vector representing uncertain load and structural parameters, $f(\mathbf{x})$ in Eq. (4) stands for the joint probability density function (PDF) of \mathbf{x} and $g(\mathbf{x})$ is known as a limit state function (performance function) formulated such that $g(\mathbf{x}) \leq 0$ defines structural failure, $g(\mathbf{x}) > 0$ its survival.

The limit-state functions [10], computed herein by means of sensitivity methods [3] express exceedance either of a maximum vertical reaction at the platform footing (important for a subsoil behaviour) or an internal force (bending, axial or shear force) capacity of the beam-column. In the paper limit state functions are assumed as follows

$$g_1(\mathbf{x}) = \frac{R_p}{R(\mathbf{x}_0)} - \frac{R(\mathbf{x})}{R(\mathbf{x}_0)} \quad (5)$$

$$g_2(\mathbf{x}) = 1 - \left[\left(\frac{M(\mathbf{x})}{M(\mathbf{x}_0)} \right)^2 + \left(\frac{N(\mathbf{x})}{N(\mathbf{x}_0)} \right)^2 + 2 \frac{|M(\mathbf{x})N(\mathbf{x})|}{M(\mathbf{x}_0)N(\mathbf{x}_0)} \right]^{\frac{1}{2}} \quad (6)$$

where \mathbf{x} denotes the vector of basic random variables from the basic space \mathbf{X} , $\mathbf{x}_0 = E\{\mathbf{x}\}$ is the vector of mean values of \mathbf{x} ; R_p expresses the maximum allowable reaction in the contact of the subsoil and foundation of the platform (assumed deterministic); $R(\mathbf{x})$ is the response (horizontal, vertical reactions, bending and torsional moments) function of all basic random variables; $R(\mathbf{x}_0)$ expresses the reaction of a chosen leg (for a Y-load direction here) computed for the mean values of \mathbf{x} ; $M(\mathbf{x}), N(\mathbf{x})$ are the bending moment and axial force (i.e. response functions of basic random variables), respectively; $M(\mathbf{x}_0) = WR, N(\mathbf{x}_0) = AR$ are the limit moment at pure bending and the limit force in simple tension or compression, respectively; W, R, A denote the mean values of section modulus, yield stress and cross-sectional, respectively.

In numerical calculations the following load and structural parameters were taken into account (Table 1)

Tab. 1. Data parameters (symbols with dash denote mean values)

Source: The Author's material

Wind parameters	$\bar{V}_{10} = 43 \text{ m/s}$ the mean speed at a height of 10m, $C_{DA} = 1.0$
Wave parameters	$\bar{H} = 11.5\text{m}, \bar{T} = 9.9\text{s}, C_M = C'_M + C''_M = 2.0, C'_M = 1.0, C_D = 1.0$
Current velocity	$\bar{v}_0 = 1 \text{ m/s}$
Subsoil parameters	$\nu = 0.25, \bar{E} = 40\text{MPa}, \bar{G} = 16\text{MPa}, \sigma_G = 8\text{MPa}$
Platform's leg diameter	$D = 4\text{m}$
Footing disc radius	$r_0 = 7.3\text{m}$
Footing penetration depth	$2,5\text{m}$

Probabilities of failure modes and β indices ($P_f = \Phi(-\beta)$, where $\Phi(\cdot)$ the Gaussian CDF) calculated using FORM, i.e. Hasofer Lind and Rackwitz-Fiessler index (HL-RF), Monte Carlo Directional Simulation (MCDS) and SORM represented by an asymptotic Improved Breitung formula (IB), for non-

correlated and correlated wind and wave fields (R_{Zp} is the maximum allowable vertical reaction) are shown in Table 2.

Concluding remarks

Cyclic loading generated by sea waves causes degradation of the subsoil expressed as variation of Young's modulus with a number of load cycles. The solution method to the problem of interaction between the subsoil and the circular foundation (footing disc) of the platform has been proposed. Based on the FEM solution for an elastic half space as a model of soil (including laboratory verification tests) the range of load cycles was established.

