

**THE SYNCHROSQUEEZING METHOD  
IN BEARING ESTIMATION OF STATIONARY SIGNALS  
FOR PASSIVE SONAR WITH TOWED ARRAY**

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*In this paper, a novel method of bearing estimation in a passive sonar system with a towed array is introduced. The classical approach of bearing estimation based on the spatial spectrum [1] is extended by using the synchrosqueezing method that is a part of the reassignment method introduced by Koderá et al. [2]. Using this method leads to a clear bearing estimation. The proposed method requires a relatively small amount of computation, because of the possibility of using the FFT algorithm. Moreover, the immunity of the method against AWGN is tested for a selected sonar array with respect to the direction of arrival and the signal frequency.*

## INTRODUCTION

Bearing estimation is an important issue in the sonar and radar technique [3–6]. In underwater acoustic systems, the direction of arrival (DOA) of signal can be estimated using a spatial spectrum [1, 7]. In general, the precision of the estimation depends on spectrum unambiguity and can be improved by increasing the geometric dimension of a sonar array, among other possibilities. Unfortunately, such an approach leads to increased costs of the whole system and cannot be realized boundlessly. Therefore, various numerical methods of increasing of the spatial spectrum distinguishability are proposed [8, 9]; for example, those based on an autoregressive model such as Burg's method [10], non-parametric methods such as the eigenvector algorithm [11, 12] or the multiple signal classification (MUSIC) method [13].

In 1976, Koderá *et al.* proposed a novel method [2] which is recommended for obtaining highly concentrated energy distributions in the joint time-frequency (TF) domain. This method is dedicated especially for the analysis of multicomponent non-stationary signals whose components are sparsely distributed on the TF plane. Until today, variants of the method, having a variety of names, have been used in many applications [14–22]. We propose to use a variant

of this method, referred to as "the synchrosqueezing", in order to obtain clear bearing estimates. In general, each of the reassignment vectors consists of two orthogonal components: time and frequency [23]. The synchrosqueezing exploits the frequency component only [24]. Thus, the synchrosqueezing method can be especially useful if the analyzed signals are stationary. In this case, the time components are usually insignificant.

In this paper, the utility of the synchrosqueezing method is shown in order to estimate the clear DOA of signals received by the towed array of a passive sonar system in the underwater environment. For this purpose, the method is adapted by using a four-dimensional (short-)time-(local-)spatial Fourier transform. Thus, we assume that components of arrival signals are locally stationary and have constant frequencies. This is the reason that the introduced method is suited for the analysis of signals derived from sonar operating at a fixed frequency or from propellers, turbines, or marine engines that work at a constant or slow changeable rotation speed. Some results of using the reassignment method for bearing estimation are also presented in [25].

Let us consider a hypothetical passive sonar system with a towed array of omnidirectional hydrophones. Its antenna consists of hydrophones that are uniformly distributed along a straight line. The antenna is towed along the surface of the water. It is assumed that each acoustic signal received by the array is uniformly sampled in both space and time. Therefore, the FFT algorithm can be used to significantly improve the numeric performance of the method. The two-dimensional input signal is formed using samples stored in buffers linked to appropriate hydrophones. The first dimension is time and the second is the hypothetical signal delay along the array. Such sonar systems are usually used by offshore warships and submarines for the underwater object detection, identification, localization, and tracking. This method can contribute to the bearing estimation as well as to all of these operations [26–28].

This paper is organized as follows. In Section 1, a novel method of the bearing estimation, based upon the synchrosqueezing, is introduced. Then, in Section 2, a description, the initial conditions, and the results of the computer simulation in the AWGN channel are presented. The method is also tested for various frequencies and directions of arrival.

## 1. SHORT-TIME-LOCAL-SPATIAL FOURIER TRANSFORM

The classical two-dimensional Fourier transformation is an operation which transforms the signal of two variables, both representing time, into another function of two variables, both representing frequency. Let us consider the following variant of the short-time Fourier transform (STFT) of a function  $u(t, \tau)$  where both  $t$  and  $\tau$  have the dimension of time:

$$U(t, \tau, \omega, \varpi) = A(t, \tau, \omega, \varpi) \exp(j\phi(t, \tau, \omega, \varpi)) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u(\zeta, \xi) h^*(\zeta - t, \xi - \tau) e^{-j(\zeta\omega + \xi\varpi)} d\zeta d\xi. \quad (1)$$

$U(t, \tau, \omega, \varpi)$  is complex-valued function of four variables: time  $t$ , geometrical location expressed by propagation delay along the antenna  $\tau$ , angular frequency  $\omega$  and angular frequency related to signal distribution in space  $\varpi$ .  $h^*(t, \tau)$  expresses the conjugated function of the analyzing tempering window. The Blackman-Harris window is used [29] that is limited in both time and delay. For analysis of constant frequency signals, the window in both dimensions should be as long as possible [30]. In the situation considered the range of the window depends on the number and distribution of hydrophones and of the length of their buffers.