Tab. 2. Probabilities of failure modes P_{f_1} , P_{f_2} , for $R_{Zp} = 18.0 MN$

Source: The Author's material

Method	Correlation coefficient between wind and wave loads ρ			
	0.0		0.20	
	P_{f_1}/β	P_{f_2}/β	P_{f_1}	P_{f_2}
HL-RF	$(\bar{E}_0, N_0): 1.843 \cdot 10^{-3}/2.904$ $(\bar{E}_1, N_1): 1.603 \cdot 10^{-4}/3.598$ $(\bar{E}_2, N_2): 0.372 \cdot 10^{-5}/4.481$ $(\bar{E}_3, N_3): 0.210 \cdot 10^{-6}/5.060$	$(\bar{E}_0, N_0): 1.412 \cdot 10^{-3}/2.986$ $(\bar{E}_1, N_1): 0.956 \cdot 10^{-4}/3.730$ $(\bar{E}_2, N_2): 0.227 \cdot 10^{-5}/4.585$ $(\bar{E}_3, N_3): 0.110 \cdot 10^{-6}/5.595$	$(\bar{E}_0, N_0):$ $2.855 \cdot 10^{-3}$	$(\bar{E}_0, N_0):$ $1.606 \cdot 10^{-3}$
MCDS	$(\bar{E}_0, N_0): 2.048 \cdot 10^{-3}/2.871$	---	$(\bar{E}_0, N_0):$ $3.037 \cdot 10^{-3}$	---
IB	$(\bar{E}_0, N_0): 2.226 \cdot 10^{-3}/2.840$	$(\bar{E}_0, N_0): 1.607 \cdot 10^{-3}/2.946$	$(\bar{E}_0, N_0):$ $3.248 \cdot 10^{-3}$	$(\bar{E}_0, N_0):$ $1.711 \cdot 10^{-3}$

The following conclusions can be drawn.

1. Loading due to sea waves generates cyclic changes transferred to the subsoil through the platform structure. The influence of such changes, expressed in variation of Young's modulus with a number of cycles (Fig. 2), on the support reactions of the structure was the objective of the analysis.
2. For the assumed data in the numerical example, the limit-state function representing exceedance of the reaction barrier appeared decisive in safety evaluation of the structure.
3. The probability of the first failure mode increases when the correlation coefficient between wind and wave loads becomes higher
4. The differences between results obtained by FORM and SORM are low, thus limit state functions used here are not highly non-linear. For such cases the first-order sensitivity methods is recommended.

Abstract

Structural reliability analysis is considered, by FORM and SORM applied to a certain idealized soil-platform structure interaction problem due to cyclic water wave and wind loads. Wave and wind loads are random variables, whereas dead load is deterministic. Load parameters are typical for storm conditions in the Baltic Sea. The soil-structure interaction is idealized as a set of linear translational and rotational springs. Due to a cyclic type of wave loads, the contact area between the structure and subsoil reveals a degradation phenomenon expressed as Young's modulus degradation. Numerical calculations of β index provide practical conclusions.



Efekt degradacji podłoża w analizie niezawodności konstrukcji platformy typu jack-up

Streszczenie

W pracy przedstawiono wyniki obliczeń niezawodnościowych FORM i SORM w odniesieniu do wyidealizowanego modelu interakcji konstrukcja platformy – podłoże gruntowe. Na konstrukcję platformy działają fale morskie oraz wiatr wyrażone jako zmienne losowe z uwzględnieniem parametrów sztormowych a także ciężar własny konstrukcji przyjęty jako obciążenie deterministyczne. Przyjęto dyskretny model konstrukcji. Podłoże gruntowe uwzględnia cechy sprężyste, które ulegają degradacji w wyniku cyklicznego obciążenia falami sztormowymi. Wykonane obliczenia niezawodnościowe indeksu β dla przyjętych stanów granicznych użytkowania pozwalają sformułować praktyczne wnioski.

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