Referring to the Kodera's *et al.* works and regarding (1), channelized instantaneous angular frequency can be defined as the partial derivative of the STFT phase with respect to time:

$$\Omega(t, \tau, \omega, \varpi) = \frac{\partial \phi(t, \tau, \omega, \varpi)}{\partial t}; \quad (2)$$

analogously local angular frequency related to the signal distribution in space is here defined as the partial derivative of the STFT phase with respect to delay:

$$\Pi(t, \tau, \omega, \varpi) = \frac{\partial \phi(t, \tau, \omega, \varpi)}{\partial \tau}. \quad (3)$$

If  $u(t, \tau)$  represents values of an acoustic signal received by towed array sonar, then both of these distributions can be used in order to estimate the direction of signal arrival. The DOA can be obtain for varying  $t, \tau, \omega$ , and  $\varpi$  as follows:

$$\Phi(t, \tau, \omega, \varpi) = \arccos(\Pi(t, \tau, \omega, \varpi) / \Omega(t, \tau, \omega, \varpi)). \quad (4)$$

$\Pi(t, \tau, \omega, \varpi)$  and  $\Omega(t, \tau, \omega, \varpi)$  have the same units: rad/s. This is the reason for not using explicitly the spatial frequency that is expressed by 1/m.

At this point, the reassignment or, in fact, the two-dimensional synchrosqueezing process can be performed from the frequency-frequency ( $\omega$ - $\varpi$ ) domain into the frequency-angle domain for each time  $t$  and delay  $\tau$  in the following way:

$$E(t, \tau, \omega, \varpi) \rightarrow E(t, \tau, \Omega(t, \tau, \omega, \varpi), \Phi(t, \tau, \omega, \varpi)). \quad (5)$$

The desired result of the reassignment is to increase energy concentration locally near the frequency of signal components and near the direction from signal arrives (bearings). The directions are represented by angles with respect to the axis of the antenna. Such a situation is presented in Fig. 1, where, in Subfigure B, sharp peaks occur very extremely close to the assumed directions ( $45^\circ$  and  $35^\circ$ ) and the frequencies (435 and 445 Hz). The gray scales are selected in order to the effectively use of the full range. The concentrated distributions in Fig. 1.B has a range greater (by 30 dB) than the classical spectrogram in Fig. 1.A.

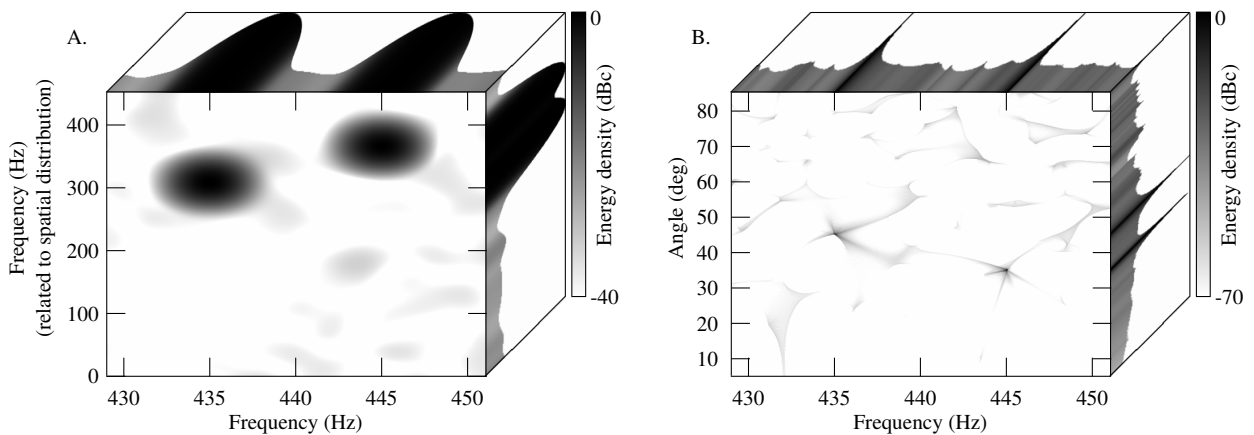


Fig. 1. A. Level of the classical two-dimensional Fourier transform (spectrogram) of signal  $u(t, \tau)$  stored in the buffers of a passive sonar with the towed array B. Level of concentrated energy density after the replacement. Two sinusoidal signals in the presence of AWGN (SNR  $\approx -8$  dB) are received. Signals have frequencies equal to 435 and 445 Hz and arrive from directions corresponding to, respectively,  $45^\circ$  and  $35^\circ$  relative to the axis of the antenna. Additional margin distributions are presented above and the right.

## 2. COMPUTER SIMULATIONS

The performance of the method introduced is tested by intentional degradation of a received signal by adding white Gaussian noise (AWGN) and by measurement of the difference between an estimated and an assumed direction of arrival. This process is entirely realized by computer simulation. The sampling rate is assumed equal to 1 kSa/s. A simulated antenna consists of 768 hydrophones distributed uniformly every each  $d_0 = 12$  cm, whereby the whole antenna length is equal to  $767 \cdot d_0 = 92.04$  m. The propagation velocity of acoustic wave in the water is assumed to be equal to  $v_{\text{water}} = 1488$  m/s and the signal delay (lag) along the longitudinal section of the antenna is equal to  $\tau_{\text{along}} = 767 \cdot d_0 / v_{\text{water}} \approx 62$  ms. It is assumed that the buffer of each hydrophone can store 1024 samples and that the sampling interval is equal to  $t_0 = 1$  ms.

In Figs. 2 and 3, selected characteristics concerning the method are presented. These characteristics include the standard error of DOA vs. DOA and signal frequency together. The received signal is degraded by the AWGN where SNR is approximately equal to  $-12$  dB. Each measurement point of each curve has been estimated by averaging 7000 realizations of AWGN.

## 3. CONCLUSION

In this paper, a fast, efficient, and clear method of bearing estimation is presented. The method is based upon both classical and spatial spectrum estimation and – what is new – on the synchrosqueezing, necessitating the use of both the magnitude and the phase of the Fourier transform. The method is referred to fast and efficient because its implementation is based upon the FFT algorithm. Moreover, the method is clear because the concentrated energy distribution is estimated after the reassignment process. In this step the energy peaks corresponding to individual received signals are sharp and clear. The relatively small error obtained in the computer simulation confirm the utility of the method for bearing estimation.

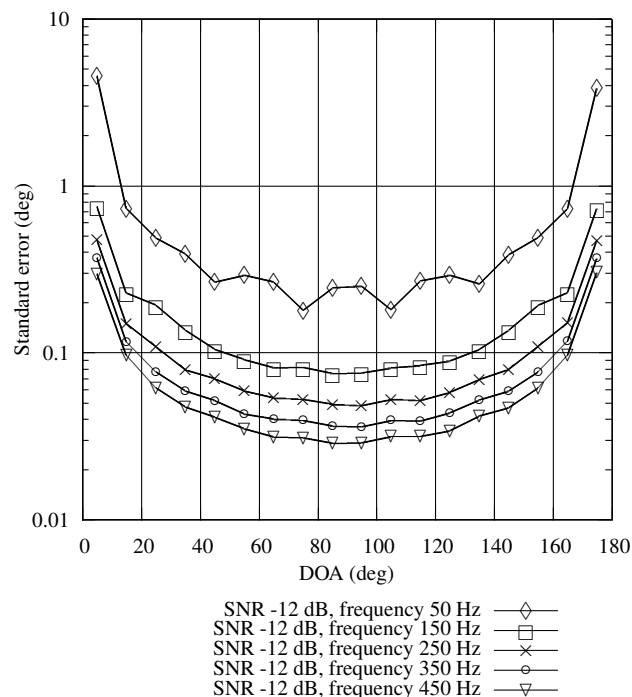


Fig. 2. Results of computer simulations of the method for DOA estimation. Standard error versus DOA for selected signal frequencies. SNR is assumed equal to  $-12$  dB.

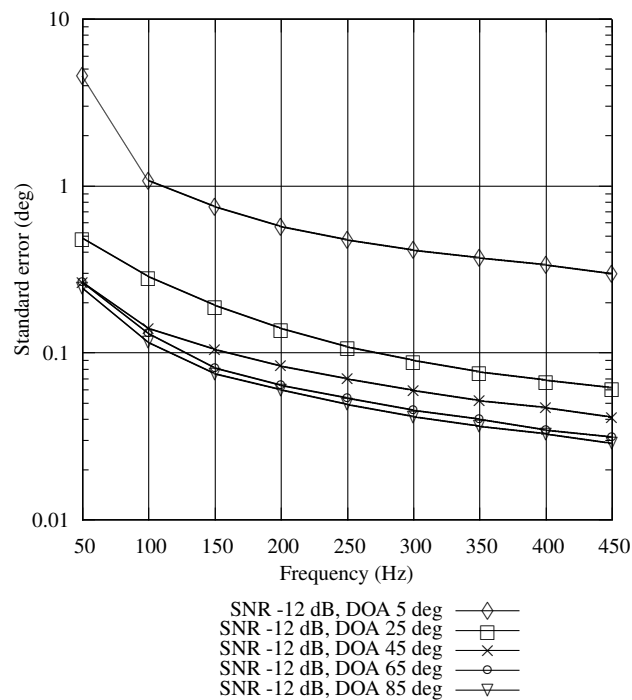


Fig. 3. Results of computer simulations of the method for DOA estimation. Standard error versus signal frequency for selected directions of signal arrival. SNR is assumed equal to -12 dB.

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